Karlsruher Institut für Technologie

Exercise Sheet 7

Submission due by July 30, 2026

Problem 1: Cycles of length 4 — 6 points

Consider the following problem. Given a graph $G = (V, E)$, find a smallest possible set $X \subseteq V$ such that $G[V \setminus X]$ contains no cycle of length 4 as a subgraph.

Use Courcelle's Theorem to show that this problem is in FPT when treewidth is used as the parameter.

Problem 2: Chordal Graphs — $6 + 6 = 12$ points

Part (a) Let $G = (V, E)$ be a chordal graph and let $a, b \in V$ be two distinct non-adjacent vertices. Let S be a minimal separator that separates a and b (i.e., no proper subset of S separates a from b). Show that S forms a clique.

Part (b) A vertex is called *simplicial* if its neighbourhood forms a clique. Show that every chordal graph is either a clique, or contains two non-adjacent simplicial vertices. You may use part (a).

Problem 3: Induced Matchings — 10 points

Let $G = (V, E)$ be a graph. A matching $M \subseteq E$ is an *induced matching* if there exists a vertex set $V' \subseteq V$ such that V' induces the matching M (i.e., $G[V'] = (V', M)$). The problem Induced Matching asks, given a graph G and a parameter k, whether there exists an induced matching of size k.

Show that Induced Matching is $W[1]$-complete.

Problem 4: Functional Dependencies **4 + 8 = 12 points**

(This exercise builds on Lecture 13 from July 20)

Part (a) Provide a parameterized reduction from UNIQUE to FD_{FIXED}.

Part (b) Provide a parameterized reduction from FD_{FIXED} to FD.

Problem 5: INDEPENDENT FAMILY **6 + 6 + 6 = 18 bonus points**

In the following, we want to prove the $W[3]$-completeness of a natural problem from the database domain. The problem INDEPENDENT FAMILY can be viewed informally as INDEPENDENT SET on hypergraphs, except that we select sets of vertices rather than individual (hopefully non-connected) vertices.

Formally, an instance of INDEPENDENT FAMILY consists of two sets of sets $\mathcal{S}, \mathcal{T} \in 2^U$ over a universe U and a parameter k. Each collection can be seen as the edge set of a hypergraph. The question is whether there exist k hyperedges $S_1, \ldots, S_k \in \mathcal{S}$ such that no hyperedge $t \in \mathcal{T}$ is contained in $\bigcup_{i=1}^k S_i$.

We also define MULTICOLORED INDEPENDENT FAMILY as follows: Given the parameter k and the collections $\mathcal{S}_1, \ldots, \mathcal{S}_k$ as well as $\mathcal{T}$ of sets over a universe U (i.e., $\mathcal{S}$ is partitioned into k colors), decide whether there exist hyperedges $S_1 \in \mathcal{S}_1, \ldots, S_k \in \mathcal{S}_k$ such that no hyperedge $t \in \mathcal{T}$ is contained in $\bigcup_{i=1}^k S_i$.

Part (a) Show that INDEPENDENT FAMILY and MULTICOLORED INDEPENDENT FAMILY are equivalent under parameterized reduction.

Part (b) Show that INDEPENDENT FAMILY is in $W[3]$.

Part (c) Show that MULTICOLORED INDEPENDENT FAMILY is $W[3]$-hard.

Hint: WEIGHTED ANTIMONOTONE 3-NORMALIZED SATISFIABILITY (see the lecture from July 20) is well suited for the reduction.