Parameterized Algorithms

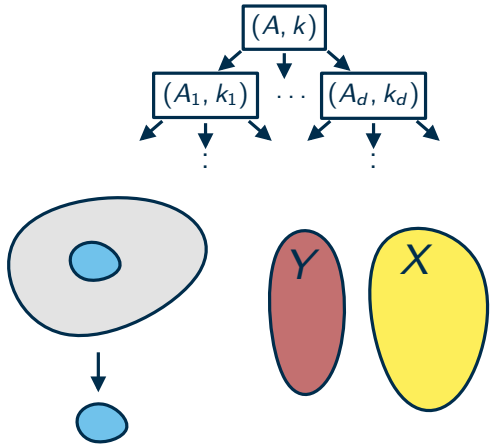
Lower Bounds: Reductions and the W-Hierarchy

Thomas Bläsius

Content

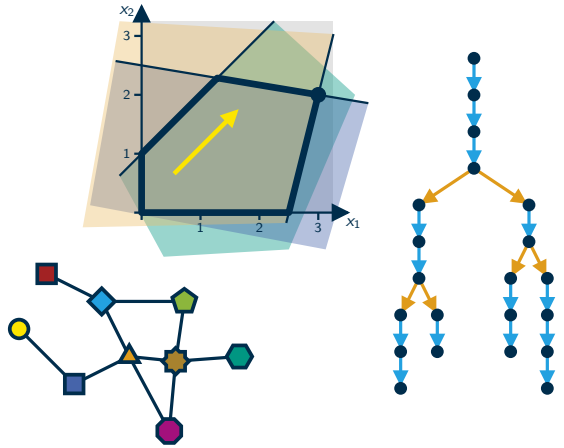
Basic toolbox

- bounded search trees
- kernelization
- iterative compression



Extended toolbox

- linear programs
- branch-and-reduce
- color coding

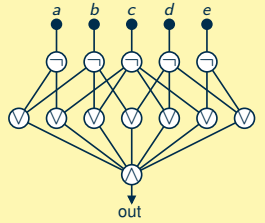


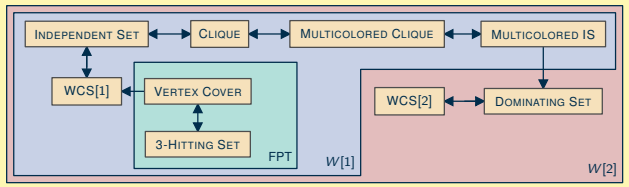
Tree width

- dynamic programming
- chordal and planar graphs
- Courcelle's theorem

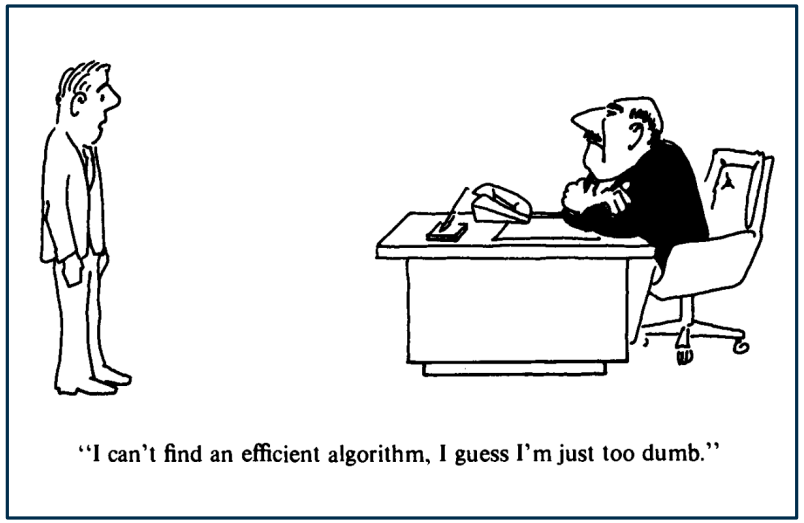
Lower bonds

- kernel lower bounds
- parameterized reductions
- circuits and the W-hierarchy
- ETH and SETH





Why do we want lower bounds?



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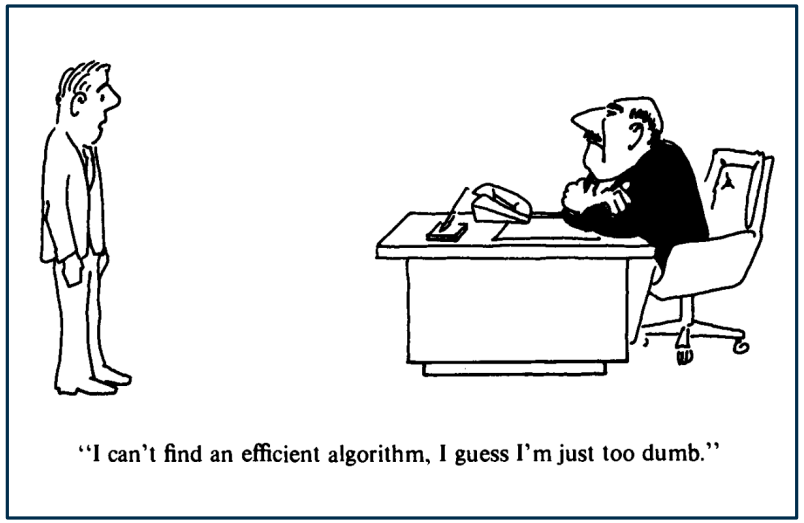
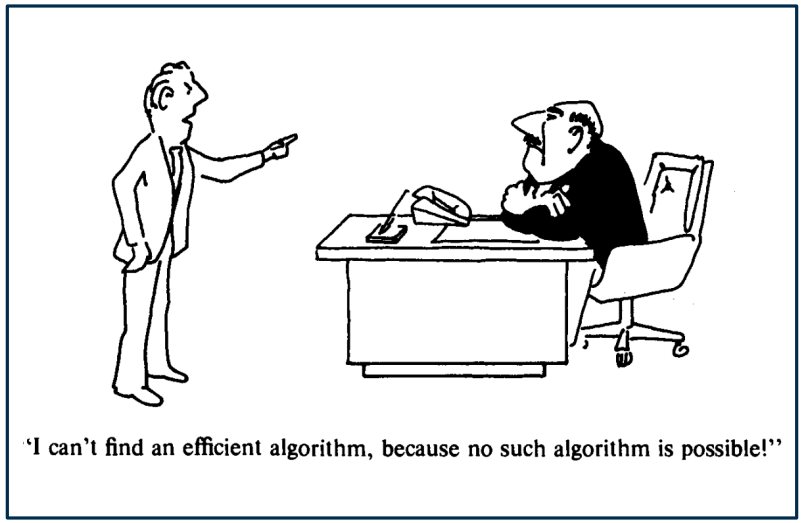
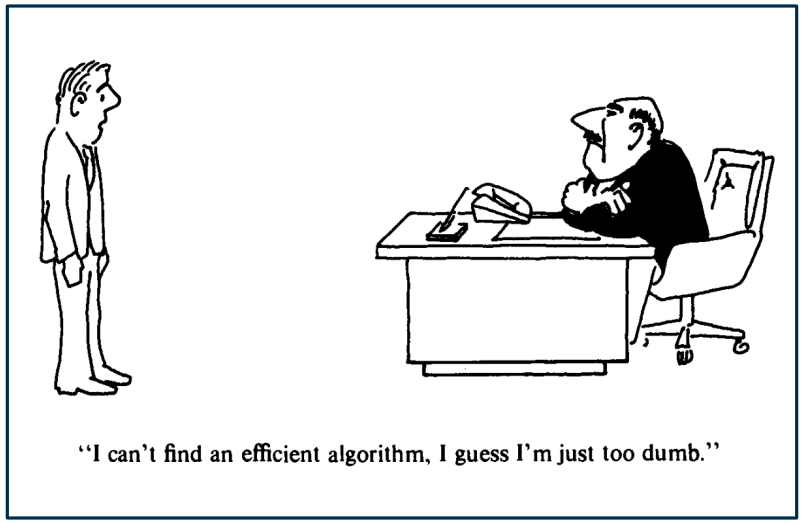


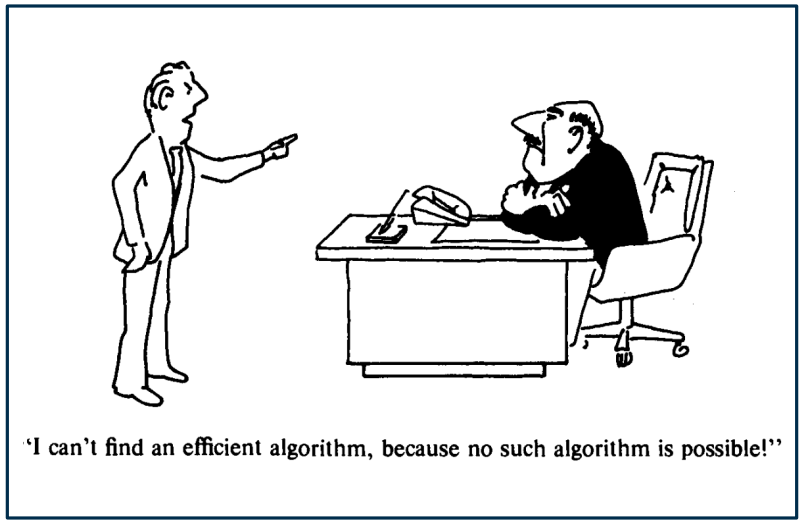
"I can't find an efficient algorithm, I guess I'm just too dumb."



"I can't find an efficient algorithm, because no such algorithm is possible!"

Why do we want lower bounds?





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- non-existence of an algorithm is difficult to prove

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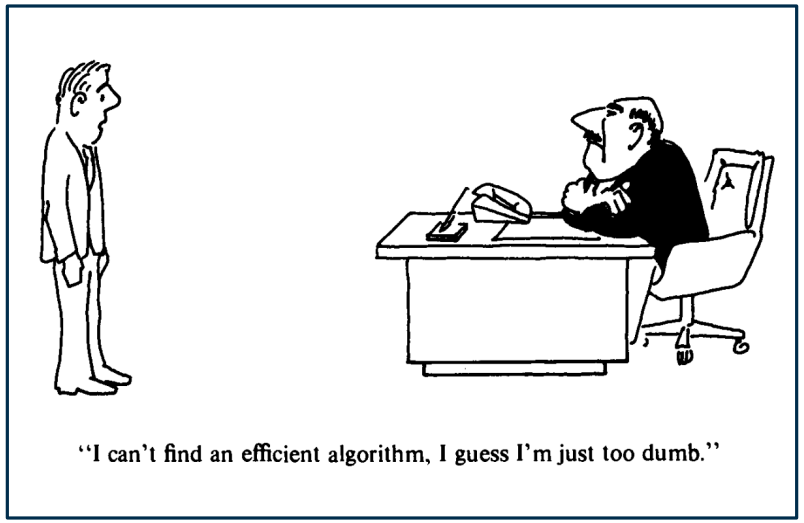

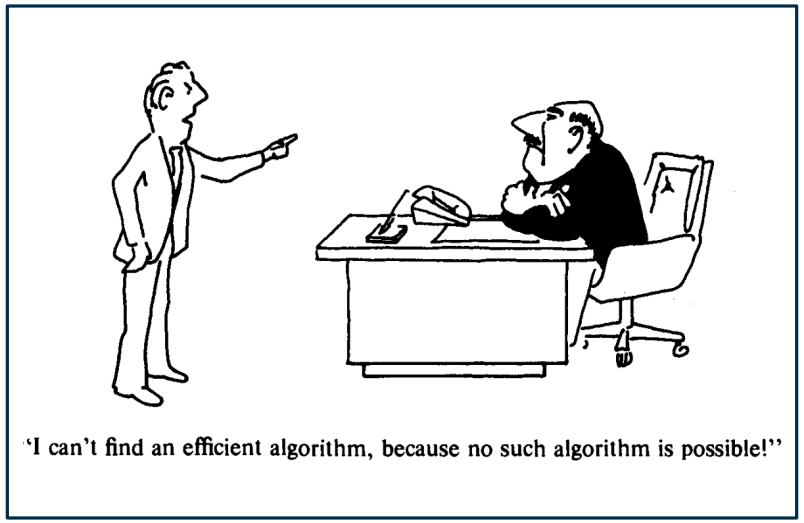

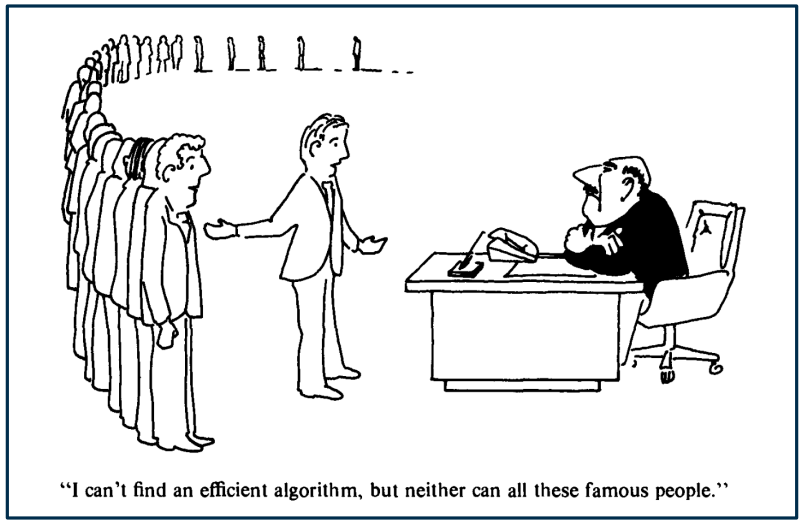

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- show: my problem efficiently solvable $\Rightarrow$ a known hard problem efficiently solvable

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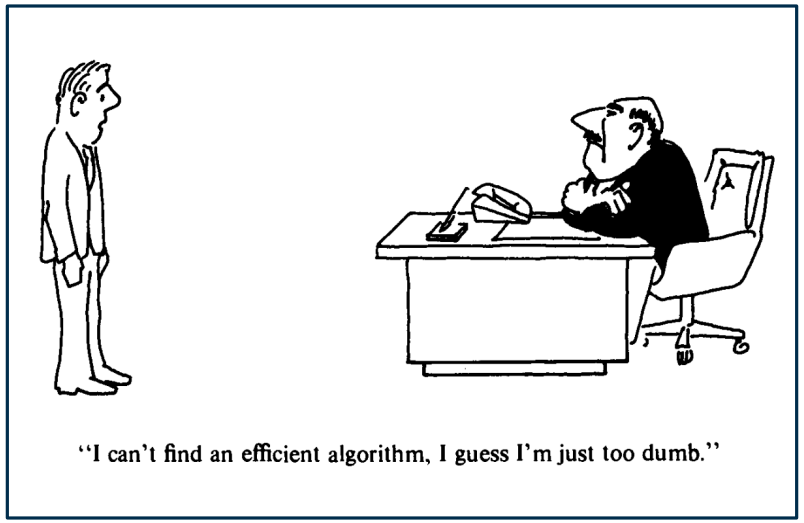

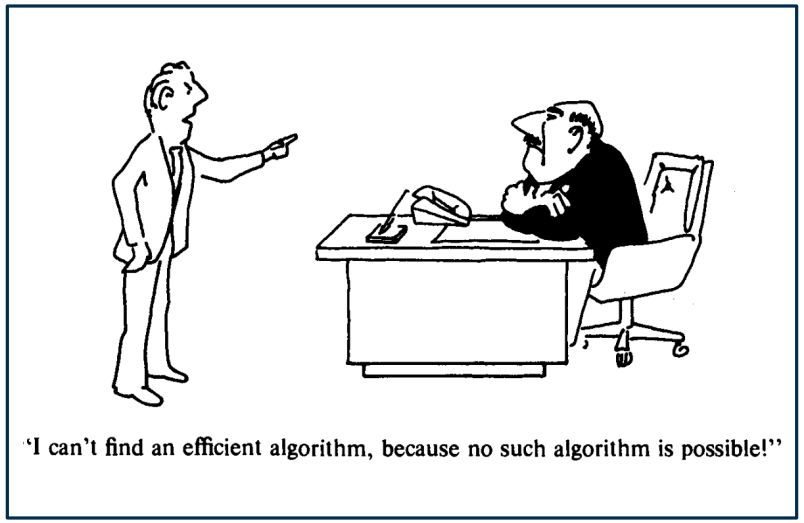

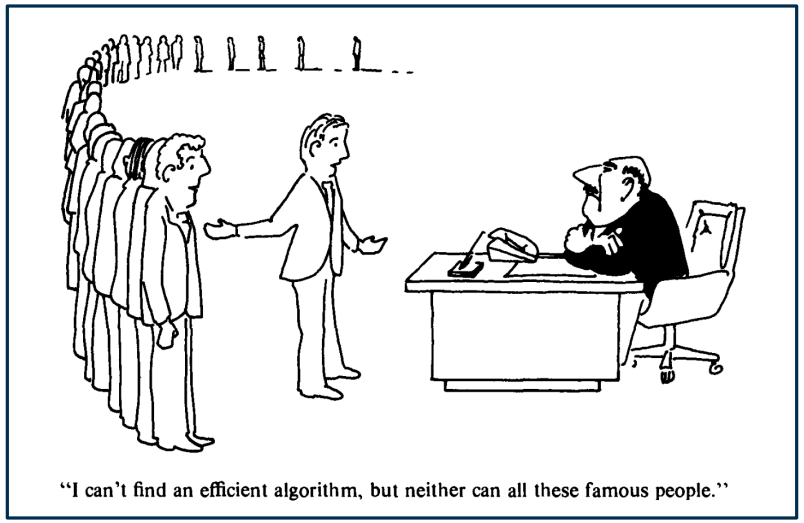

Problem

- non-existence of an algorithm is difficult to prove

Solution

- show: my problem efficiently solvable $\Rightarrow$ a known hard problem efficiently solvable
- tool: reduction between problems

Polynomial reductions

Reduction from problem $\mathcal{L}$ to problem $\mathcal{L}'$

- map each instance I of $\mathcal{L}$ to an instance I' of $\mathcal{L}'$
- I is yes-instance $\Leftrightarrow$ I' is yes-instance

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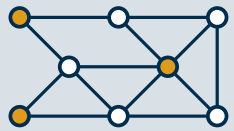
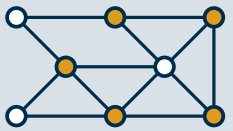
Example: Independent Set to Vertex Cover

Independent Set



$V' \subseteq V$, such that
$\forall e \in E : |e \cap V'| \leq 1$

Vertex Cover



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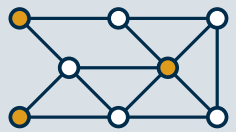	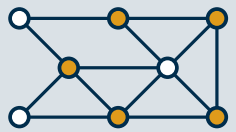
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Example: Independent Set to Vertex Cover

- reduction: G has IS of size k $\Leftrightarrow$ G has VC of size $n - k$
- thus: VC $\in$ P $\Rightarrow$ IS $\in$ P

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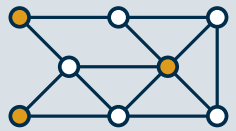	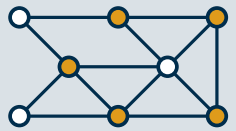
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- reduction: G has IS of size k $\Leftrightarrow$ G has VC of size $n - k$
- thus: VC $\in$ P $\Rightarrow$ IS $\in$ P
- And "VC $\in$ FPT $\Rightarrow$ IS $\in$ FPT"?

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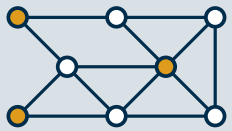

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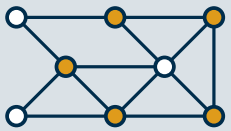

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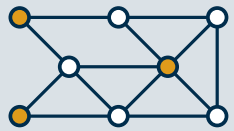

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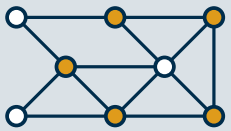

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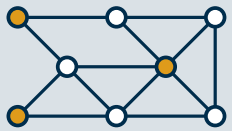
- And "VC $\in$ FPT $\Rightarrow$ IS $\in$ FPT"? $\rightarrow$ parameter too large
- for "efficient" = "FPT", we need a different reduction

Independent Set



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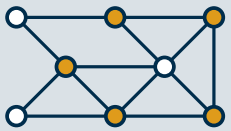

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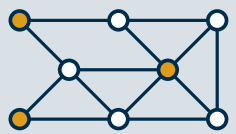
Example: Independent Set to Clique

Independent Set



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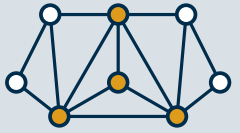
Clique



$V' \subseteq V$, such that $G[V']$ complete

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- G has IS of size k $\Leftrightarrow$ complement of G has k-clique

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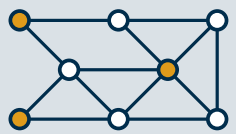	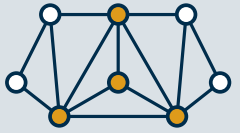
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- thus: Clique $\in$ FPT $\Rightarrow$ IS $\in$ FPT

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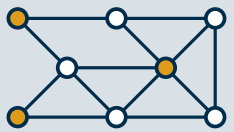	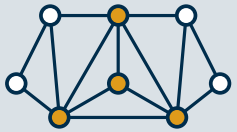
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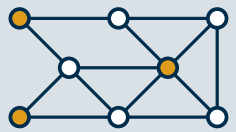
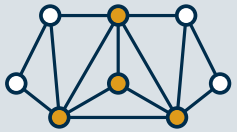
- G has IS of size k $\Leftrightarrow$ complement of G has k-clique
- thus: Clique $\in$ FPT $\Rightarrow$ IS $\in$ FPT
- inverse also holds: Clique $\in$ FPT $\Leftrightarrow$ IS $\in$ FPT

Independent Set



$V' \subseteq V$, such that $\forall e \in E : |e \cap V'| \leq 1$

Clique



$V' \subseteq V$, such that $G[V']$ complete

Parameterized reduction

Reduction from problem $\mathcal{L}$ to problem $\mathcal{L}'$

- map each instance (I, k) of $\mathcal{L}$ to instance (I', k') of $\mathcal{L}'$ with $k' \leq g(k)$
- (I, k) yes-instance $\Leftrightarrow$ (I', k') yes-instance
- the map must be computable in FPT-time $f(k) \cdot |I|^{O(1)}$ (f and g computable)

Implication

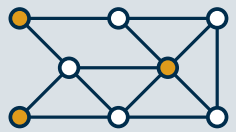
- FPT algorithm for $\mathcal{L}'$ $\Rightarrow$ FPT algorithm for $\mathcal{L}$
- my problem $\mathcal{L}'$ efficiently solvable $\Rightarrow$ known hard problem $\mathcal{L}$ efficiently solvable

(for "efficient" = "FPT")

Example: Independent Set to Clique

- G has IS of size k $\Leftrightarrow$ complement of G has k-clique
- thus: Clique $\in$ FPT $\Rightarrow$ IS $\in$ FPT
- inverse also holds: Clique $\in$ FPT $\Leftrightarrow$ IS $\in$ FPT
- conjecture: Clique, IS $\notin$ FPT

Independent Set



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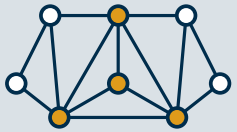

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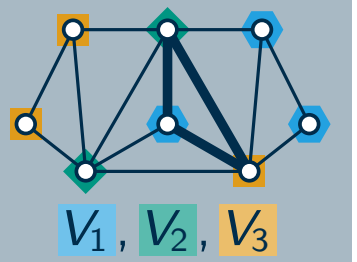
Clique $\rightarrow$ Multicolored Clique

Problem: Multicolored Clique
Given a graph $G = (V, E)$, a parameter k and a partition $(V_1, \ldots, V_k)$ of the vertices. Is there a clique with exactly one vertex from each V_i?



CLIQUE $\rightarrow$ MULTICOLORED CLIQUE

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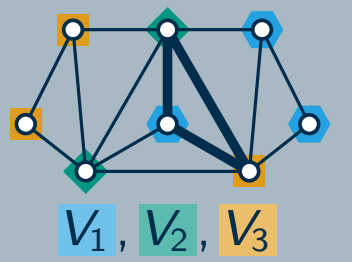
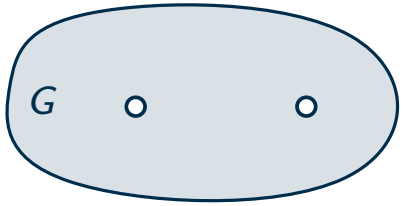


instance $(G, 3)$ of CLIQUE



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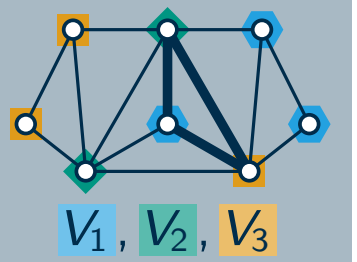

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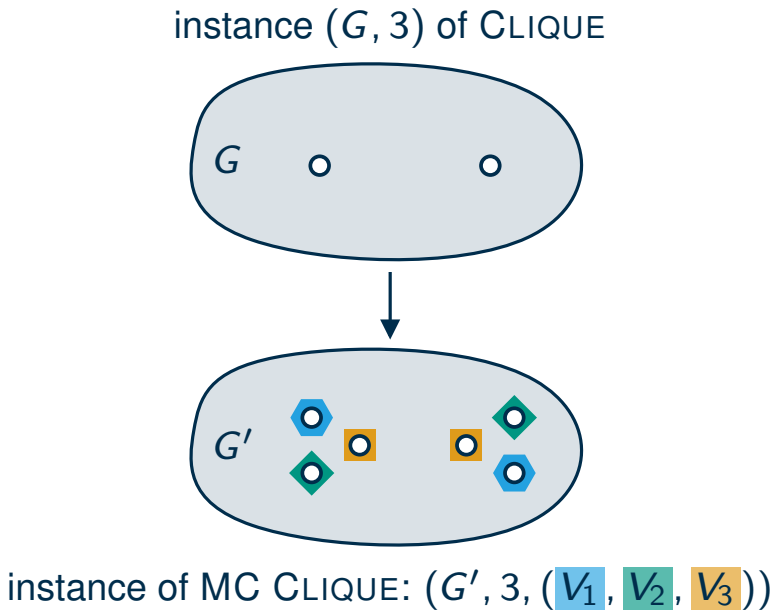

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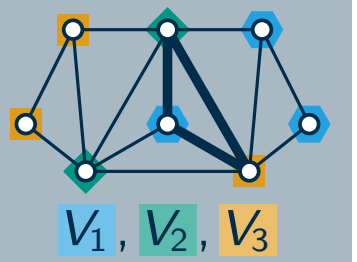

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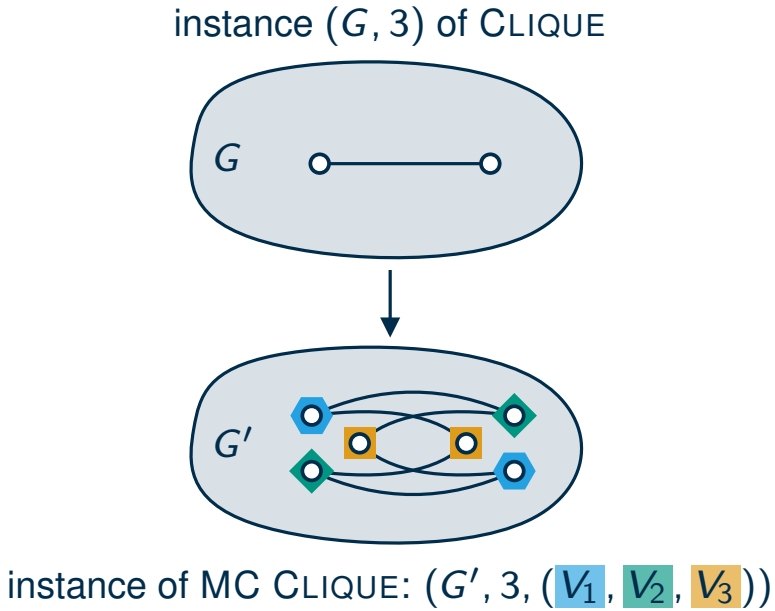

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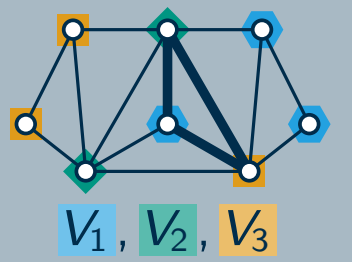

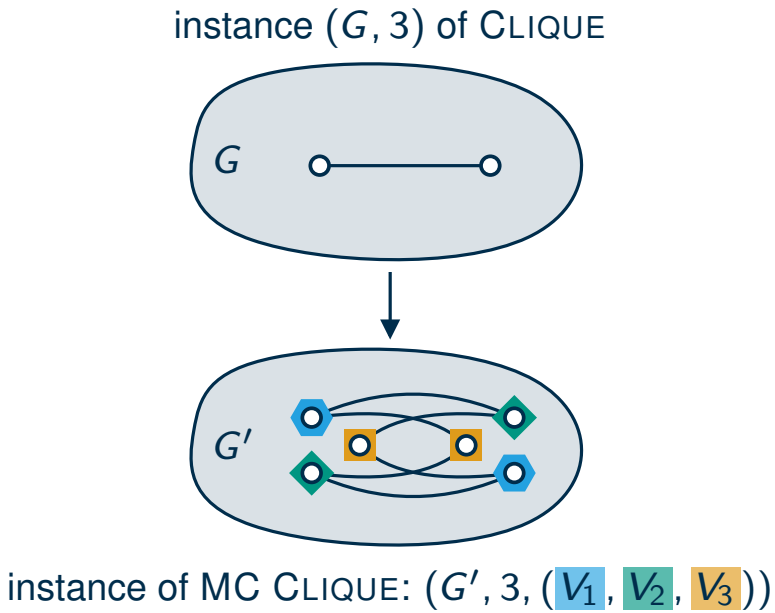

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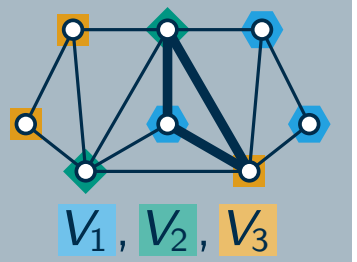

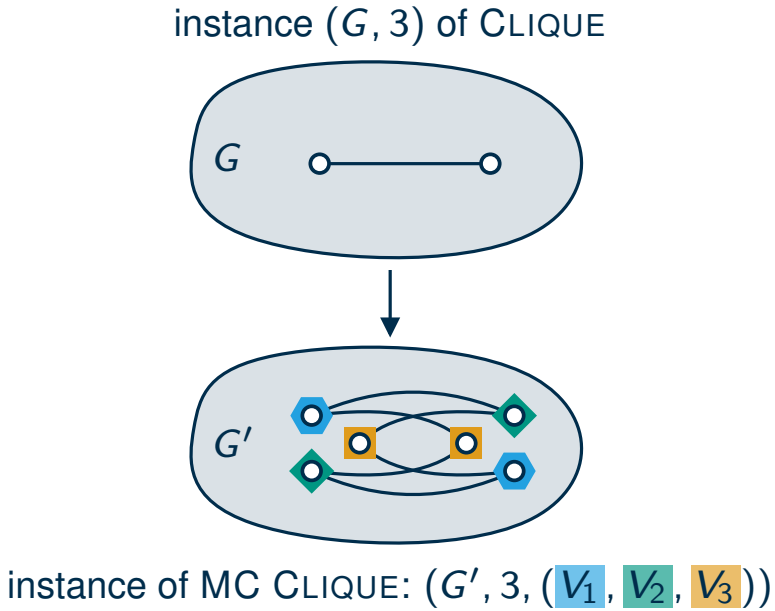

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- all vertices have different colors

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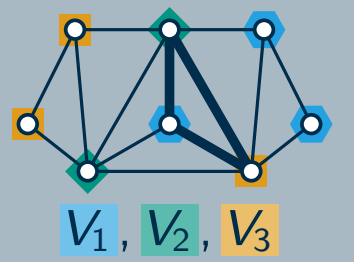

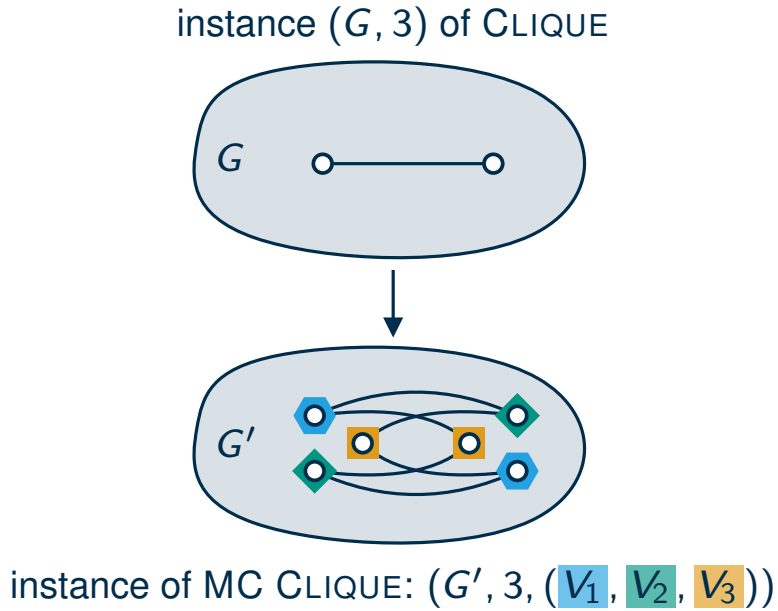

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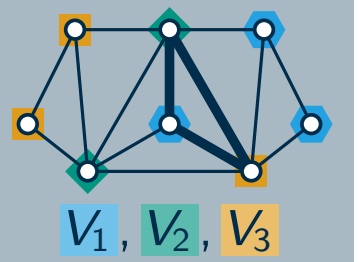

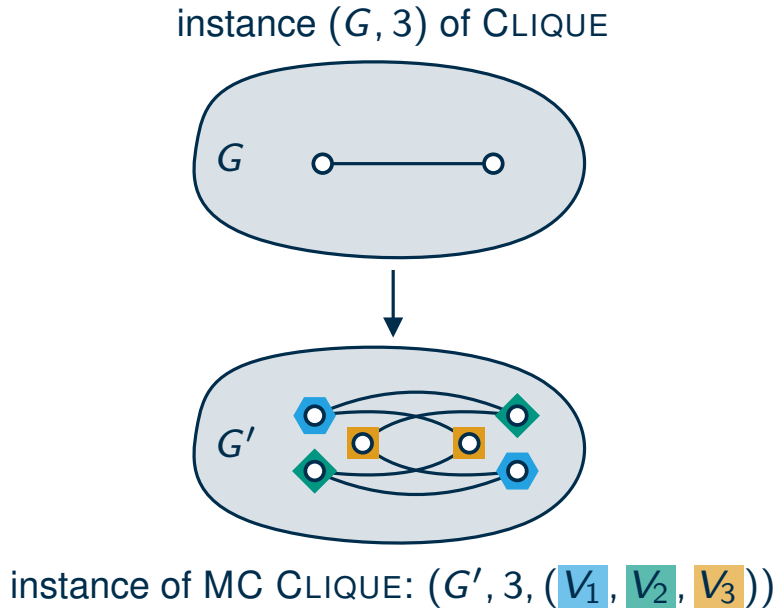

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- colorful clique: $v^1_{\pi(1)}, \ldots, v^k_{\pi(k)}$ with $\pi : [k] \to [n]$
- $v_{\pi(1)}, \ldots, v_{\pi(k)}$ form a clique in G
- it has size k as π is injective

MULTICOLORED CLIQUE → CLIQUE

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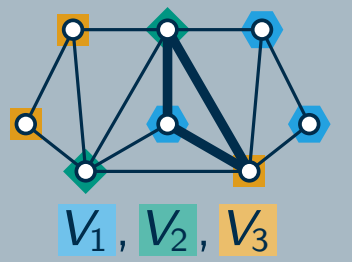
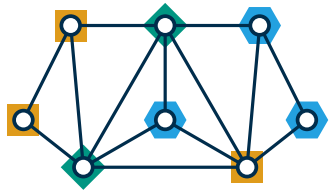


instance of MC CLIQUE: $(G, 3, (V_1, V_2, V_3))$



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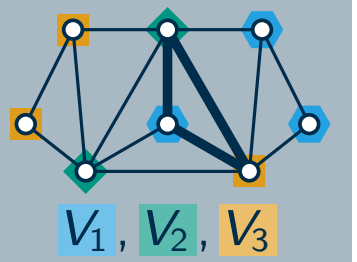

Reduction: Multicolored Clique to Clique

- delete edges between equally colored vertices
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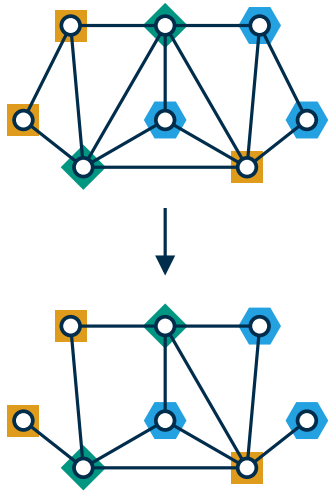

instance $(G', 3)$ of Clique

MULTICOLORED CLIQUE → CLIQUE

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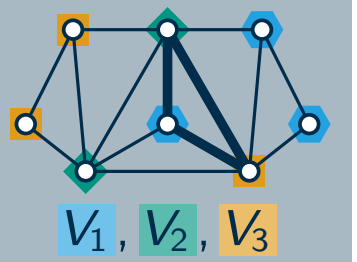

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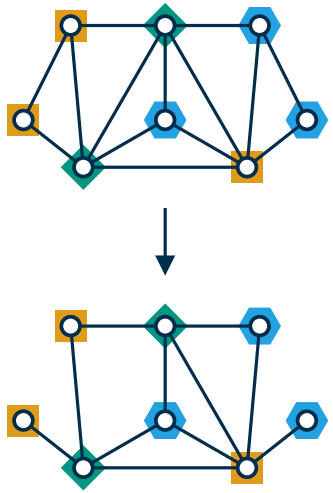

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MULTICOLORED CLIQUE → CLIQUE

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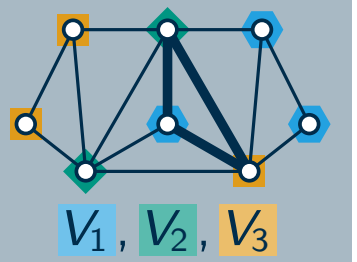

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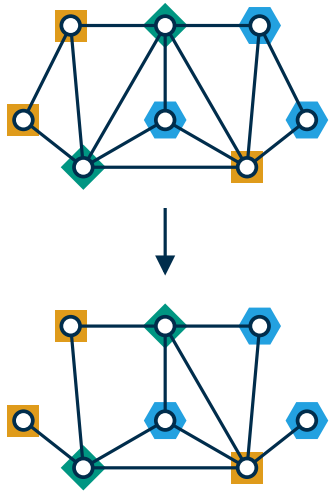

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Reduction: MULTICOLORED CLIQUE to CLIQUE

- delete edges between equally colored vertices
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colorful k-clique in G $\Rightarrow$ k-clique in G'

- no edges of the colorful clique are deleted
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MULTICOLORED CLIQUE → CLIQUE

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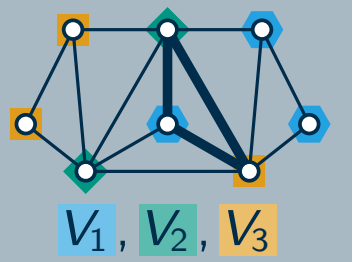

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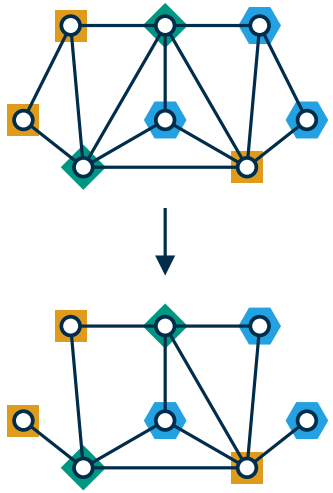

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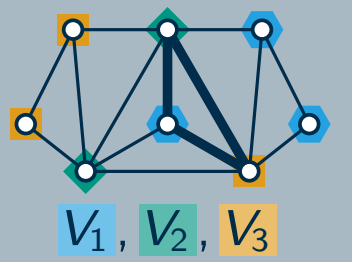

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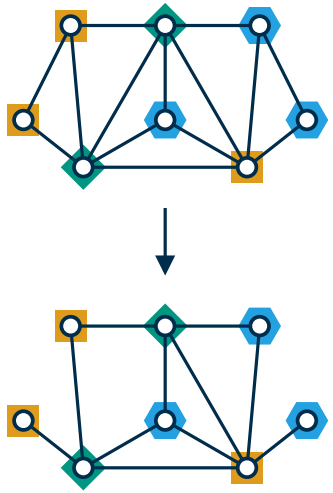

instance $(G', 3)$ of CLIQUE

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- delete edges between equally colored vertices
- set $k' = k$

colorful k-clique in G $\Rightarrow$ k-clique in G'

- no edges of the colorful clique are deleted
- it remains a clique of size k in G'

k-clique in G' $\Rightarrow$ colorful k-clique in G

- no vertices with equal color are adjacent in G'
- each clique in G' is colorful in G

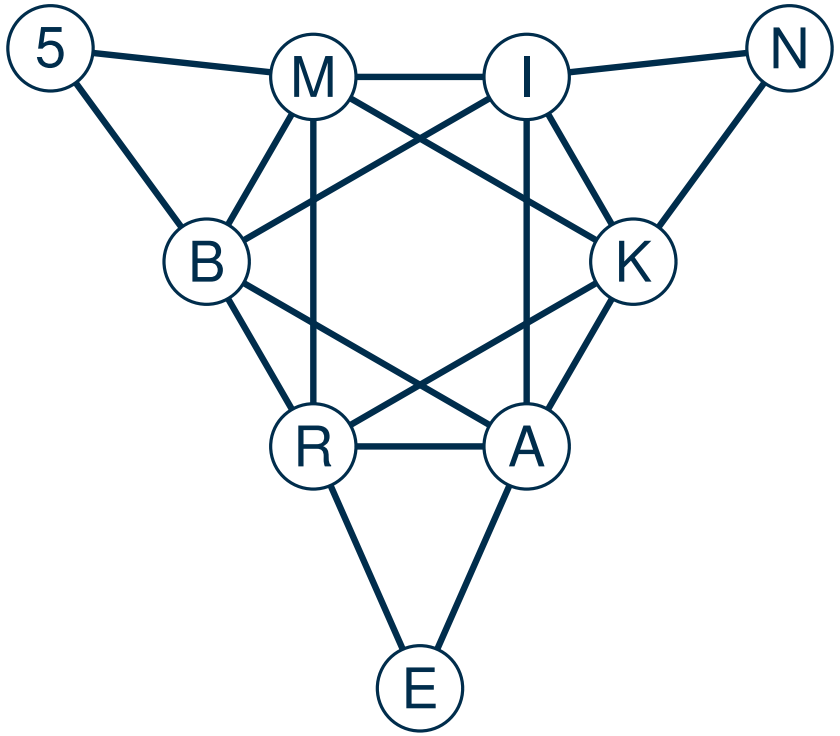
Reductions so far

Reductions so far

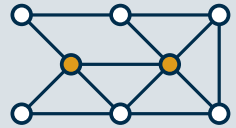
Dominating Set

Goal

- select as few vertices as possible
- for every vertex: the vertex or one of its neighbors is selected



Dominating Set



$V' \subseteq V$, such that
$\forall v \in V : |N[v] \cap V'| \geq 1$

Dominating Set

Reduce Multicolored Independent Set to Dominating Set

MC Independent Set

$V' \subseteq V$ colorful, such that
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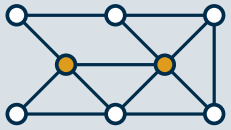
Dominating Set



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Dominating Set

Reduce Multicolored Independent Set to Dominating Set

- enforce the following properties in an instance of Dominating Set
  - exactly one vertex of each color must be chosen
  - forbid the choice of certain vertex pairs

MC Independent Set

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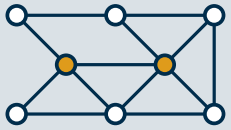
Dominating Set



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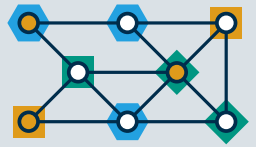
Exactly one vertex for each color

MC Independent Set



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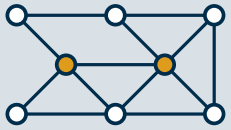

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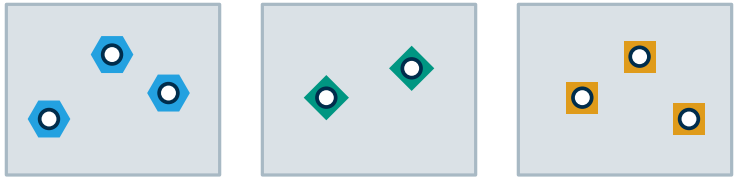

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Reduce Multicolored Independent Set to Dominating Set

- enforce the following properties in an instance of Dominating Set
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Exactly one vertex for each color

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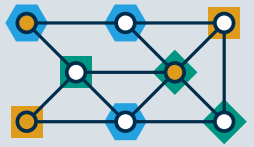

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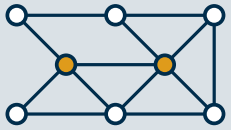

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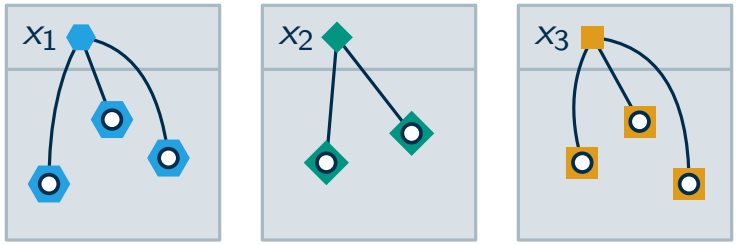

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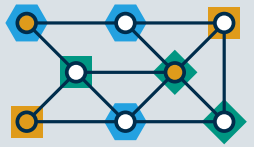

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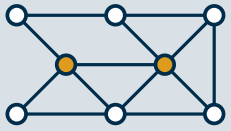

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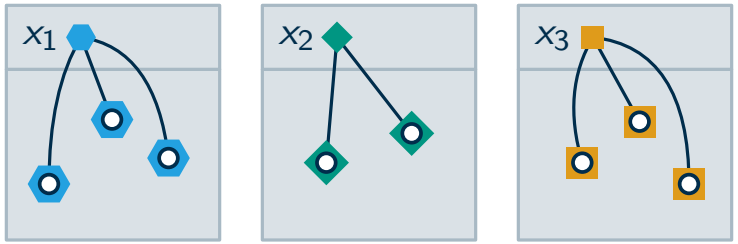

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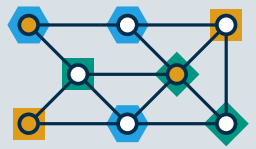

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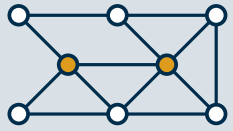

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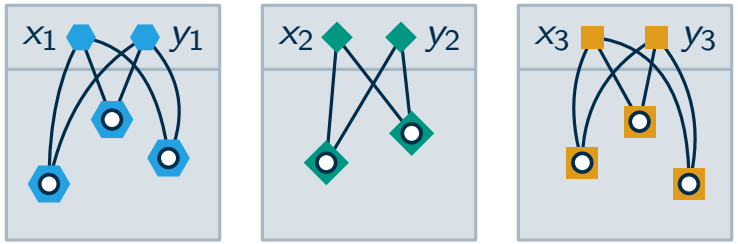

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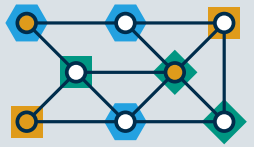

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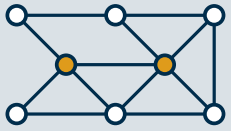

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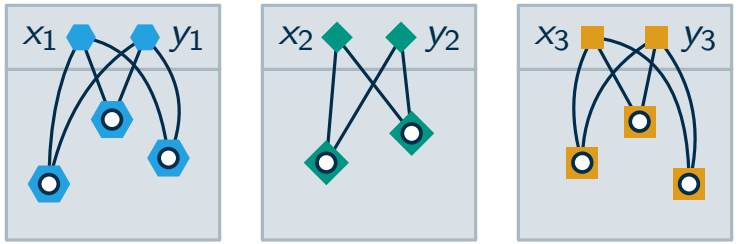

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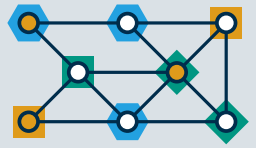

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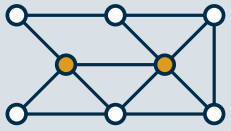

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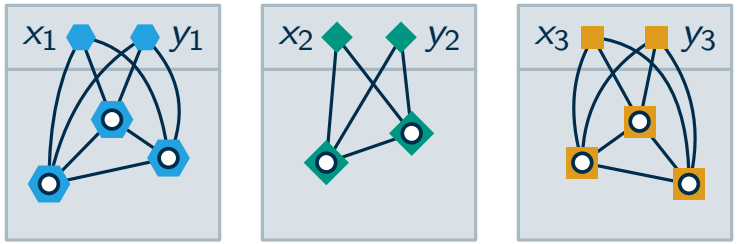

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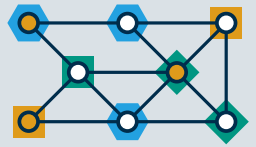

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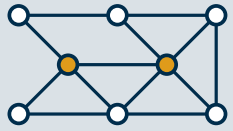

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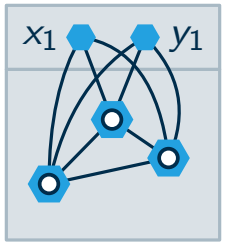

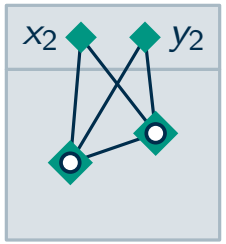

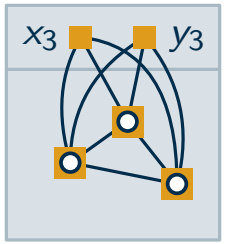

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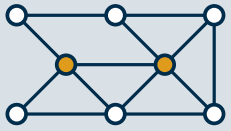

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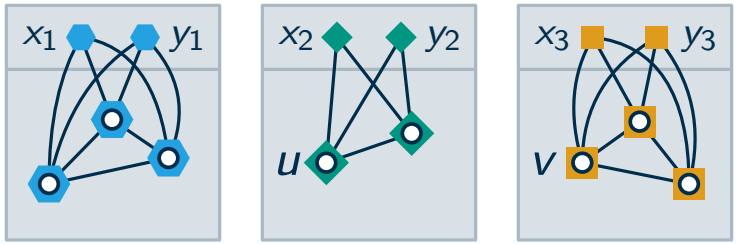

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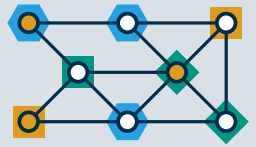

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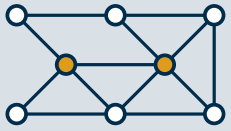

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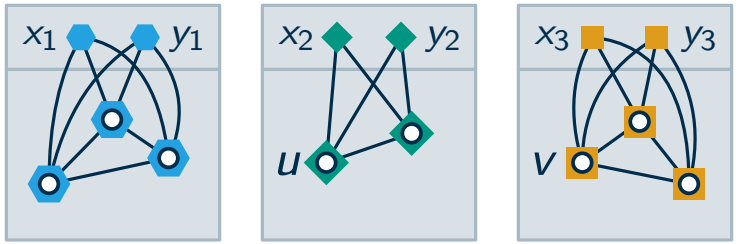

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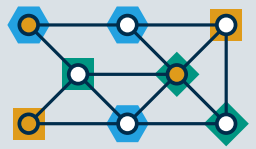

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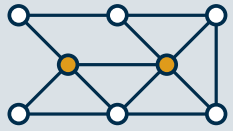

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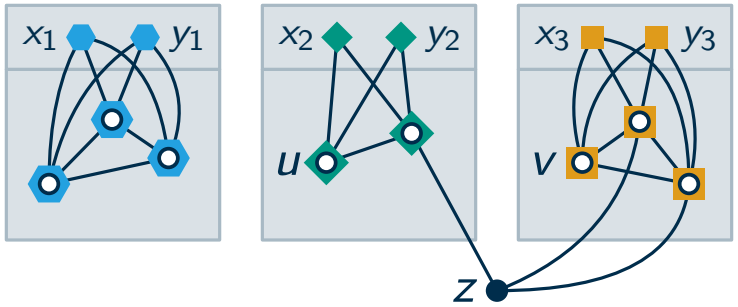

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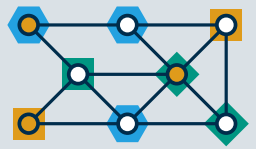

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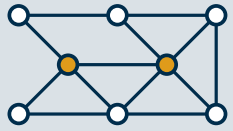

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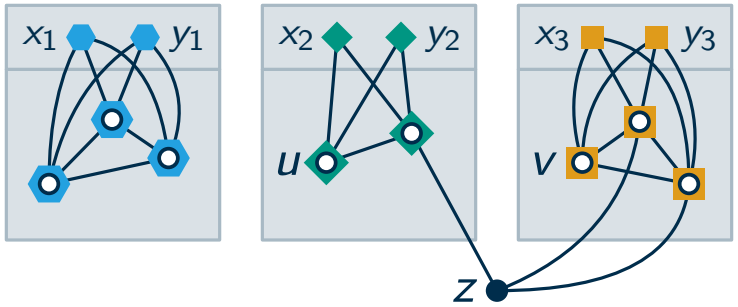

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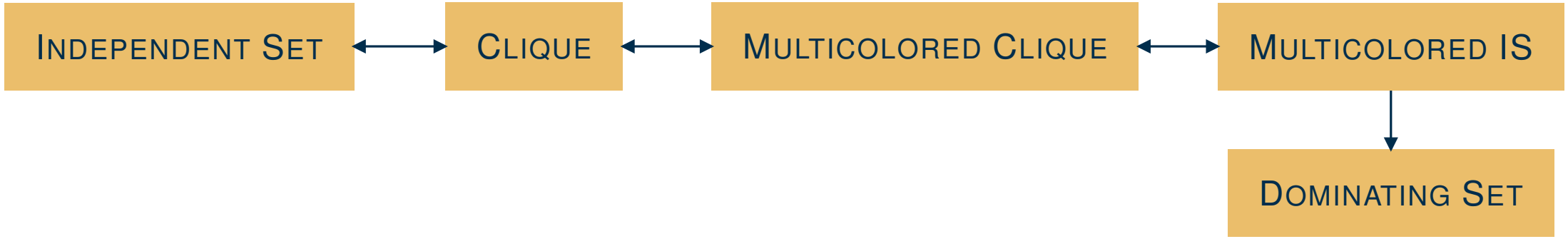

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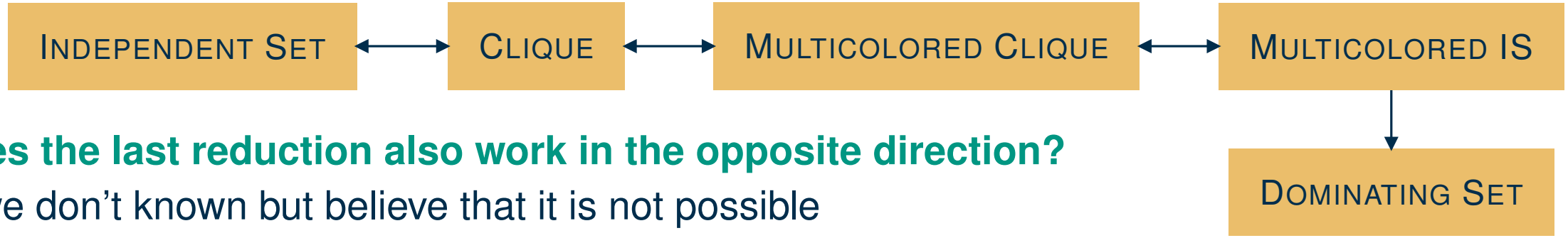

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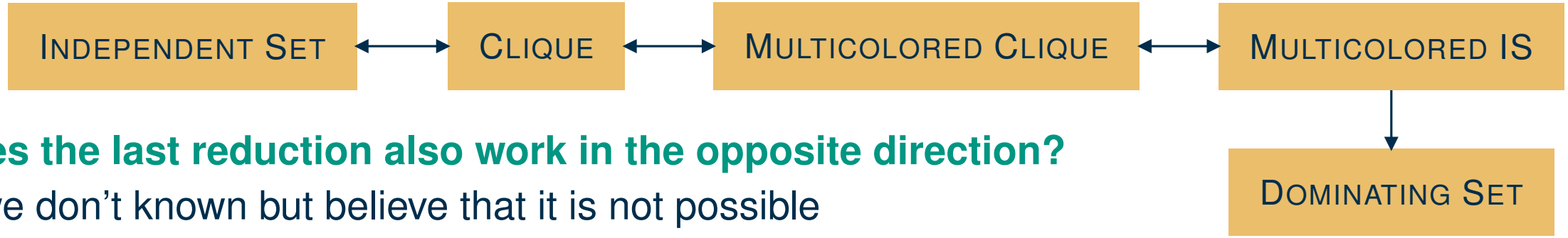

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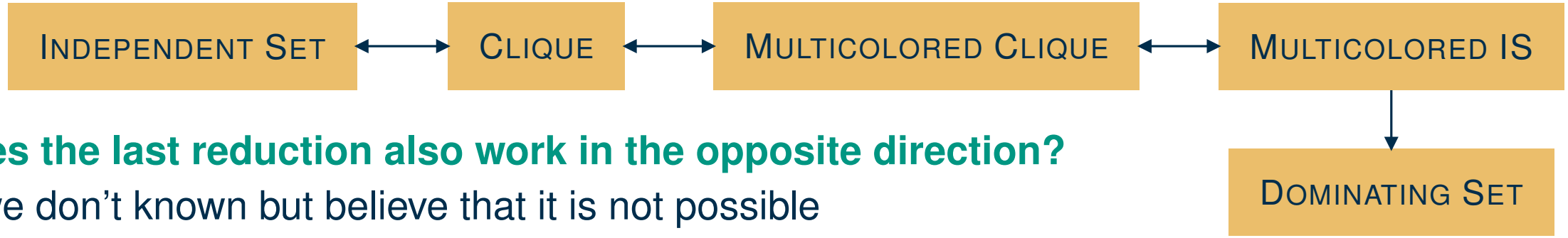

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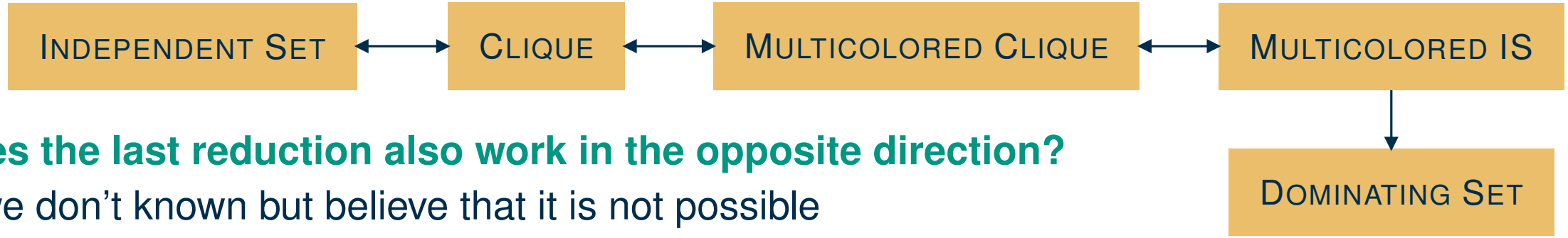

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Comparison with complexity class P

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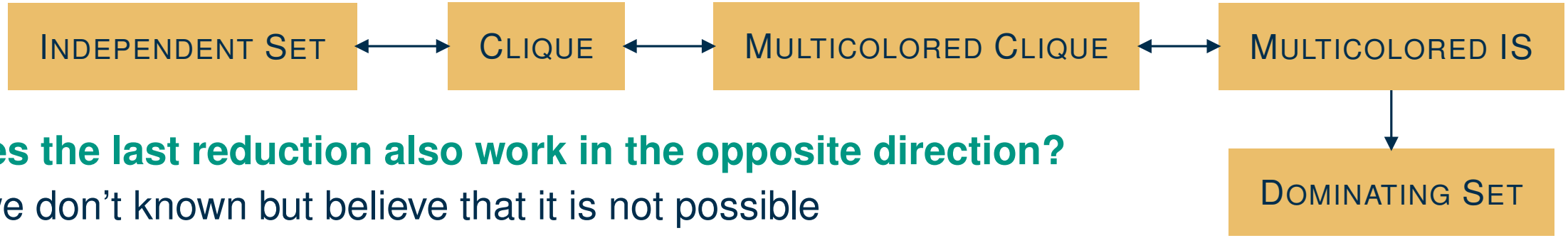

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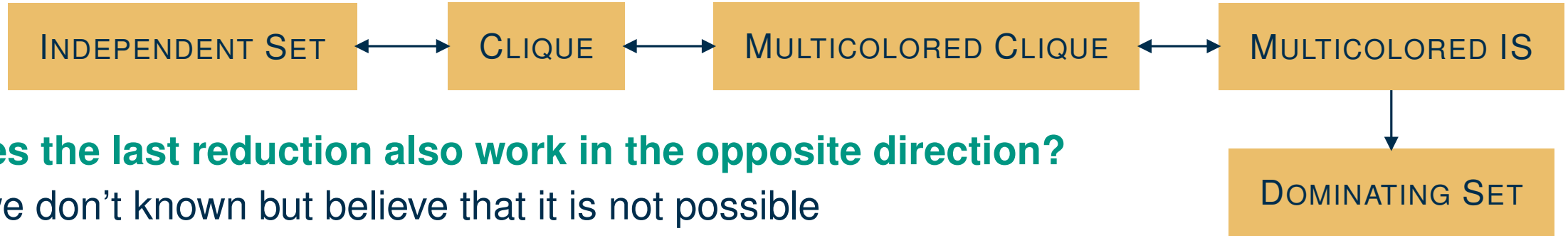

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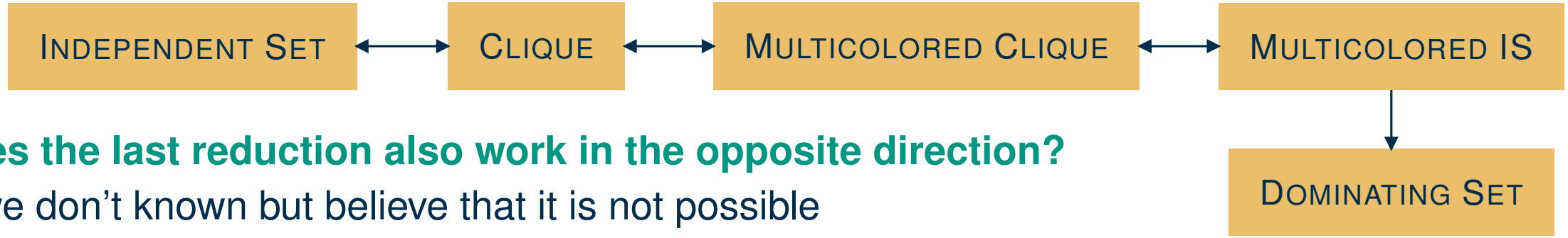
What now?

- define hierarchy of complexity classes

Reductions so far



Does the last reduction also work in the opposite direction?

- we don't known but believe that it is not possible
- DS seems strictly harder than CLIQUE or IS (DS $\in$ FPT $\Rightarrow$ IS $\in$ FPT, but not the other way round)
- we need a more refined notion of hardness

Comparison with complexity class P

- here usually one notion of hardness suffices: NP-hardness
- there are intermediate problems (assuming P $\neq$ NP), but we don't know any natural cases
  (candidates: prime factorization, graph isomorphism)

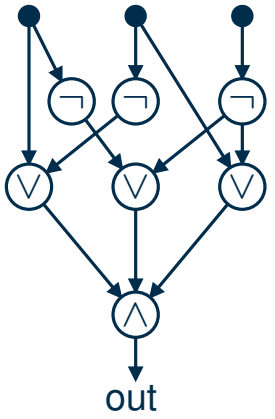
What now?

- define hierarchy of complexity classes
- use prototypical complete problems for each level of the hierarchy

Weighted Circuit Satisfiability

Boolean circuit

- directed acyclic graph (DAG) with the following types of nodes:



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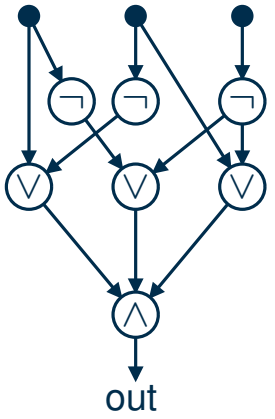

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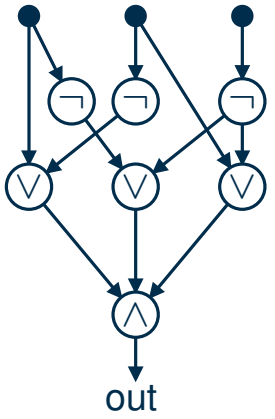

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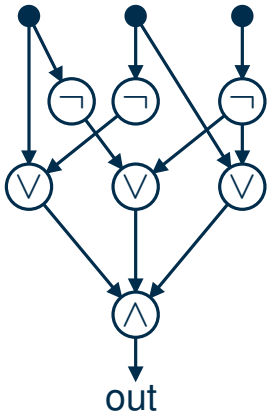

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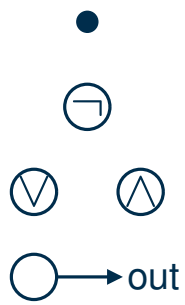

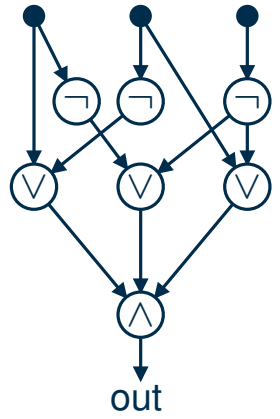

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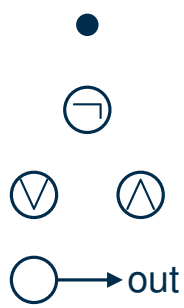

- a assignment of the input nodes to 0 or 1 yields values for all other nodes (in the obvious way)
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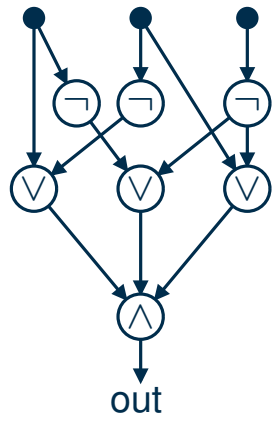

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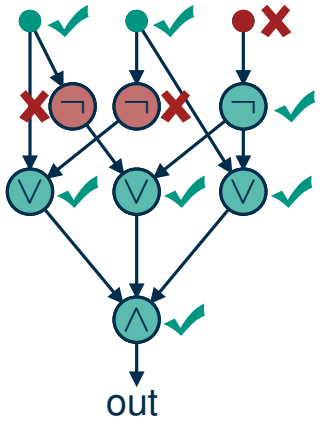

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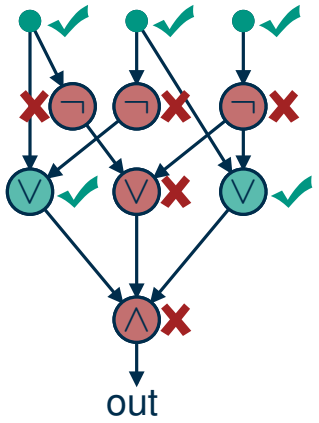

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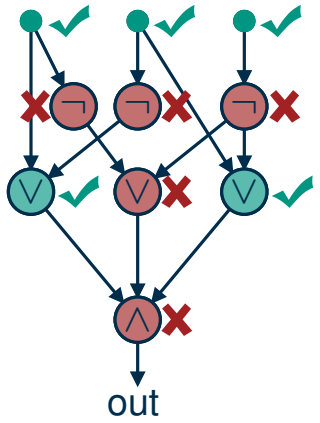

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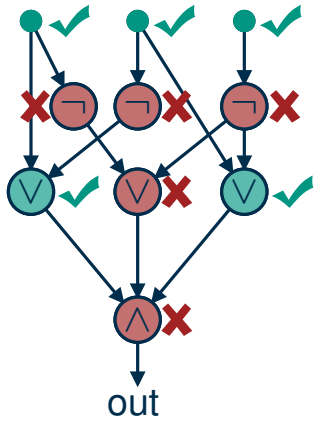


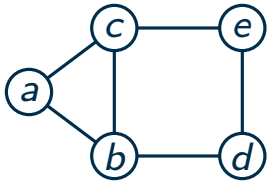
Problem: Weighted Circuit Satisfiability (WCS)
Given a Boolean circuit and a parameter k, is there a satisfying assignment of weight exactly k?

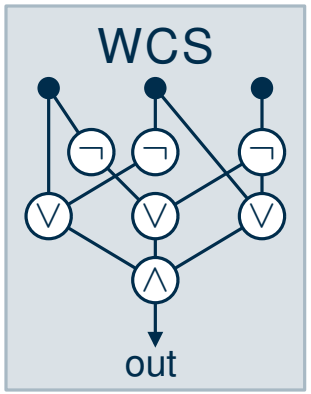
Reductions as far as the eye can see

Independent Set → WCS

Independent Set

$V' \subseteq V$, such that
$\forall e \in E : |e \cap V'| \leq 1$





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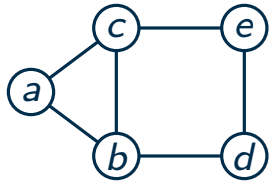

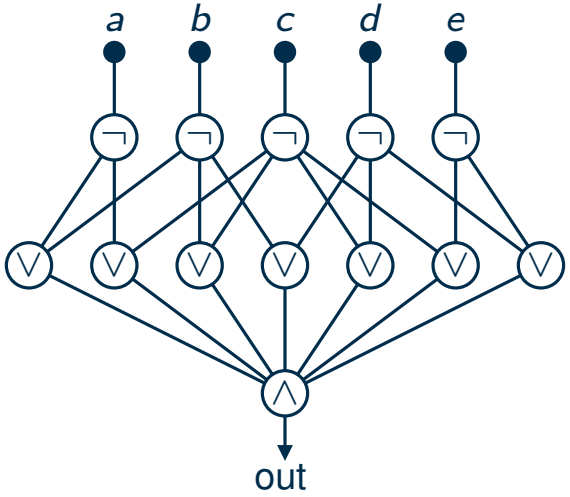

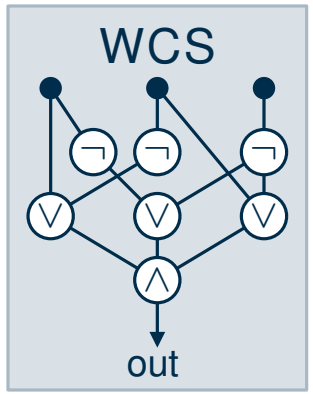

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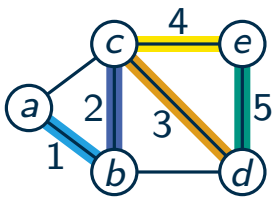

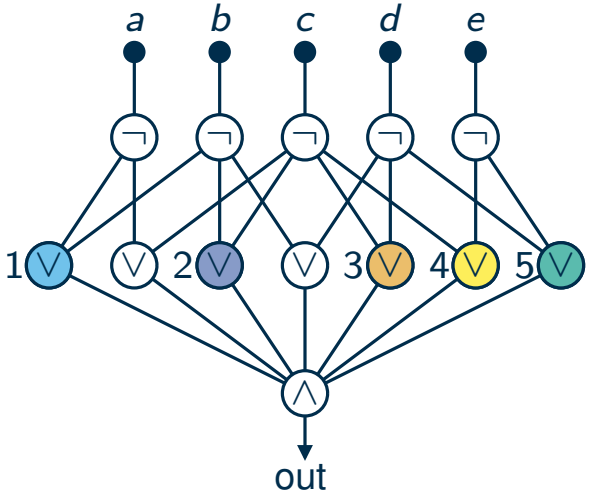

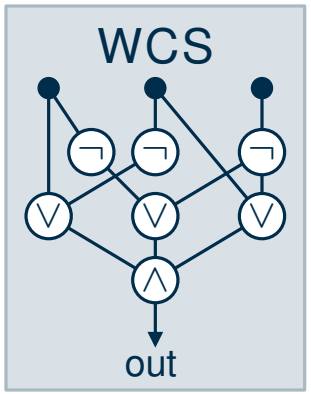

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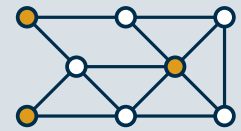

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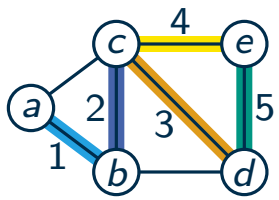

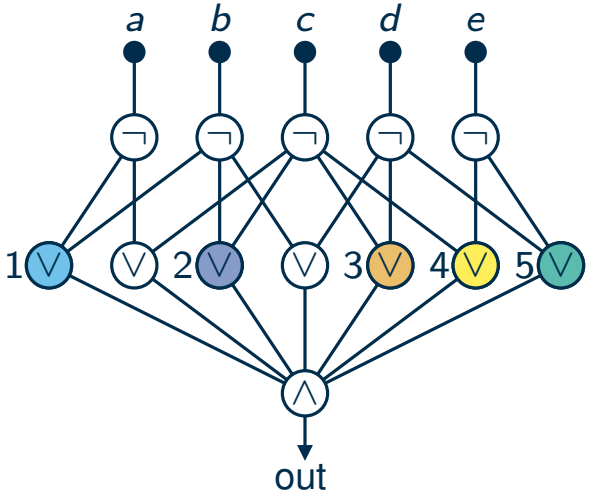

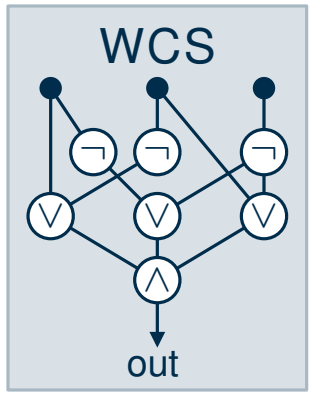

Dominating Set → WCS

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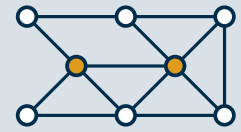

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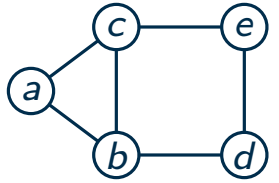

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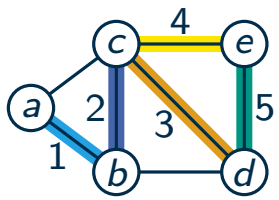

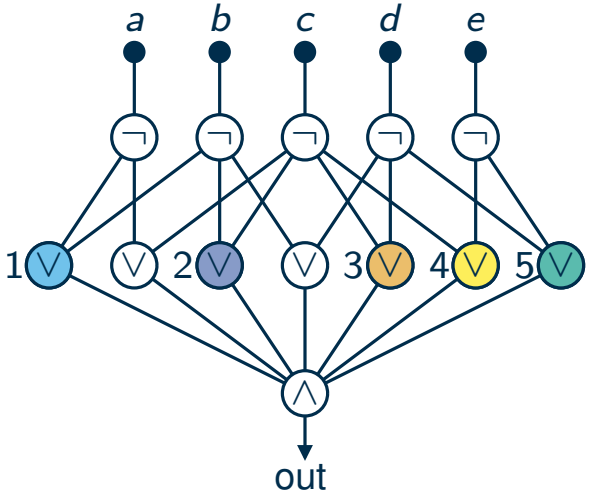

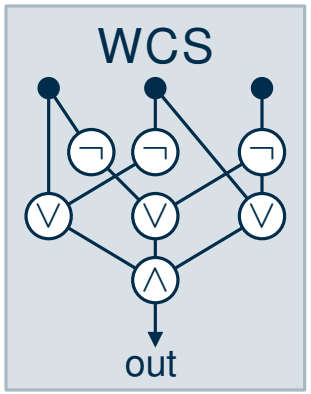

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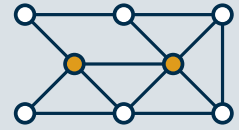

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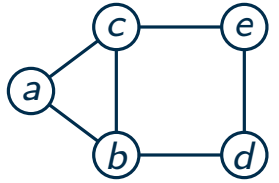

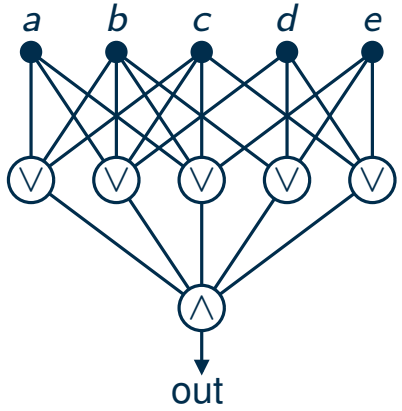

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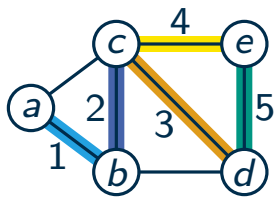

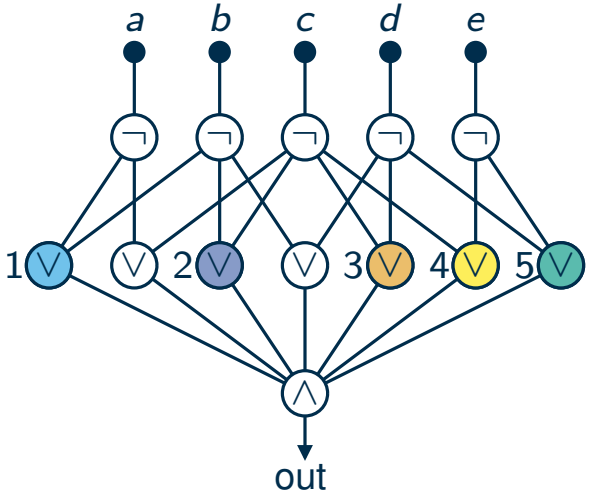

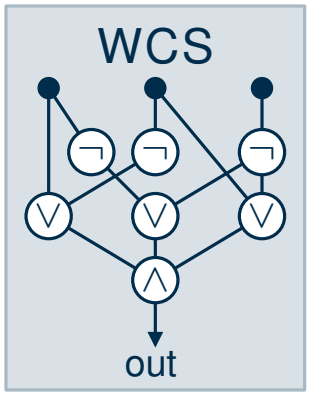

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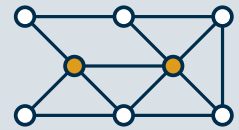

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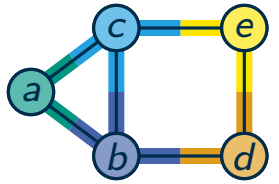

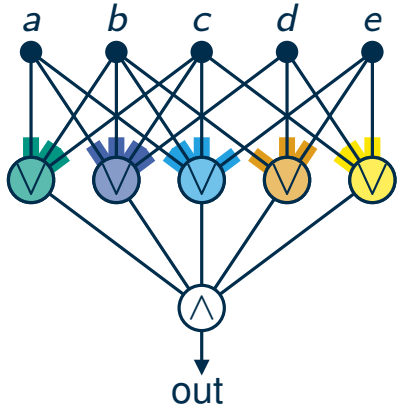

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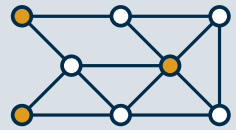

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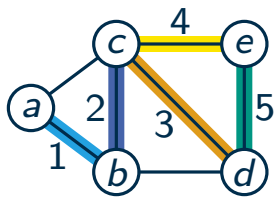

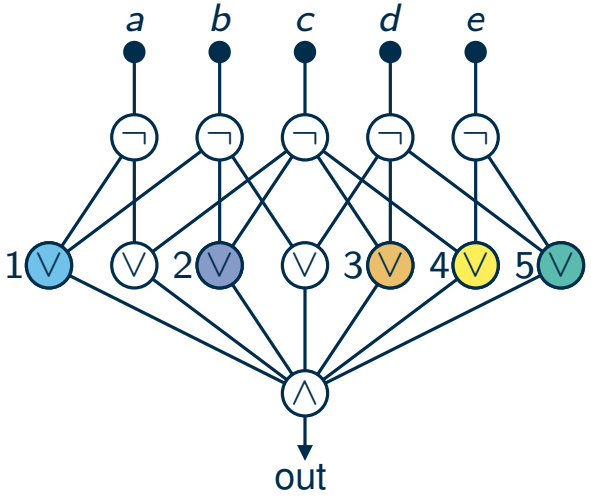

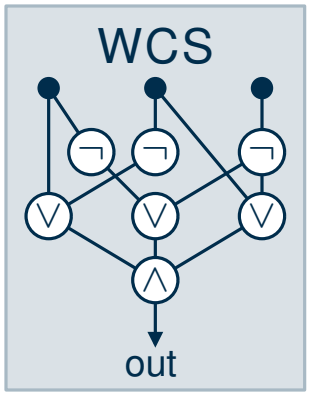

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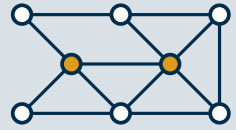

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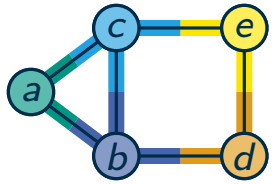

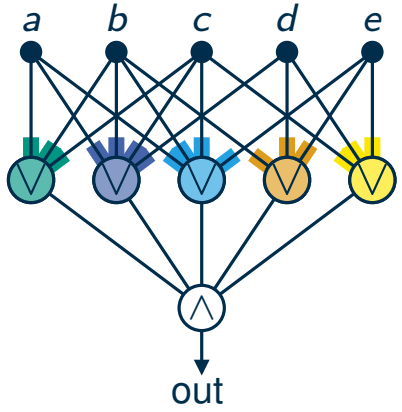

Observations

- both circuits: constant height

Reductions as far as the eye can see

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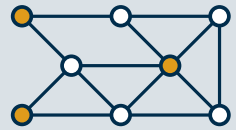

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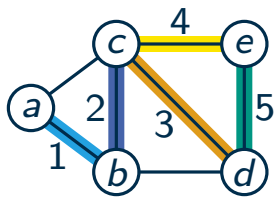
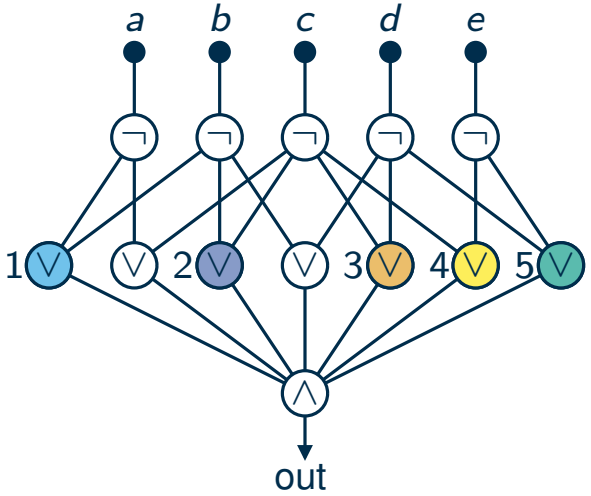
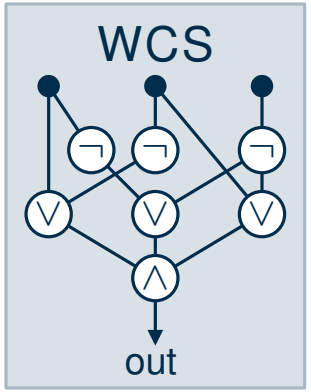
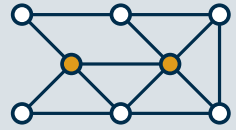






Dominating Set → WCS

Dominating Set



$V' \subseteq V$, such that
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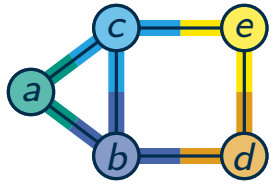

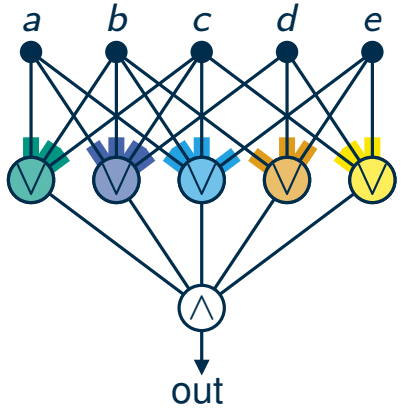

Observations

- both circuits: constant height
- the DS circuit has more nodes with in-degree > 2

Reductions as far as the eye can see

Independent Set → WCS

Independent Set

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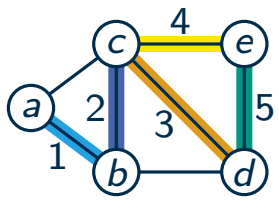
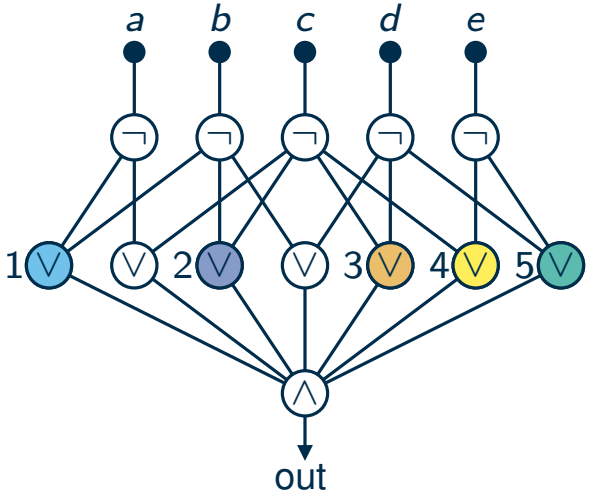
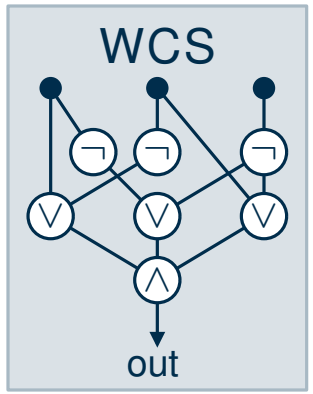
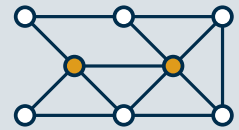






Dominating Set → WCS

Dominating Set



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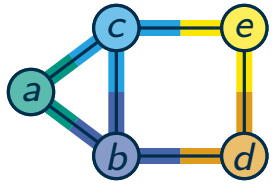
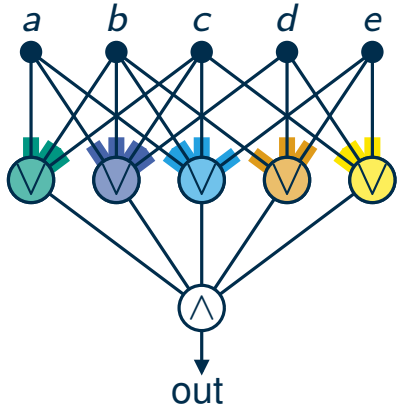


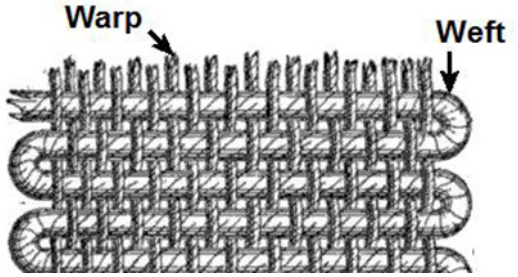


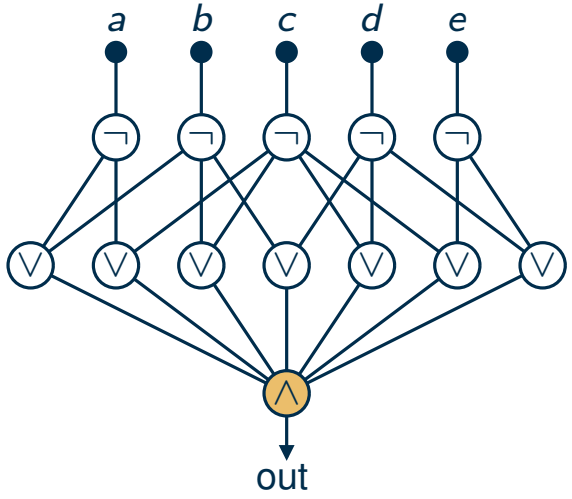
Observations

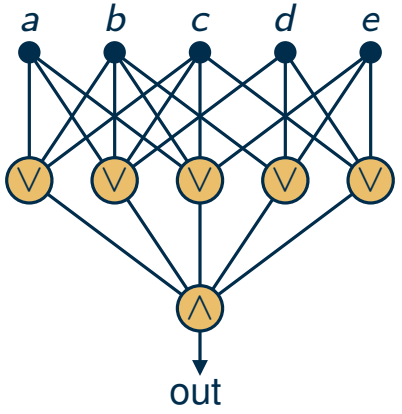
- both circuits: constant height
- the DS circuit has more nodes with in-degree > 2
- Does this make DS harder (with respect to FPT) than IS?

Weft

Definition
The largest number of nodes with in-degree > 2 on a directed path is called **weft** of the circuit.







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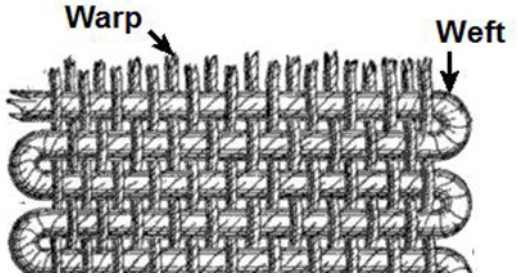

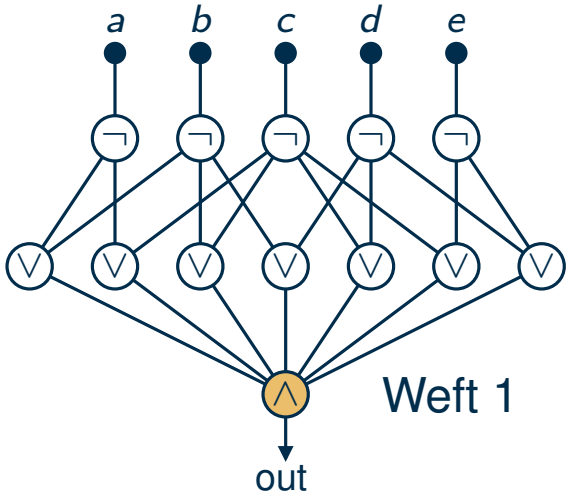

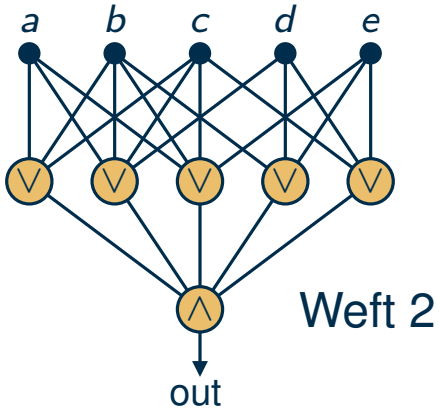

Weft

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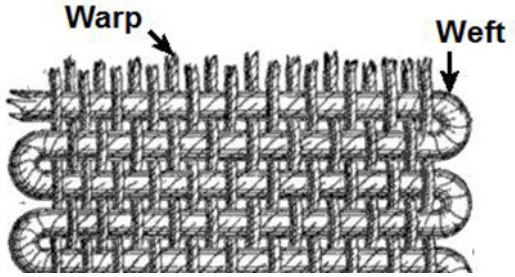
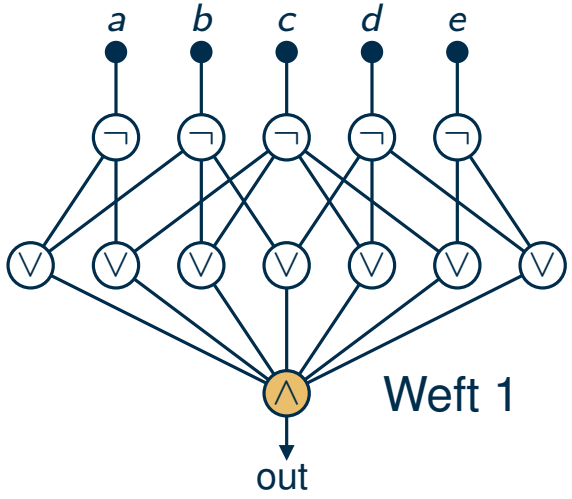
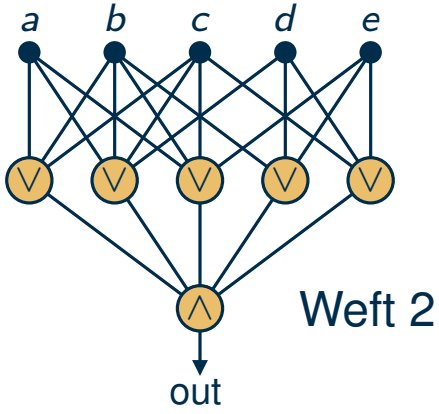
Problem
WCS[t] restricts WCS to circuits of constant height and weft $\leq t$.







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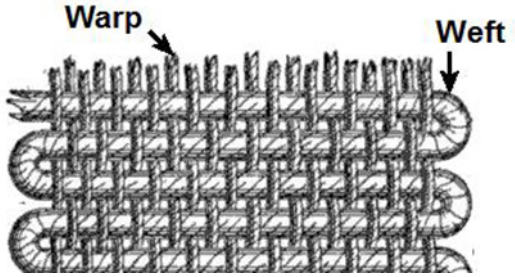
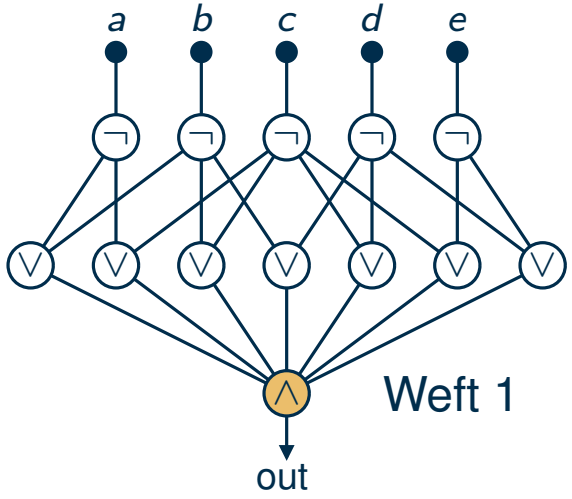
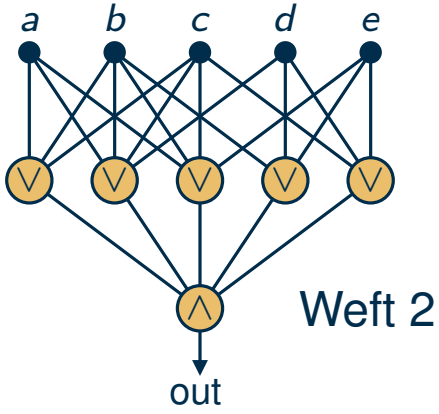


Definition
The class $\boldsymbol{W[t]}$ is the set of problems that have a parameterized reduction to WCS[t].

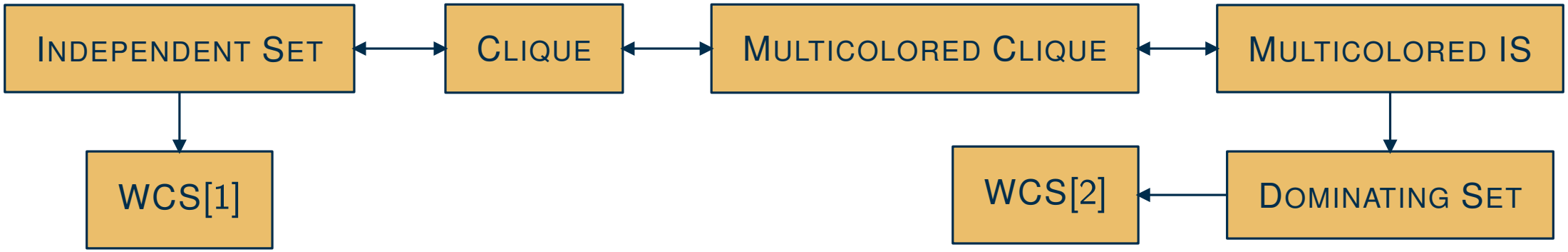




Just seen

- INDEPENDENT SET $\in W[1]$
- DOMINATING SET $\in W[2]$

Bisherige FPT-Reduktionen



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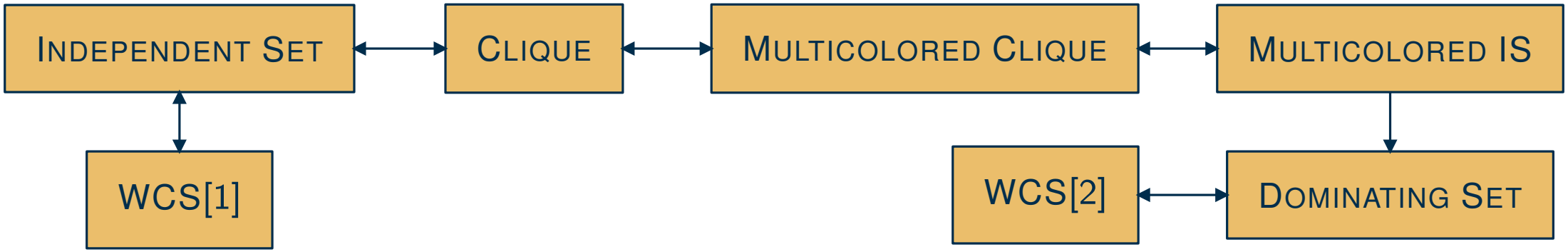

Additional reductions

- WCS[1] $\rightarrow$ IS and WCS[2] to DS

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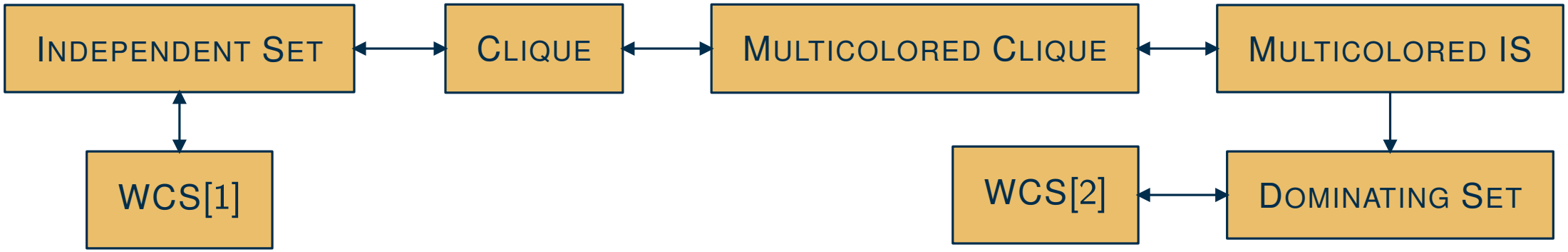

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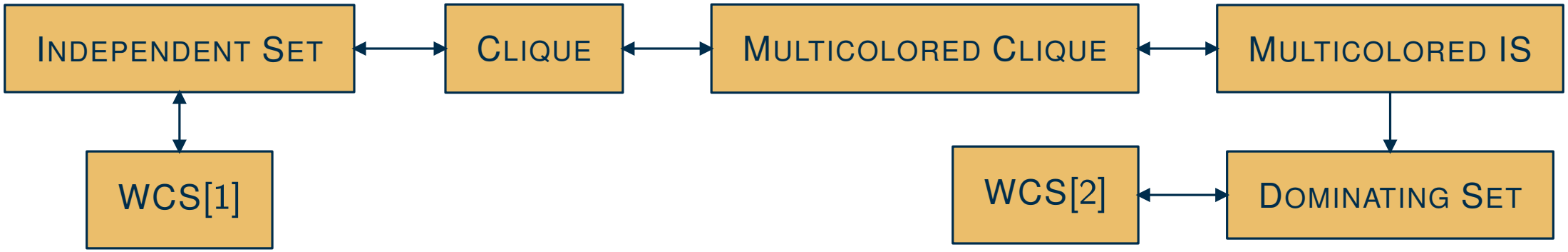

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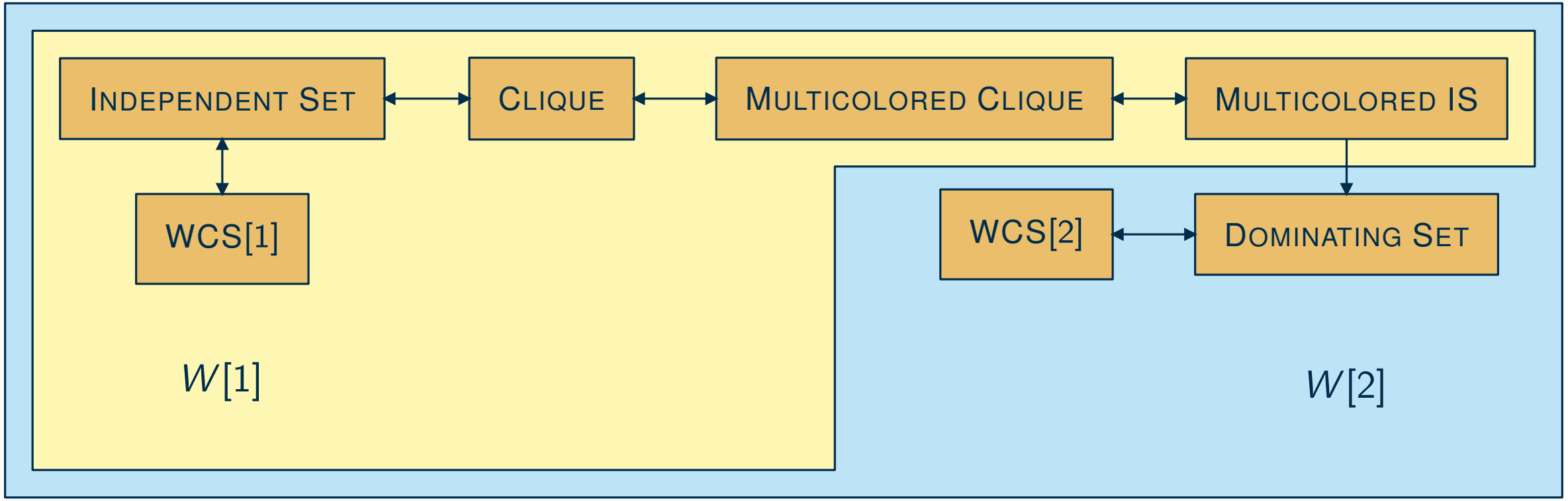

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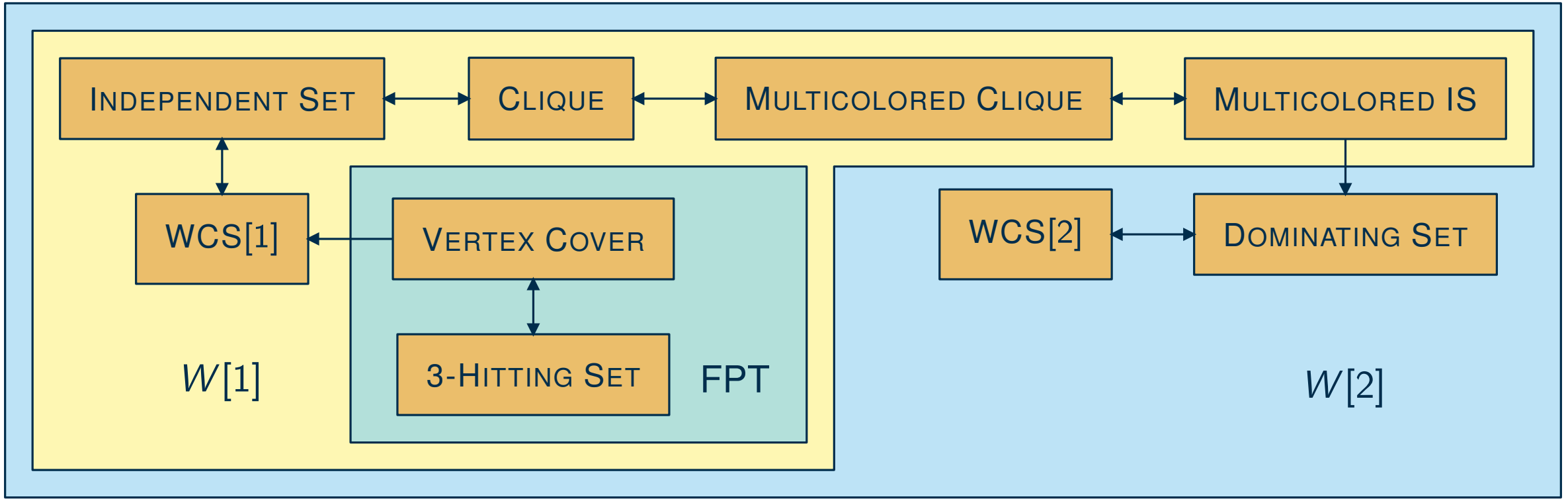
Relations between classes

- $W[1] \subseteq W[2]$

Why?

Bisherige FPT-Reduktionen



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Relations between classes

- $W[1] \subseteq W[2]$
- FPT $\subseteq W[1] \subseteq W[2]$

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Why?

Wrap-Up

The W-hierarchy

- complexity classes FPT $\subseteq W[1] \subseteq W[2] \subseteq W[3] \subseteq \cdots$
- $W[t]$ defined via prototypical complete problem WCS$[t]$: $\mathcal{L} \in W[t] \Leftrightarrow \mathcal{L}$ reducible to WCS$[t]$

(via parameterized reductions)

Wrap-Up

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- still true: you don't find an FPT-algo due to a fundamental problem, not because you are stupid

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- reduce from other hard problems

Wrap-Up

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How do I prove that my problem is hard?

- reduce from other hard problems
- reductions from Independent Set or Clique prove $W[1]$-hardness

Wrap-Up

The W-hierarchy

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How do I prove that my problem is hard?

- reduce from other hard problems
- reductions from Independent Set or Clique prove $W[1]$-hardness
- reductions from Dominating Set or Hitting Set prove $W[2]$-hardness