Exercise Sheet 4

Submission due by June 18th 2026

Problem 1: Application of Lenstra — 4 + 8 points

Formulate an ILP to show that the following problems are in FPT:

Part (a) Variety Subset Sum: Given a multiset A of integers in $\mathbb{N}$ (unlike in a normal set, elements in a multiset can occur multiple times) and a value $b \in \mathbb{N}$. Is there a sub-multiset $X \subseteq A$ with $\sum_{x \in X} x = b$? Consider the number of distinct numbers in A as the parameter.

Part (b) Makespan Scheduling: Given m machines, n jobs with processing times $p_1, \dots, p_n \in \mathbb{N}$, and an upper bound for the maximum processing duration $k \in \mathbb{N}$. Is there an assignment of the jobs to the machines such that no machine requires more than k time to process all its jobs? Consider the bound k on the processing duration as the parameter.

Hint: How can the possible job assignments to a machine look like?

Problem 2: Max Sat — 8 + 8 = 16 points

Given a boolean formula (in CNF) with n variables and m clauses. Given a parameter k, the parameterized Max Sat problem asks for a variable assignment that satisfies at least k clauses.

Part (a) Provide safe reduction rules that yield a kernel with at most $2k$ clauses and k variables.

Hint: Use Hall's Theorem to reduce the number of variables.

Satz (Hall's Theorem). *Let $G = (V_1 \cup V_2, E)$ be a bipartite graph. There exists a matching in G that covers all vertices of V_1 if and only if $|X| \leq |N(X)|$ for every subset $X \subseteq V_1$. Otherwise, an inclusion-minimal set $X \subseteq V_1$ with $|X| > |N(X)|$ can be efficiently found.*

Part (b) Provide an FPT algorithm for the following parameterization "above $\frac{m}{2}$" with parameter k: Is there a variable assignment that satisfies at least $\frac{m}{2} + k$ clauses?

Hint: Consider clauses with exactly one variable separately from larger clauses and show first that many larger clauses lead to a large solution.

Problem 3: Weighted Vertex Cover on Trees — 12 points

Given an undirected graph G with weight function $w: V \to \mathbb{N}$ for the vertices. The goal of Weighted Vertex Cover is to find a vertex cover of G with minimum total weight. In this task we only consider trees. Formulate a dynamic program that determines the minimum weight of a vertex cover for a given tree. What is the running time of your DP?