Parameterized Algorithms

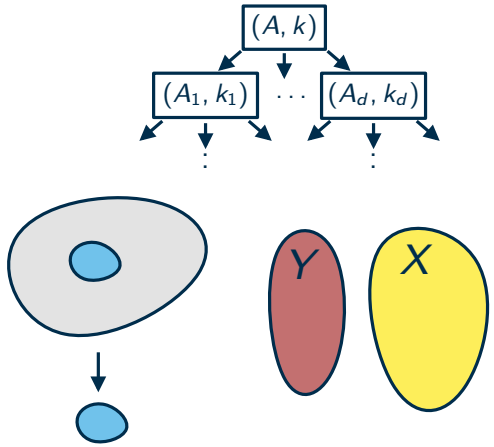
Parameterized Complexity and Databases

Thomas Bläsius

Content

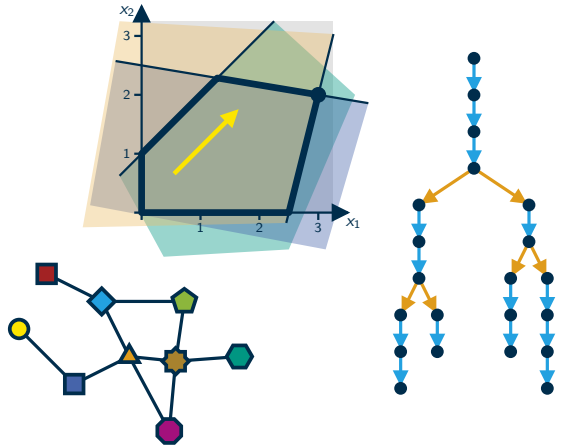
Basic toolbox

- bounded search trees
- kernelization
- iterative compression



Extended toolbox

- linear programs
- branch-and-reduce
- color coding

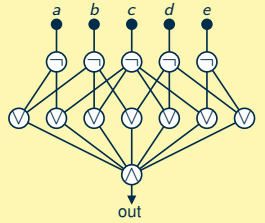


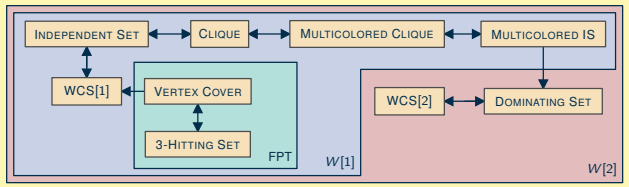
Tree width

- dynamic programming
- chordal and planar graphs
- Courcelle's theorem

Lower bonds

- kernel lower bounds
- parameterized reductions
- circuits and the W-hierarchy
- ETH and SETH





Reminder: Boolean Circuits

Definition
The largest number of nodes with in-degree > 2 on a directed path is called **weft** of the circuit.

Problem
WCS[t] restricts WCS to circuits of constant height and weft $\leq t$.

Definition
The class $\boldsymbol{W[t]}$ is the set of problems that have a parameterized reduction to WCS[t].

Remarks

- problem $\mathcal{L}$ is $W[t]$-complete if
  - $\mathcal{L} \in W[t]$ (parameterized reduction from $\mathcal{L}$ to WCS[T])
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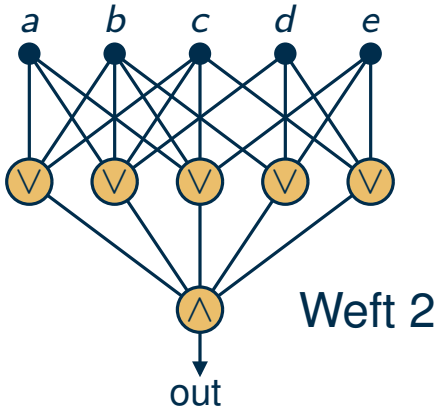

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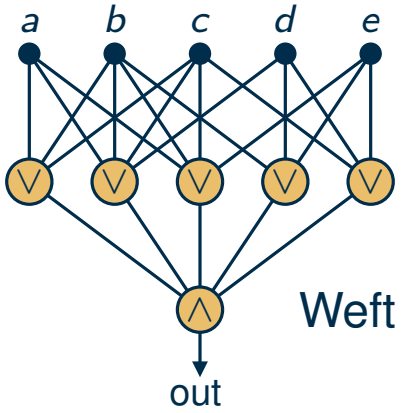

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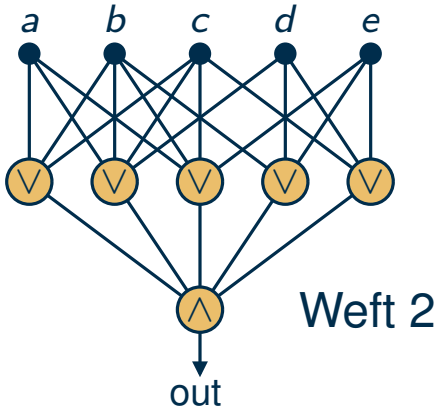
- we believe: FPT $\subset W[1] \subset W[2] \subset W[3] \subset \cdots$



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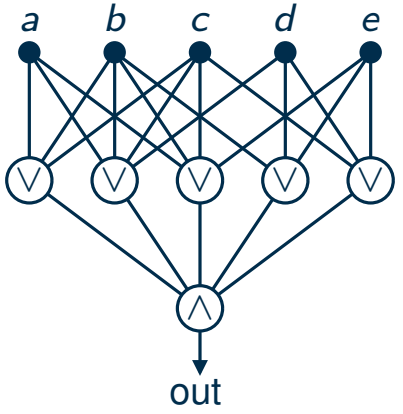

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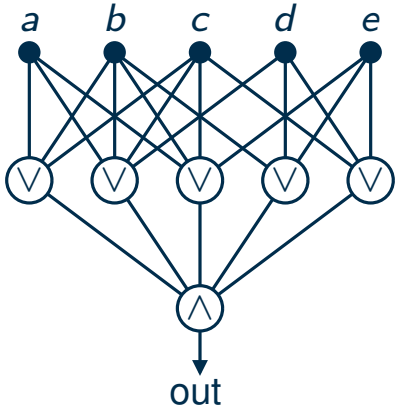

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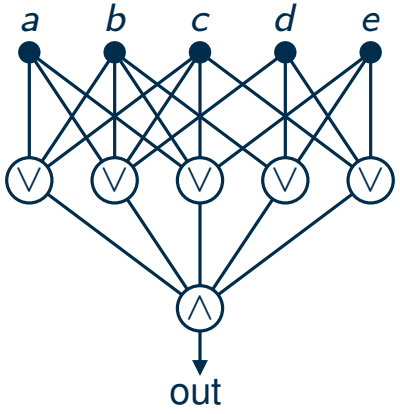

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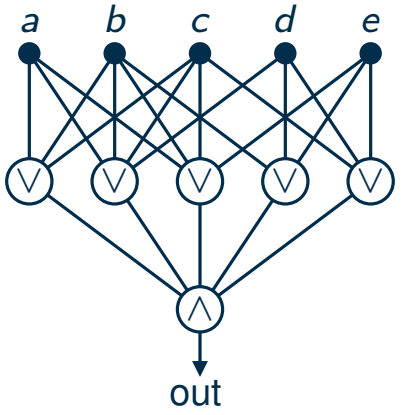

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- show $W[t]$-hardness for normalized circuits

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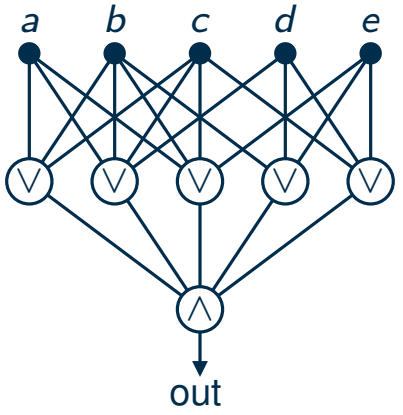
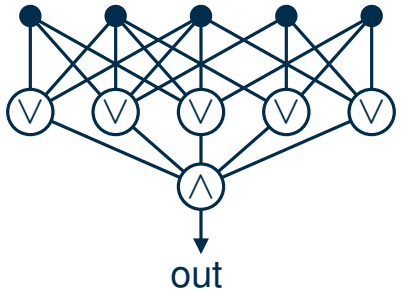


Normalized circuits

Properties of the circuit for DOMINATING SET

- on every path $\wedge$ and $\vee$ nodes alternate and output is $\wedge$ node



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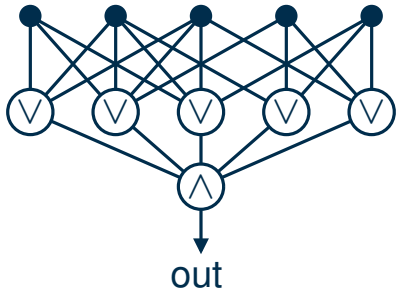

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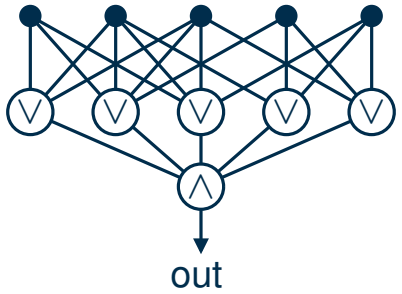

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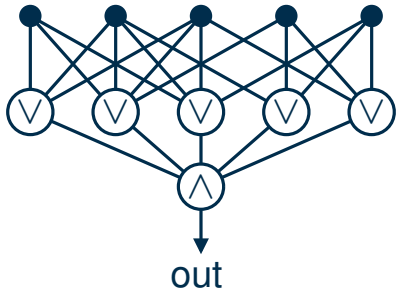
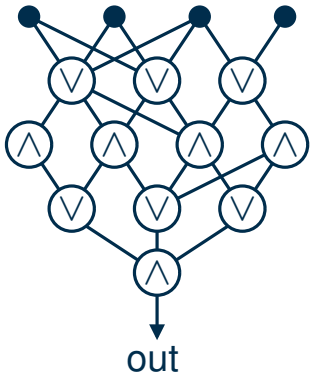


WEIGHTED MONOTONE t-NORMALIZED SATISFIABILITY

- t layers of alternating $\wedge$ and $\vee$, output is $\wedge$, no $\neg$ nodes



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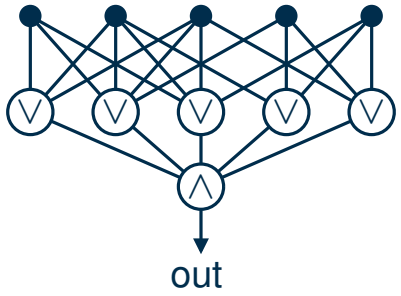

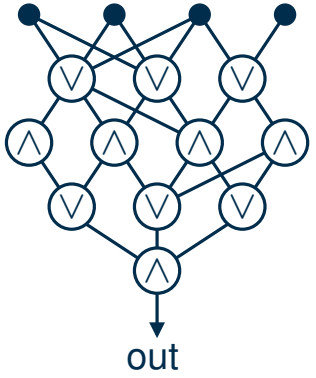

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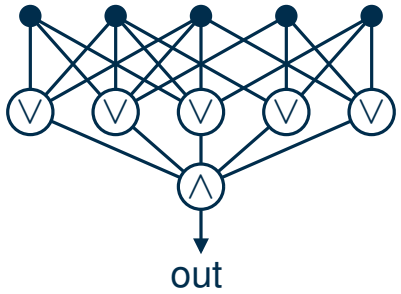

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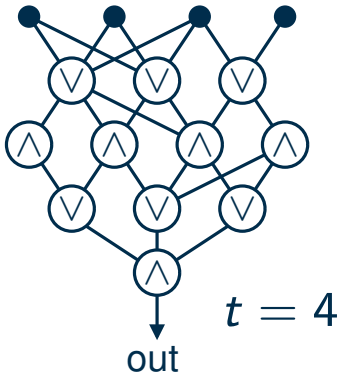

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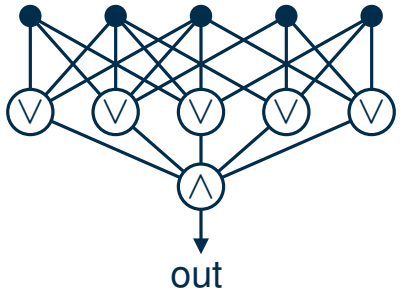

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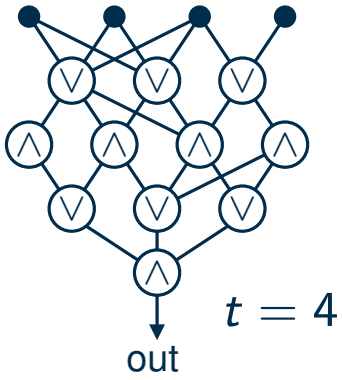

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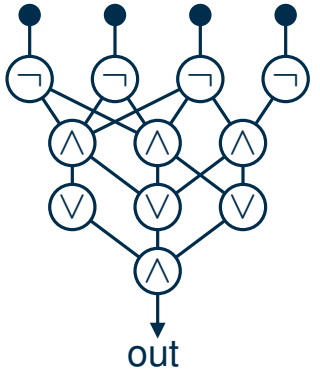

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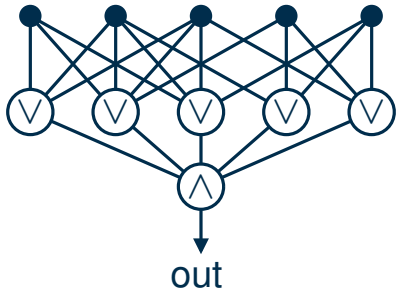

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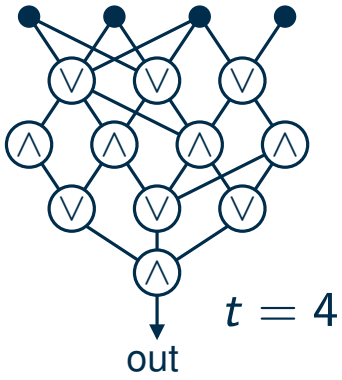

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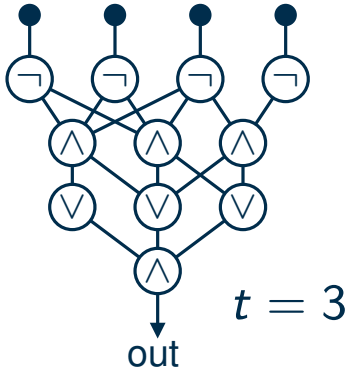

Completeness of normalized circuits

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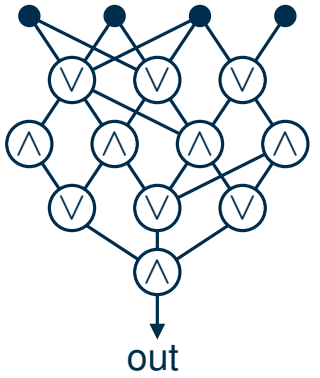

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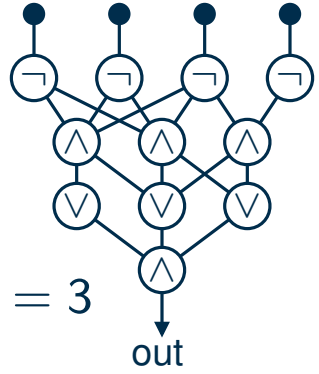

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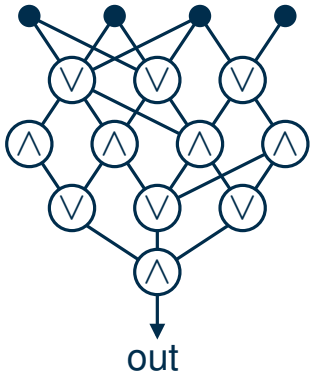

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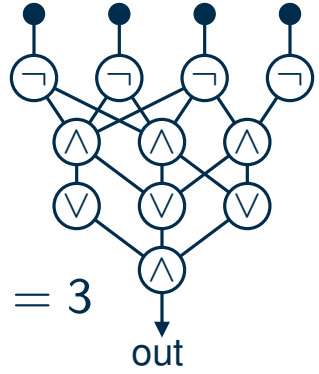

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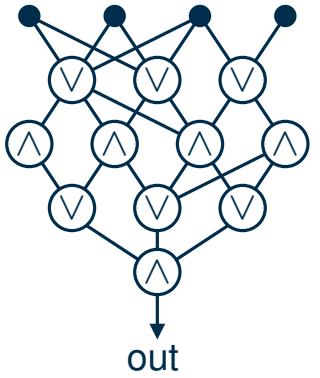

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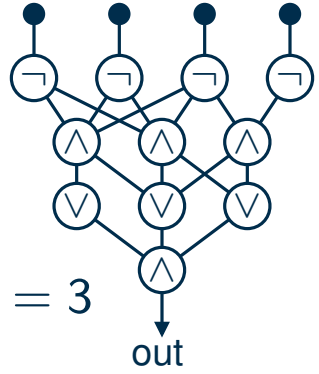

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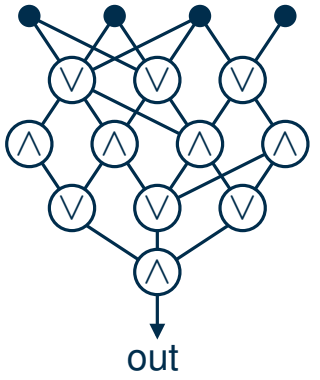

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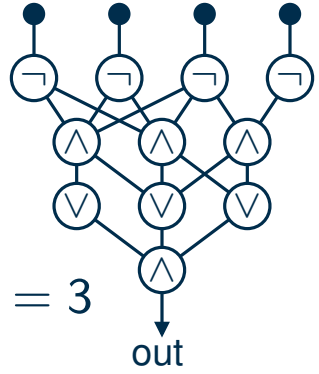

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- no single column is unique

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Do you know this problem? $\rightarrow$ HITTING SET

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Just seen

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Now: Reduction from HITTING SET to UNIQUE

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Functional dependencies

Observation

- {name} is not a unique column combination
- the name alone does not uniquely identify the row
- but: the name identifies the place

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Which of the following are functional dependencies?

- {first name} $\to$ birthday
- {birthday} $\to$ first name
- {birthday} $\to$ place
- {first name} $\to$ place
- {first name, birthday} $\to$ place
- {place, birthday} $\to$ first name

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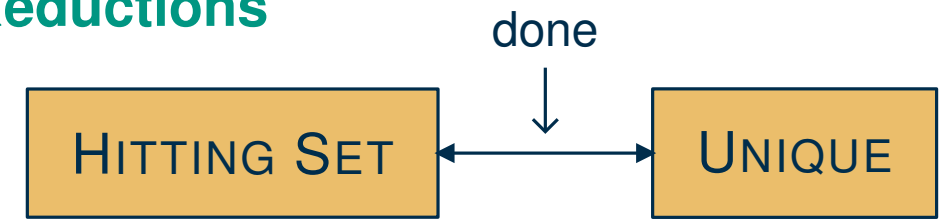

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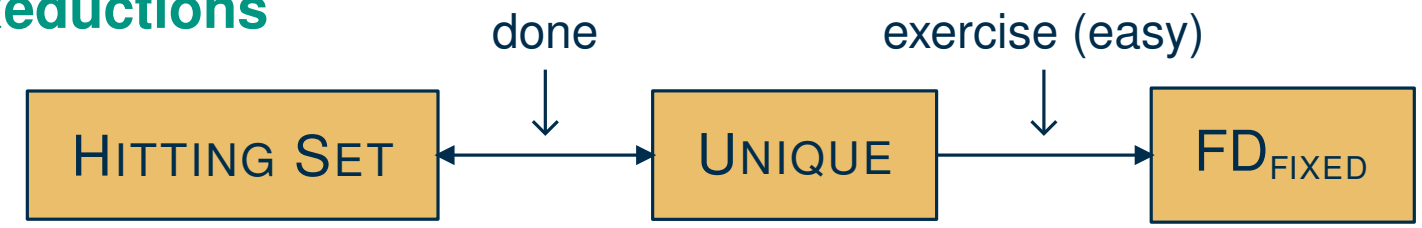

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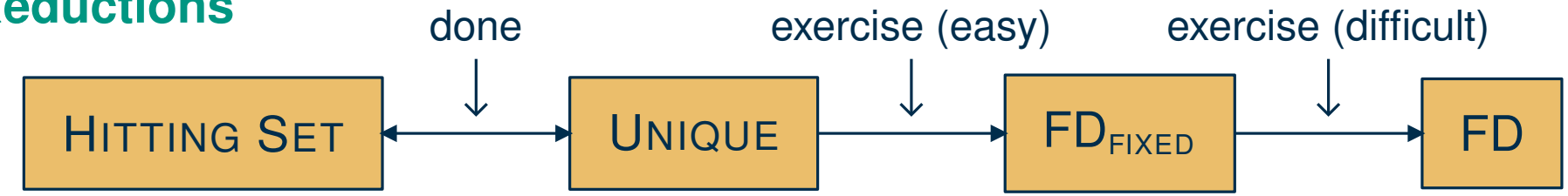

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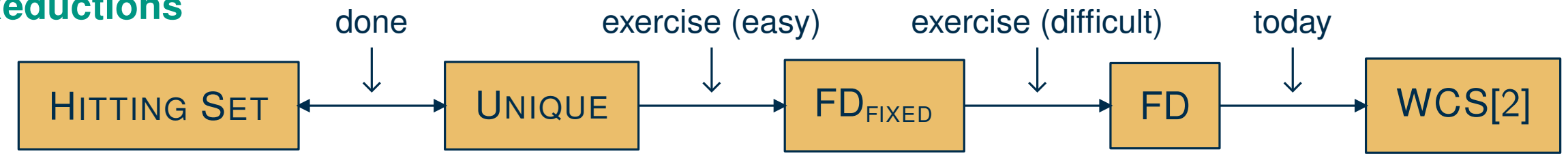

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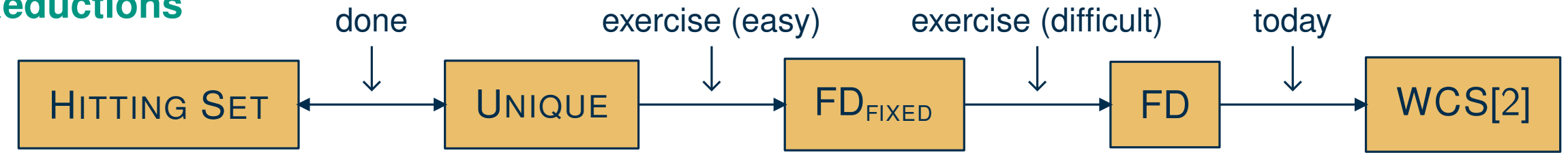

- HITTING SET and WCS[2] are $W[2]$-complete $\Rightarrow$ FD_{FIXED} and FD are $W[2]$-complete

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- RHS consists of exactly one column
- LHS and RHS are disjoint column sets
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How does the parameter change?

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Rough plan: model conditions of FD using Boolean formulas

- use only $\vee$ for the conditions and then one $\wedge$ in the end (CNF) $\Rightarrow$ weft 2

What are the conditions? What variables do we use?

- RHS consists of exactly one column
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} indicator variables x_A and y_A for $A \in$ LHS and $A \in$ RHS

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- at least one: $\bigvee_{\text{column } A} y_A$

FD $\rightarrow$ WCS[2]

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Theorem: UNIQUE, FD_{FIXED}, and FD are $W[2]$-complete.

Inclusion dependencies

Goal: find data of one database in another database

schema R

A	B	C
1	1	1
3	1	2
1	2	1
2	3	3
3	4	4

schema S

D	E	F	G
4	1	3	1
3	3	4	1
3	3	3	2
4	2	1	3
2	4	1	3
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Problem: $\text{IND}_{\text{FIXED}}$
Given two databases r, s with the same schema and a parameter k. Is there an inclusion dependency of size k with σ being the identity?

Find the largest inclusion dependency

5	M	I	N
1	1	4	3
4	1	2	4
2	4	3	2
1	3	4	4
2	3	4	1
4	1	1	4
2	3	1	1

B	R	E	A	K
3	3	1	1	3
4	3	3	1	3
3	1	3	4	2
2	3	2	2	4
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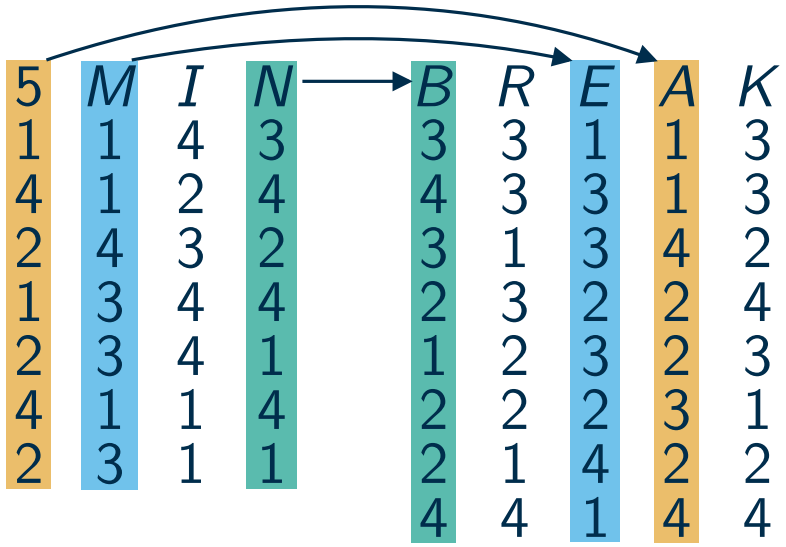

Solution

- $X = \{5, M, N\}$
- $\sigma(5) = A$, $\sigma(M) = E$, and $\sigma(N) = B$

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IND is $W[3]$-complete

Reduction from $\text{IND}_{\text{FIXED}}$ to IND

- easy

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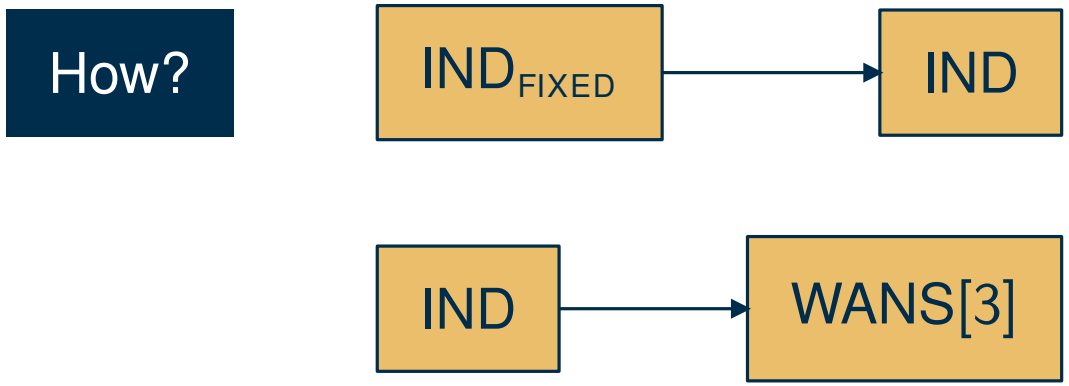
Reduction from IND to WANS[3]



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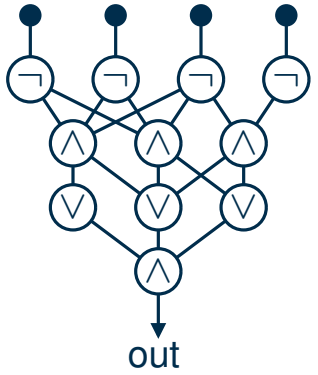
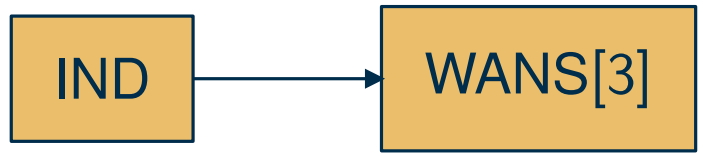
How?

Reduction from IND to WANS[3]

- model IND-conditions as Boolean formulas
- ensure: conj. of disj. of conj. of negative literals





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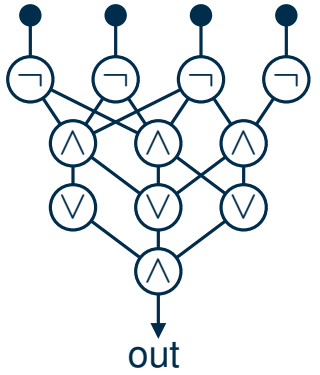

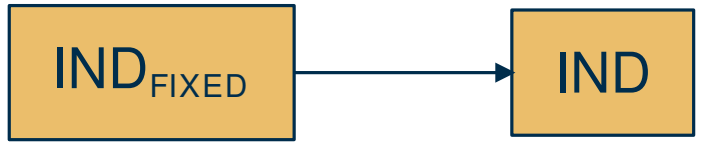

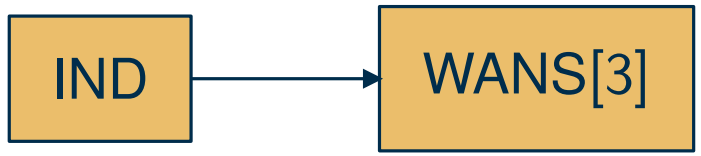

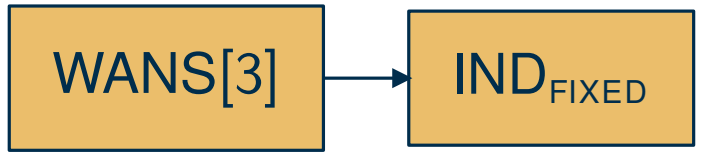

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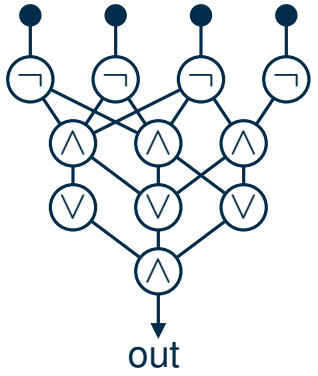
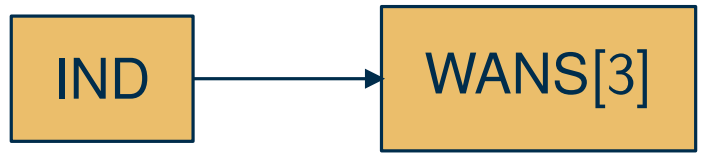
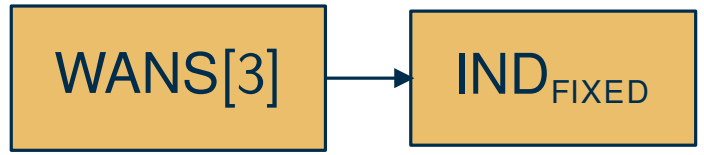




Reduction from WANS[3] to $\text{IND}_{\text{FIXED}}$

- model antimonotone 3-normalized Boolean formula with two databases



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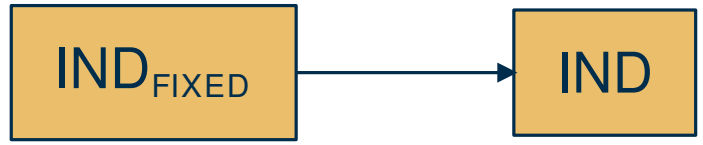

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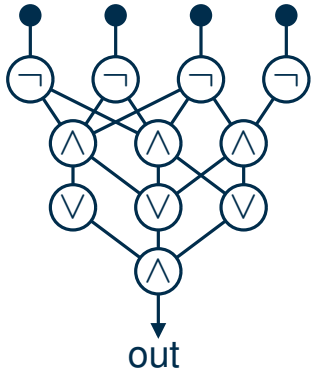

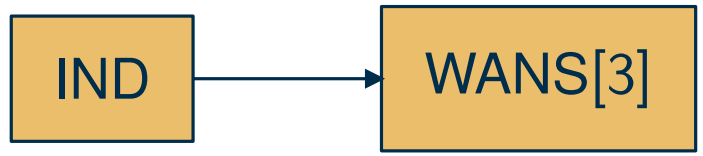

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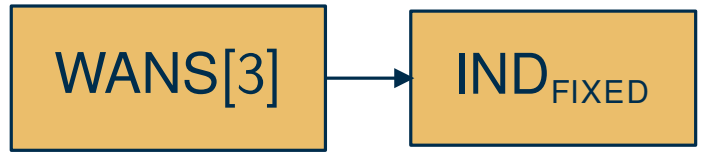

Overall plan:

WANS[3] $\rightarrow$ $\text{IND}_{\text{FIXED}}$: the final “and”

Viewing $\text{IND}_{\text{FIXED}}$ as Boolean function

- formalize set of columns as 01-string

(e.g., $\{A, C\} \equiv 101$)

database r

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1	1	1
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3	4	4

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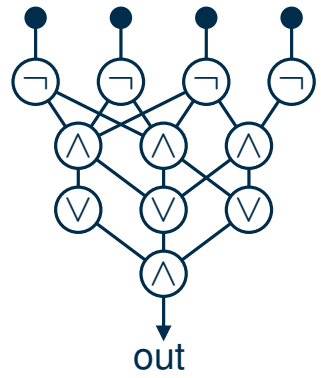

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Viewing $\text{IND}_{\text{FIXED}}$ as Boolean function

- formalize set of columns as 01-string (e.g., $\{A, C\} \equiv 101$)
- Boolean function f: $f(X) = 1 \Leftrightarrow X$ is IND (e.g., $f(101) = 1$, $f(110) = 0$)

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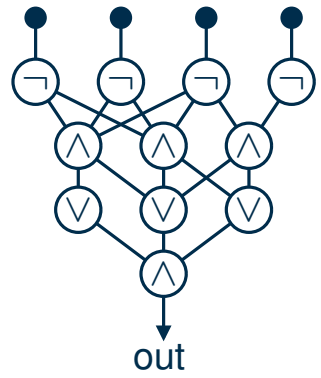

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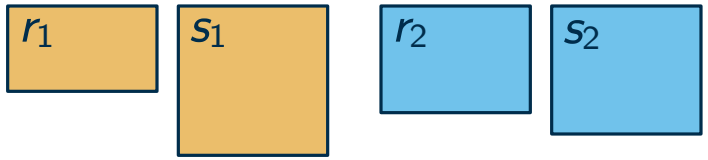
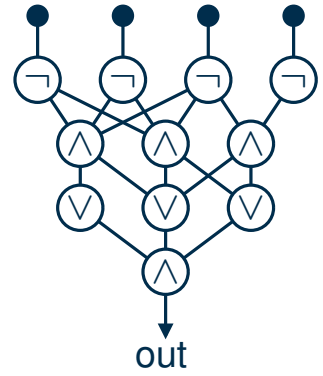
"And"ing two instances (on the same Schema)

- consider two instances (r_1, s_1) and (r_2, s_2) with functions f_1 and f_2





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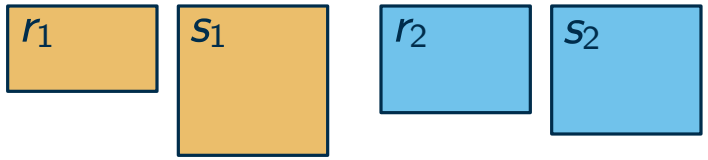
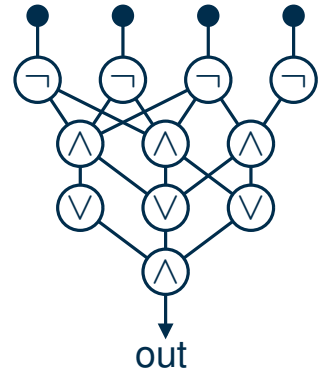
- Is there an instance (r, s) such that $f = f_1 \wedge f_2$?





WANS[3] $\rightarrow$ $\text{IND}_{\text{FIXED}}$: the final "and"

Viewing $\text{IND}_{\text{FIXED}}$ as Boolean function

- formalize set of columns as 01-string (e.g., $\{A, C\} \equiv 101$)
- Boolean function f: $f(X) = 1 \Leftrightarrow X$ is IND (e.g., $f(101) = 1$, $f(110) = 0$)

database r

A	B	C
1	1	1
3	1	2
1	2	1
2	3	3
3	4	4

database s

A	B	C
1	3	1
1	4	3
2	3	3
3	1	2
3	1	4
3	4	4

"And"ing two instances (on the same Schema)

- consider two instances (r_1, s_1) and (r_2, s_2) with functions f_1 and f_2
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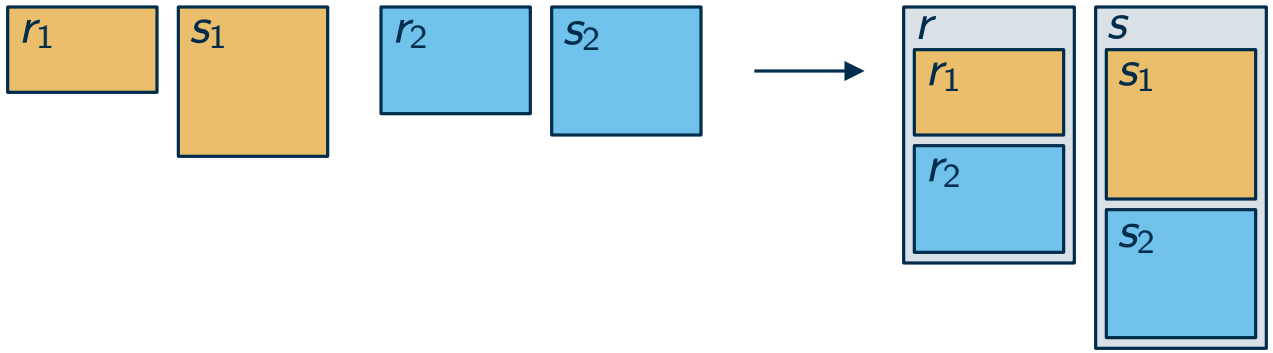

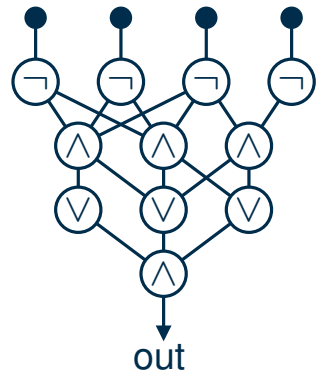

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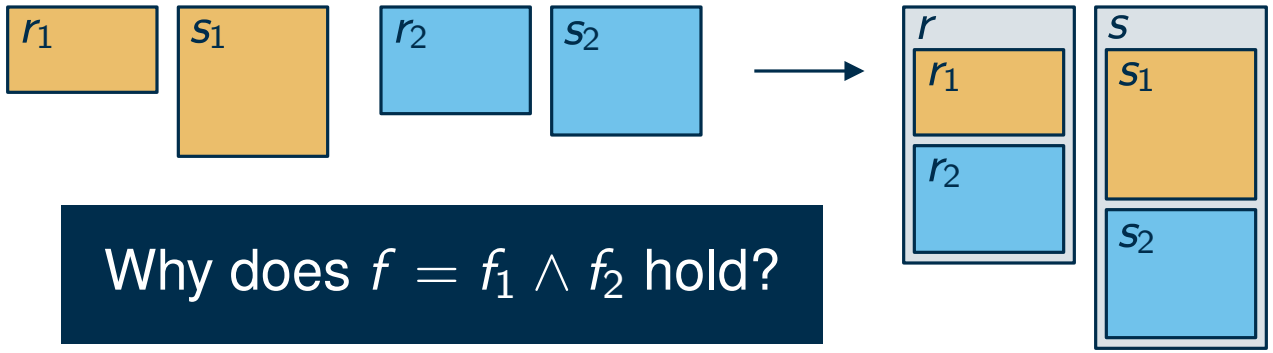
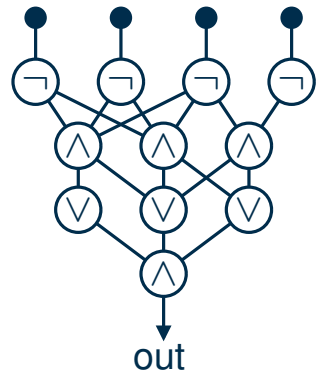


Why does $f = f_1 \wedge f_2$ hold?



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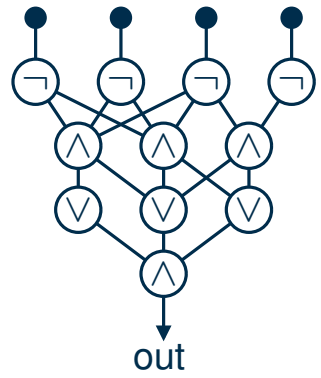

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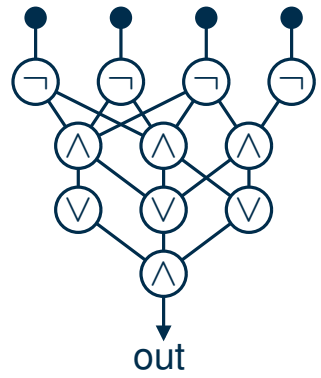

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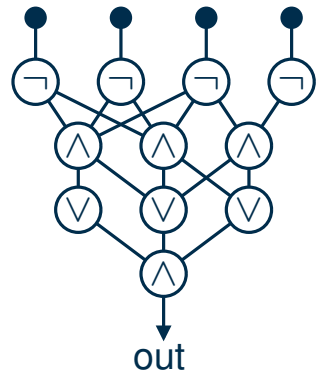

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Disjunction of conjunction of negative literals

$\varphi = c_1 \vee c_2 \vee c_3$

$c_1 = (\neg x_1 \wedge \neg x_2 \wedge \neg x_3)$

$c_2 = (\neg x_2 \wedge \neg x_4 \wedge \neg x_5)$

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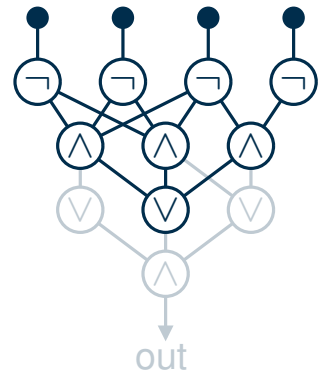

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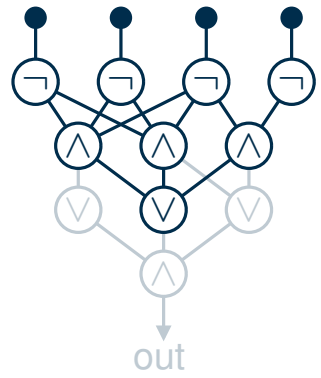
- modeling a single c_i is easy

r:

A_1	A_2	A_3	A_4	A_5	A_6
0	0	0	0	0	0

s:

A_1	A_2	A_3	A_4	A_5	A_6
1	1	1	0	0	0



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0	0	0	0	0	0

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A_1	A_2	A_3	A_4	A_5	A_6
1	1	1	0	0	0
0	1	0	1	1	0
1	0	1	1	0	1

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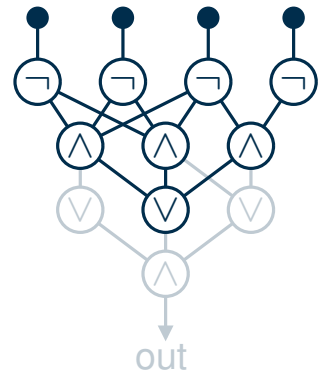
- for IND it suffices to find the data from r in one row of s
  multiple rows in $s \rightarrow$ naturally models “or” between them



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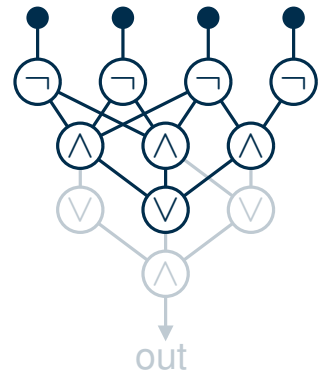
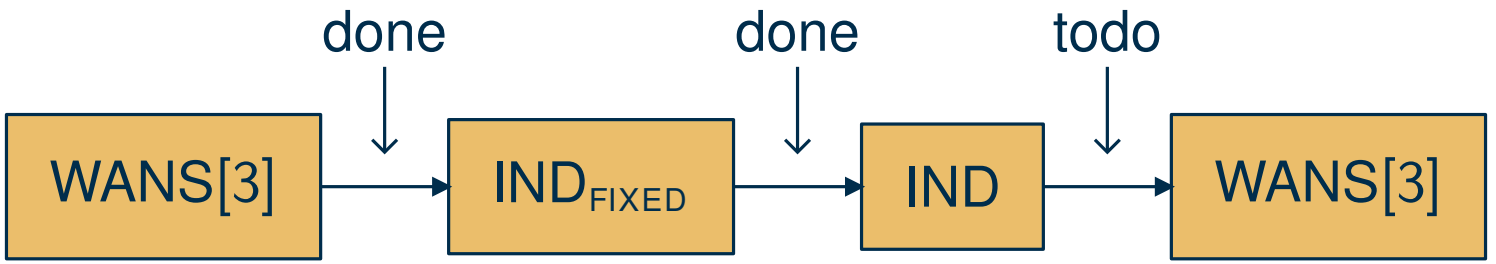


Check: solution for φ yields solution for (r, s) and vice versa

Seen so far



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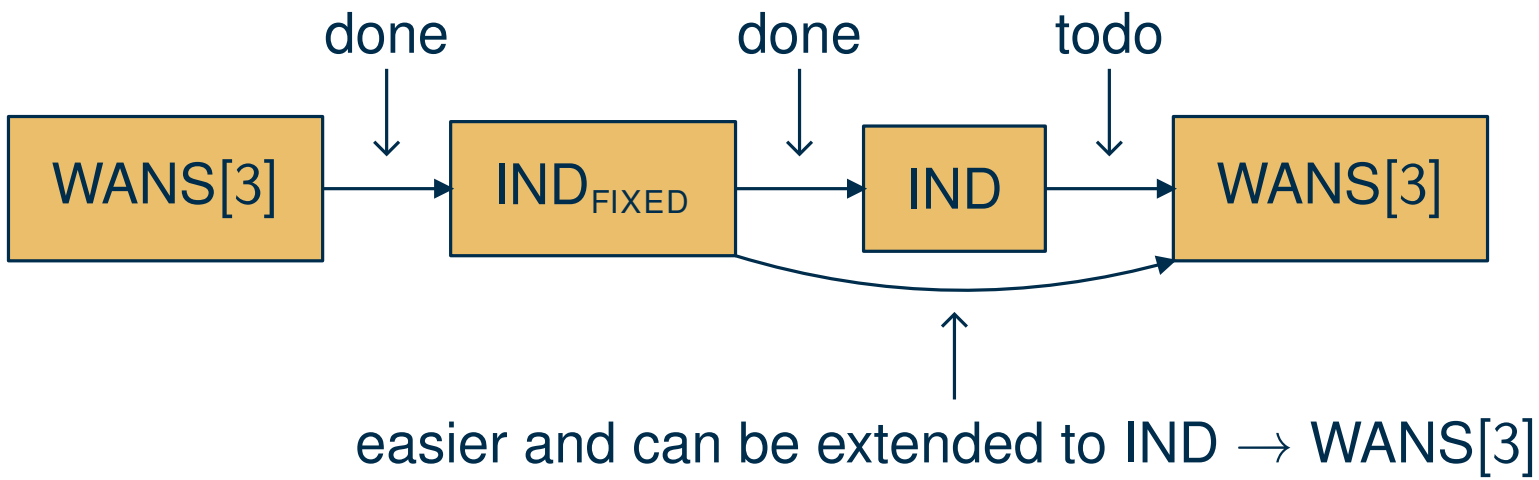
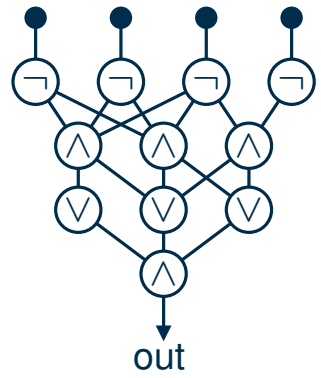

done
done
todo
WANS[3]
IND$_{\text{FIXED}}$
IND
WANS[3]
easier and can be extended to IND $\rightarrow$ WANS[3]

$\mathrm{IND}_{(\mathrm{FIXED})} \rightarrow \mathrm{WANS}[3]$

$\mathrm{IND}_{\mathrm{FIXED}}$ as Boolean formula



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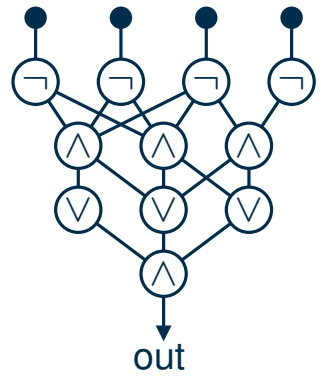
- single pair of rows (r_i, s_j): exclude certain columns

	A_1	A_2	A_3	A_4
r_i	1	0	1	0

	A_1	A_2	A_3	A_4
s_j	1	1	0	0

$\longrightarrow \quad \neg x_2 \wedge \neg x_3$



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s_1	1	1	0	0
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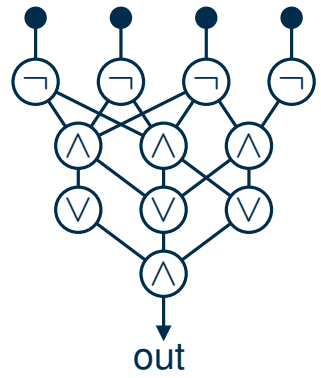
$\rightarrow (r_j \mapsto s_1)\ \neg x_2 \wedge \neg x_3 \ \vee\ (r_j \mapsto s_2)\ \neg x_1 \wedge \neg x_2$



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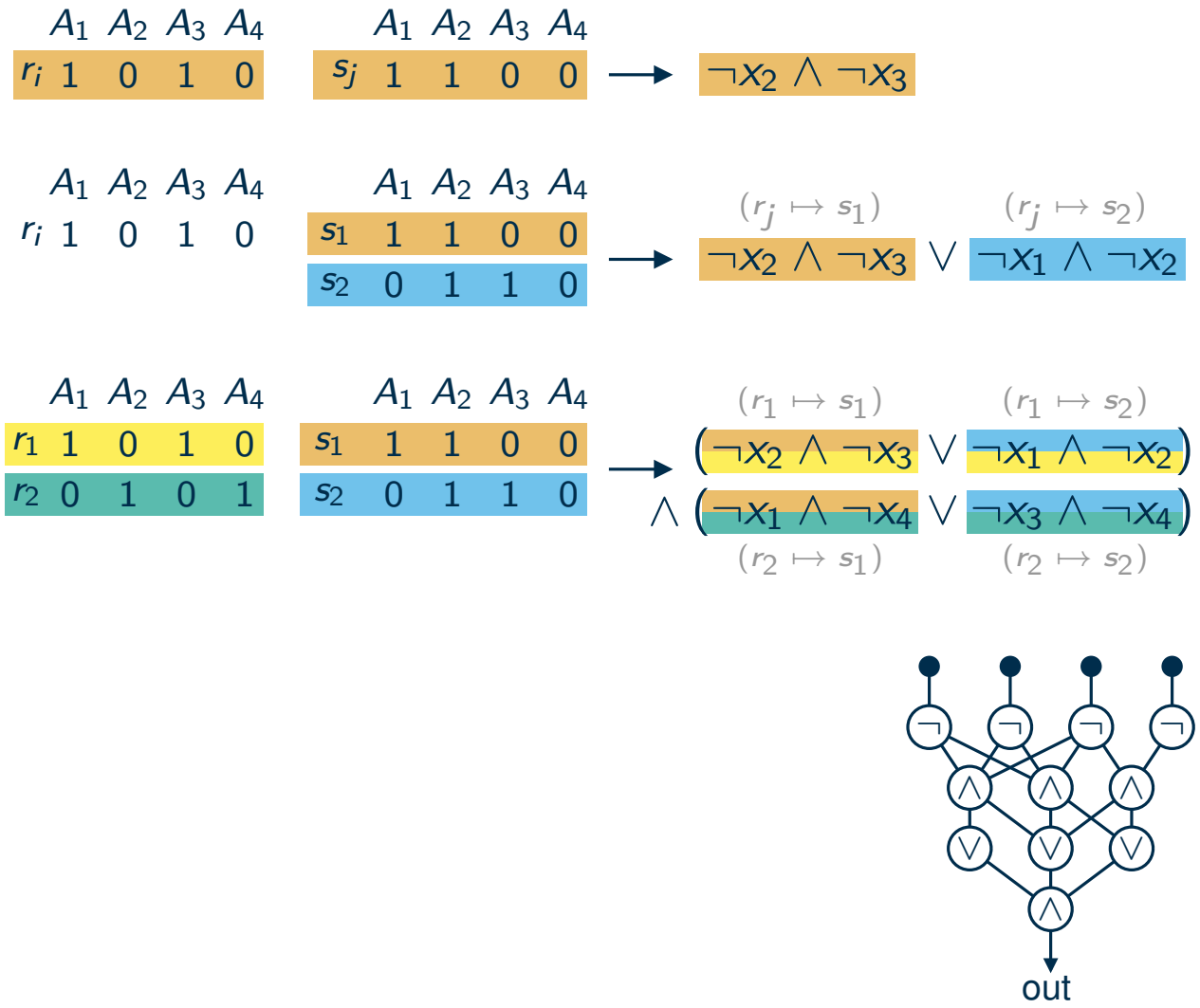

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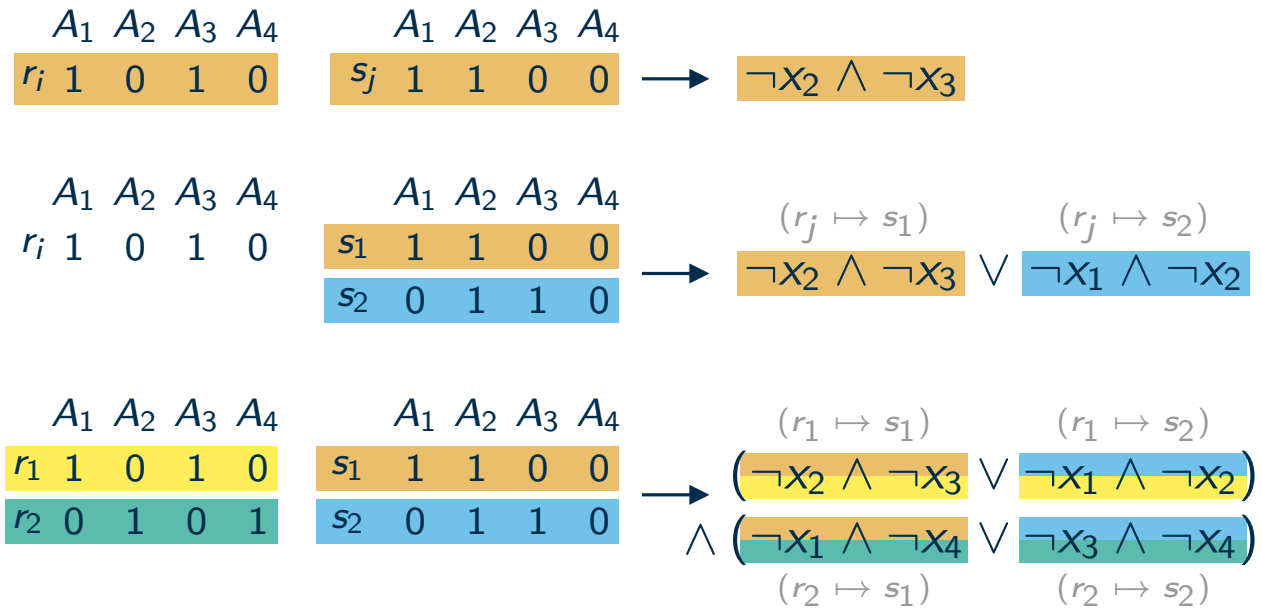
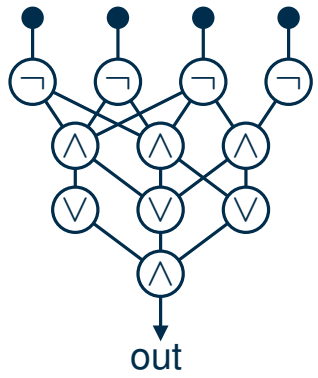


Extension to IND $\to$ WANS[3]



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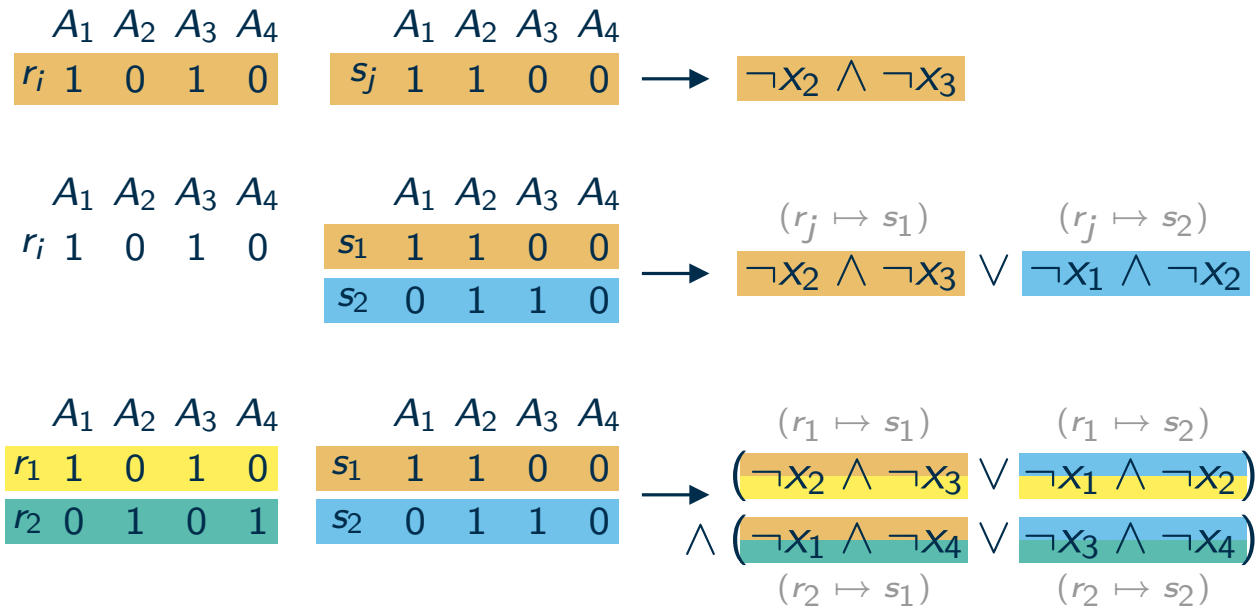

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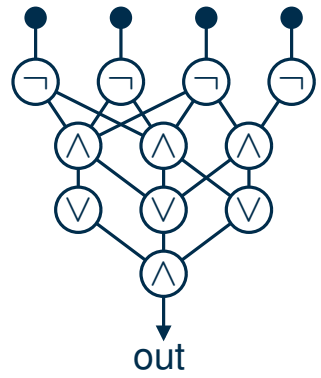

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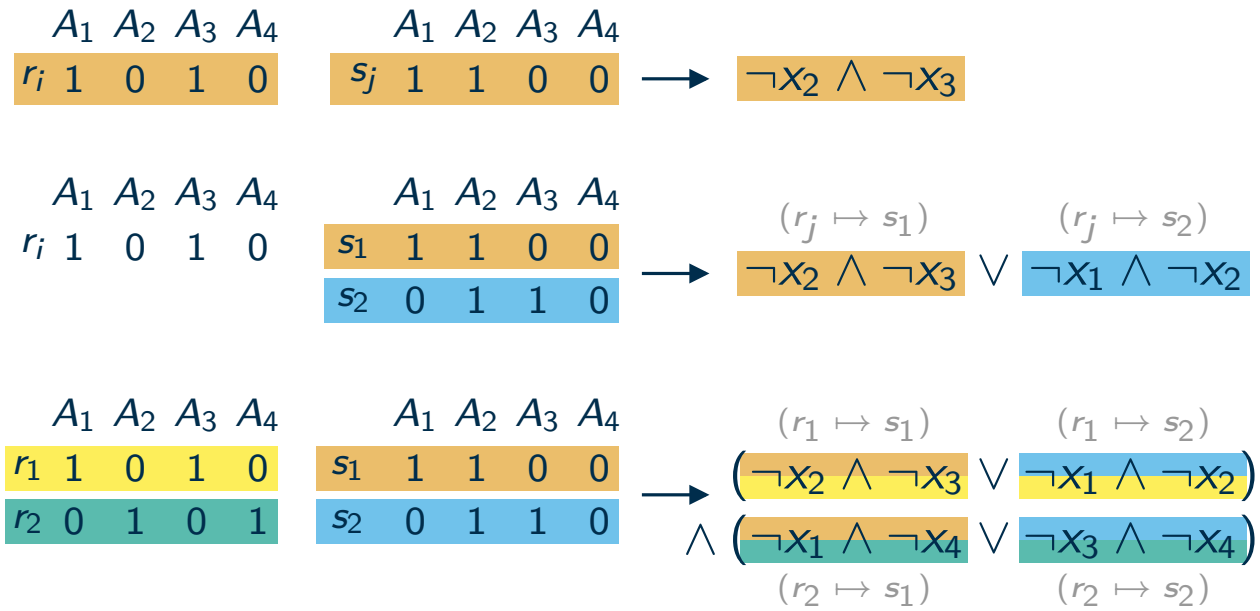

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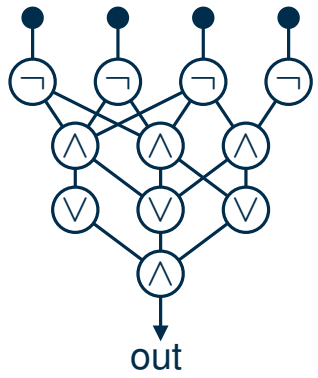

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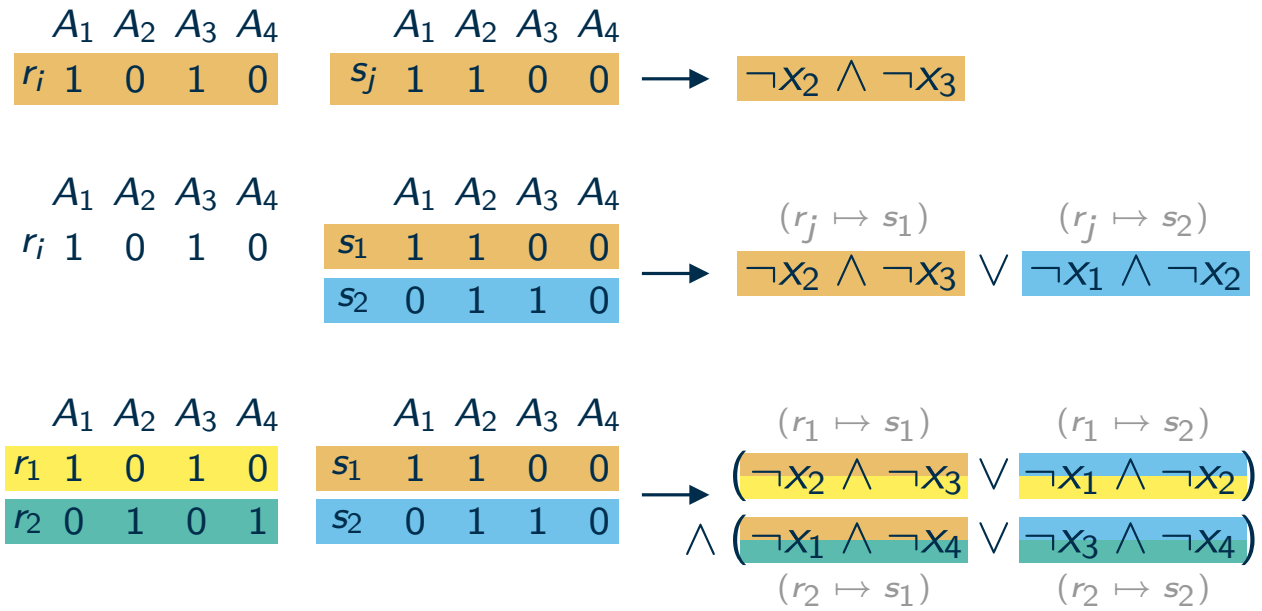

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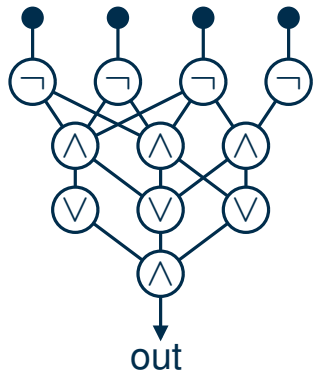

Theorem
$\text{IND}_{\text{FIXED}}$ and IND are $W[3]$-complete.

Wrap-Up

Normalized circuits

- we can start reductions from WCS with rather clean circuits
- no need to look at weft; just count the layers

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- rare case of a non-graph problem
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Dependency detection

- rare case of a non-graph problem
- there are natural $W[3]$-complete problems
- does not explain why functional dependencies can be computed efficiently in practice

  (one can actually enumerate all inclusion-wise minimal UCCs/FDs rather quickly)

Literature

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- Thomas Bläsius, Tobias Friedrich, Martin Schirneck [2016]
- contains the reduction from today doi.org/10.4230/LIPIcs.IPEC.2016.6

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Discovering Functional Dependencies through Hitting Set Enumeration

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