

Computational Geometry

Exercise 2: Sheet 1, Sheet 2, Museum Guards

Elly, Wendy

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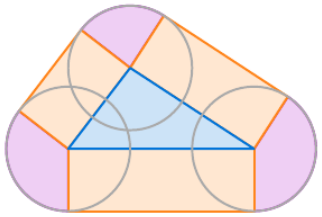
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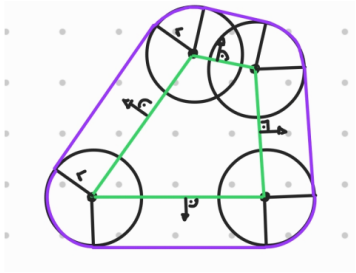
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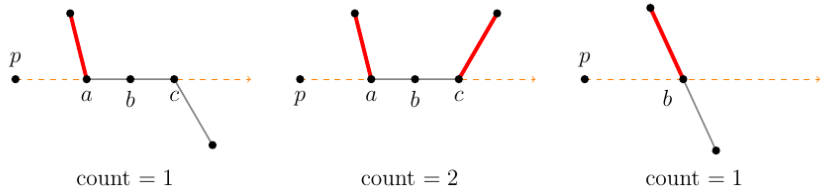
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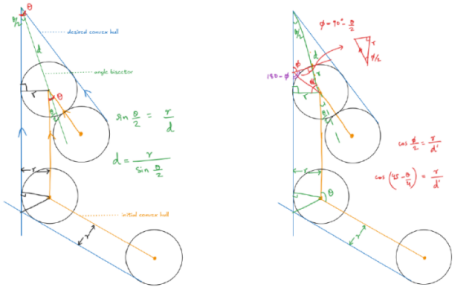
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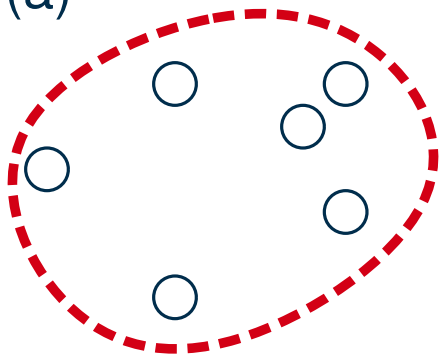




Sheet 1

Mysterious Meteorites

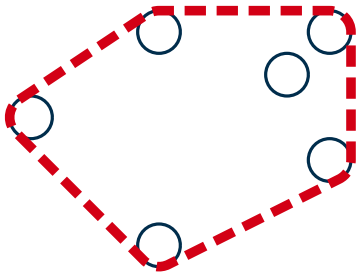
(a)



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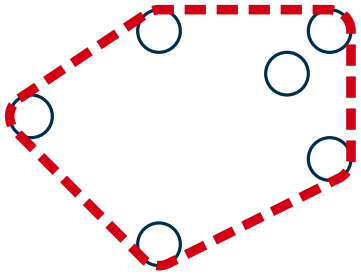
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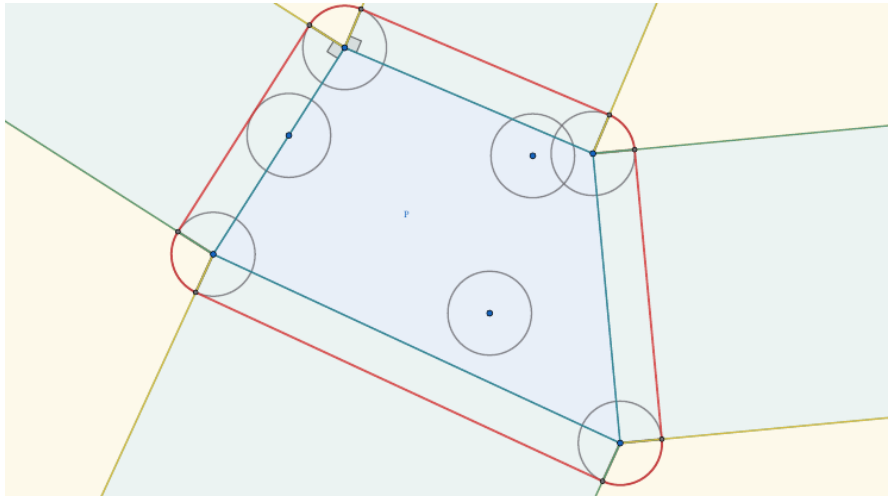


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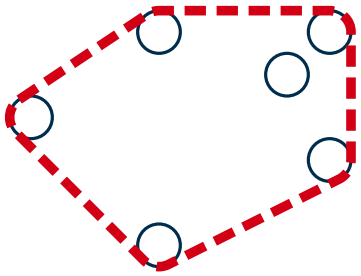


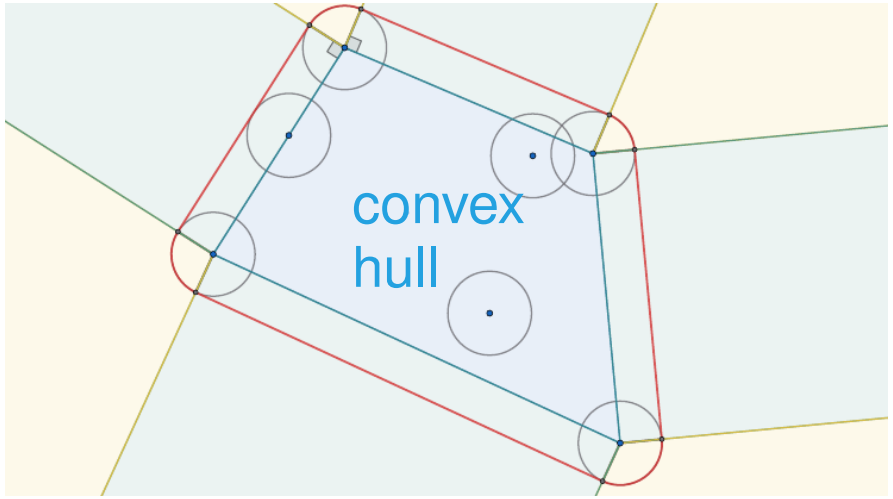


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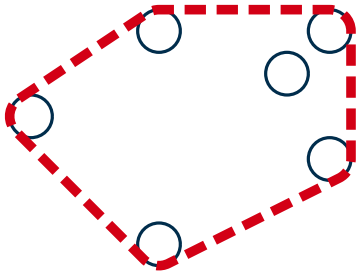


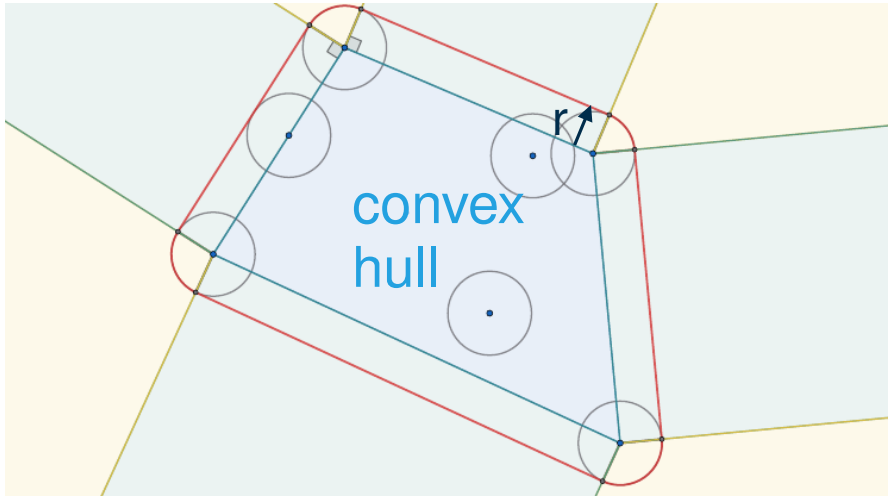


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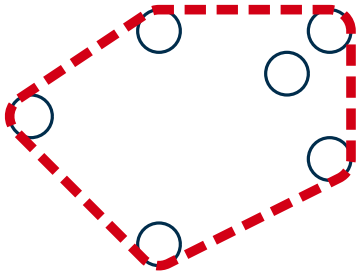


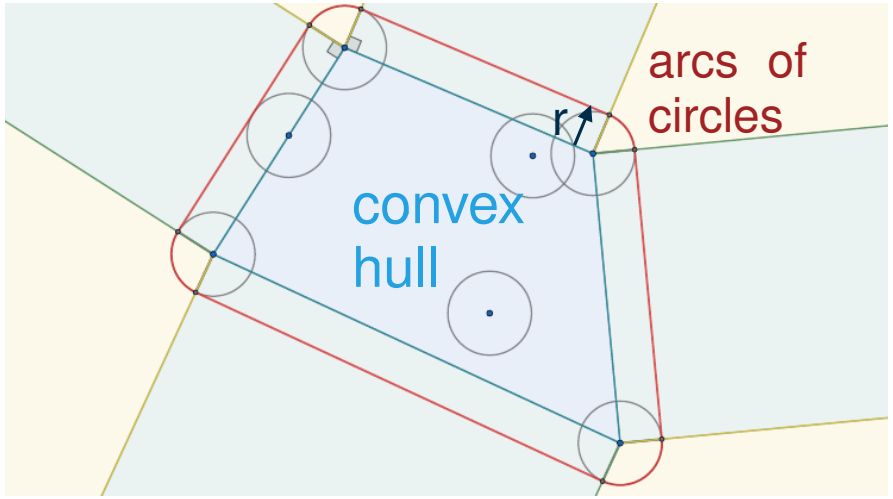


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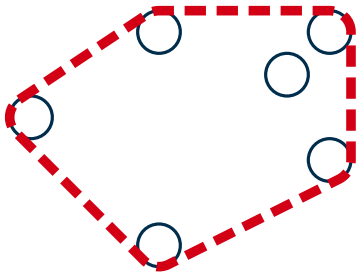




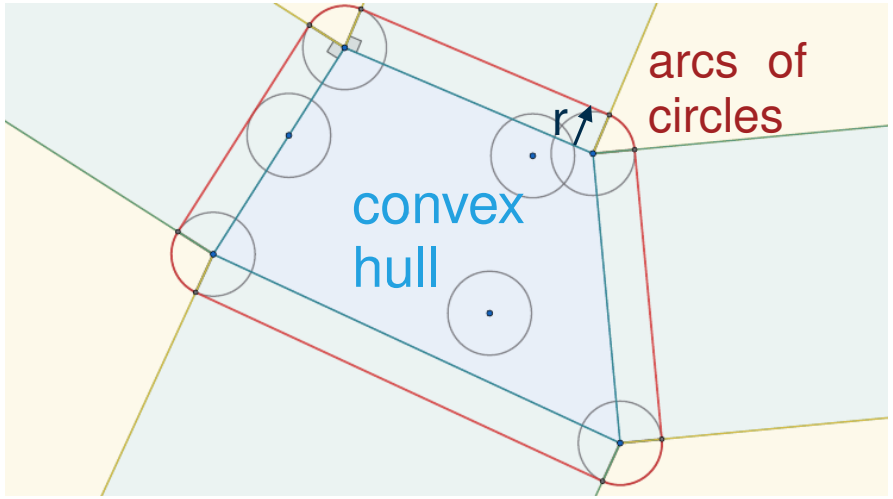
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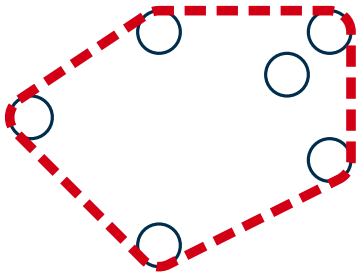
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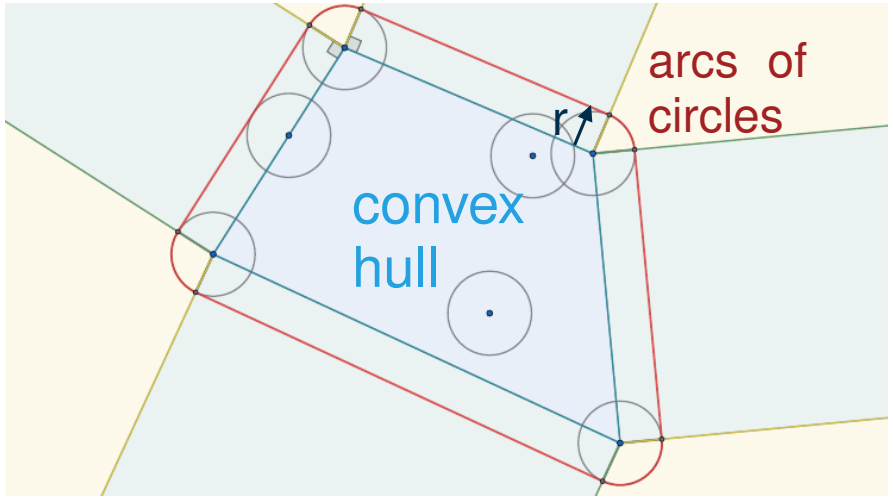
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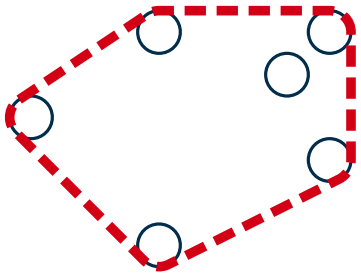
- show convex hull is the correct object: convex and minimal
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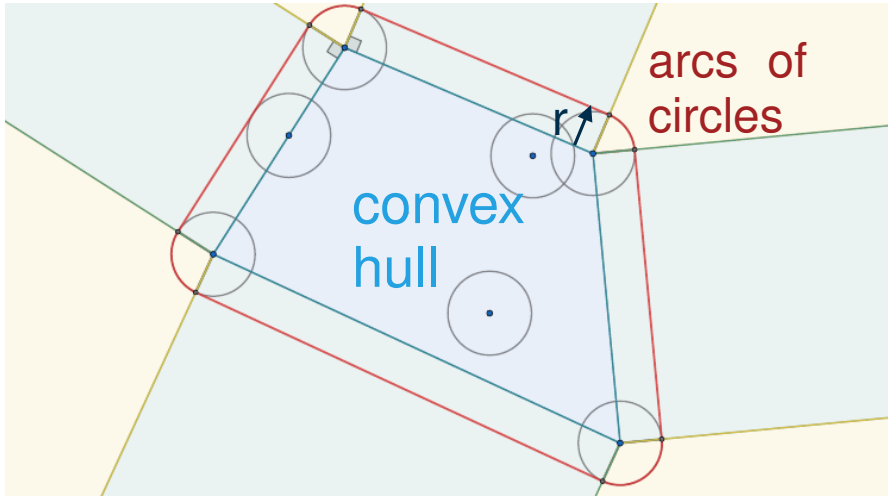
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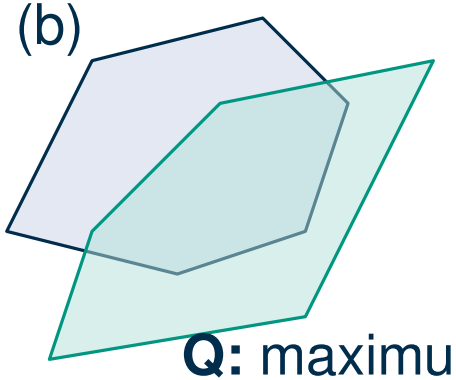
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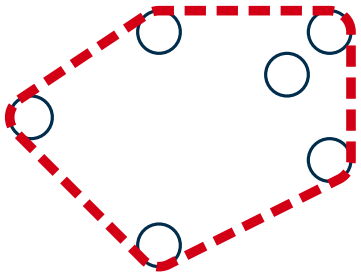


Q: maximum number of corners?

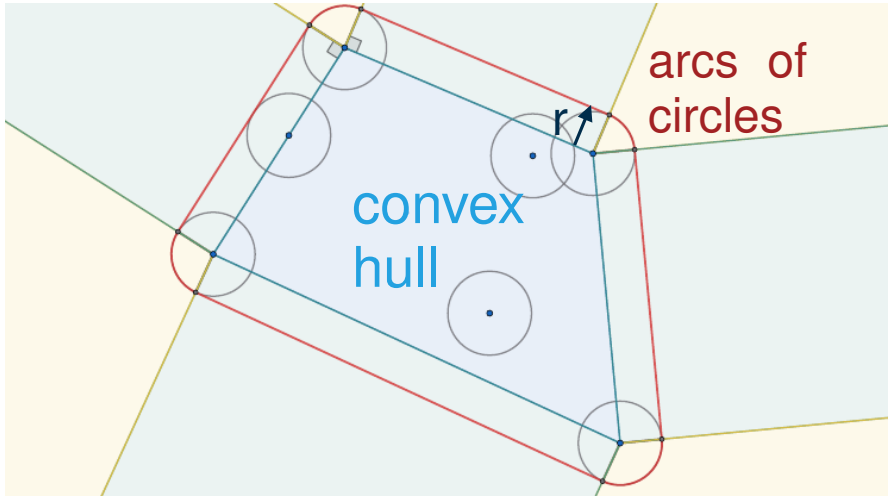
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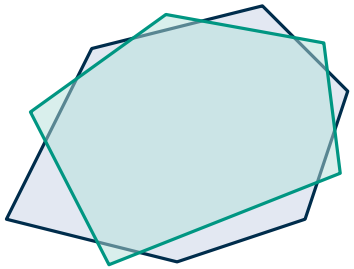
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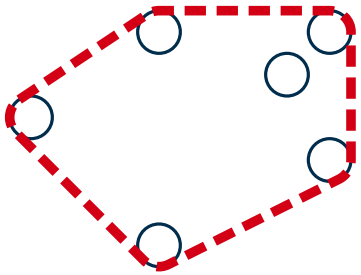


$m + n + 2\min\{n, m\}$

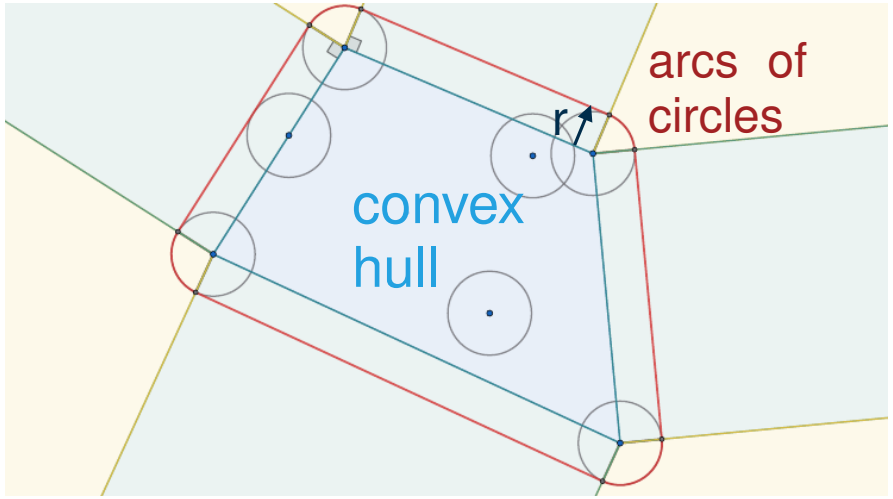
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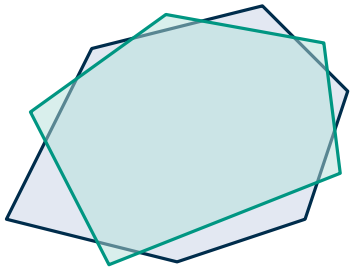


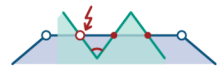
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(b)

- show upper bound



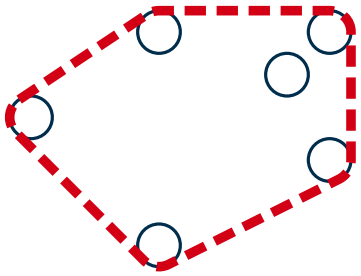


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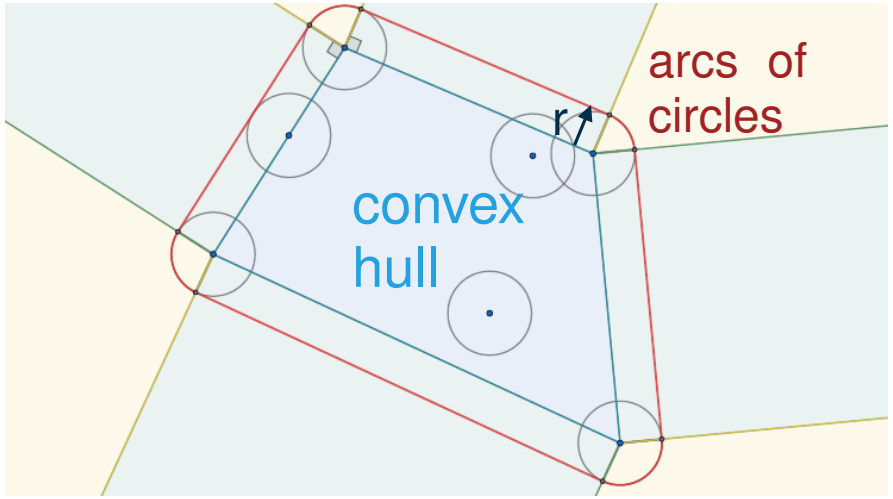
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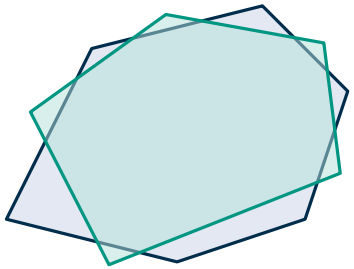
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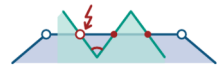


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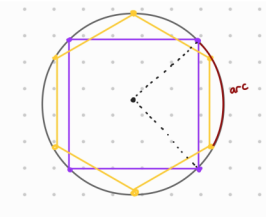


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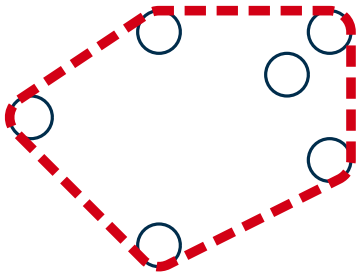
- show lower bound: construct worst case



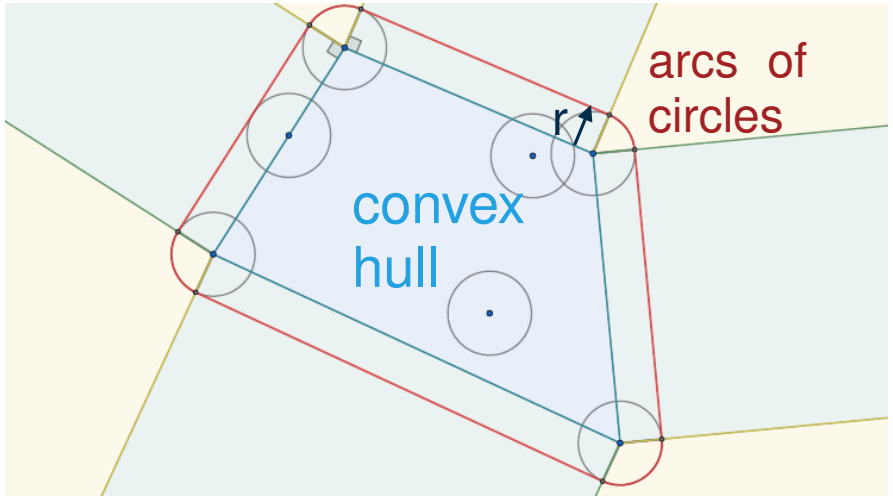
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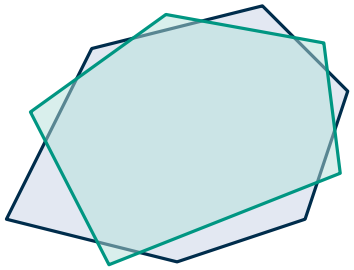
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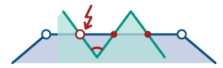
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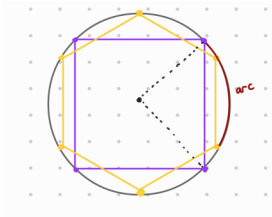
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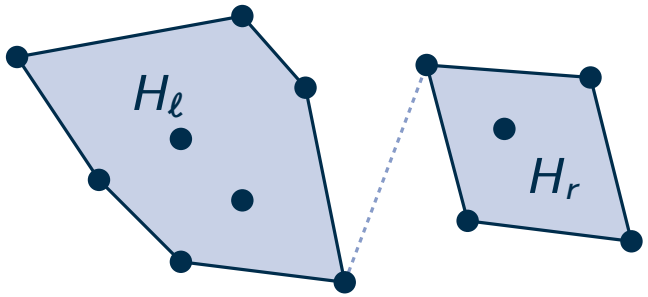


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Convex Hull via Divide and Conquer



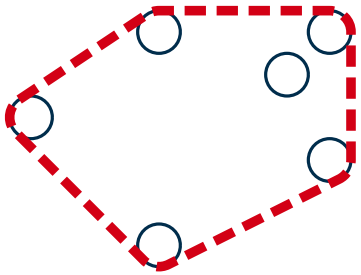
- find line between polygons

How?

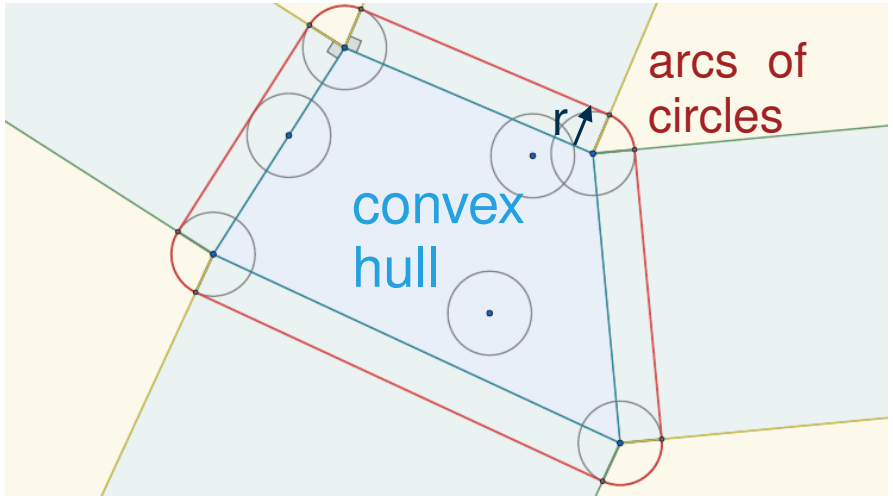
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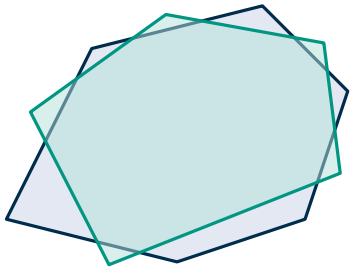
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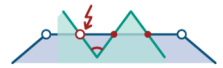


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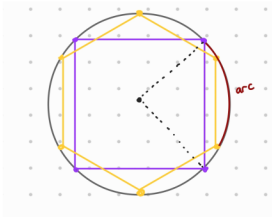


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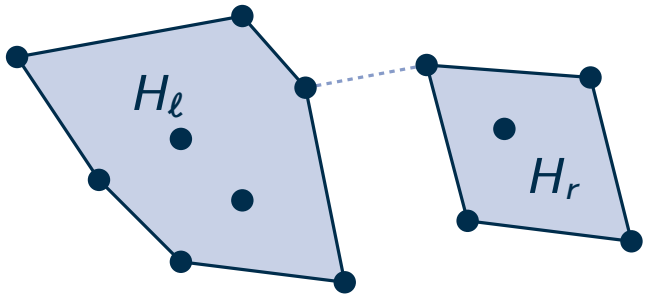
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Convex Hull via Divide and Conquer

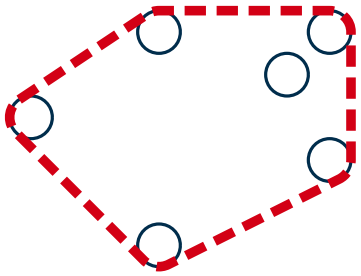


- find line between polygons How?
- move line iteratively (no wrong bends)

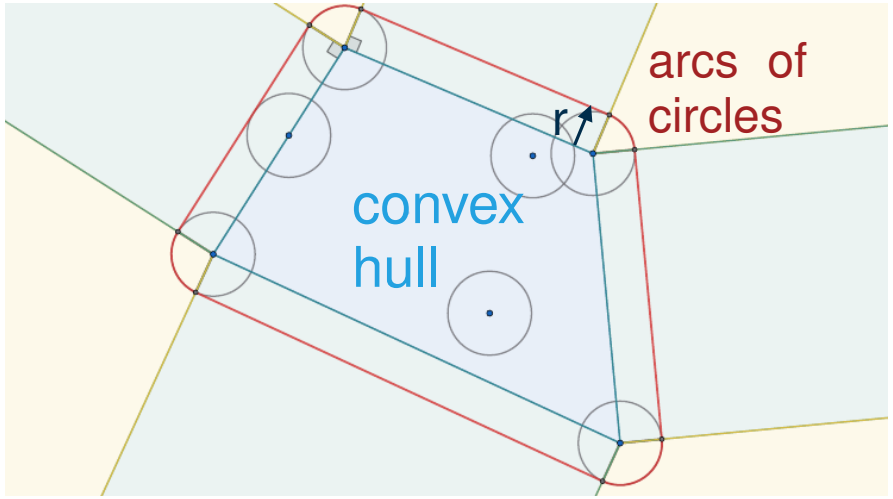
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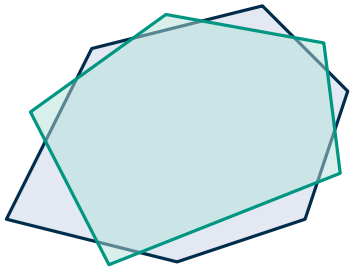
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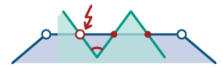


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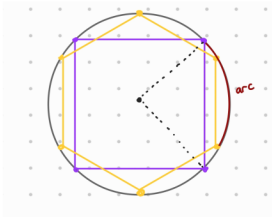


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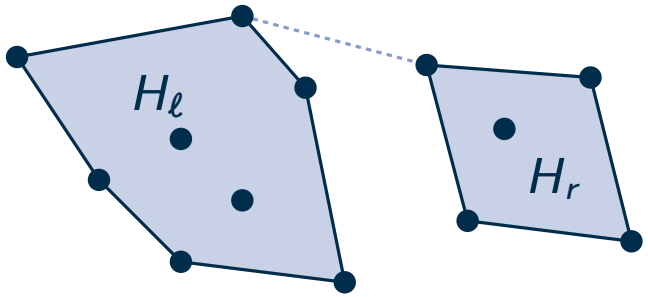
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- show lower bound: construct worst case



Convex Hull via Divide and Conquer

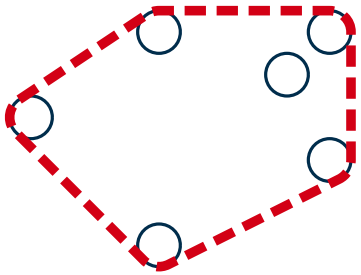


- find line between polygons How?
- move line iteratively (no wrong bends)

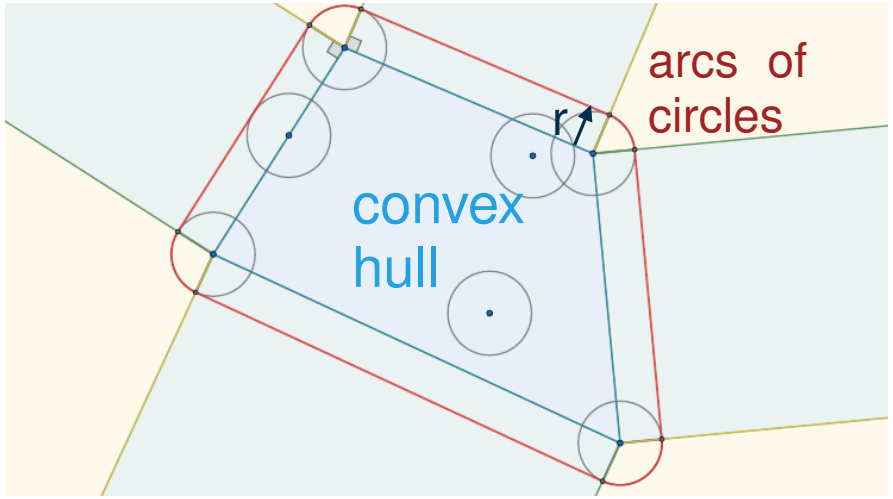
Sheet 1

Mysterious Meteorites

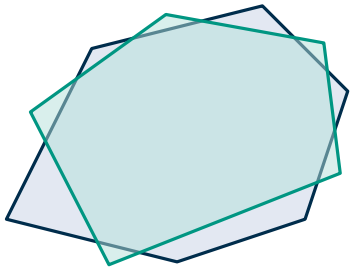
(a)



- show convex hull is the correct object: convex and minimal
- find good abstraction level

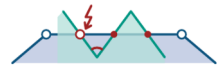


(b)

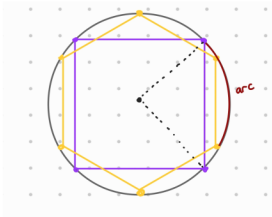


$m + n + 2\min\{n, m\}$

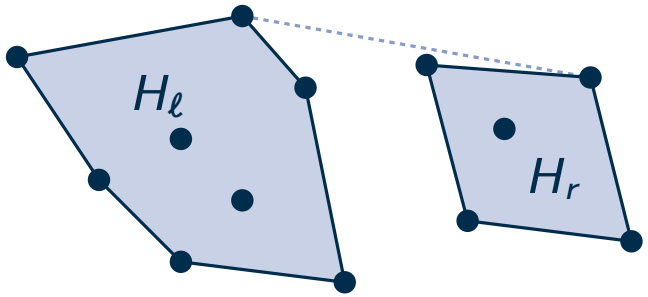
- show upper bound



- show lower bound: construct worst case



Convex Hull via Divide and Conquer

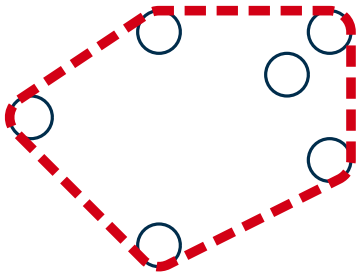


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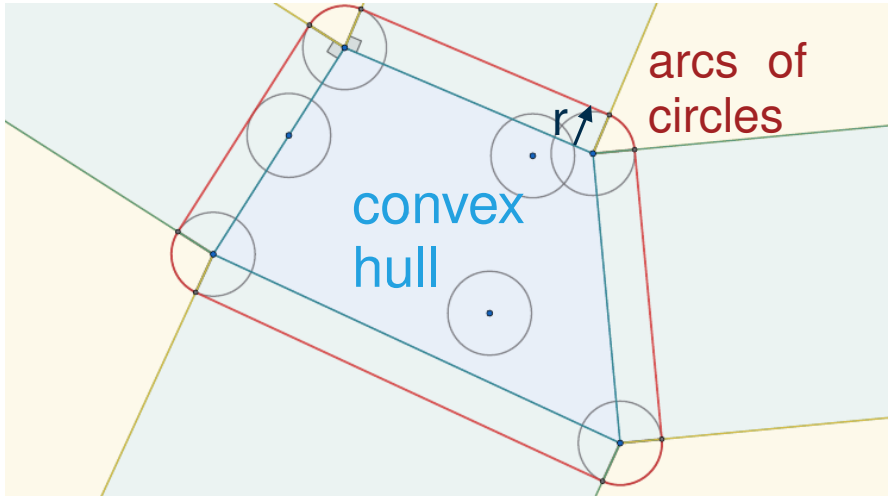
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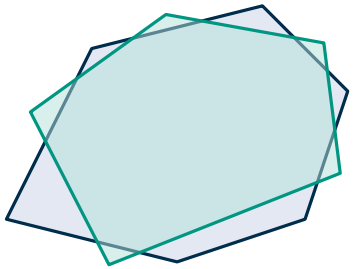
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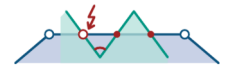


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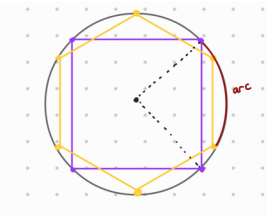


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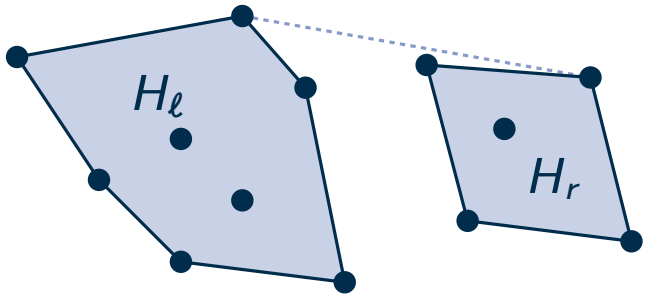
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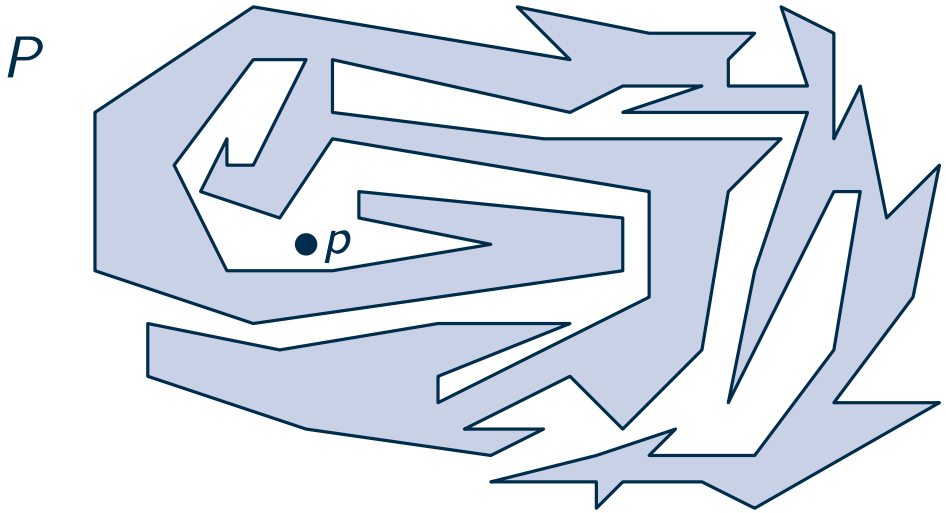
Convex Hull via Divide and Conquer



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- cheesy solutions?

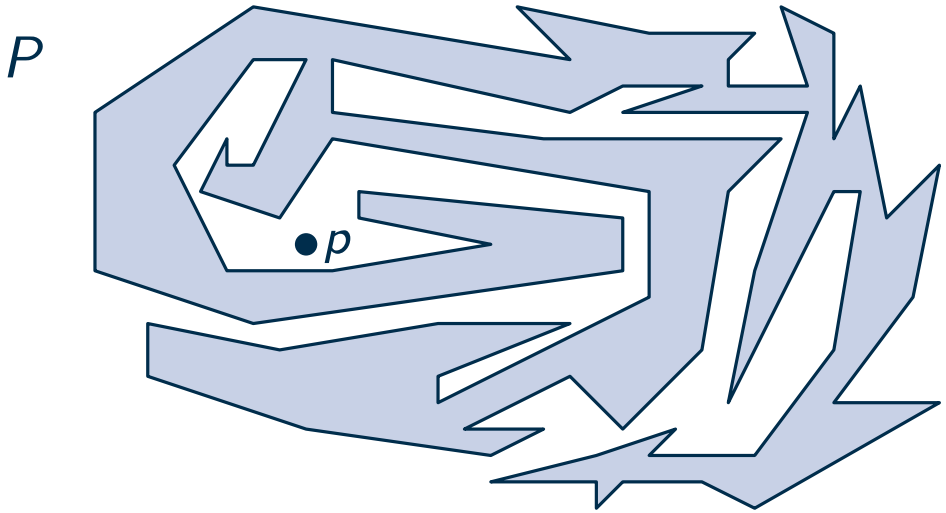
Sheet 1

Aerial Photography Is $p \in P$?



Sheet 1

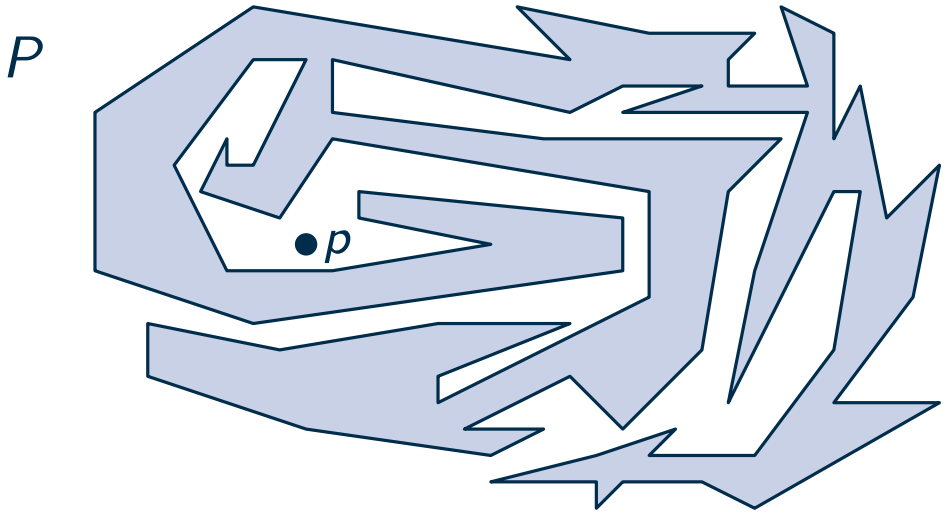
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- many variants

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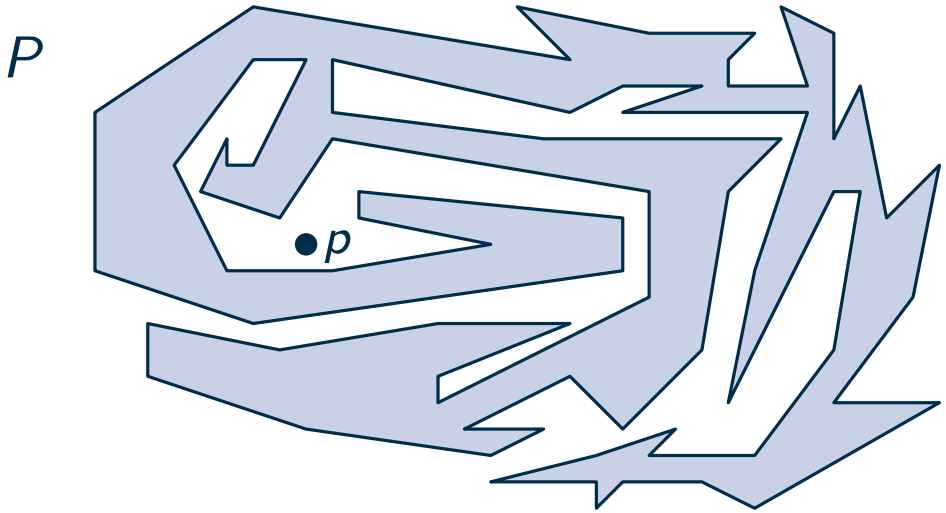
Aerial Photography Is $p \in P$?



- many variants
- triangulate, check for each triangle

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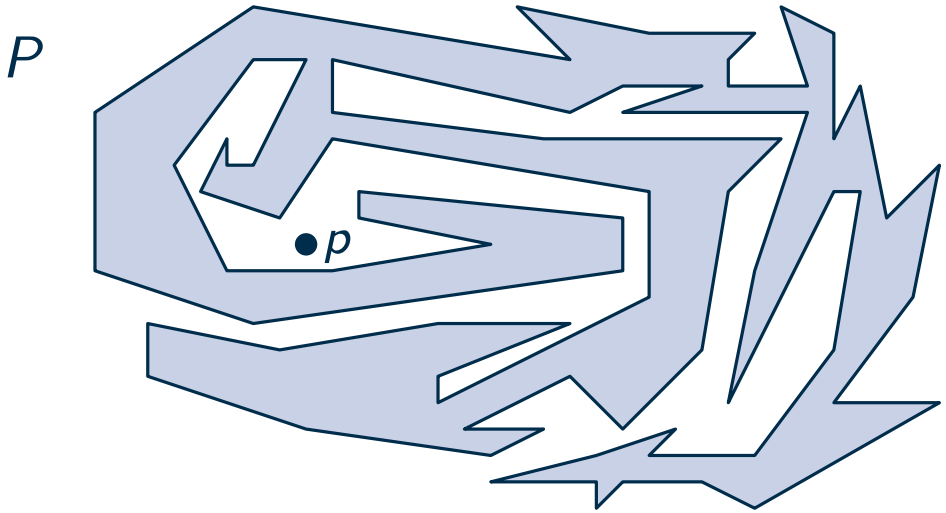
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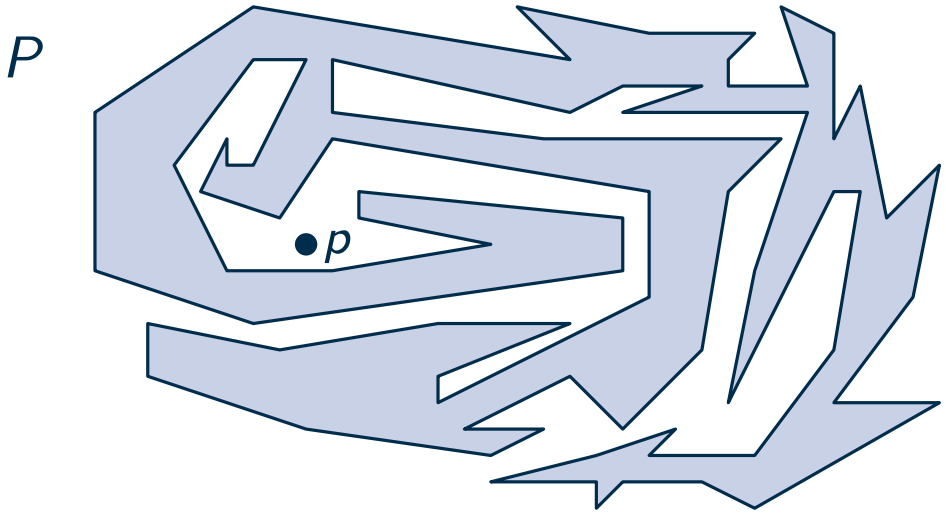
Aerial Photography Is $p \in P$?



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- triangulate, check for each triangle How?
- determine using closest segment

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Aerial Photography Is $p \in P$?

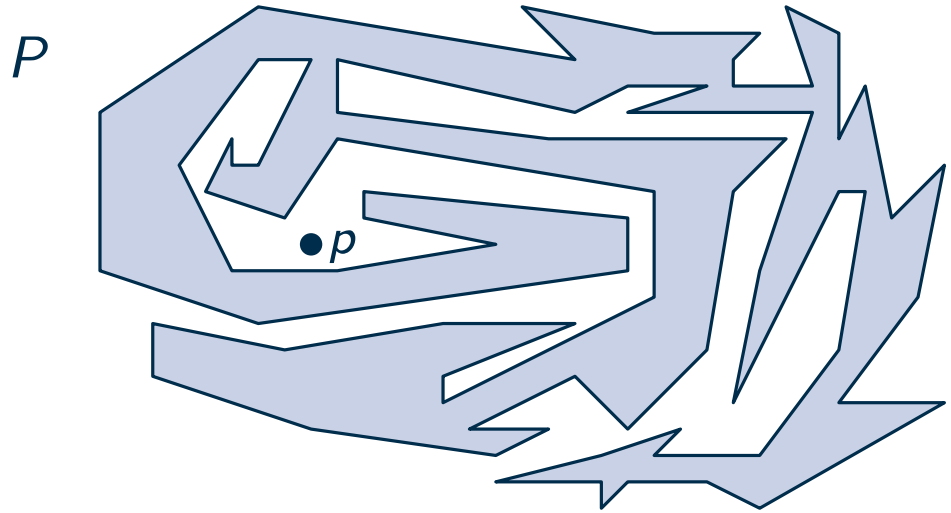


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Aerial Photography

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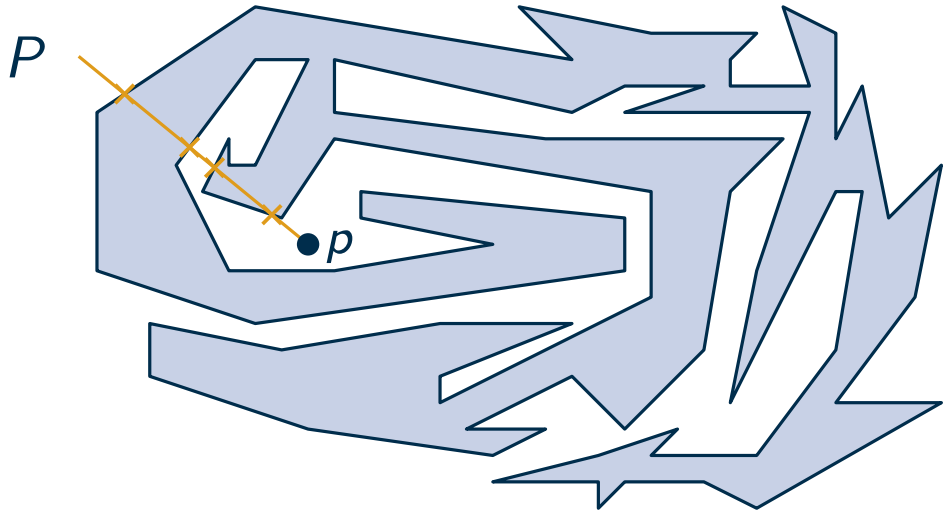
Ray Shooting

- shoot a ray, count intersections

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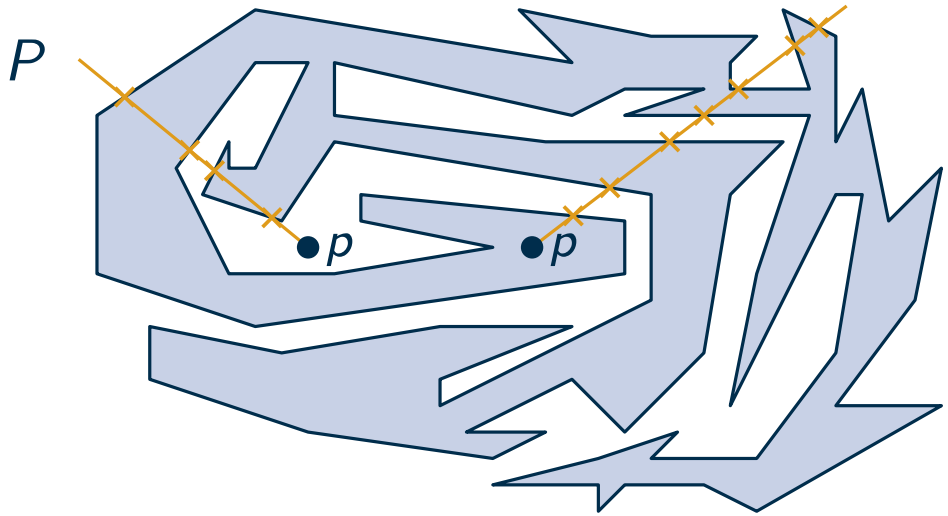
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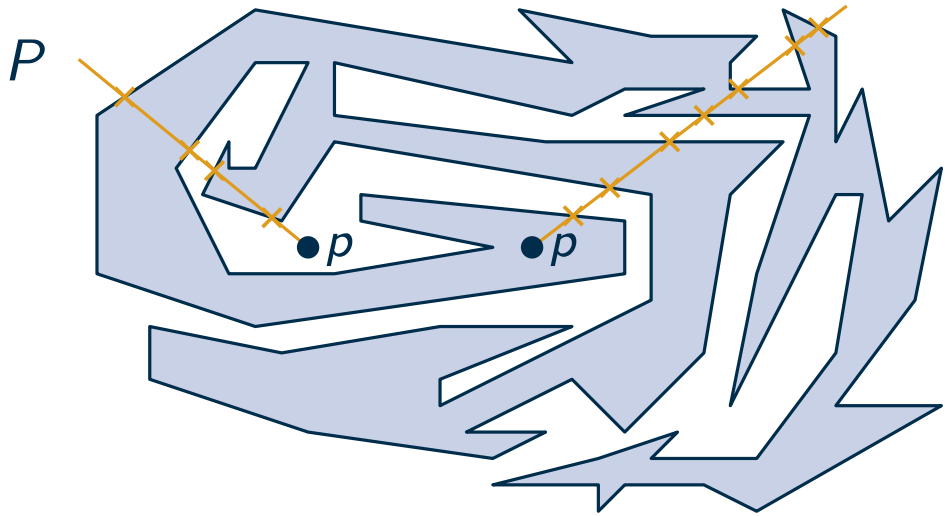
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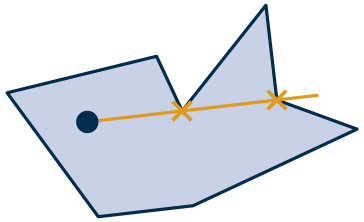
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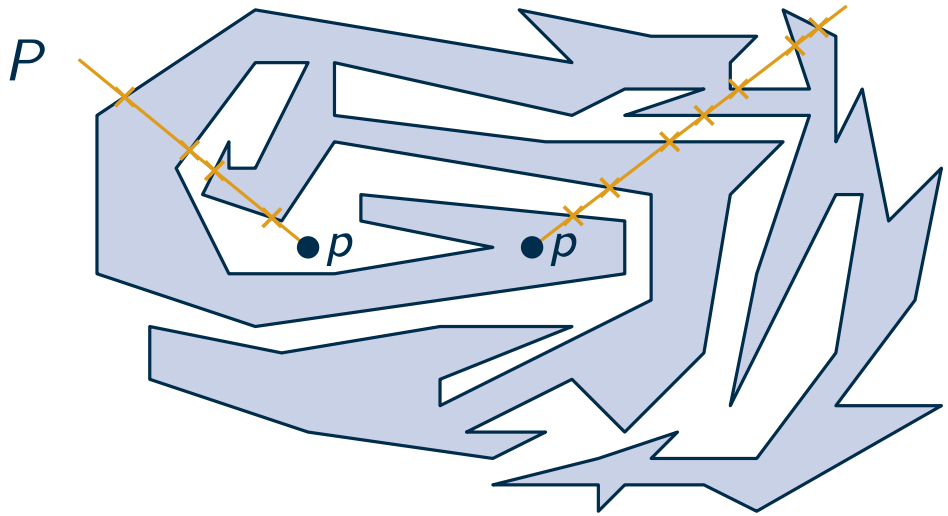
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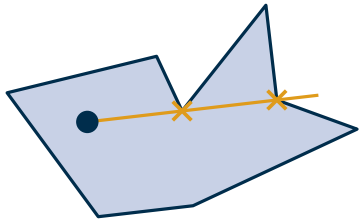
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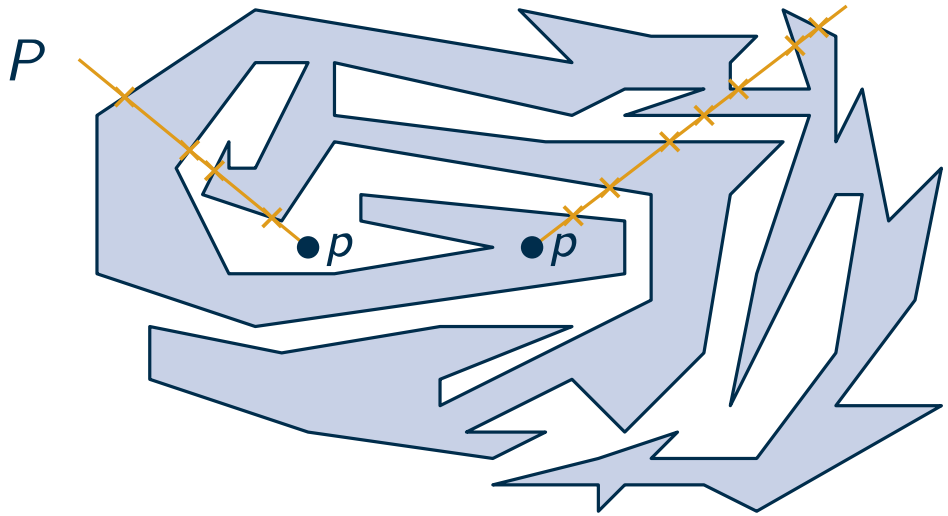
- shoot a ray, count intersections
- odd $\rightarrow$ inside, even $\rightarrow$ outside — Edge cases?
  - shoot another ray



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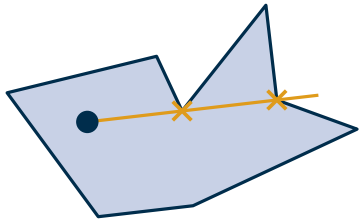
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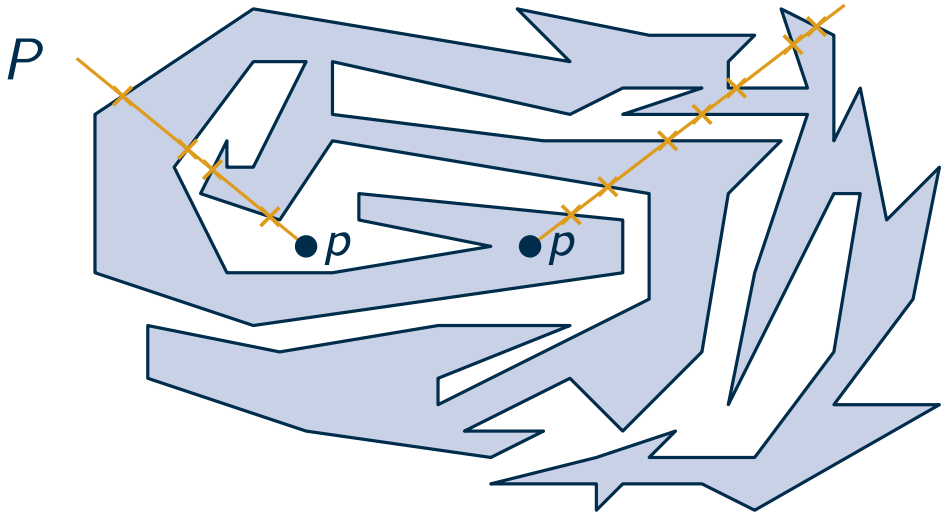
- shoot a ray, count intersections
- odd $\rightarrow$ inside, even $\rightarrow$ outside — Edge cases?
  - shoot another ray
  - count more carefully



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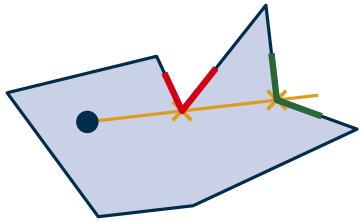
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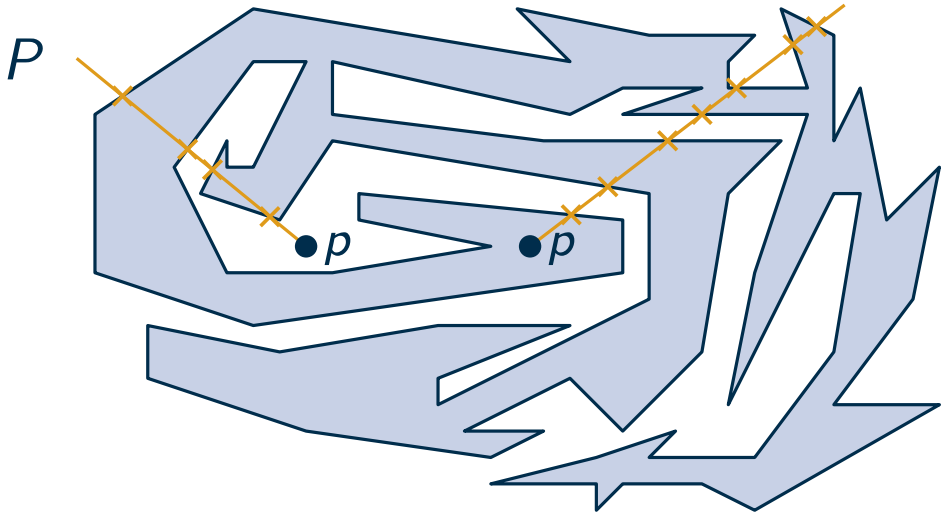
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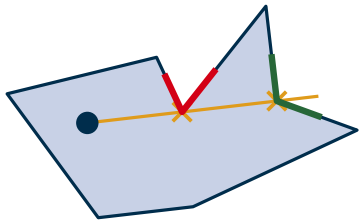
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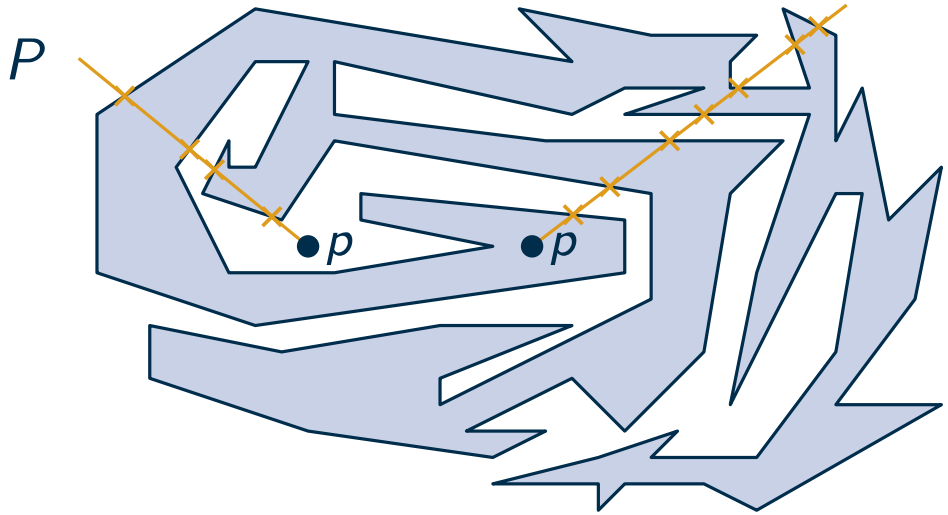


  - shoot another ray
  - count more carefully
  - shoot a ray with no edge case — How?

Sheet 1

Aerial Photography

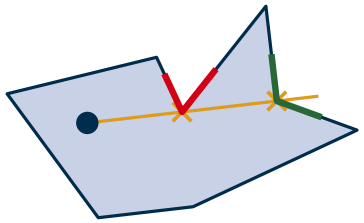
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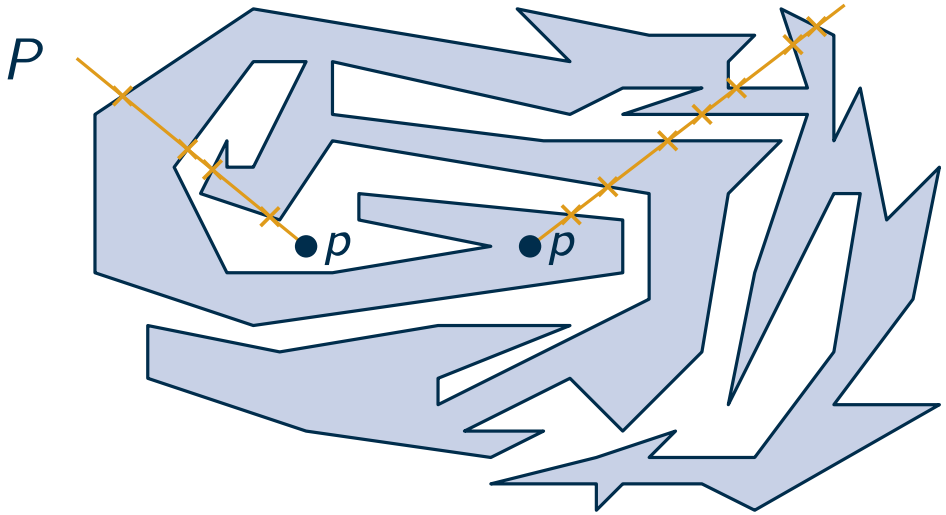
Robustness

to which extent do we care?

Sheet 1

Aerial Photography

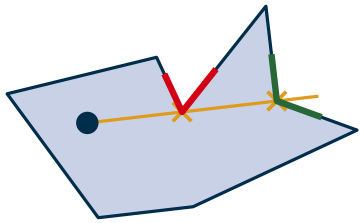
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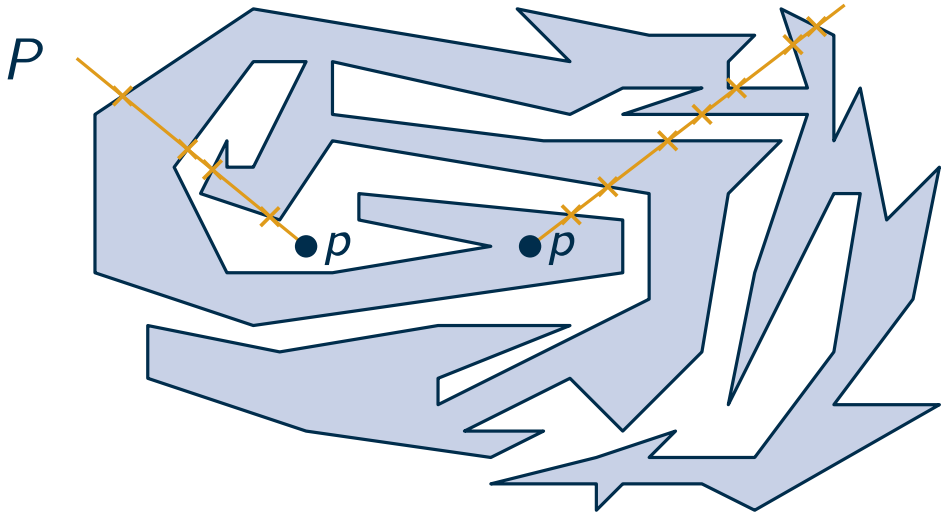
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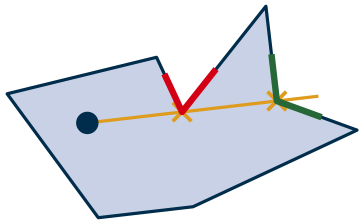
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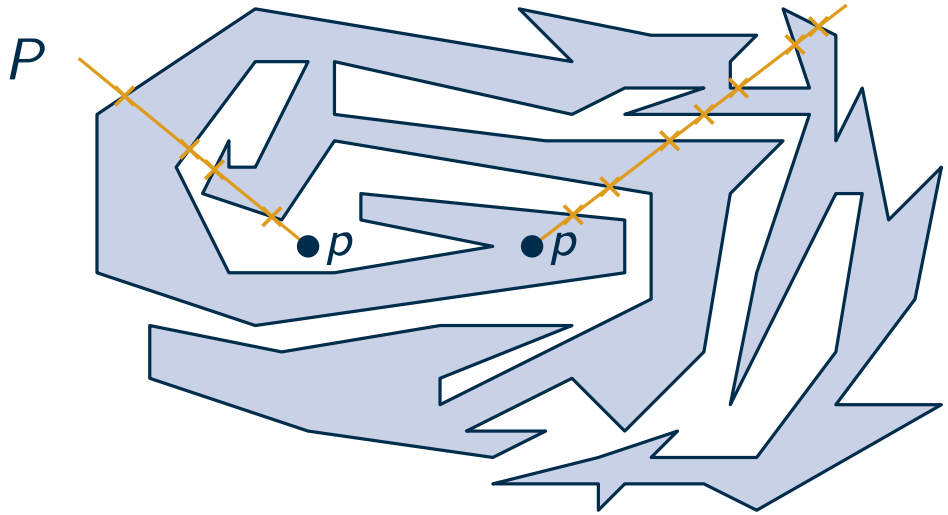
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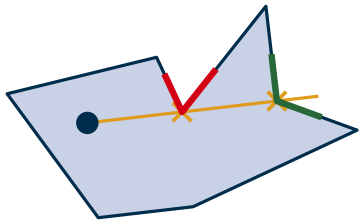
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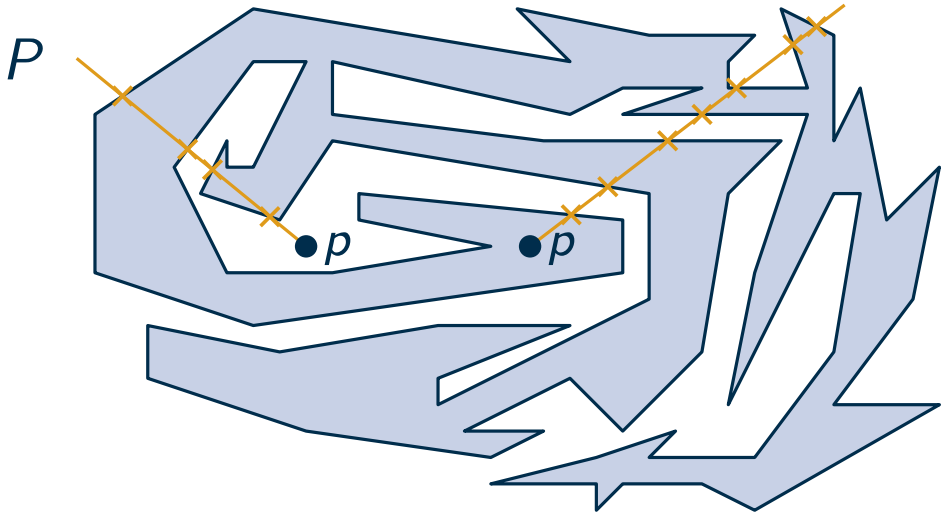
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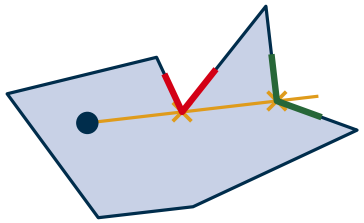
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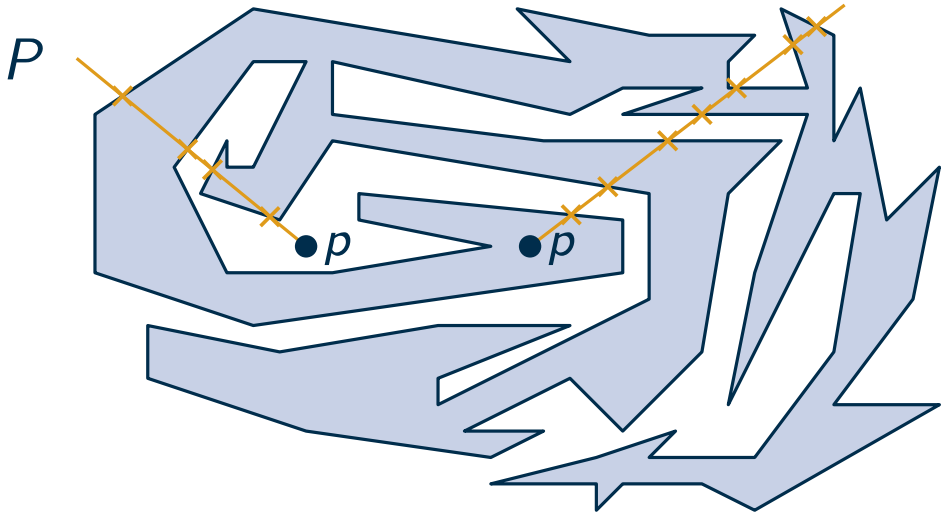


- has correct output format (e.g polygon for CH)

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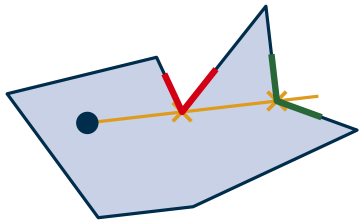
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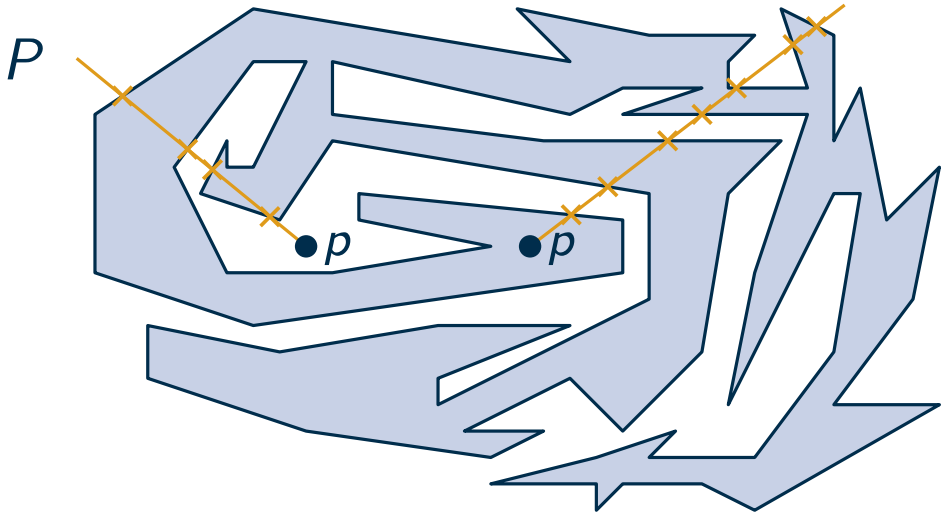


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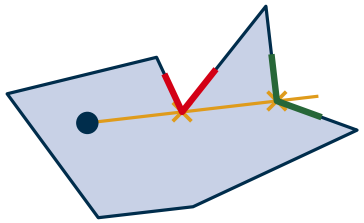
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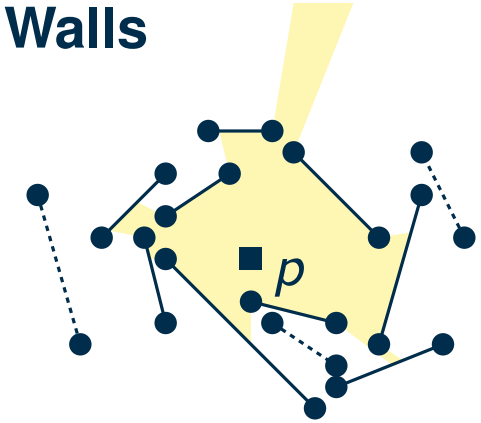
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- has correct output format (e.g polygon for CH)
- usually assume integer input — Why does this help?

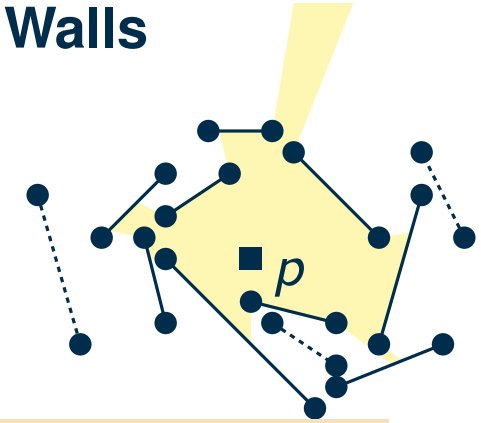
Assignment Sheet 2

Dangerous Walls



Assignment Sheet 2

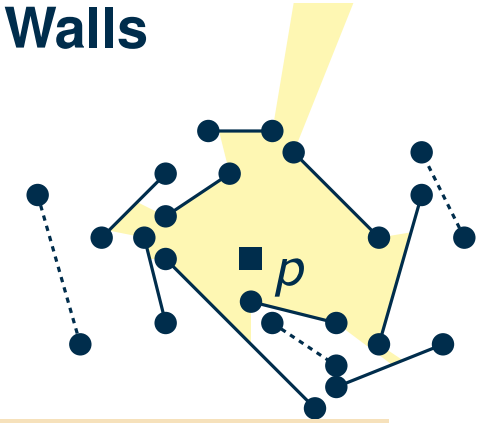
Dangerous Walls



use nice technique from lecture

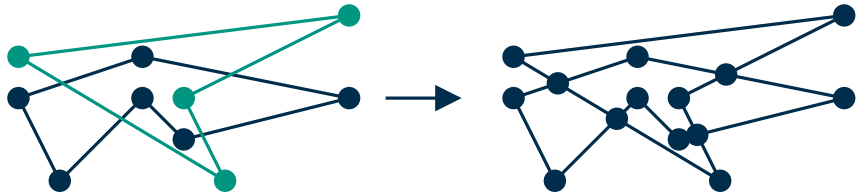
Assignment Sheet 2

Dangerous Walls



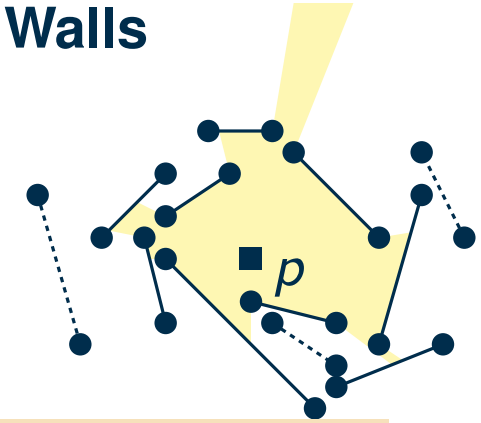
use nice technique from lecture

Map Overlay



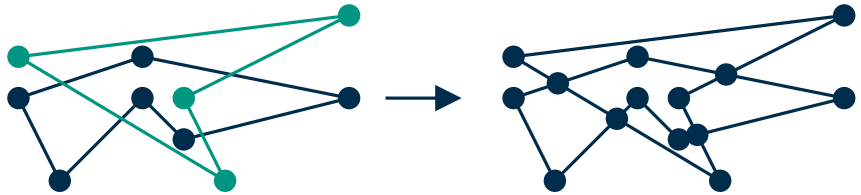
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Dangerous Walls



use nice technique from lecture

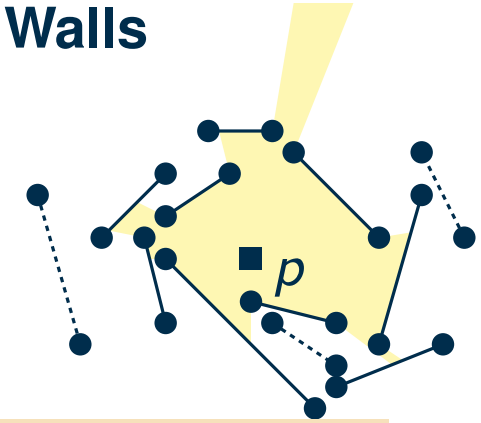
Map Overlay



annoying pointers, use a figure

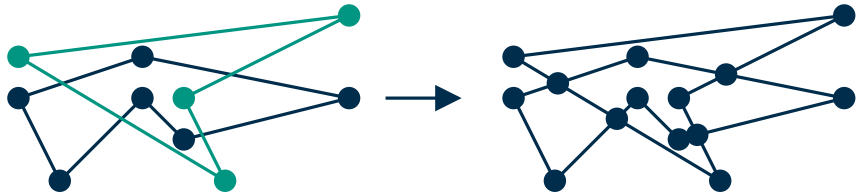
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Dangerous Walls



use nice technique from lecture

Map Overlay



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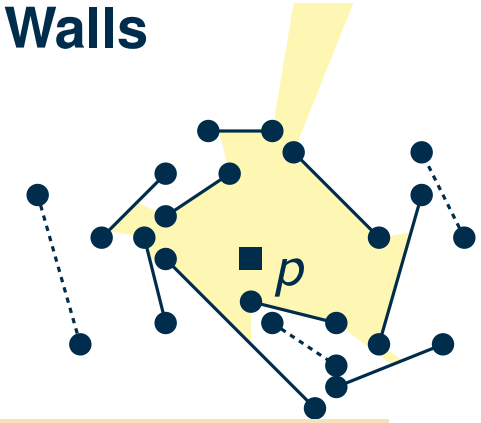
Boolean Operations

Given: Polygons P, Q

Task: compute $P \cap Q$, $P \cup Q$, and $(P \cup Q) \setminus (P \cap Q)$

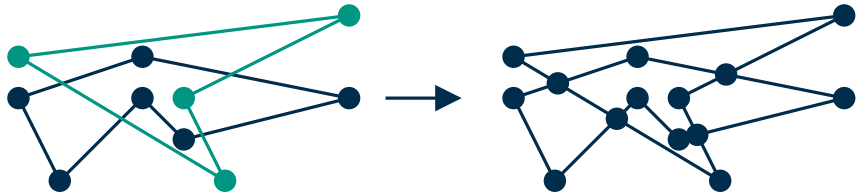
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Boolean Operations

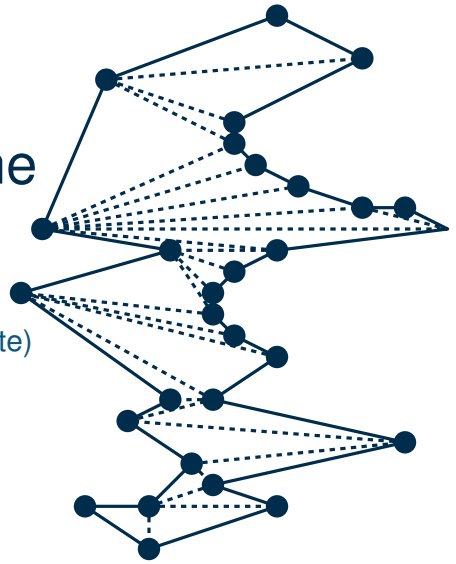
Given: Polygons P, Q

Task: compute $P \cap Q$, $P \cup Q$, and $(P \cup Q) \setminus (P \cap Q)$

y-Monotone Triangulation

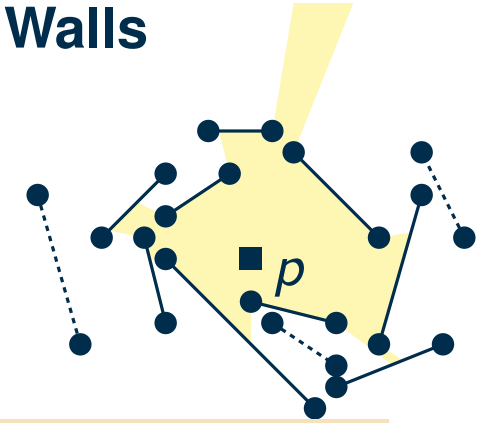
How to triangulate y-monotone Polygon in $O(n)$?

(easier: assume no two vertices have same y-coordinate)



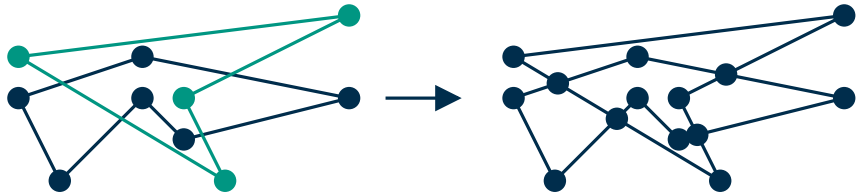
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use nice technique from lecture

Map Overlay



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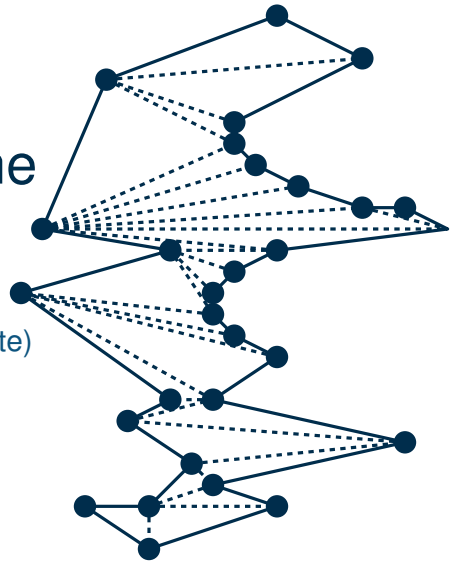
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y-Monotone Triangulation

How to triangulate y-monotone Polygon in $O(n)$?

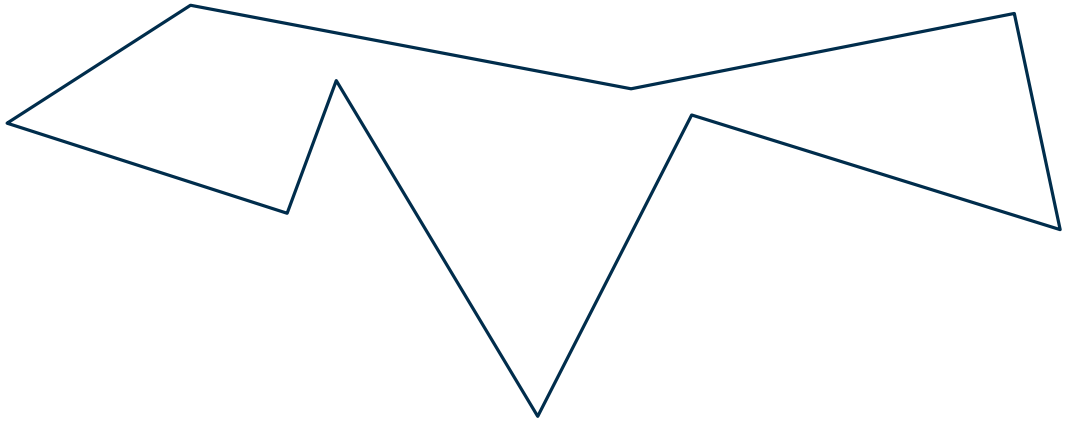
(easier: assume no two vertices have same y-coordinate)

find invariant for your algo



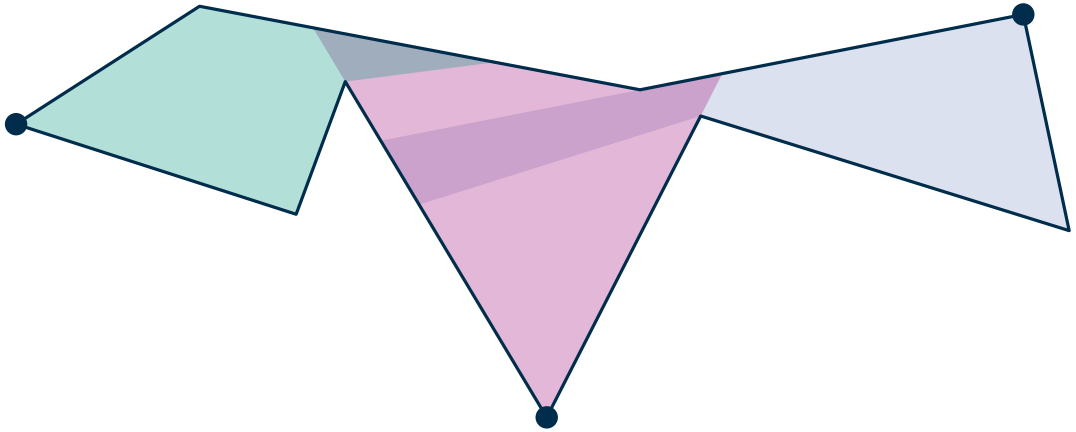
Art Gallery Problem

How many vertex guards are needed to guard a museum?



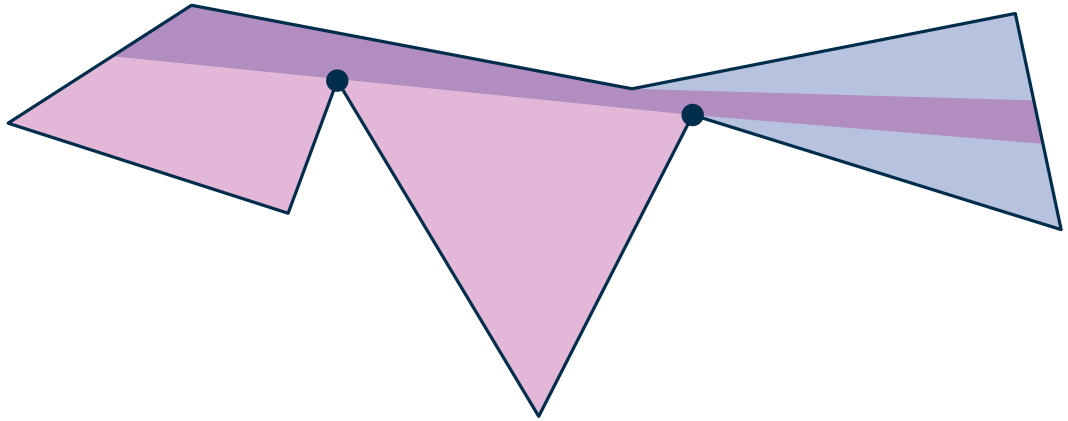
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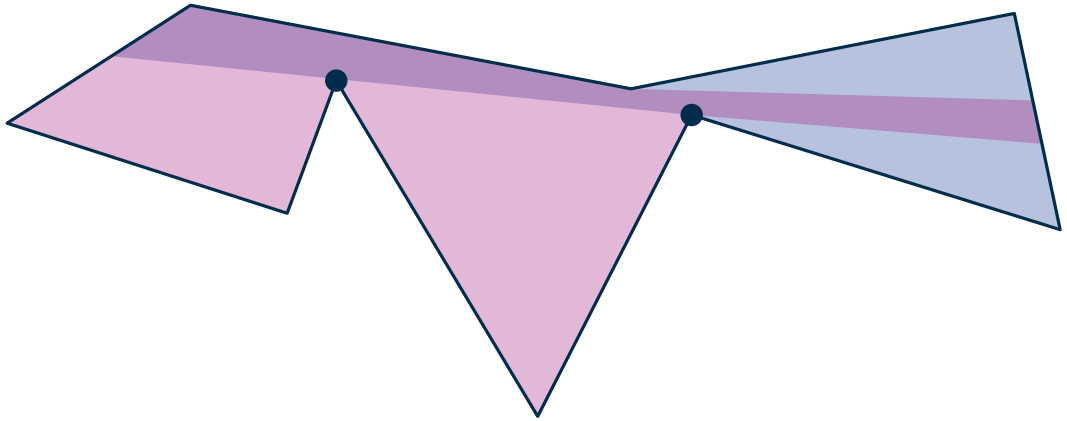
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Art Gallery Problem

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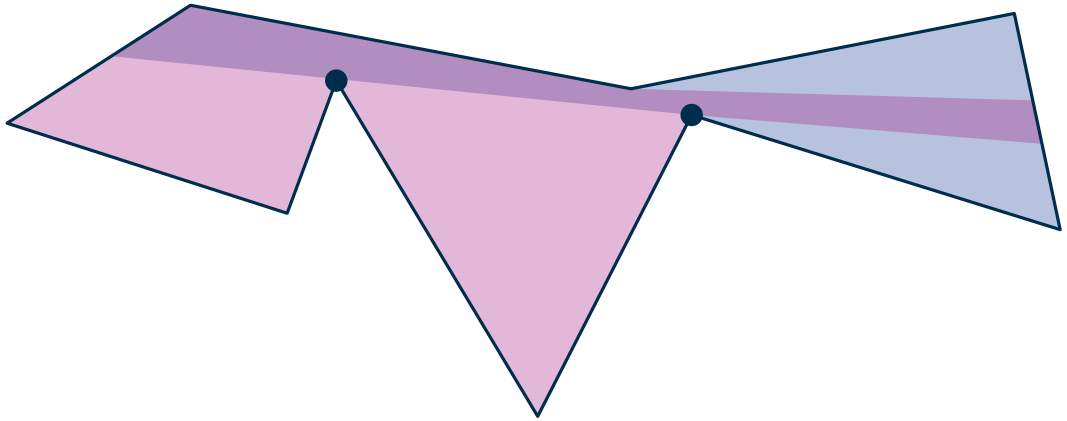


Lower Bounds

- min #vertices that needs 2 (3) guards?
- construct bad (worst?) cases for n vertices

Art Gallery Problem

How many vertex guards are needed to guard a museum?



Lower Bounds

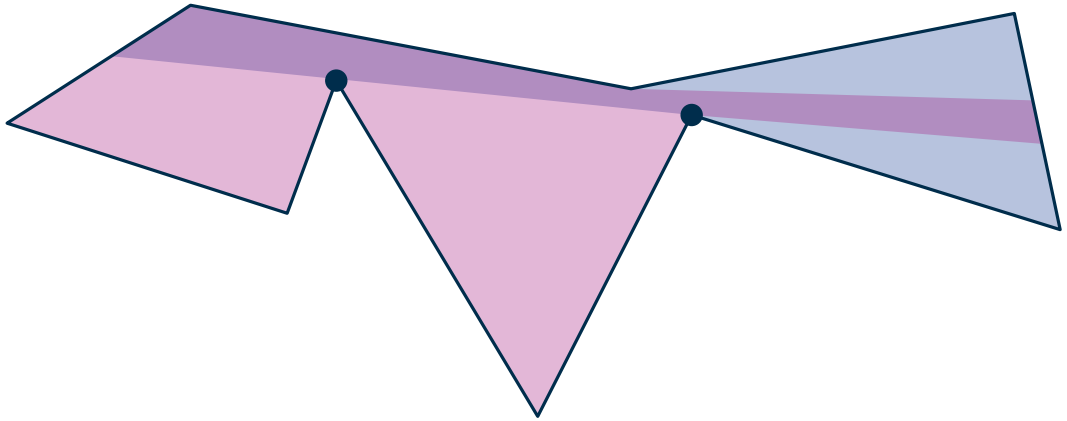
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- construct bad (worst?) cases for n vertices

Upper Bounds

- partition museum into easier shapes

Art Gallery Problem

How many vertex guards are needed to guard a museum?



Lower Bounds

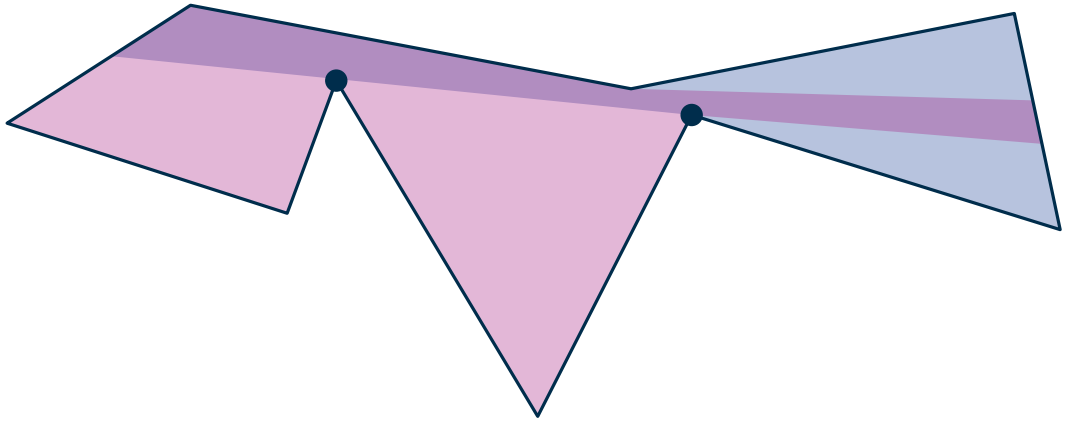
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Upper Bounds

- partition museum into easier shapes
- find out how it helps

Art Gallery Problem

How many vertex guards are needed to guard a museum?



Lower Bounds

- min #vertices that needs 2 (3) guards?
- construct bad (worst?) cases for n vertices

Upper Bounds

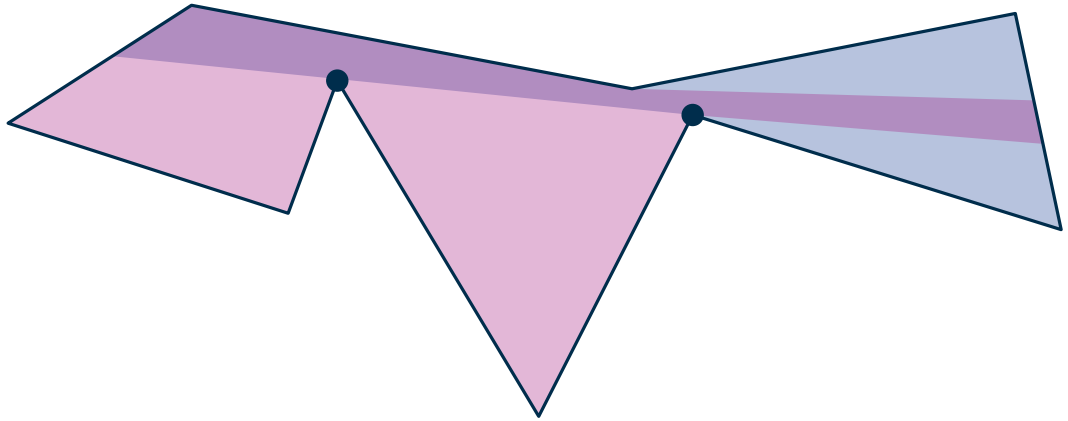
- partition museum into easier shapes
- find out how it helps

Bonus

- Research other fun variants

Art Gallery Problem

How many vertex guards are needed to guard a museum?



Lower Bounds

- min #vertices that needs 2 (3) guards?
- construct bad (worst?) cases for n vertices

Upper Bounds

- partition museum into easier shapes
- find out how it helps

convex polygons
bound using #reflex vts

triangles (a little trickier)
3-coloring

Bonus

- Research other fun variants

Art Gallery Problem – Solution Sketch

How many vertex guards are needed to guard a museum?

Art Gallery Problem – Solution Sketch

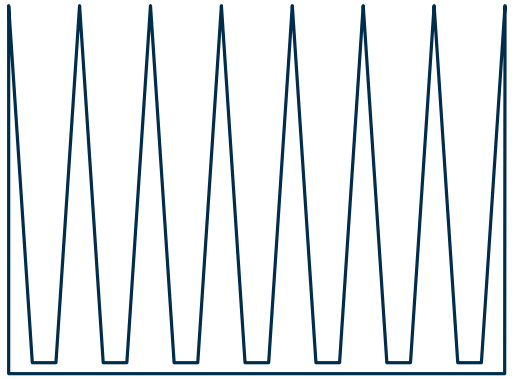
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Lower Bounds

Art Gallery Problem – Solution Sketch

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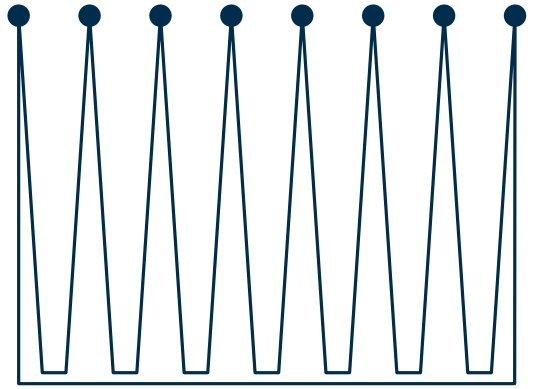
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How many vertex guards are needed to guard a museum?

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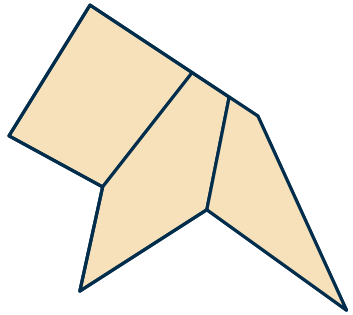


$\lfloor \frac{n}{3} \rfloor$ guards

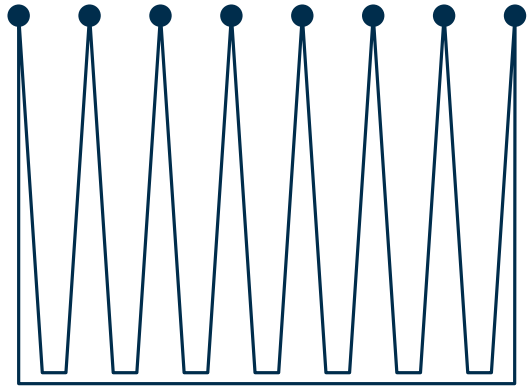
Art Gallery Problem – Solution Sketch

How many vertex guards are needed to guard a museum?

Partition into Convex Polygons



Lower Bounds



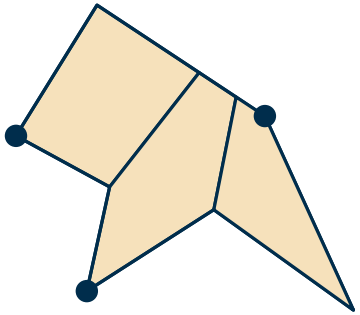
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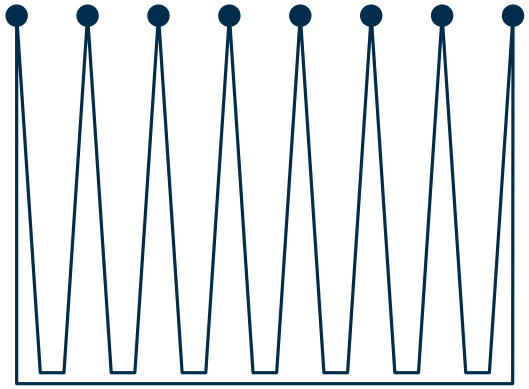
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Partition into Convex Polygons

- one guard per convex polygon



Lower Bounds



$\lfloor \frac{n}{3} \rfloor$ guards

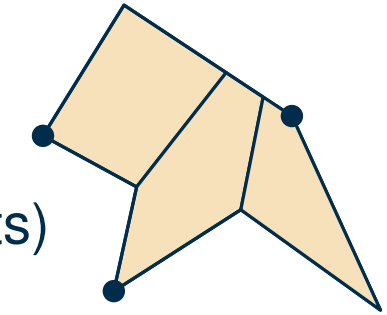
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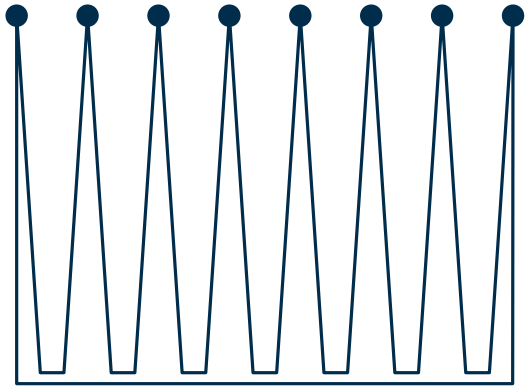
Partition into Convex Polygons

- one guard per convex polygon
- $r + 1$ many convex polygons (r: #reflex vts)

Why?



Lower Bounds



$\lfloor \frac{n}{3} \rfloor$ guards

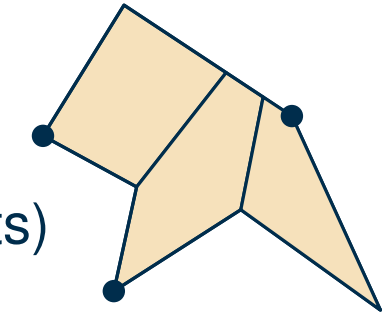
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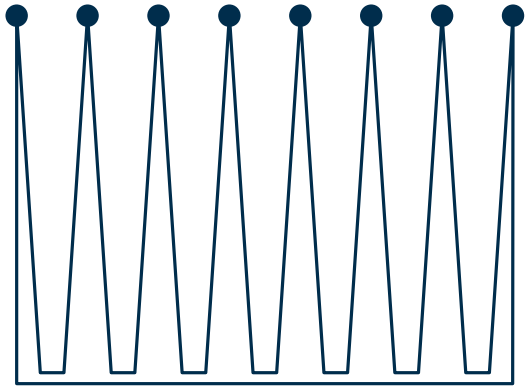
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Lower Bounds



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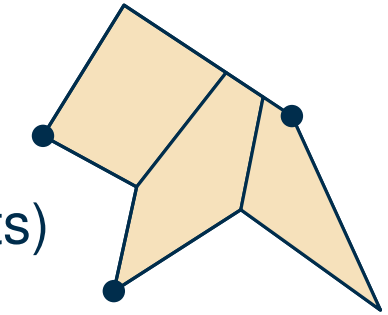
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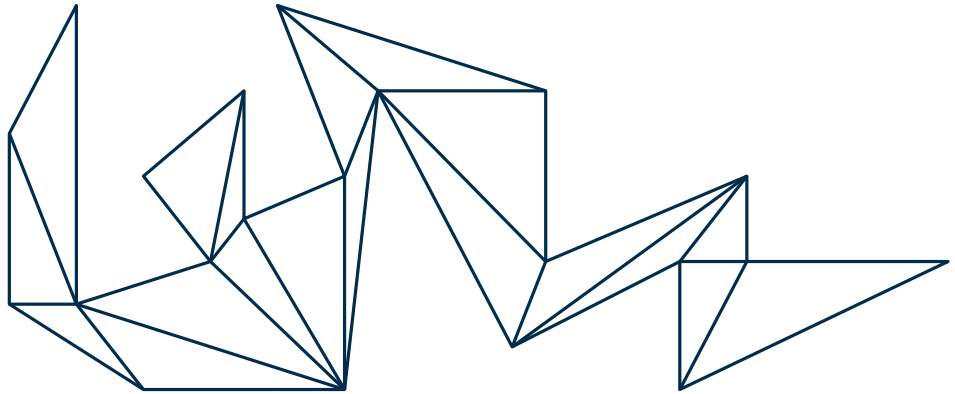
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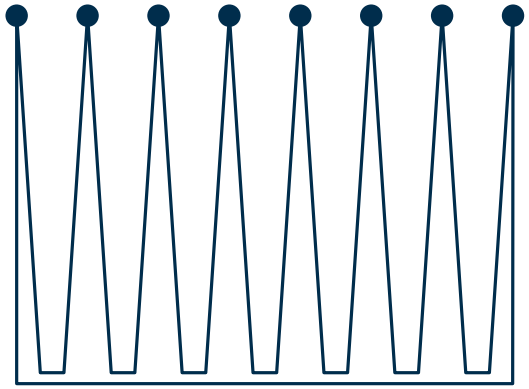
Why?



Partition into Triangles



Lower Bounds



$\lfloor \frac{n}{3} \rfloor$ guards

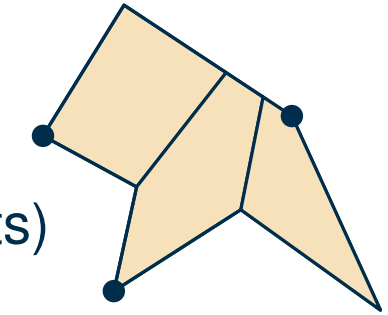
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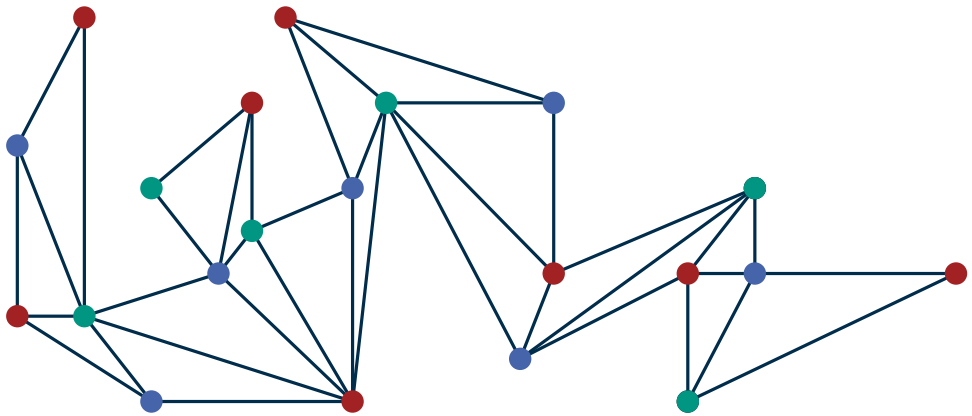
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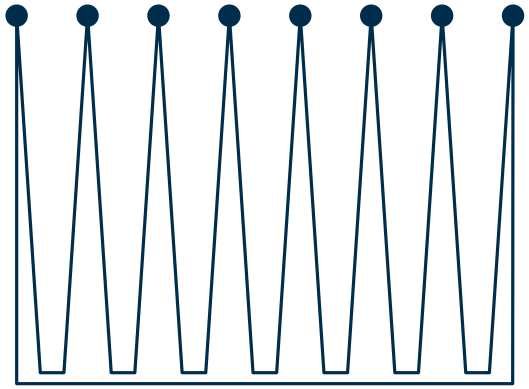
Partition into Triangles

- planar triangulations can be 3-colored

Why?



Lower Bounds



$\lfloor \frac{n}{3} \rfloor$ guards

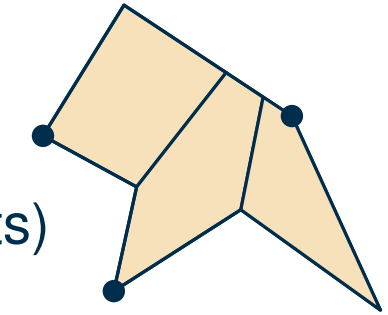
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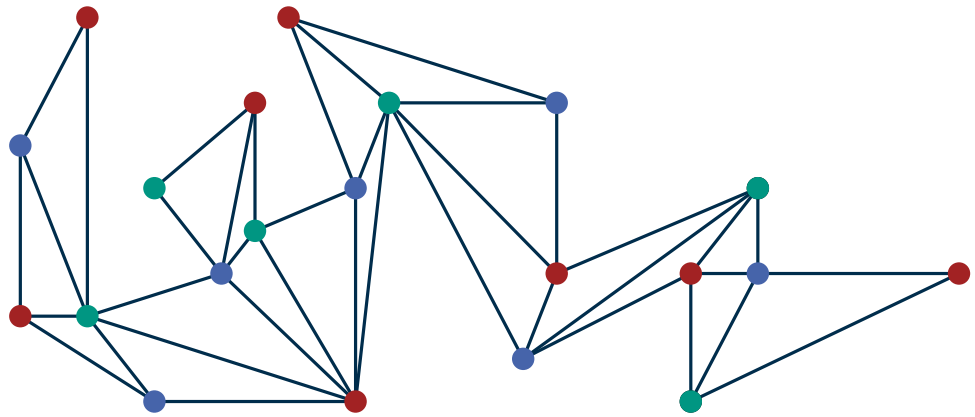
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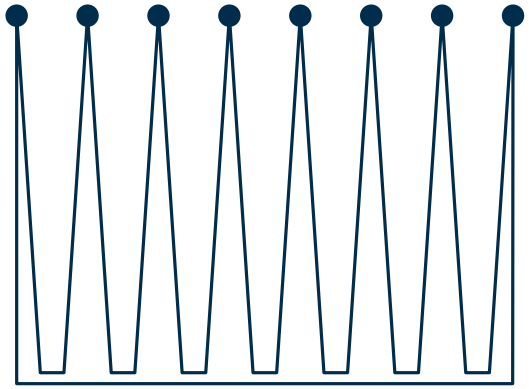
Why?

- put guards on smallest color class

Why valid?



Lower Bounds

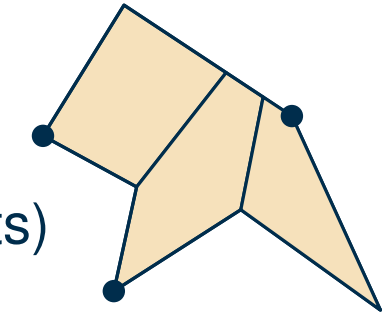


$\lfloor \frac{n}{3} \rfloor$ guards

Art Gallery Problem – Solution Sketch

How many vertex guards are needed to guard a museum?

Partition into Convex Polygons



- one guard per convex polygon
- $r + 1$ many convex polygons (r: #reflex vts)
- $\rightarrow r + 1$ guards needed

Why?

Partition into Triangles

- planar triangulations can be 3-colored

Why?

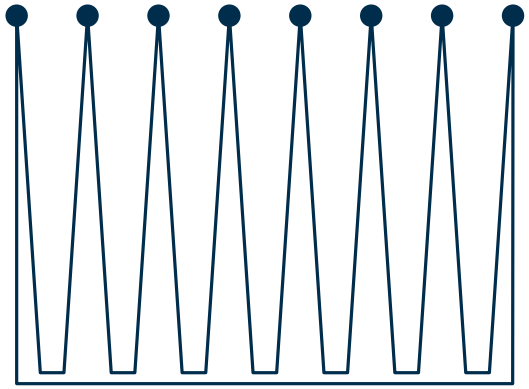
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Why valid?

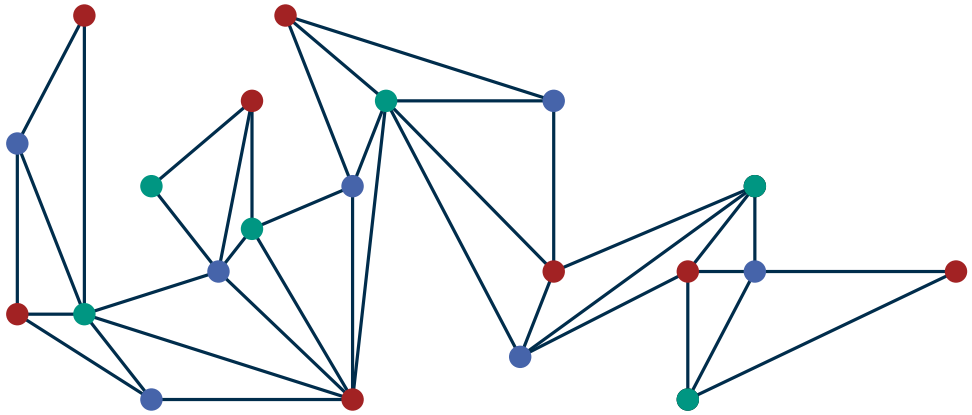
- at most $\lfloor \frac{n}{3} \rfloor$ guards needed

Why?

Lower Bounds



$\lfloor \frac{n}{3} \rfloor$ guards



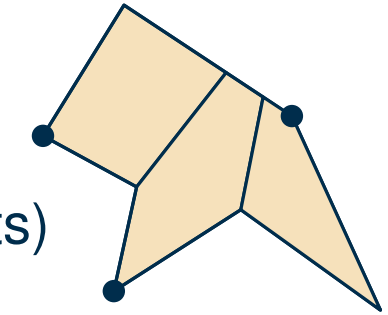
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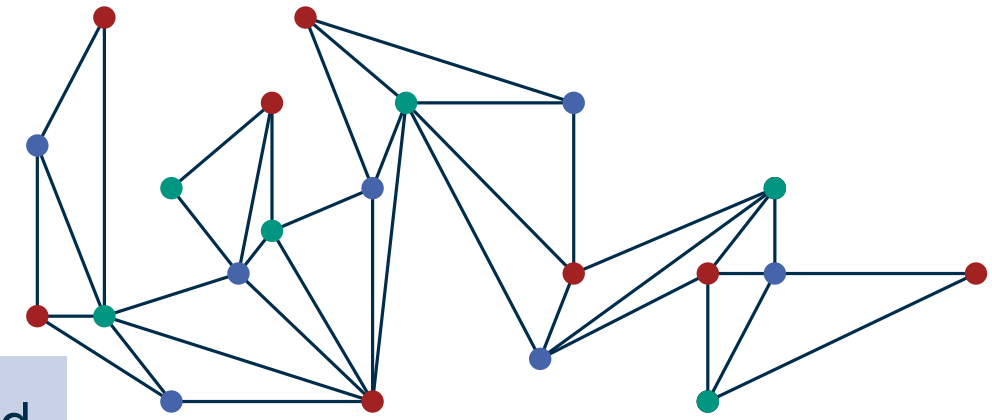
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Why valid?

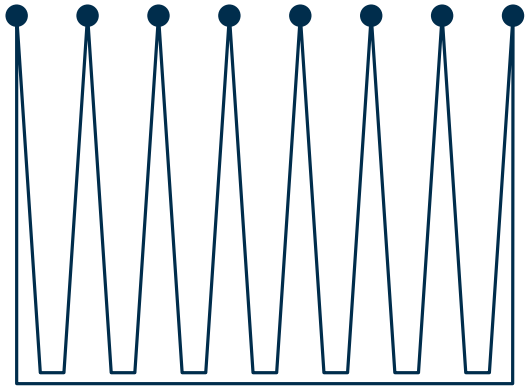
- at most $\lfloor \frac{n}{3} \rfloor$ guards needed

Why?



$\lfloor \frac{n}{3} \rfloor$ guards are always sufficient and sometimes needed

Lower Bounds

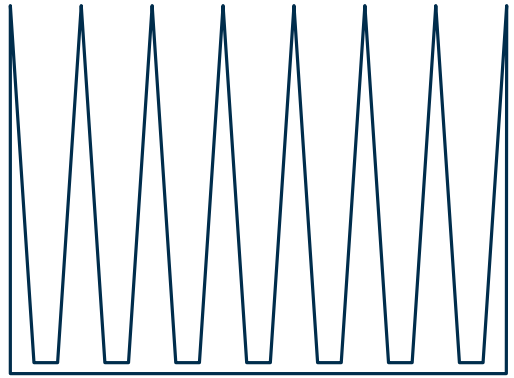


$\lfloor \frac{n}{3} \rfloor$ guards

More Fun Stuff

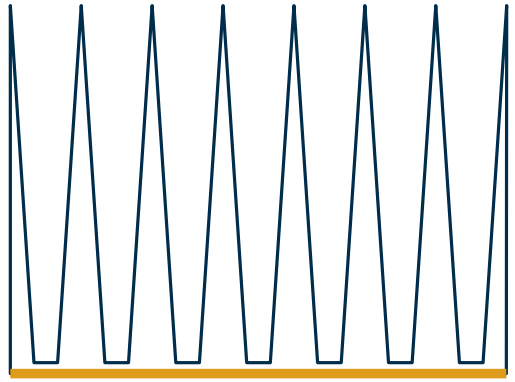
More Fun Stuff

- mobile guards: guards can move along edges



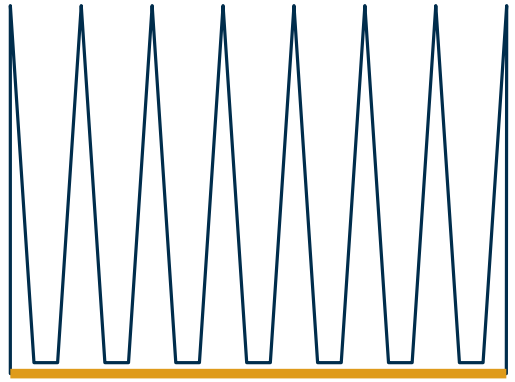
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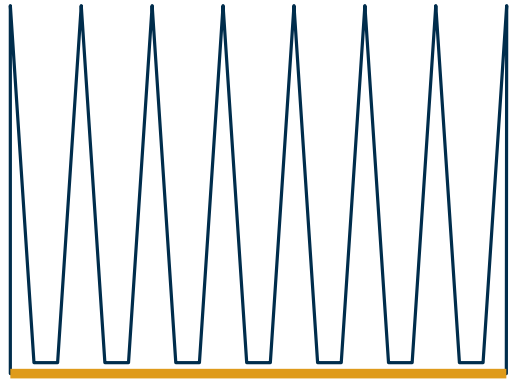
More Fun Stuff

- mobile guards: guards can move along edges
- need to guard exterior of polygon



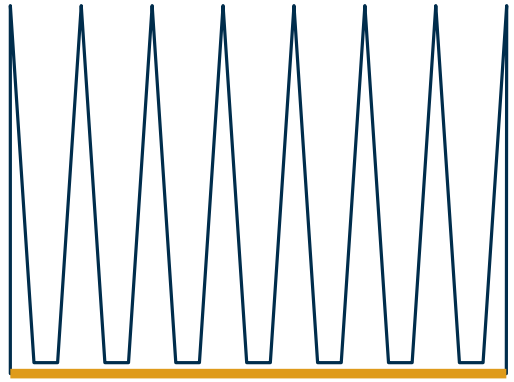
More Fun Stuff

- mobile guards: guards can move along edges
- need to guard exterior of polygon
- prison guards: need to guard interior and exterior of polygon



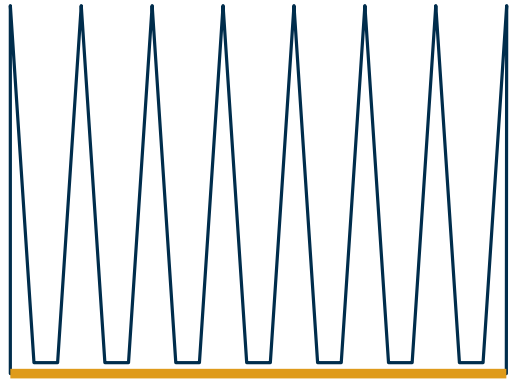
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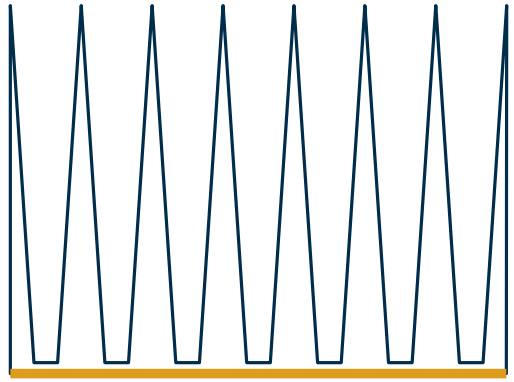
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296 pages of more fun stuff: Art Gallery Theorems and Algorithms by J. O'Rourke