Parameterized Algorithms

Summary

Thomas Bläsius

General notes

Goal of the exam

- we need to certify that you
  - know the content of the lecture
  - understand it
  - can use the learned techniques to design and analyze new algorithms
- oral exam: easiest to check first two
- focus on the second (which implies the first)
- I'll make an effort to also check the third

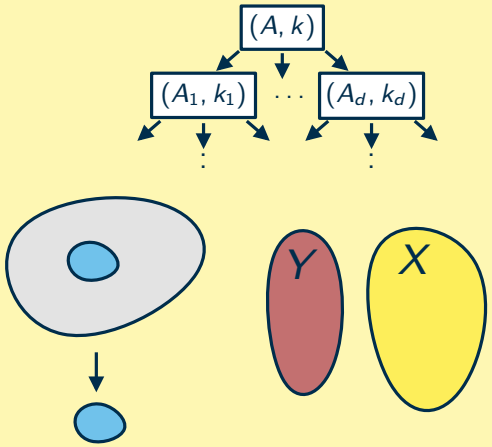
Preparation

- step 1: recap and understand the content
- step 2: explain the content to someone
- **important:** Step 2 is exactly what happens in the exam. You should train that!

Content

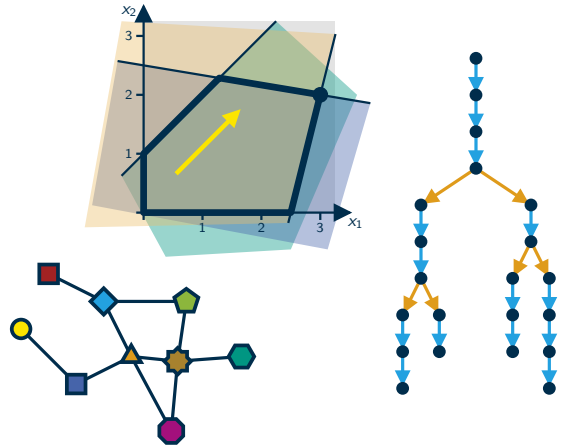
Basic toolbox

- bounded search trees
- kernelization
- iterative compression



Extended toolbox

- linear programs
- branch-and-reduce
- color coding

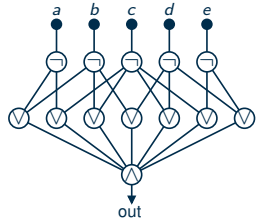


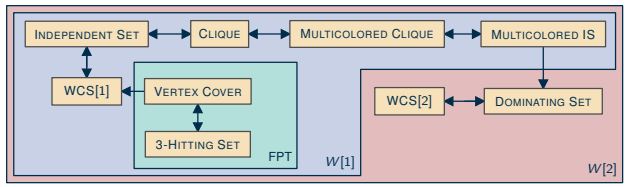
Tree width

- dynamic programming
- chordal and planar graphs
- Courcelle's theorem

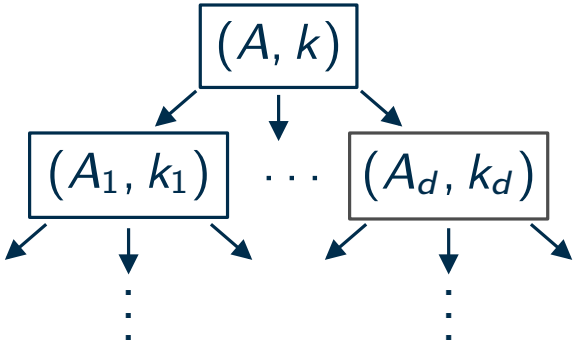
Lower bonds

- kernel lower bounds
- parameterized reductions
- circuits and the W-hierarchy
- ETH and SETH





FPT basic techniques



basic terms: parameterization

- What is a parameterized problem?
- What is FPT?

Search trees

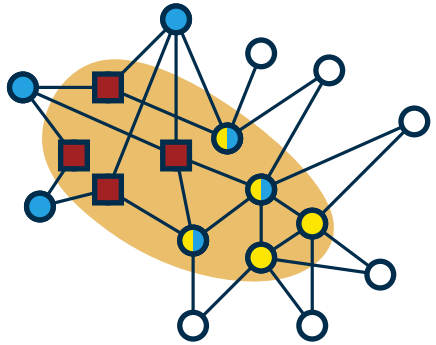
- How do search trees work?
- How do we get FPT time?
- How do we usually bound the height?

Kernelization

- What is a kernel?
- How do we compute it?
- When is kernelization possible?

Iterative compression

- What is the core idea of this technique?
- Where do we get the too-large solution from?
- What is the compression problem?



How can we apply these techniques to VERTEX COVER?

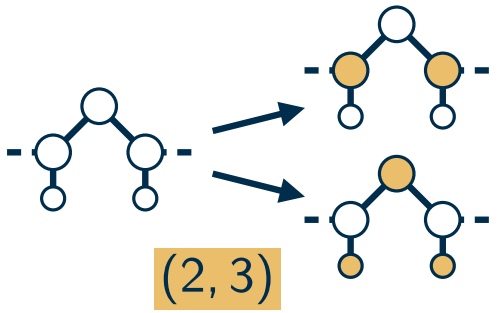
Bounded search trees

Branching in general

- What is a branching vector?
- How do we get the tree size given the branching vector?
- Why does this boil down to computing the root of a polynomial?

Better branching for VERTEX COVER

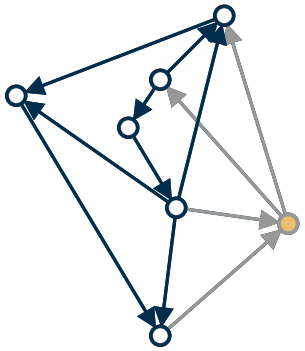
- How do vertices with high degree help?
- Is there always a vertex with degree 4?
- How do we get rid of degree-2 vertices?
- What is the idea for degree-3 vertices?
- What about degree-4 vertices?
- Why can I ignore the branching vector $(1, 4)$?



Iterative compression

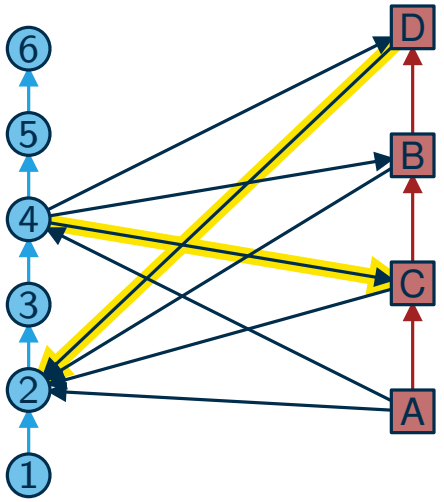
Feedback Vertex Set

- What is the problem?
- How do we reduce FVS to FVS Compression?
- How do we reduce FVS Compression to Disjoint FVS?
- What running time costs do you accumulate with this?



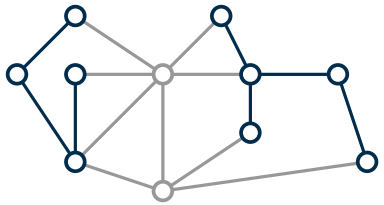
Disjoint Feedback Vertex Set on tournament graphs

- What are tournament graphs?
- How does it reduce to Longest Increasing Subsequence?



Undirected FVS

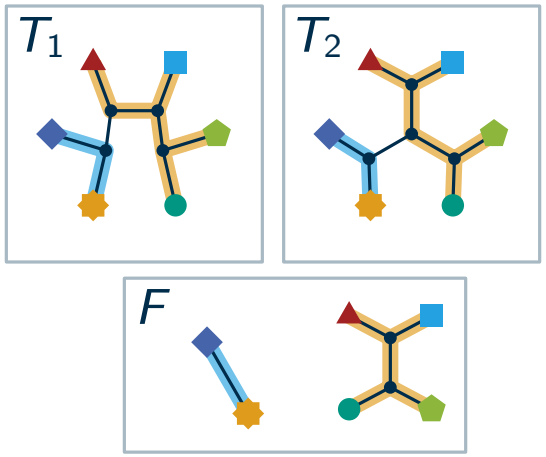
- Reduction to the disjoint variant?
- How do we solve the disjoint variant?

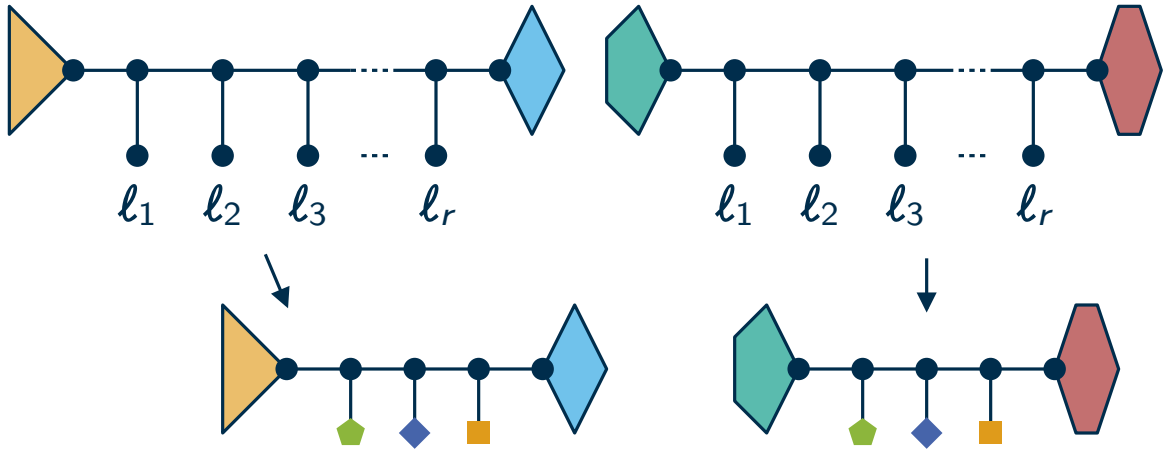


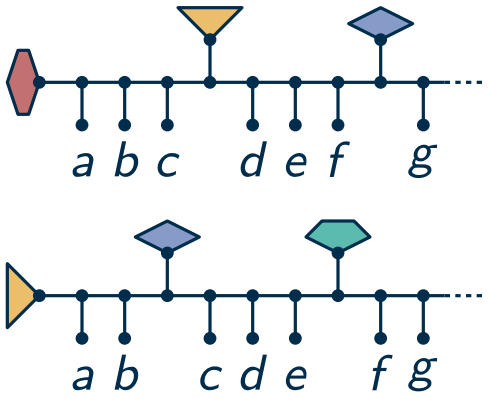
Kernelization: similar trees

Maximum Agreement Forest

- What is the problem?
- Why would it help if every tree in the solution has few leaves?
- What counter examples for this?
- How do we get rid of them?
- How does the amortized perspective help?







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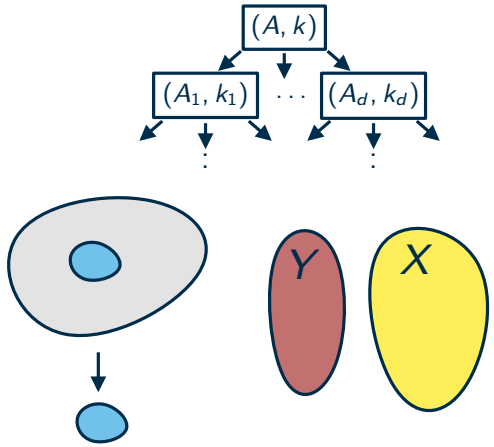
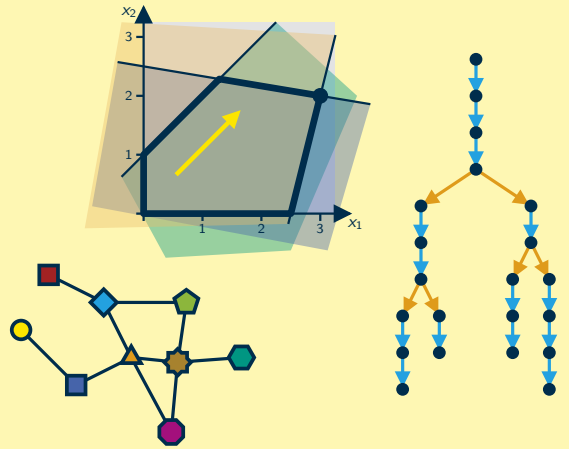


Extended toolbox

- linear programs
- branch-and-reduce
- color coding



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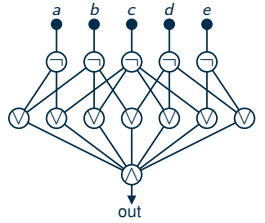
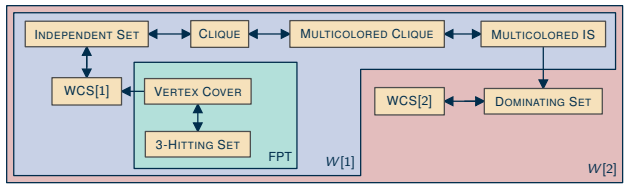
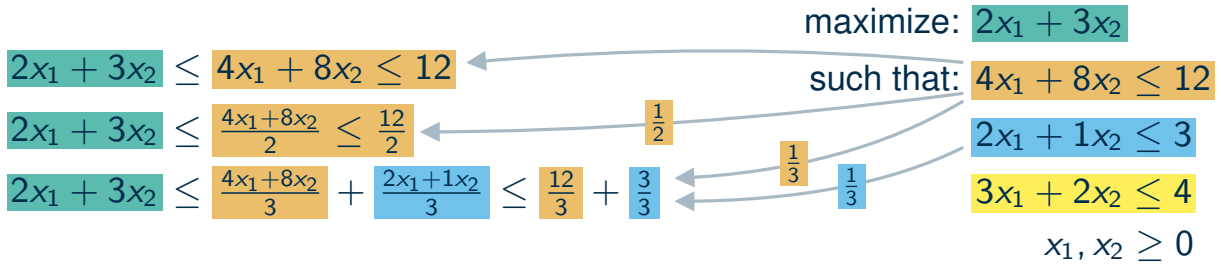




Linear programs (1)

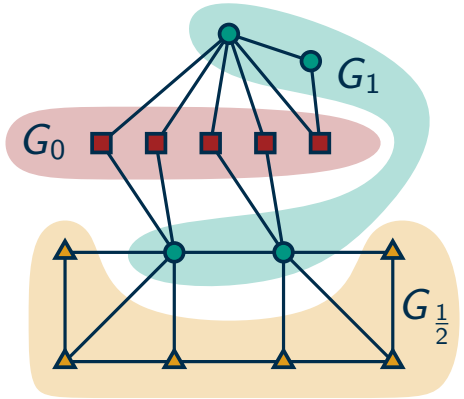
Linear programs in general

- How to deal with absolute values?
- What is the dual program linear program?



VERTEX COVER and LPs

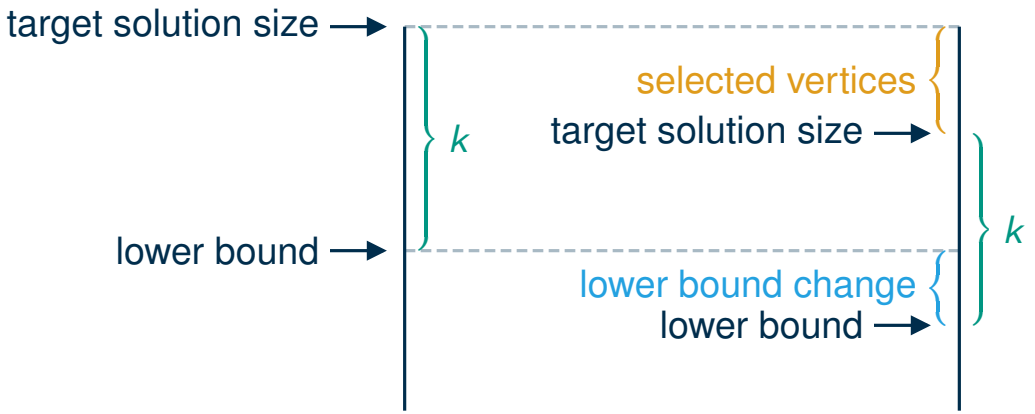
- What is special about the LP relaxation of the VC ILP?
- How can this help for kernelization?
- Do we actually need to solve an LP?



Linear programs (2)

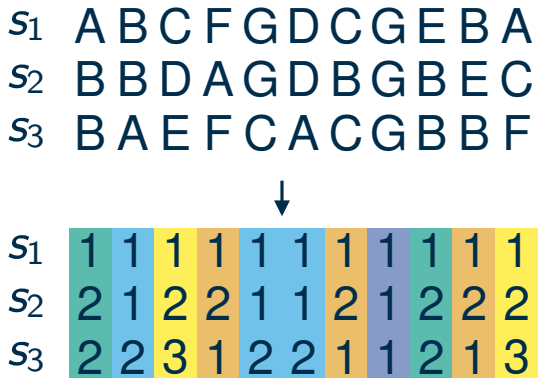
Parameterization above lower bound

- What is it?
- Where do we have to be more careful than usual?
- How does it work for VERTEX COVER?



Lenstra's theorem

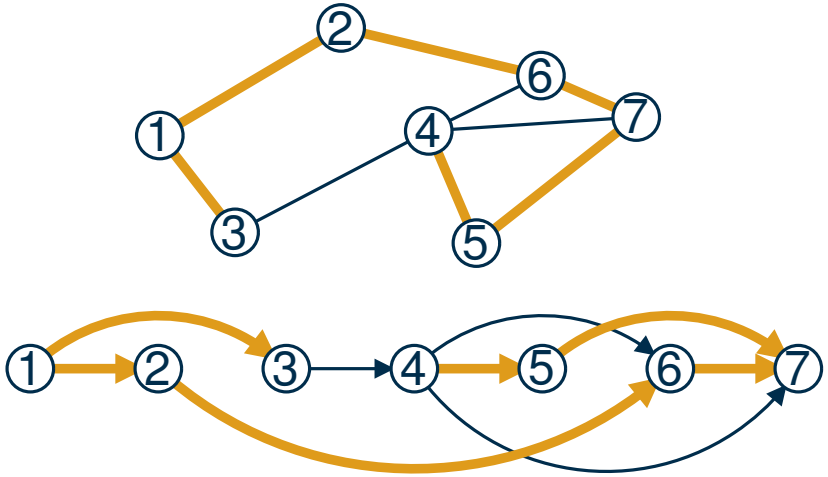
- What does it say?
- How can we use it to solve CLOSETS STRING?



Color Coding

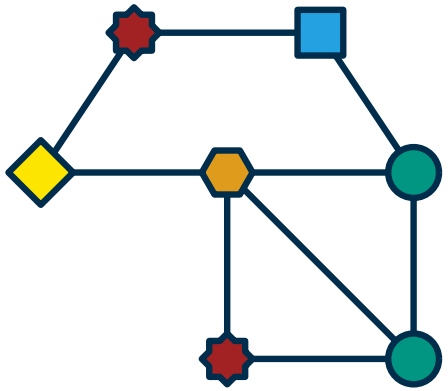
LONGEST PATH: random order

- What additional structure did we guess?
- How does this help to solve the problem?
- What do we need to show for the success probability to get FPT running time?



Color Coding

- What additional structure do we guess here?
- How does this help?
- Can we derandomize this?
- What are perfect hash families?
- What are universal sets?



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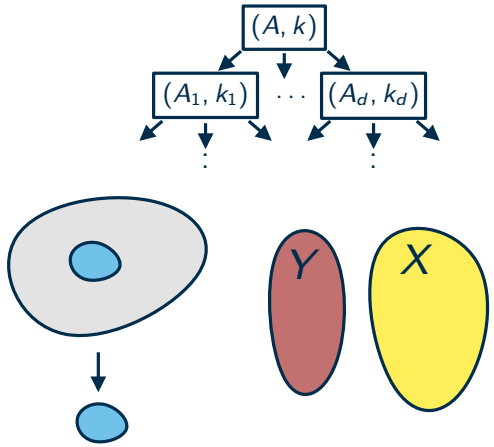

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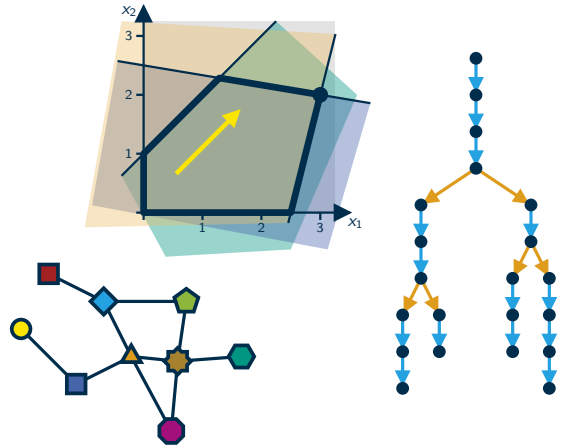

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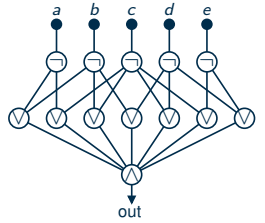
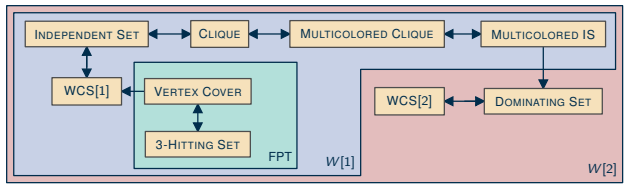
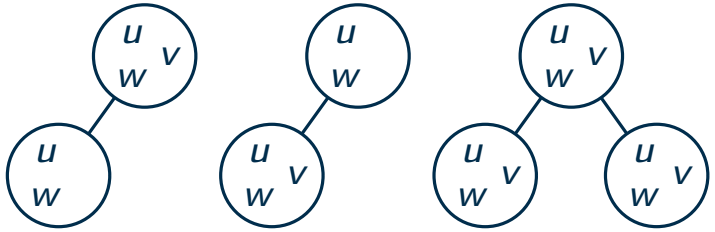




Treewidth

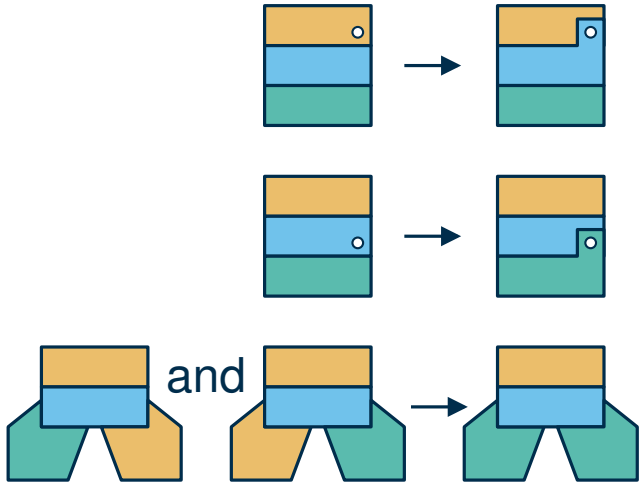
Path and tree decomposition

- What are path and tree decompositions?
- What are pathwidth and treewidth?
- When is a tree decomposition nice?
- Why can we assume to have a nice tree decomposition?



Dynamic programming

- How can we solve problems via DP on tree decompositions?
- examples: INDEPENDENT SET and HAMILTONIAN CIRCUIT
- How do we keep track of global requirements?
- Can you transfer this to other problems?



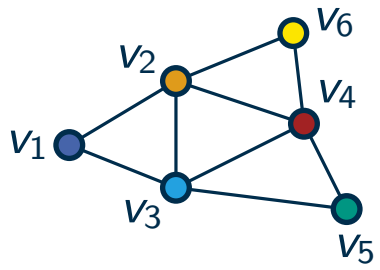
Courcelle & chordal graphs

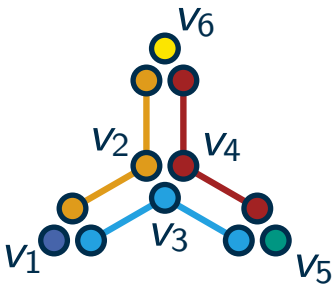
Courcelle's theorem

- What can we do with MSO_2?
- How can we model connectedness?
- How to model HAMILTONIAN CIRCUIT and VERTEX COVER?
- How about other problems?
- What does Courcelle's theorem tell us?
- Why do we need the optimization variant?

Chordal graphs

- What are interval graphs and chordal graphs?
- What is the interval/chordal width?
- What is the relation to pathwidth and treewidth?





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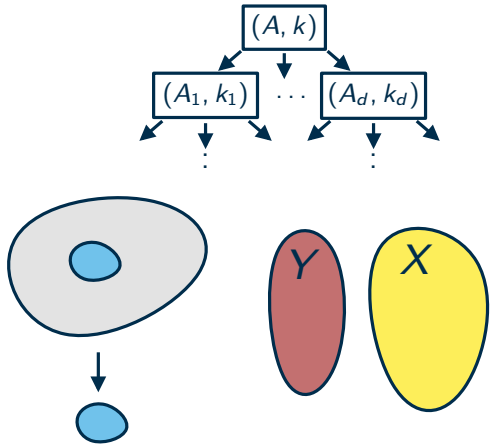

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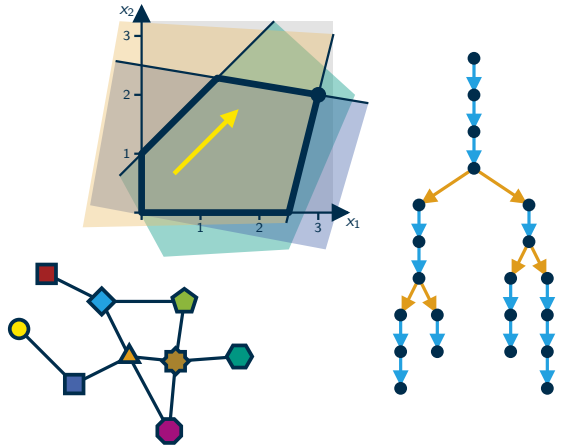

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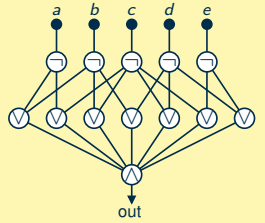
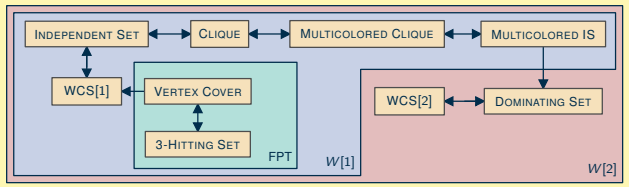
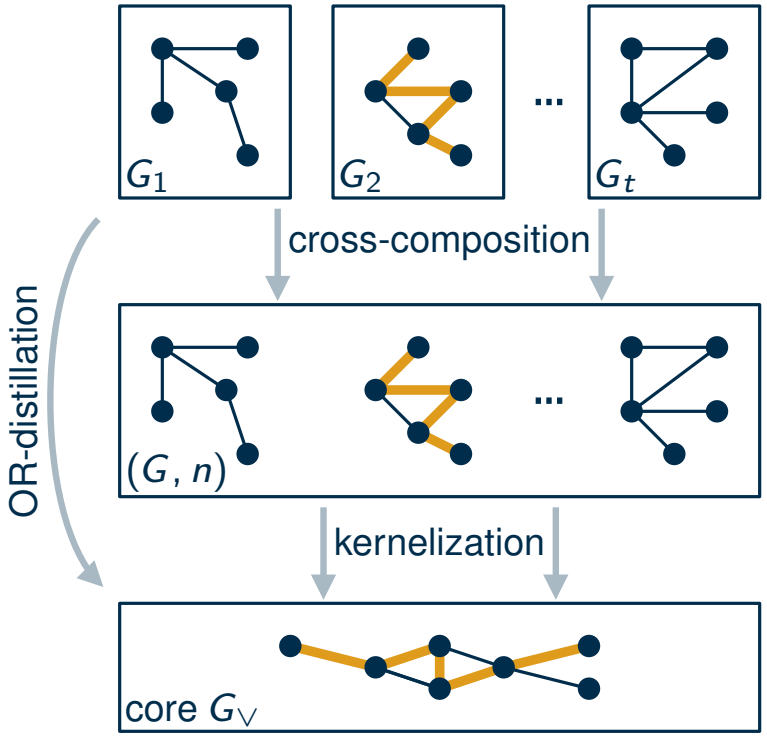




Kernel lower bounds

OR-Distillation

- What is an OR-distillation?
- How would a polynomial kernel for LONGEST PATH give an OR-distillation from HAMILTONIAN PATH into LONGEST PATH?



coNP/poly

- What is coNP/poly?
- What does polynomial advice mean?
- How can we show that a problem is in coNP/poly?

Kernel lower bounds

- Why would an OR-distillation of HAMP into LONGP imply HAMP $\in$ coNP/poly?
- What are the no-certificates? Advice strings? How does the OR-distillation help?
- How can we get an OR-distillation of SET SPLITTING into itself?

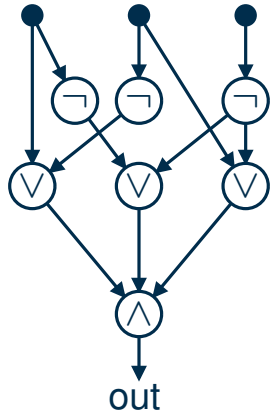
W-Hierarchie

Parameterized reductions

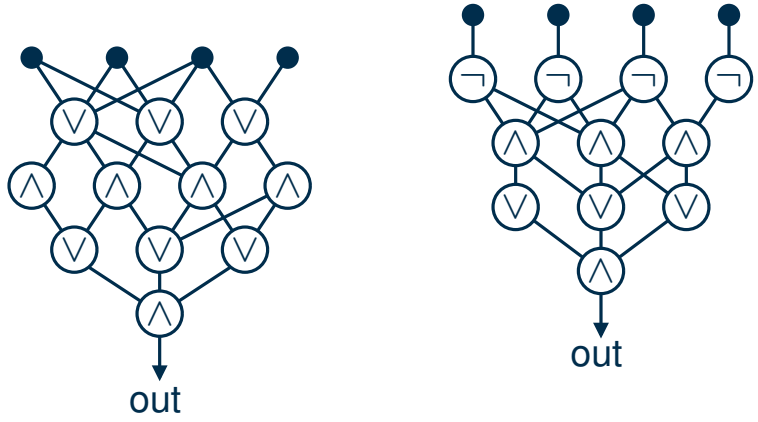
- What is it and why is it defined this way?
- How to reduce between Multicolored Clique and Clique?
- How to do these reductions?
  - Multicolored IS $\rightarrow$ Dominating Set
  - Independent Set $\rightarrow$ Weighted Circuit Satisfiability
  - Dominating Set $\rightarrow$ Weighted Circuit Satisfiability

W-Hierarchy

- What is WCS? How does it define the classes of the W-hierarchy?
- How do we show containment/hardness for W[t]?
- What are complete problems W[1] and W[2]?
- Why is FPT a subset of W[1]?



Relational databases

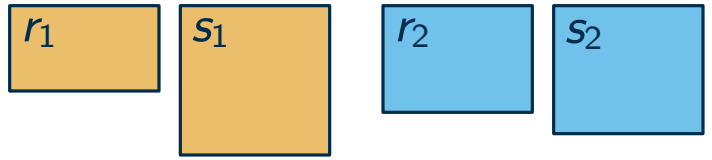


Normalized circuits

- What are they and why are they useful?

UNIQUE

- How can we reduce from and to HITTING SET?

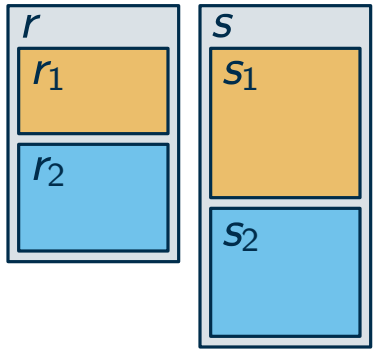


FD

- How can we reduce this to WCS[2]?

$\text{IND}_{\text{FIXED}}$

- How can we interpret instances as Boolean functions?
- How can we and them, and why does that help?
- How can we model disjunctions of conjunctions of negated literals?
- How can we reduce to a Boolean circuit?



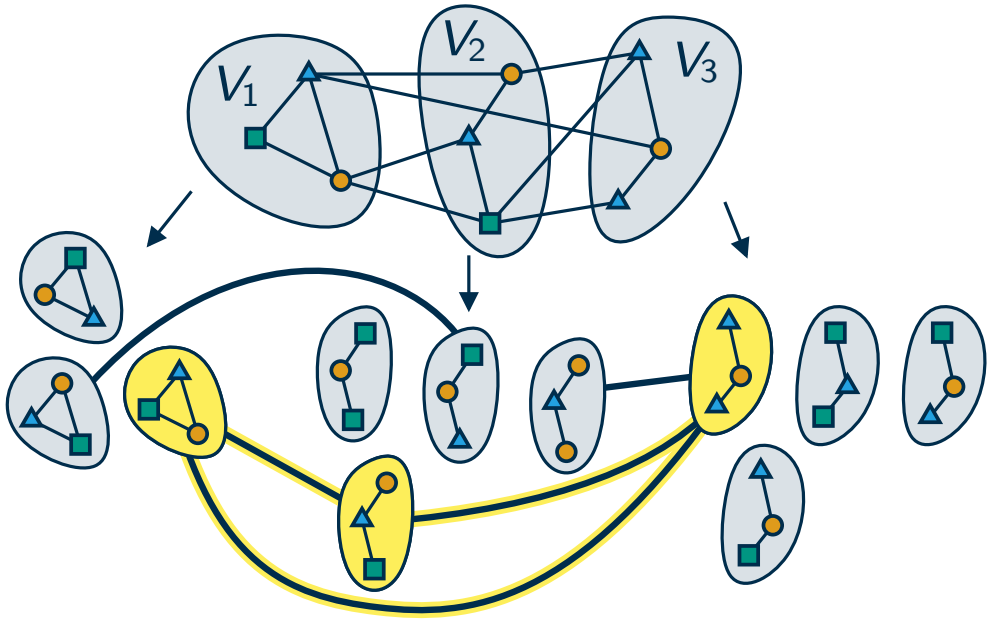
ETH and SETH

Basics

- What is ETH? What is SETH? How do they relate?
- How do we get lower bounds for other problems?
- Is a polynomial reduction enough?

So many reductions

- How do we get a lower bound for CLIQUE?
- What does that mean for $W[1]$ and FPT?
- Do we need to be careful with the parameter?
- What is GRID TILING?
- How to reduce CLIQUE to GRID TILING?
- GRID TILING to PLANAR LIST COLORING?
- GRID TILING to UNIT DISK INDPENDENT SET?



Ad break – winter 26/27

Algorithms courses offered by us

- lecture: computational geometry (we have a big room and two slots for exercises right from the start this time)
- practical course: route planning
- practical course: beating the worst-case

Courses from other algorithms groups to check out

- Marvin Künnemann
- Henning Meyerhenke
- Peter Sanders
- Torsten Ueckerdt

Also

- scientific figures and presentations in ipe

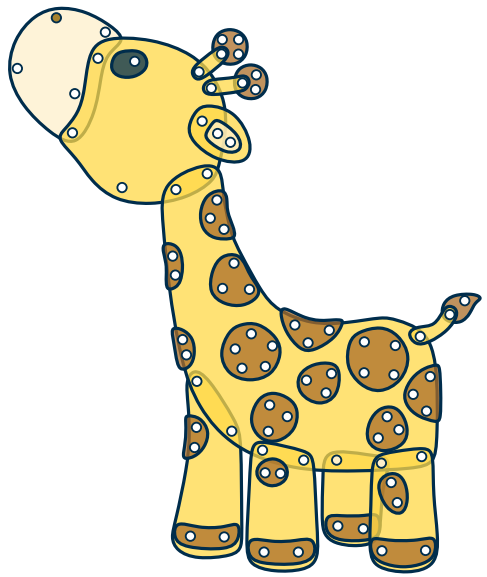
Scientific Figures and Presentations in Ipe

https://portal.wiwi.kit.edu/forms/form/988

What?

- Learn how to use Ipe to create:
  - nice figures for thesis or scientific papers
  - presentations (like this one)
- Learn how to customize Ipe to your likings
- Learn some basic design principles

Limited capacity, apply now!



When?

- Winter semester 2026/27
- Regular meetings and exercises up to Christmas
- One final submission and presentation at the end of the lecture period

Why?

- You are tired of creating presentations with Latex Beamer or PowerPoint, or
- You have already used Ipe and want to learn some cool tricks, and
- 2 ECTS Interdisciplinary Qualifications (Schlüsselqualifikation)

Evaluation

Things to mention / discuss

- many very nice comments :-)
- different activities in the exercise sessions: allocation of time
- script, references to book chapter, paper reading in active session
- video recordings
- Ilias vs. Discord, Homepage
- finding a team
- purpose of parameterized algorithms: purely theoretical vs. practically efficient algorithms
- i wish there'd be more connection to practice and real-world examples — me too buddy, me too :-(

Good Luck!