Parameterized Algorithms
Summer Term 2026
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Karlsruher Institut für Technologie

Exercise Sheet 2

Submission due by May 14th, 2026

Organizational Info

- You have *two weeks* to work on the exercise problems.
- Please work in pairs or groups of three if possible.
- Submit your solutions as a *well-formatted* PDF via ILIAS.

Problem 1: Weighted Vertex Cover

8 + 4 points

In the previous version, the desired running time was $O(1.342^k \cdot n)$, where n is supposed to be the size of the input. However, in the context of graphs this is not clear.

Given is an undirected graph G with weight function $w : V \to \mathbb{N}$ for the vertices. The goal of the Weighted Vertex Cover problem is to find a vertex cover of G with minimum total weight. In the parameterized problem, we ask if there is a weighted vertex cover with weight at most k. We show that there is an FPT algorithm with the same running time as the algorithm for Vertex Cover (parameterized by the same parameter k) from the lecture, namely $O(1.342^k \cdot (n + m))$.

Part (a) Assume for now that all weights are bounded by some constant $c \in \mathbb{N}$. Describe an FPT algorithm that solves parameterized Weighted Vertex Cover on graphs with constant weights in $O(1.342^k \cdot (n + m))$. Briefly argue its correctness.

Hint: Use an FPT algorithm for Vertex Cover as a black box.

Part (b) Describe an FPT algorithm that solves parameterized Weighted Vertex Cover on general graphs in $O(1.342^k \cdot (n + m))$.

Hint: How do we obtain instances with constant weights? How should we choose c, the upper bound for the weights?

Problem 2: 3-Hitting Set

10 points

An instance of 3-Hitting Set consists of a universe U and a family of sets $\{S_1, \ldots, S_n\}$ with $|S_i| \leq 3$ and $S_i \subseteq U$. The goal is to decide if there is a set $H \subseteq U$ with $|H| \leq k$ such that $H \cap S_i \neq \emptyset$ for all S_i.

Give an algorithm that uses a bounded search tree with running time $2.562^k \cdot n^{O(1)}$.

Problem 3: d-Hitting Set **12 points**

An instance of d-Hitting Set consists of a universe U and a family of sets $\{S_1, \ldots, S_n\}$ with $|S_i| \leq d$ and $S_i \subseteq U$. The goal is to decide if there is a set $H \subseteq U$ with $|H| \leq k$ such that $H \cap S_i \neq \emptyset$ for all S_i. Give an algorithm with $(d - 0.658)^k \cdot n^{O(1)}$ running time.

Hint: Use iterative compression to obtain an algorithm with $2.342^k \cdot n^{O(1)}$ running time for 3-Hitting Set.

Problem 4: Vertex Cover implementation **6 points + 8 bonus points**

In this task, you should implement a program (using any programming language you prefer) that computes a minimum vertex cover for a given graph. You should use the rules given in lecture 3, but you can also implement additional rules and optimizations.

Describe in the PDF submission which rules you implemented and what further optimizations you made. Also, for each graph, specify in the PDF submission the size of the vertex cover you found. Additionally, submit the source code as well as your computed solutions (in the format described below) as a ZIP file.

You get two points for every minimum vertex cover.

Input format: The first line consists of n and m, the number of vertices and edges, respectively. The following m lines each consist of two vertices, which are connected by an (undirected) edge. The vertices are numbered from 1 to n.

Output format: The first line should contain the solution size k. The following k lines should contain the IDs of k vertices that form a minimum vertex cover.

Note: You can validate your vertex cover using the file `validator.py`. Note that minimality is not checked. Please check if your vertex covers are valid before submitting.