

## Exercise Sheet 1

Submission due by April 30th, 2026

### Organizational Info

- You have *two weeks* to work on the exercise problems.
- Please work in pairs or groups of three if possible.
- Submit your solutions as a *well-formatted* PDF via ILIAS.

### Problem 1: Misc

2 + 2 + 2 points

**Part (a)** Is there a difference between  $\Theta(2^n)$  and  $2^{\Theta(n)}$ ? If yes, which one grows faster asymptotically?

**Part (b)** Let  $T$  be an unrooted binary tree, i.e.,  $T$  consists of vertices with degree 3 (inner vertices) and vertices with degree 1 (leaves). Let  $n_3$  be the number of inner vertices and  $n_1$  the number of leaves. Show that  $n_3 = n_1 - 2$ .

**Part (c)** Give two parameterized problems for which there exists an FPT algorithm if and only if  $P = NP$ .

### Problem 2: Branching vector

2 + 4 points

This exercise builds on the second lecture. For the following recurrences, give the tightest possible upper bounds of the form  $x^n$  (3-4 decimal places are sufficient) and prove their correctness using induction.

$$f(n) = \begin{cases} 3 \cdot f(n-1), & n > 5 \\ 1, & \text{else} \end{cases} \quad g(n) = \begin{cases} g(n-1) + g(n-2) + g(n-3), & n > 17 \\ 1, & \text{else} \end{cases}$$

### Problem 3: DOMINATING SET on special graphs

4 + 8 + 6 Points

Let  $G = (V, E)$  be a graph. A *dominating set* is a subset  $X \subseteq V$  such that for every vertex  $v \in V$ , at least one vertex from  $N[v] = \{v\} \cup N(v)$  is contained in  $X$ . The problem DOMINATING SET is to determine a smallest possible dominating set in a given graph.

**Part (a)** Give a polynomial-time algorithm that solves DOMINATING SET on trees. Can you achieve a running time that is linear in the size of the graph?

*Hint:* There are different approaches here (e.g. DP or greedy), there is a greedy approach that is easier to generalize to an algorithm for the next part.

**Part (b)** A subset  $X \subseteq V$  is a  $k$ -dominating set if for every vertex  $v \in V$ ,  $X$  contains a vertex that has at most distance  $k$  to  $v$ . Give a polynomial-time algorithm that determines a smallest  $k$ -dominating set for a given tree. Prove the correctness of your algorithm.

**Part (c)** We now consider DOMINATING SET on graphs with maximum degree  $d$ . Give an FPT algorithm parameterized by  $k + d$ , where  $k$  denotes the solution size.

#### **Problem 4: Points and Lines**

**10 points**

Given a set of  $n$  points  $P = \{p_1, \dots, p_n\}$  in the plane and a parameter  $k$ , the problem LINE COVER is to decide whether there exist  $k$  lines such that each point  $p_i \in P$  lies on at least one of these lines. Give an FPT algorithm for this parameterized problem.