Parameterized Algorithms

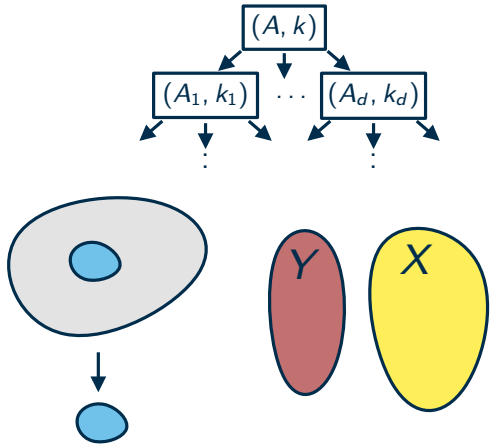
Treewidth & Dynamic Programming

Thomas Bläsius

Content

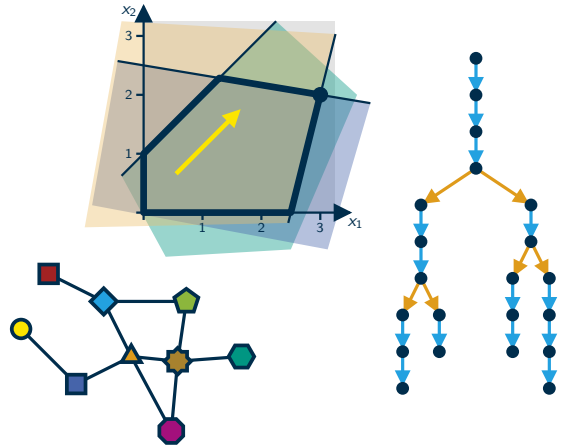
Basic toolbox

- bounded search trees
- kernelization
- iterative compression



Extended toolbox

- linear programs
- branch-and-reduce
- color coding

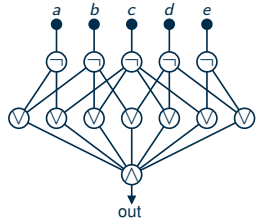


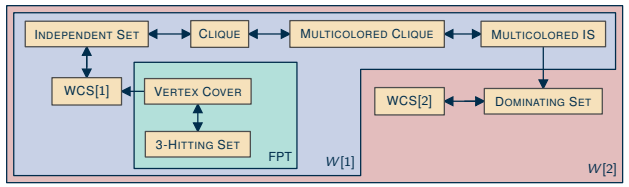
Tree width

- dynamic programming
- chordal and planar graphs
- Courcelle's theorem

Lower bonds

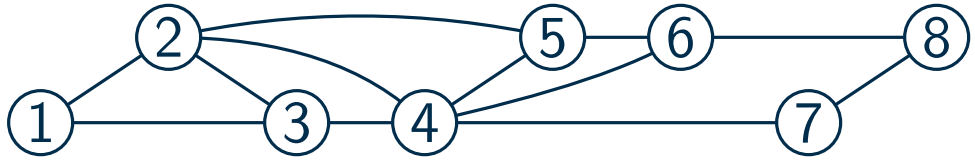
- kernel lower bounds
- parameterized reductions
- circuits and the W-hierarchy
- ETH and SETH





Graphs with order

Graph $G = (V, E)$ with vertex order $V = \{v_1, \ldots, v_n\}$



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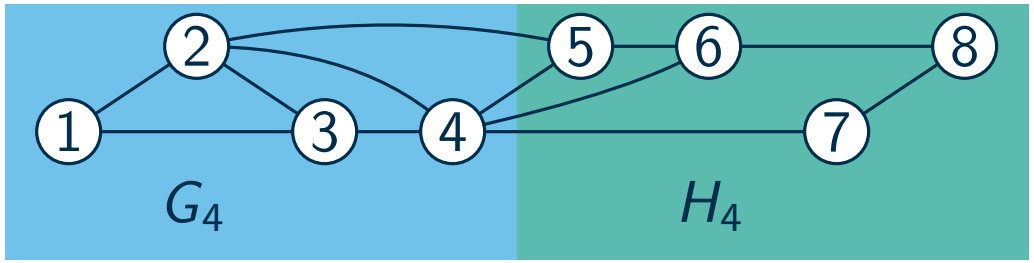

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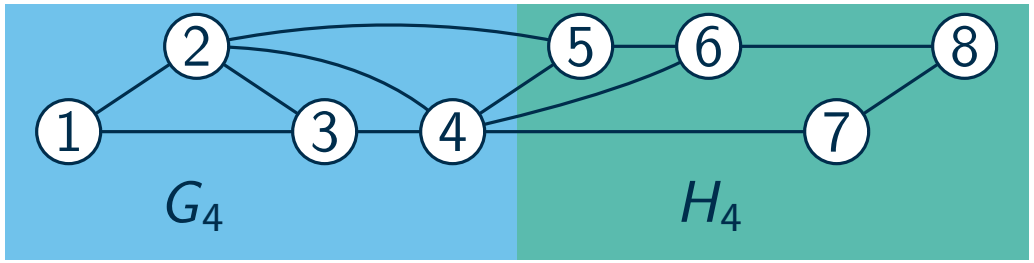

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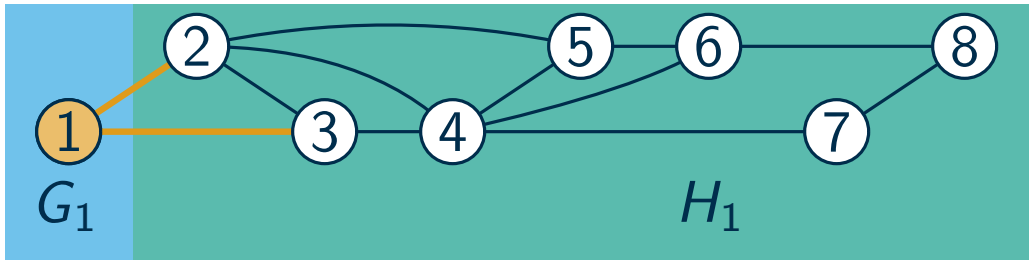
$k_1 = 1$



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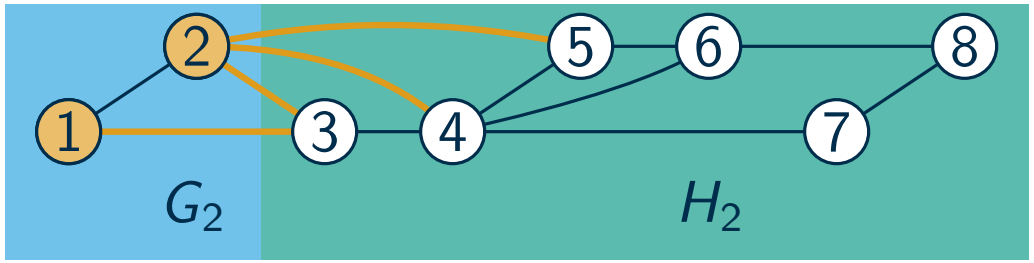
$k_2 = 2$



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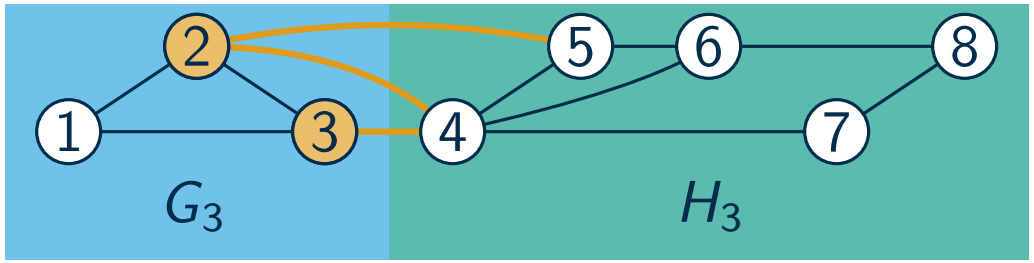
$k_3 = 2$



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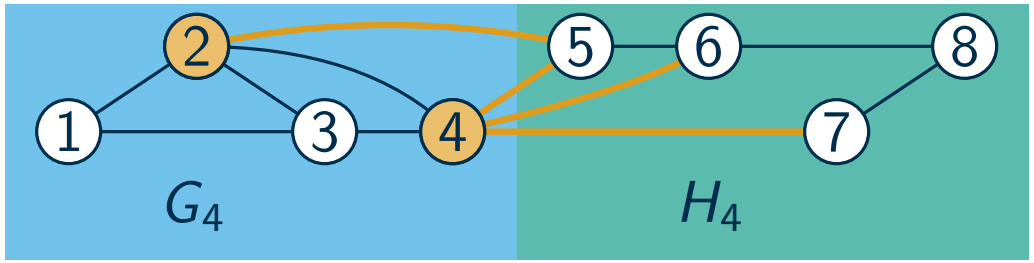
$k_4 = 2$



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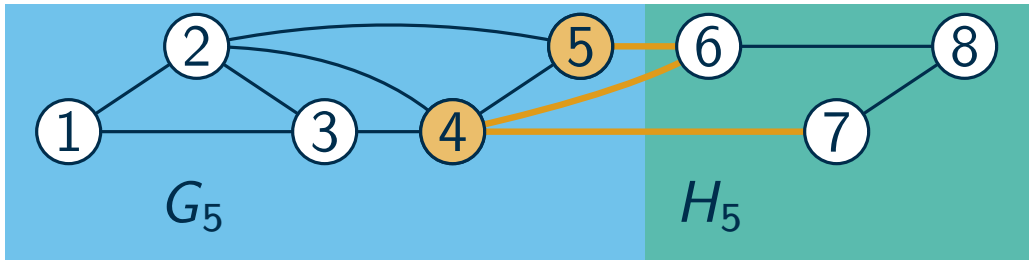
$k_5 = 2$



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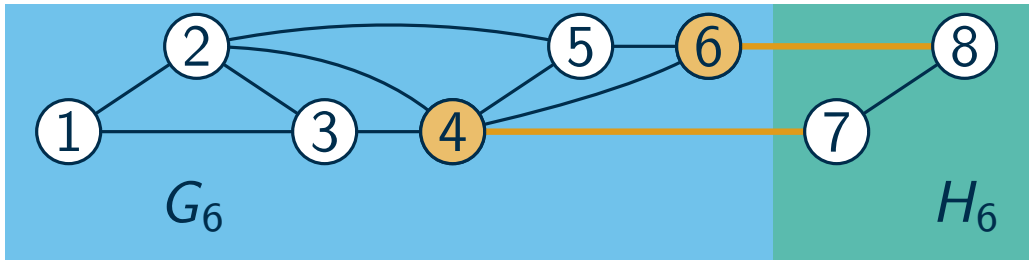
$k_6 = 2$



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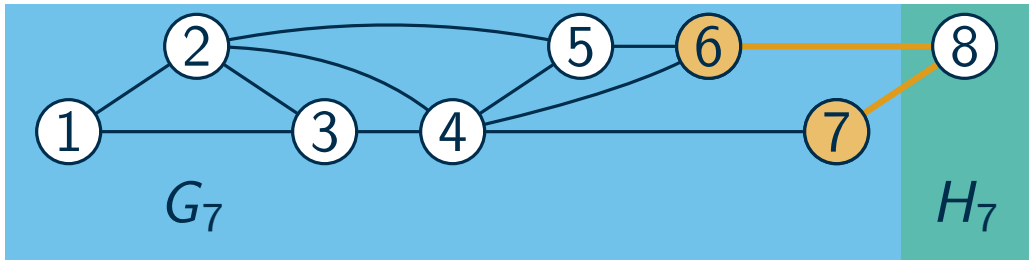
$k_7 = 2$
$\Rightarrow k = 2$



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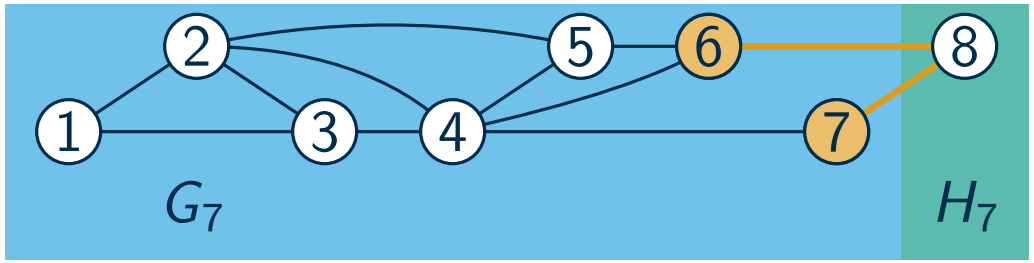
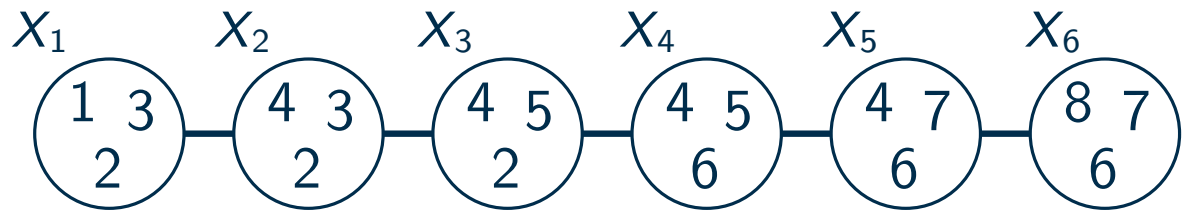


Alternative perspective: Path decomposition

- path $x_1, \ldots, x_r$ with **bags** $X_1, \ldots, X_r$ ($X_i \subseteq V$), such that:



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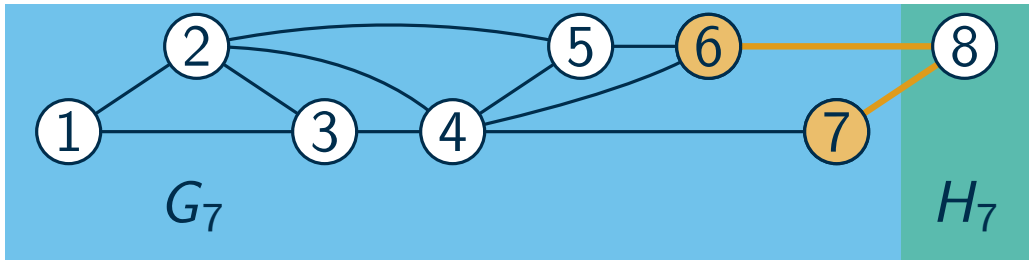
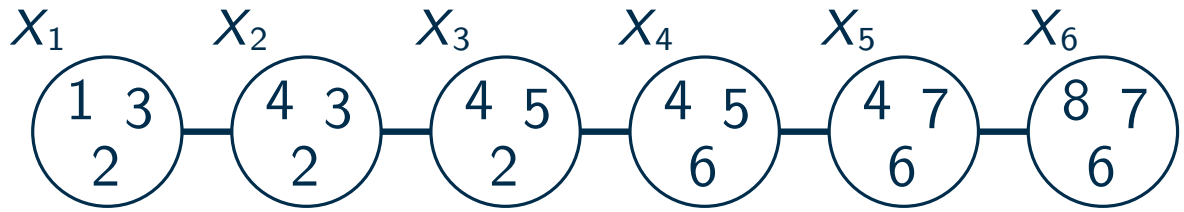


Alternative perspective: Path decomposition

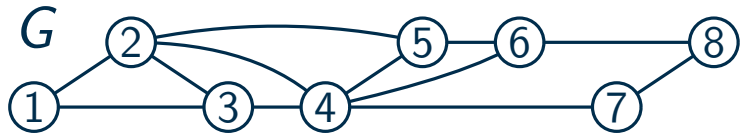
- path $x_1, \ldots, x_r$ with **bags** $X_1, \ldots, X_r$ ($X_i \subseteq V$), such that:
  - $X_1 \cup \cdots \cup X_r = V$
  - $\{u, v\} \in E \Rightarrow u, v$ share a bag
  - the bags of every vertex form a subpath

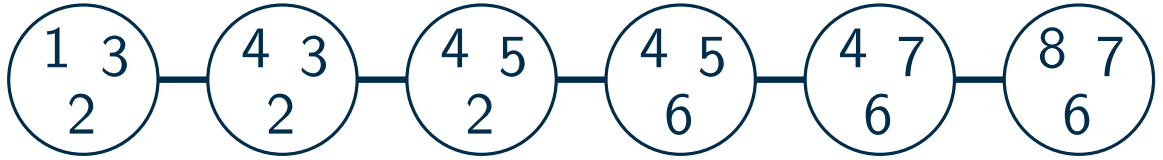


Pathwidth and nice path decompositions

Pathwidth

- width of a decomposition: $\max\{|X_i|\} - 1$
- pathwidth of a graph: minimum width of a decomposition





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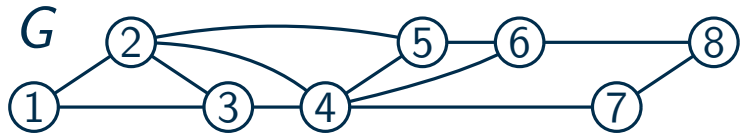

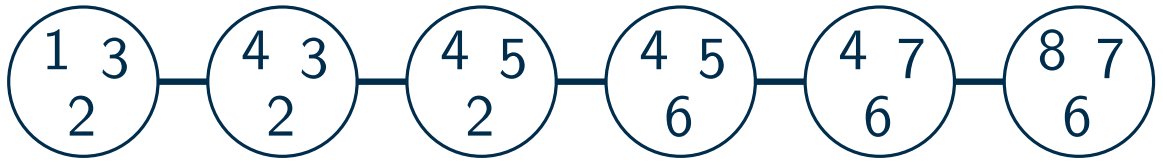

Nice path decompositions

- only two node types: introduce and forget

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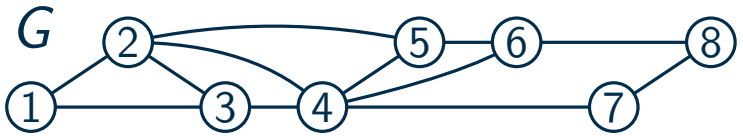

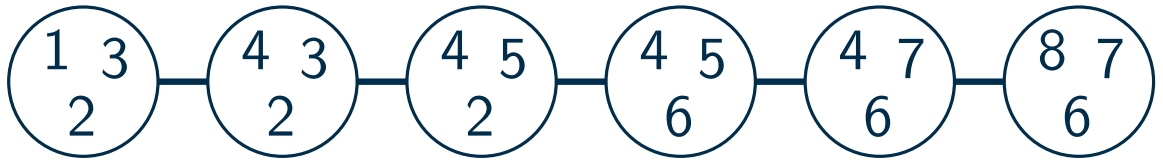

Nice path decompositions

- only two node types: introduce and forget
- x_i is introduce node if $X_i = X_{i-1} \cup \{v\}$
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Pathwidth and nice path decompositions

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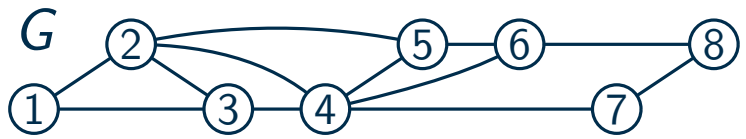

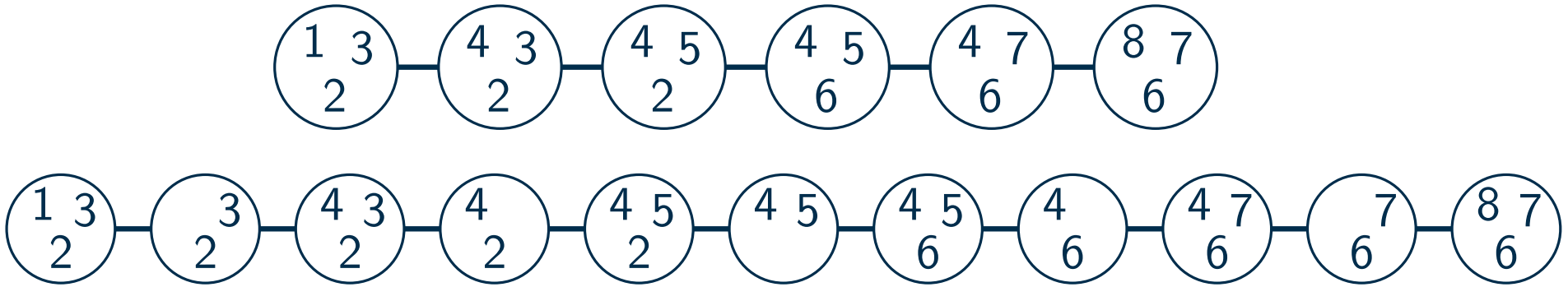

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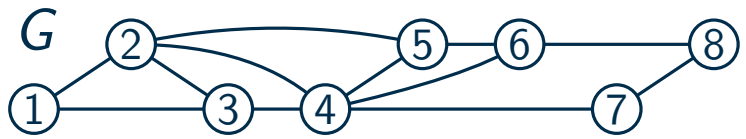

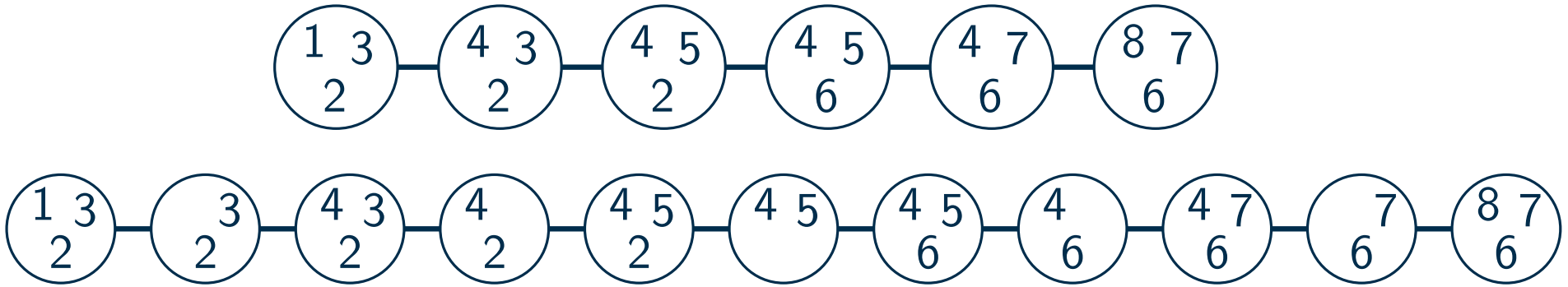

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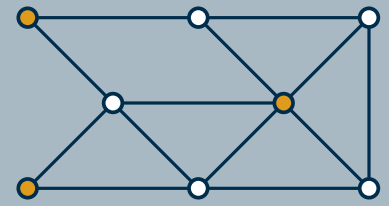
Lemma
If G has a path decomposition of width p, then G has a nice path decomposition of width p.

Dynamic programming

Problem: INDEPENDENT SET
Given a graph, a parameter p, and a path decomposition of width p. Is there an independent set of size k?

(vertices $V' \subseteq V$ with $\{u, v\} \notin E$ for $u, v \in V'$)



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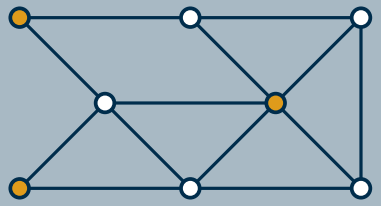

Dynamic programming over nice path decomposition

- as before: $V_i = X_1 \cup \cdots \cup X_i$ and $G_i = G[V_i]$
- step i: $\forall\, X_i' \subseteq X_i$, compute max IS U_i in G_i with $U_i \cap X_i = X_i'$

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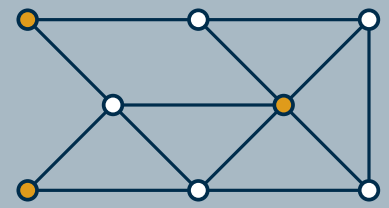

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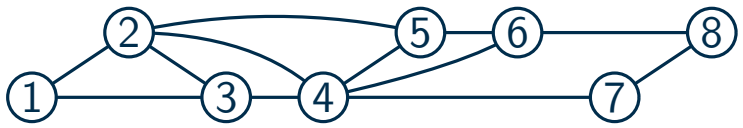

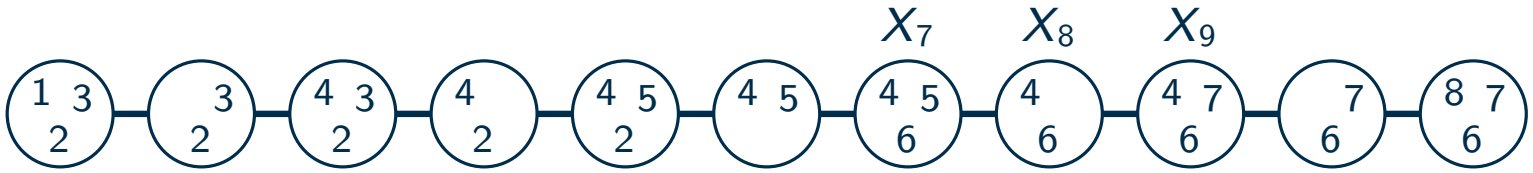

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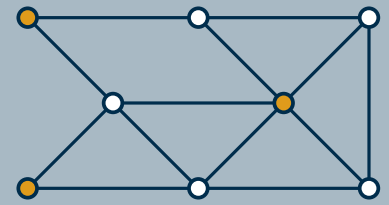

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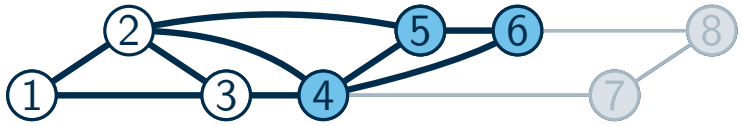
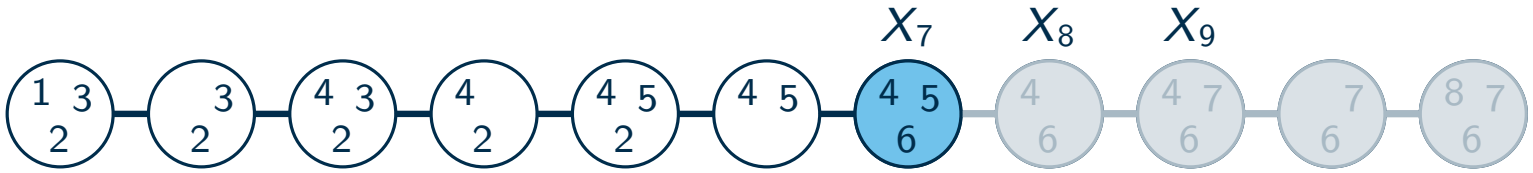
- step i: $\forall\, X'_i \subseteq X_i$, compute max IS U_i in G_i with $U_i \cap X_i = X'_i$

subset $X'_7 \subseteq X_7$	$\emptyset$	$\{4\}$	$\{5\}$	$\{6\}$	$\{4,5\}$	$\{4,6\}$	$\{5,6\}$	$\{4,5,6\}$
largest IS size in G_7	1	2	2	2	$-\infty$	$-\infty$	$-\infty$	$-\infty$





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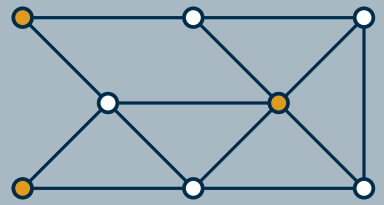

Dynamic programming over nice path decomposition

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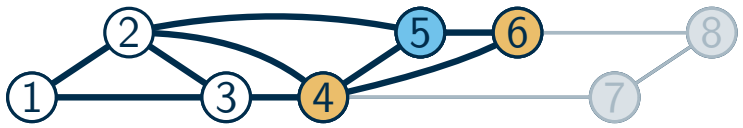
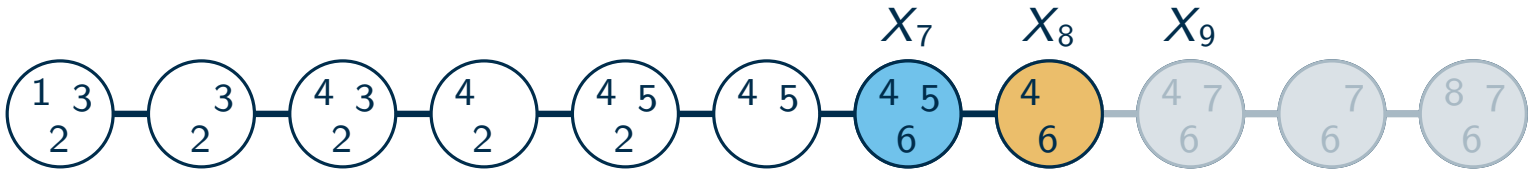
- step i: $\forall\, X'_i \subseteq X_i$, compute max IS U_i in G_i with $U_i \cap X_i = X'_i$

subset $X'_7 \subseteq X_7$	$\emptyset$	$\{4\}$	$\{5\}$	$\{6\}$	$\{4,5\}$	$\{4,6\}$	$\{5,6\}$	$\{4,5,6\}$
largest IS size in G_7	1	2	2	2	$-\infty$	$-\infty$	$-\infty$	$-\infty$
subset $X'_8 \subseteq X_8$	$\emptyset$	$\{4\}$		$\{6\}$		$\{4,6\}$		
largest IS size in G_8								





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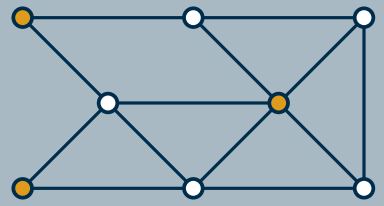

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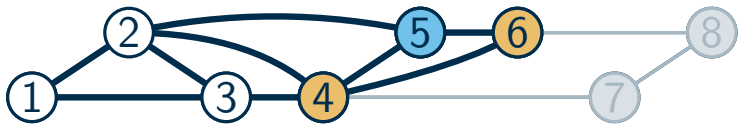
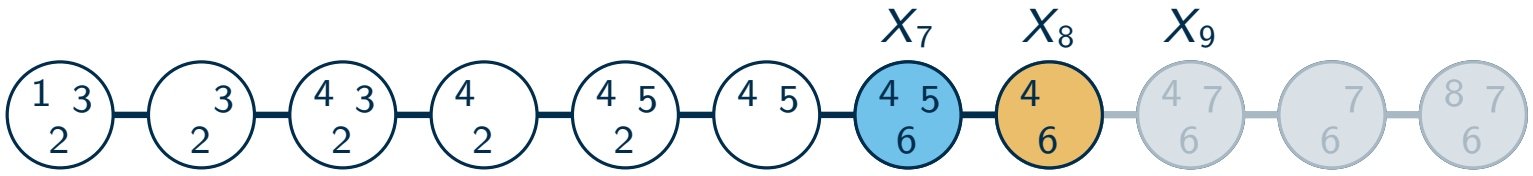
subset $X'_7 \subseteq X_7$	$\emptyset$	$\{4\}$	$\{5\}$	$\{6\}$	$\{4,5\}$	$\{4,6\}$	$\{5,6\}$	$\{4,5,6\}$
largest IS size in G_7	1	2	2	2	$-\infty$	$-\infty$	$-\infty$	$-\infty$
subset $X'_8 \subseteq X_8$	$\emptyset$	$\{4\}$		$\{6\}$		$\{4,6\}$		
largest IS size in G_8	2							





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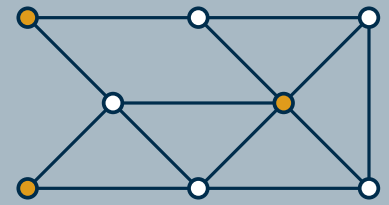

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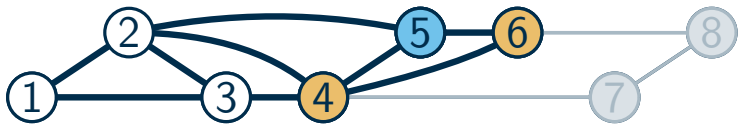

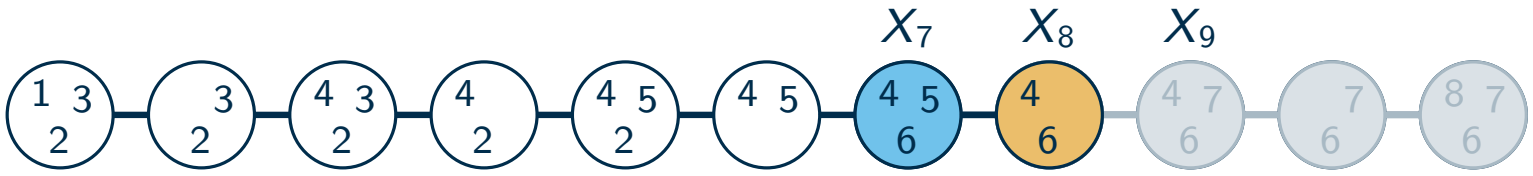

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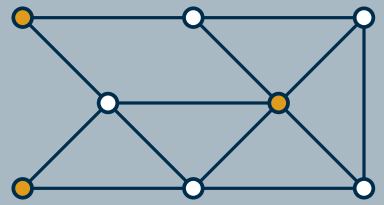

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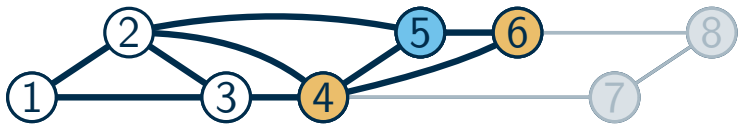

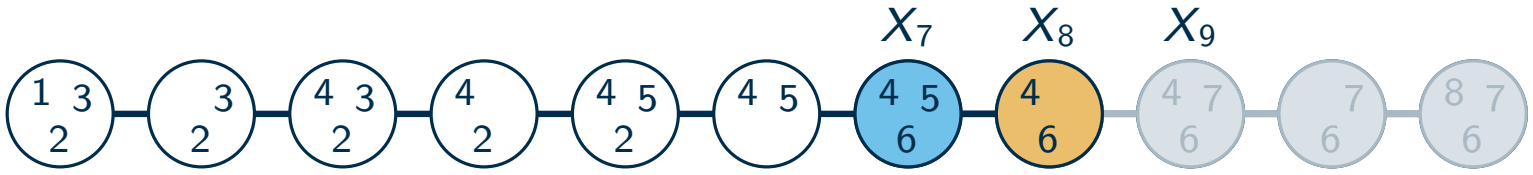

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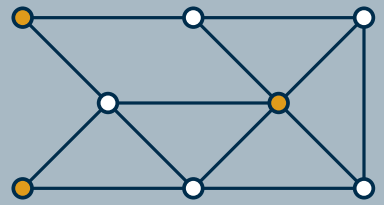

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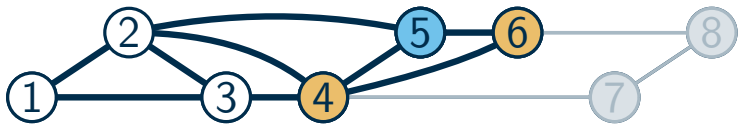
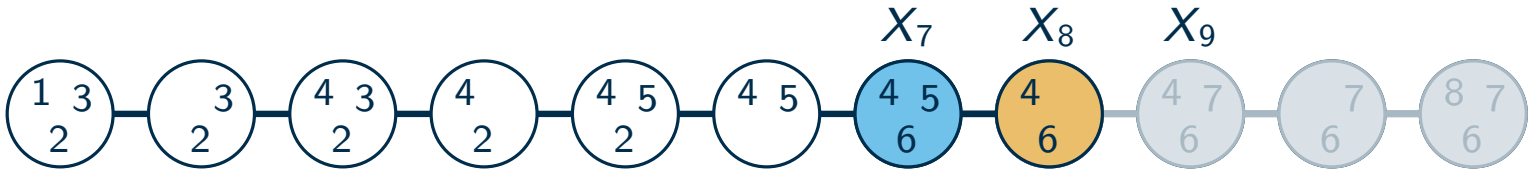
subset $X'_8 \subseteq X_8$	$\emptyset$	$\{4\}$		$\{6\}$		$\{4,6\}$		
largest IS size in G_8	2	2		2		$-\infty$		





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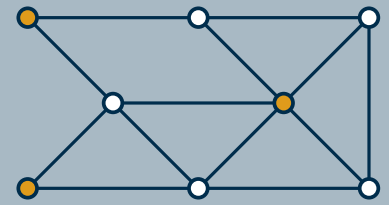

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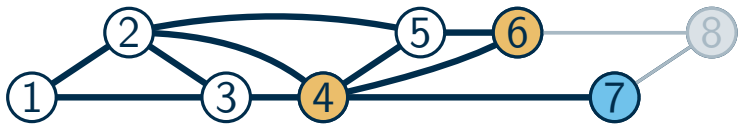
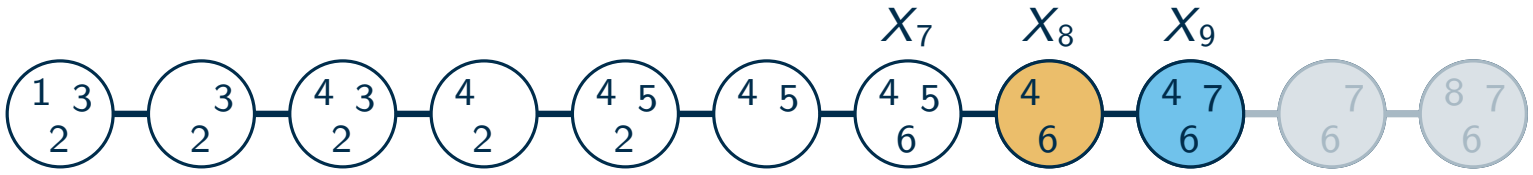
subset $X'_8 \subseteq X_8$	$\emptyset$	$\{4\}$	$\{6\}$	$\{4,6\}$
largest IS size in G_8	2	2	2	$-\infty$

subset $X'_9 \subseteq X_9$	$\emptyset$	$\{7\}$	$\{4\}$	$\{4,7\}$	$\{6\}$	$\{6,7\}$	$\{4,6\}$	$\{4,6,7\}$
largest IS size in G_9								





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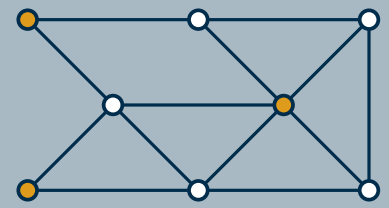

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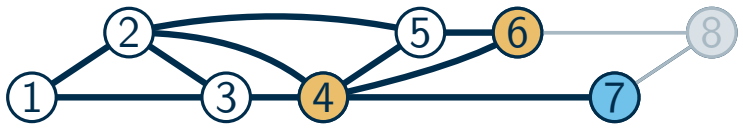
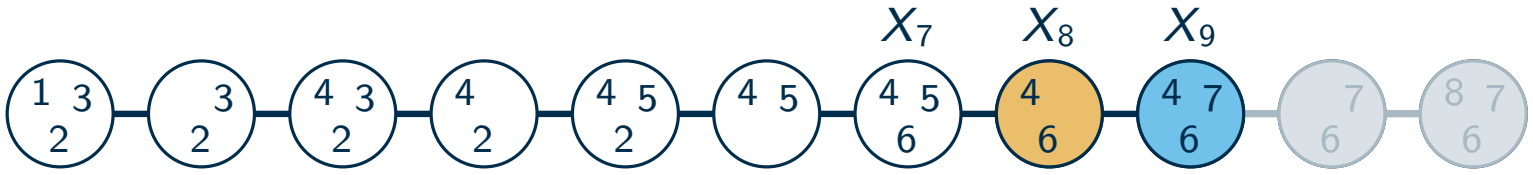
subset $X'_8 \subseteq X_8$	$\emptyset$	$\{4\}$	$\{6\}$	$\{4,6\}$
largest IS size in G_8	2	2	2	$-\infty$

subset $X'_9 \subseteq X_9$	$\emptyset$	$\{7\}$	$\{4\}$	$\{4,7\}$	$\{6\}$	$\{6,7\}$	$\{4,6\}$	$\{4,6,7\}$
largest IS size in G_9	2	3						





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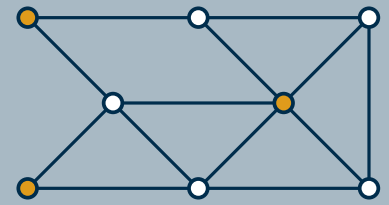

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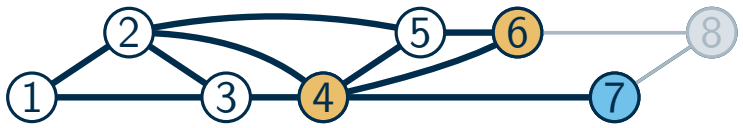
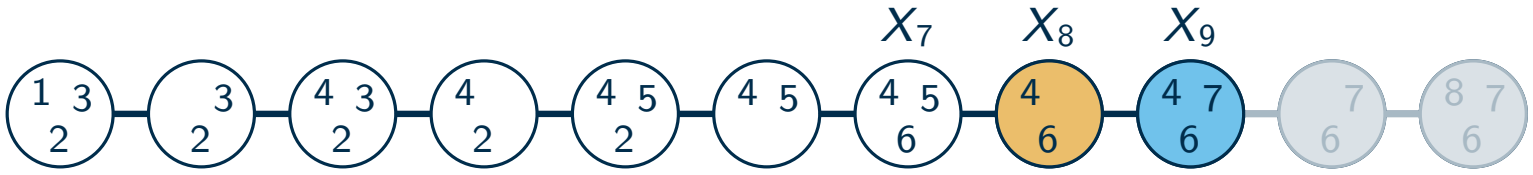
largest IS size in G_8	2	2	2	$-\infty$				
subset $X'_9 \subseteq X_9$	$\emptyset$	$\{7\}$	$\{4\}$	$\{4,7\}$	$\{6\}$	$\{6,7\}$	$\{4,6\}$	$\{4,6,7\}$
largest IS size in G_9	2	3	2	$-\infty$				





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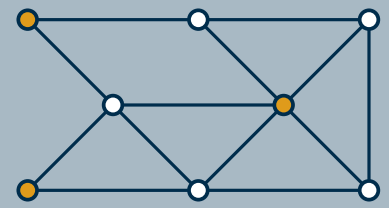

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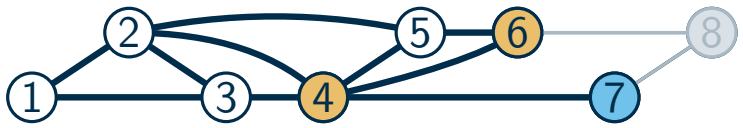

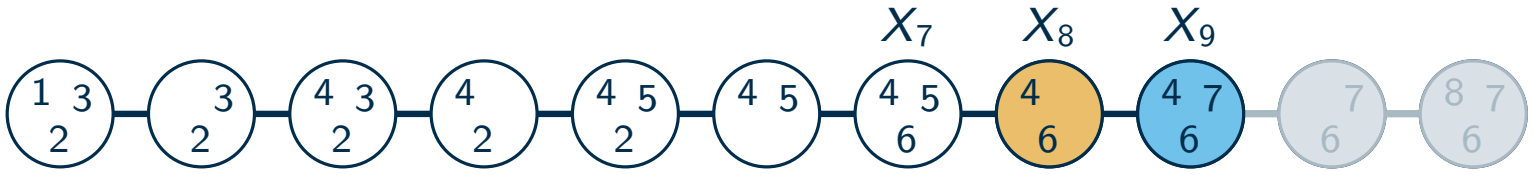

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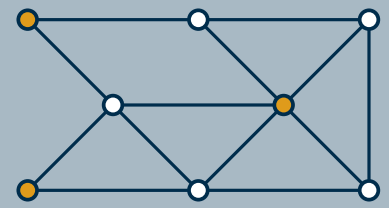

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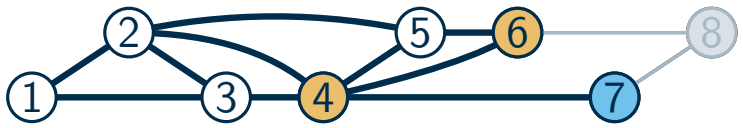

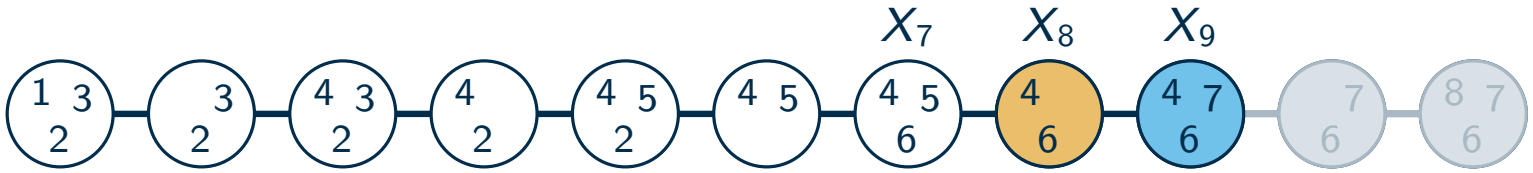

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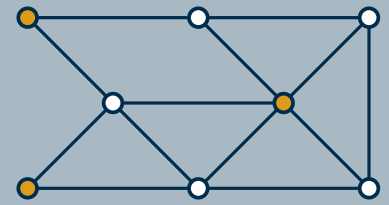

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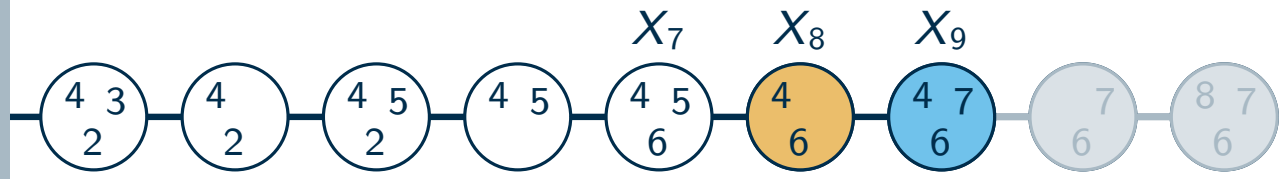
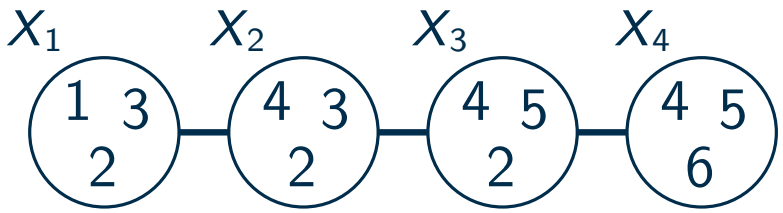
Theorem
INDEPENDENT SET is FPT with respect to the pathwidth p (the DP runs in $O(2^p n^c)$).



Treewidth

Path decomposition

- path on nodes $x_1, \ldots, x_r$ with bags $X_1, \ldots, X_r$, such that:
  - $X_1 \cup \cdots \cup X_r = V$
  - $\{u, v\} \in E \Rightarrow u, v$ share a bag
  - the bags of every vertex form a subpath



Treewidth

Tree
~~Path~~ decomposition

- ~~path~~ tree on nodes $x_1, \ldots, x_r$ with bags $X_1, \ldots, X_r$, such that:
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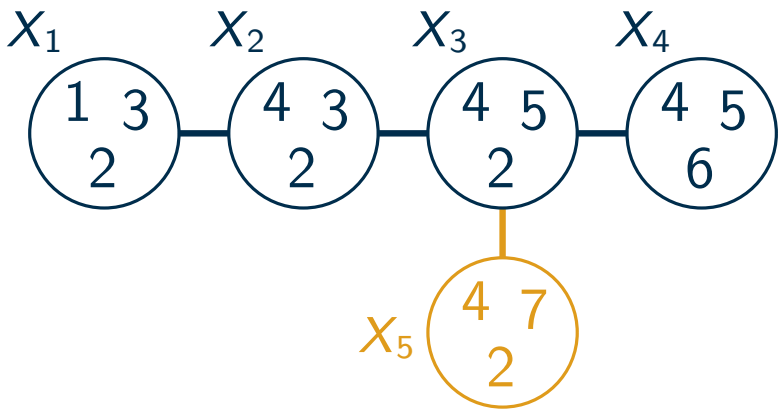
  - the bags of every vertex form a ~~sub~~tree



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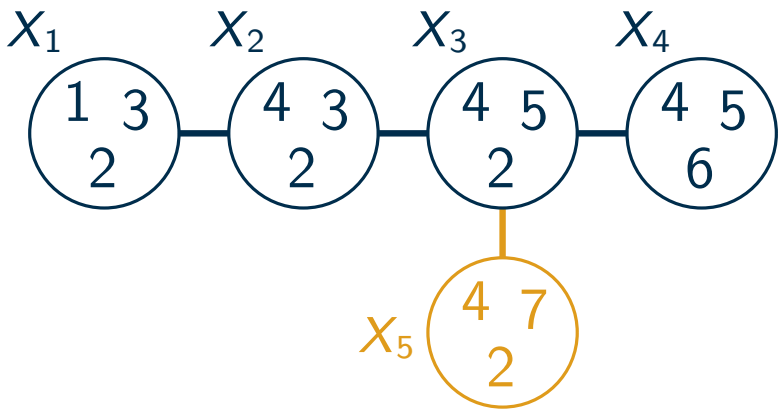

Treewidth

- width of a decomposition:
  $\max\{|X_i|\} - 1$
- treewidth of a graph:
  minimum width of a decomposition

Treewidth

Tree
~~Path~~ decomposition

- ~~path~~ tree on nodes $x_1, \ldots, x_r$ with bags $X_1, \ldots, X_r$, such that:
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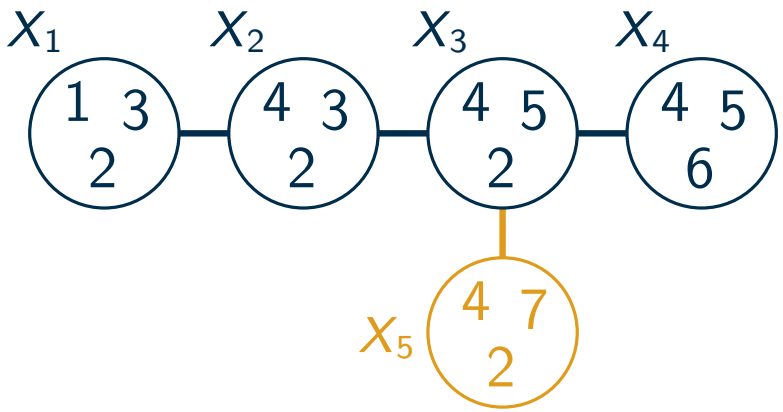
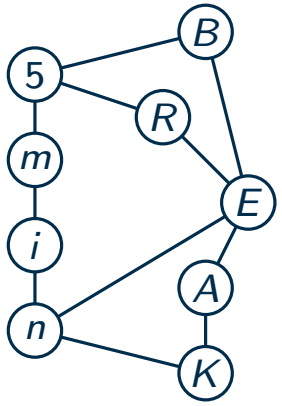


Treewidth

- width of a decomposition: $\max\{|X_i|\} - 1$
- treewidth of a graph: minimum width of a decomposition

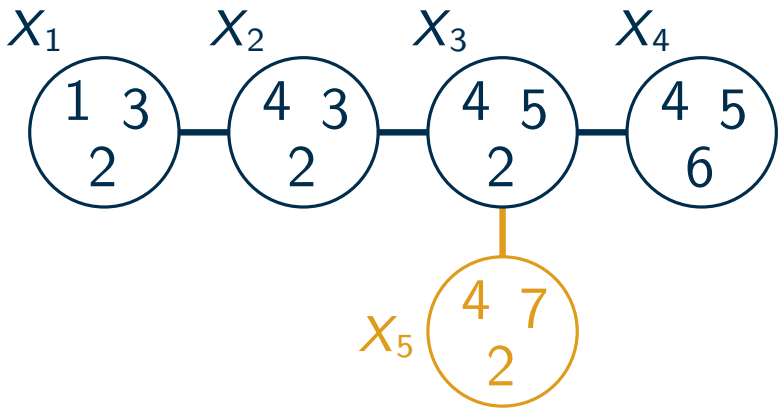
What is the treewidth of this graph?



Treewidth

Tree
~~Path~~ decomposition

- ~~path~~ tree on nodes $x_1, \ldots, x_r$ with bags $X_1, \ldots, X_r$, such that:
  - $X_1 \cup \cdots \cup X_r = V$
  - $\{u, v\} \in E \Rightarrow u, v$ share a bag
  - the bags of every vertex form a ~~subpath~~ tree



Treewidth

- width of a decomposition: $\max\{|X_i|\} - 1$
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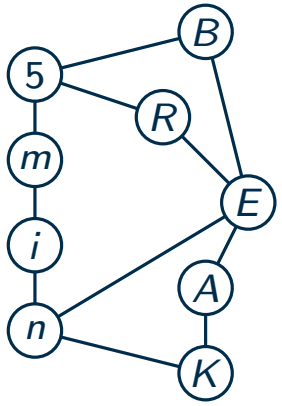

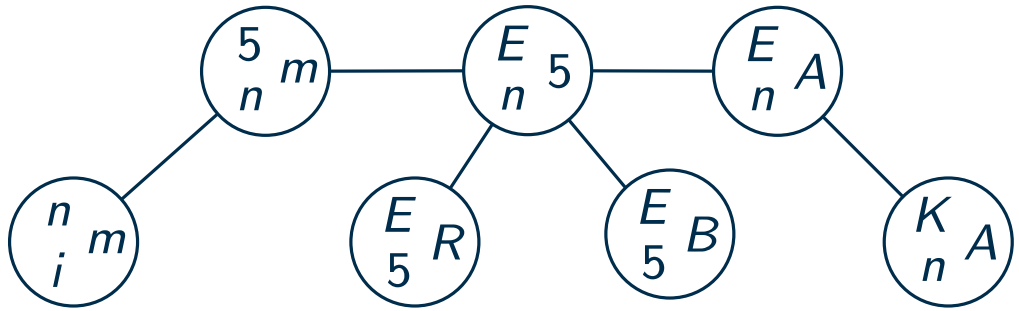

$\Rightarrow$ treewidth 2

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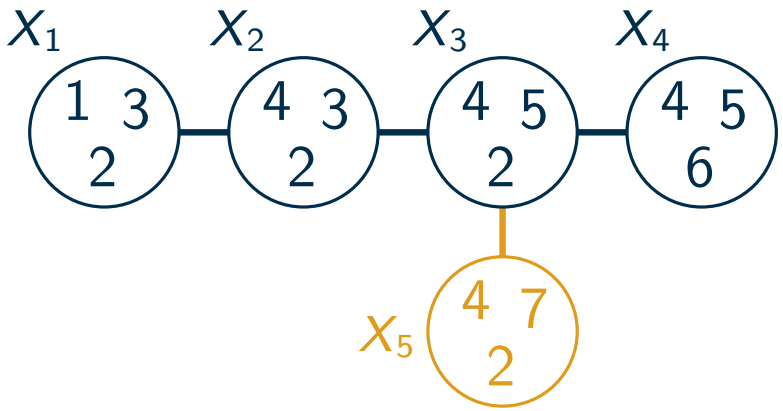

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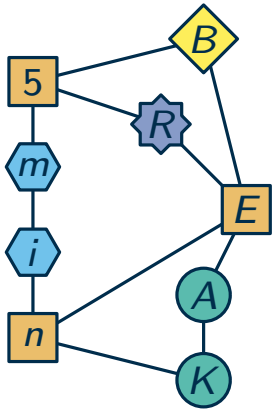
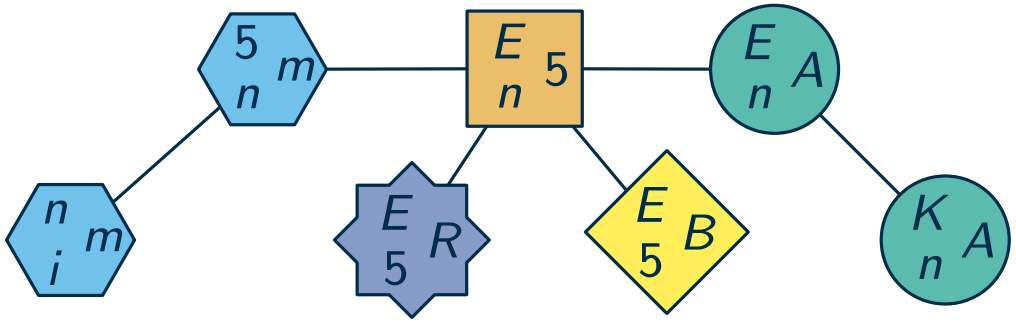




$\Rightarrow$ treewidth 2

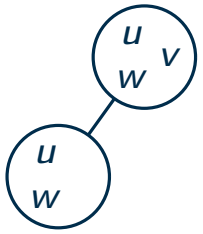
Nice tree decompositions

Three types of nodes

Nice tree decompositions

Three types of nodes

- x_i is **introduce node** if
  - x_i has exactly one child x_j and
  - $X_i = X_j \cup \{v\}$ for one vertex $v \in V$



intuition: DP runs bottom-up

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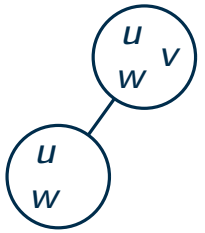
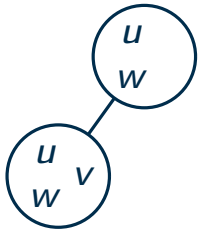


introduce v



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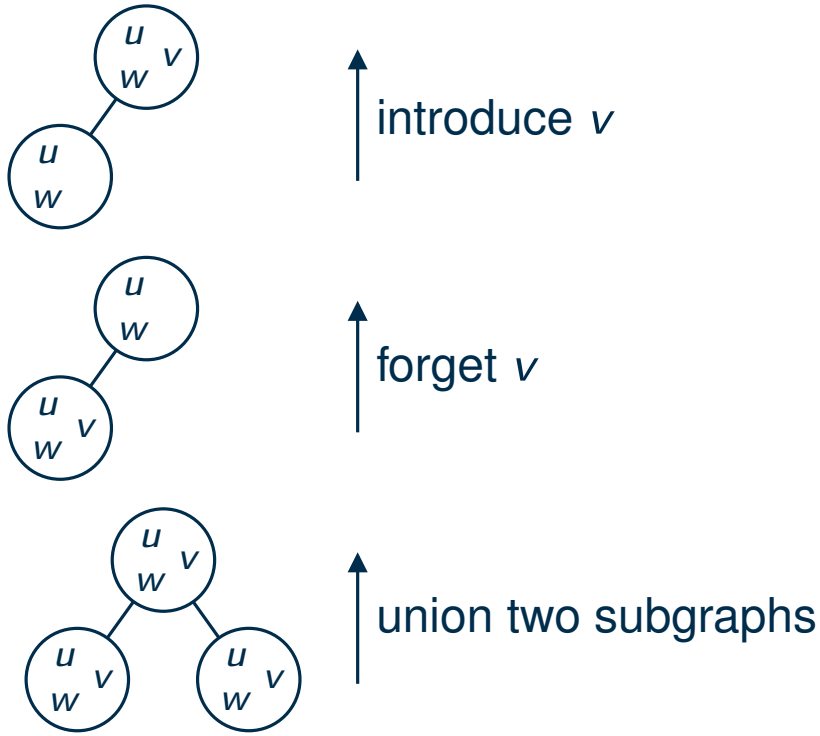

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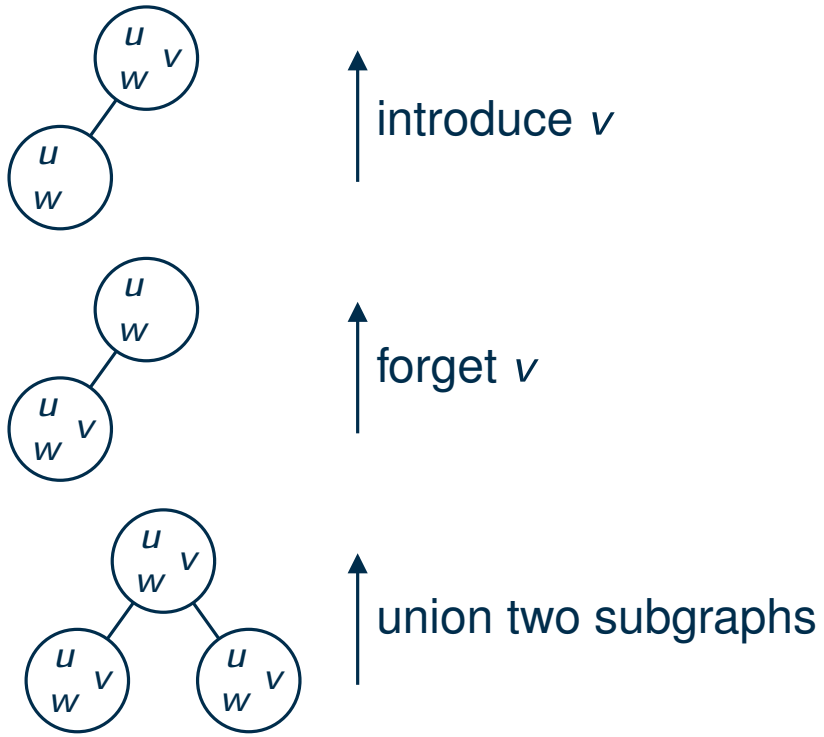

Theorem
If G has treewidth t, then it has a nice tree decomposition of width t.

Making things nice

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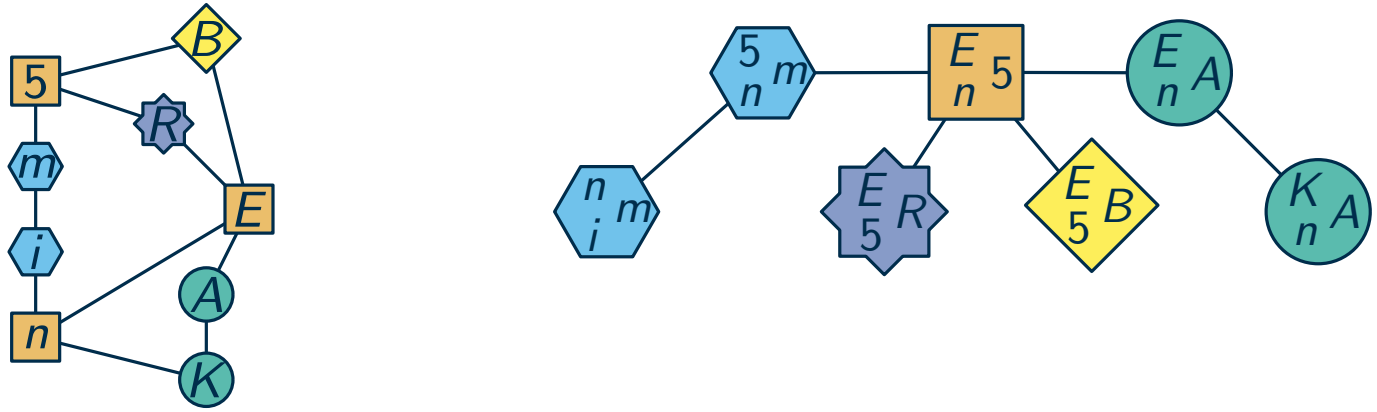

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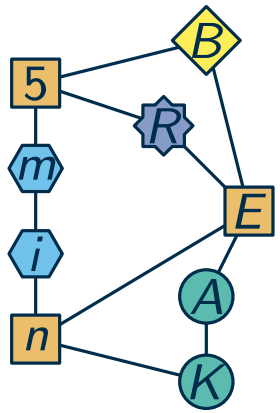

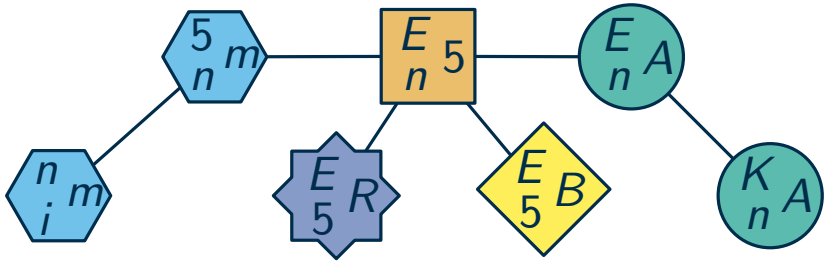

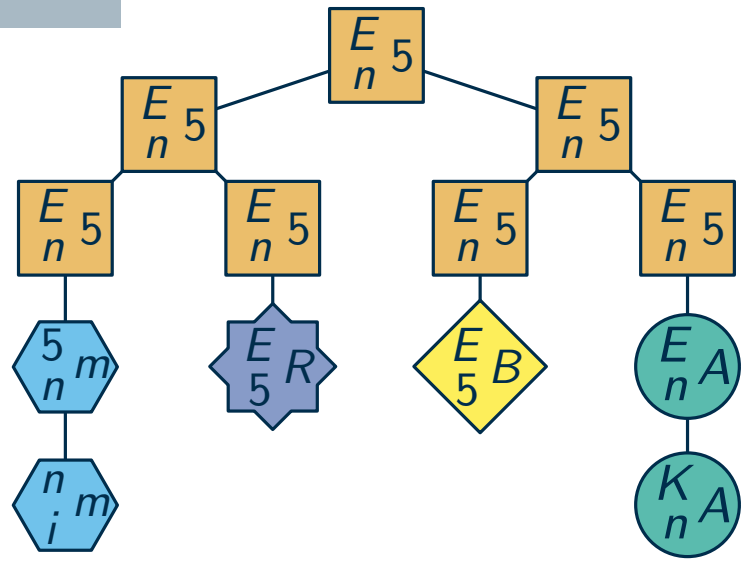

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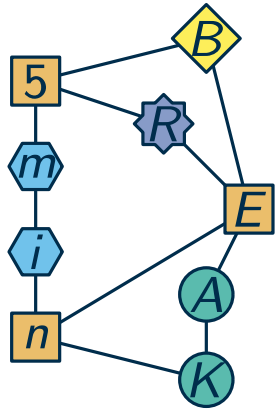

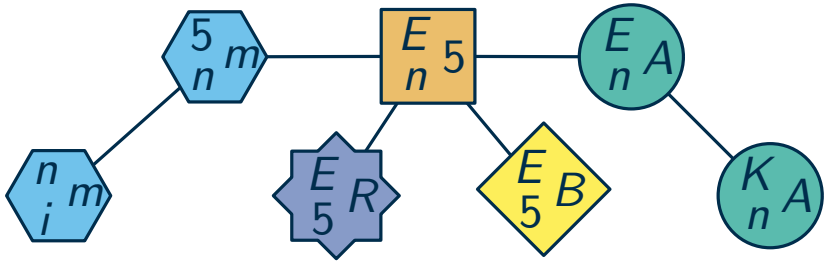

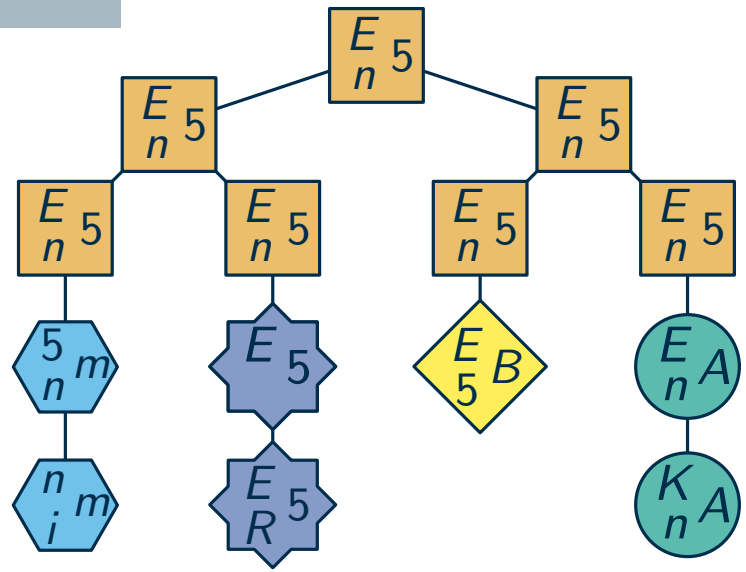

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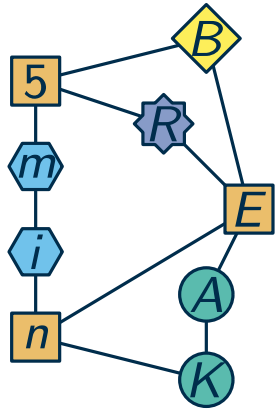

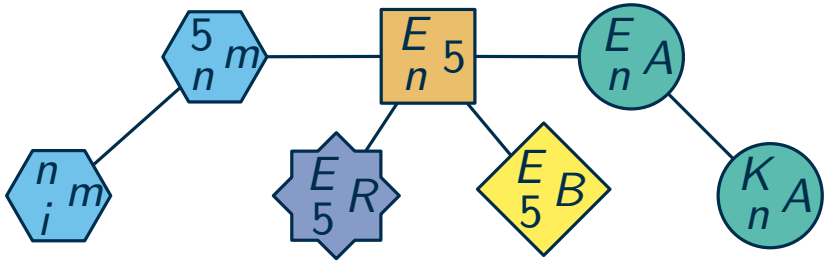

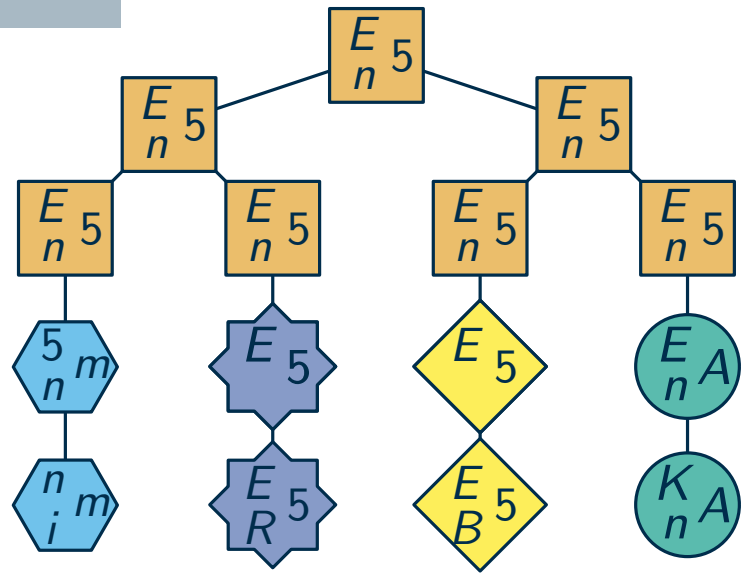

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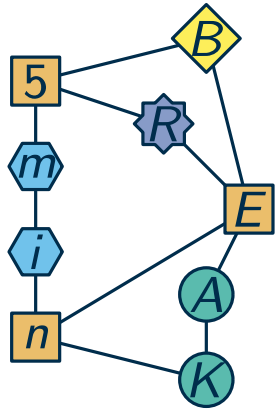

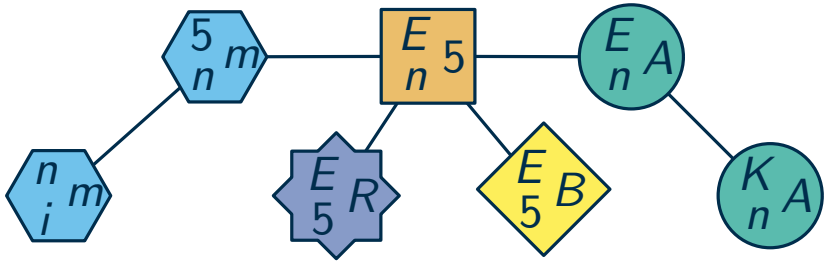

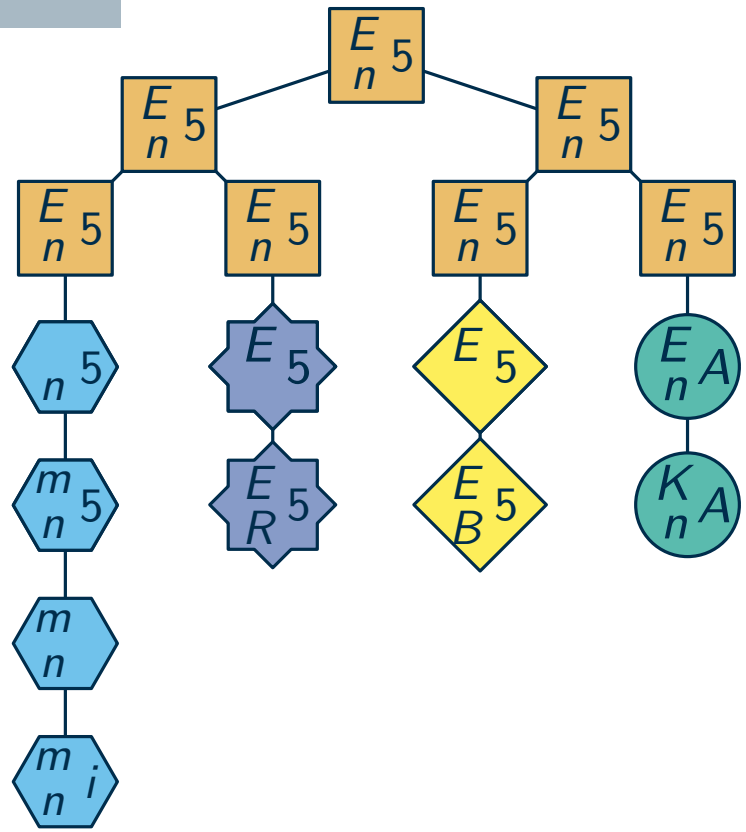

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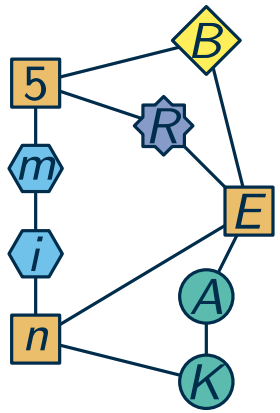
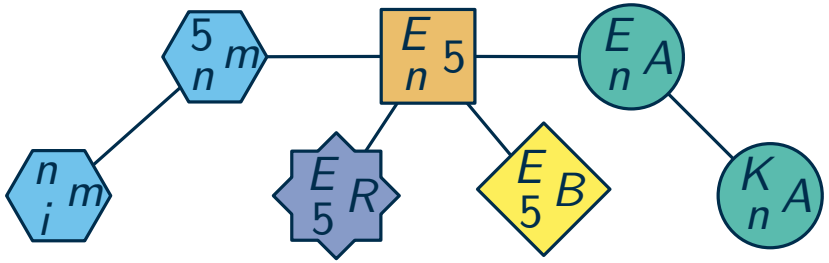
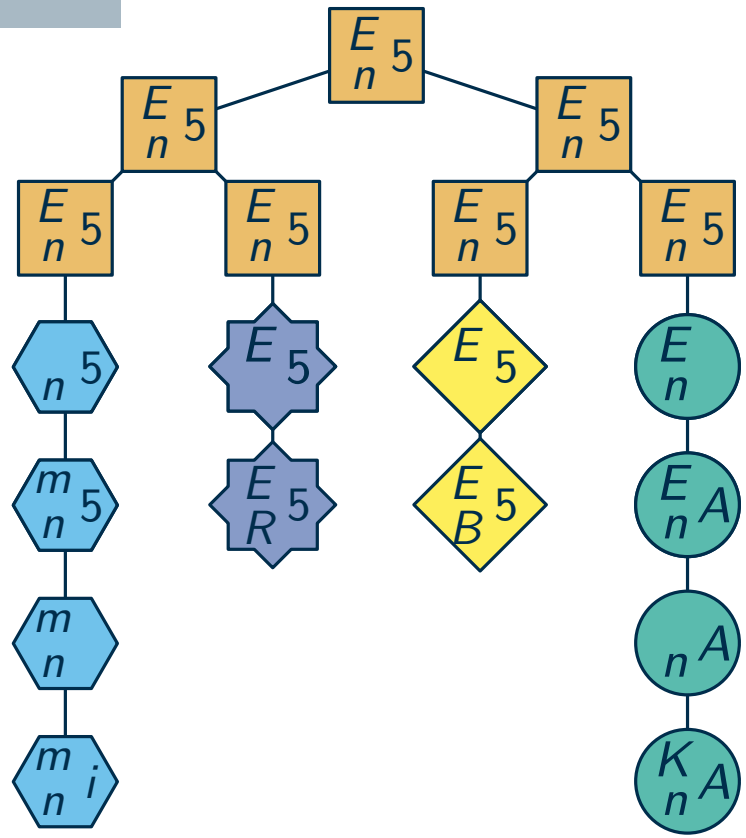






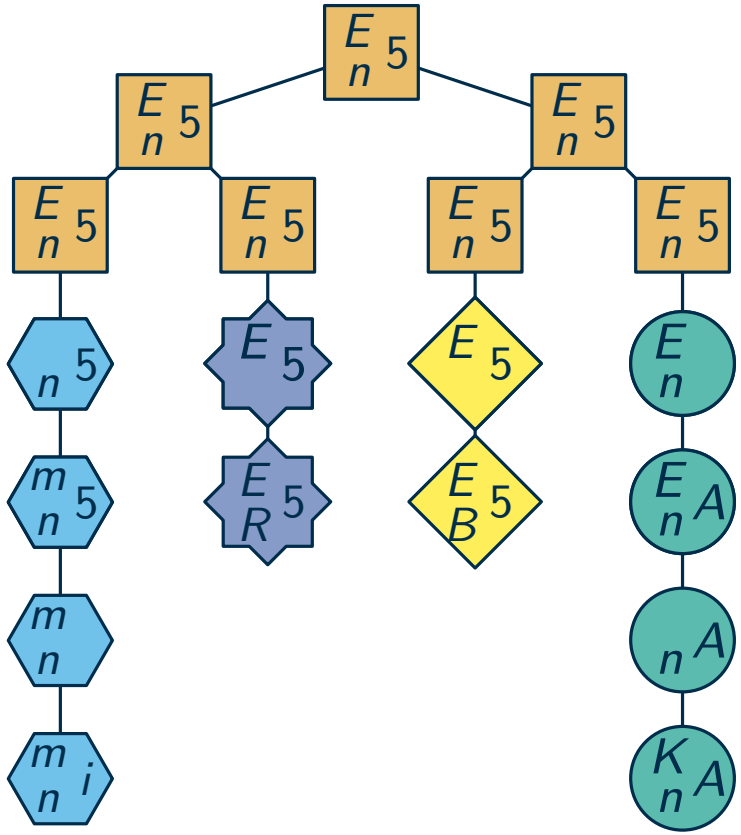
DP over nice tree decompositions

Dynamic programming over a nice tree decomposition

- introduce/forget node: as before for the path decomposition

Reminder: Independent Set

- vertex set with no edges
- maximize size
- parameter: treewdith t



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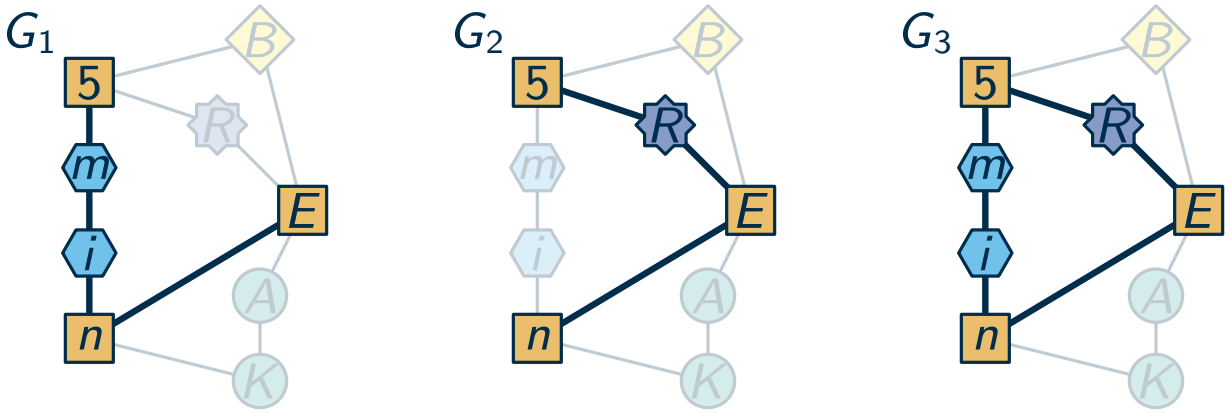
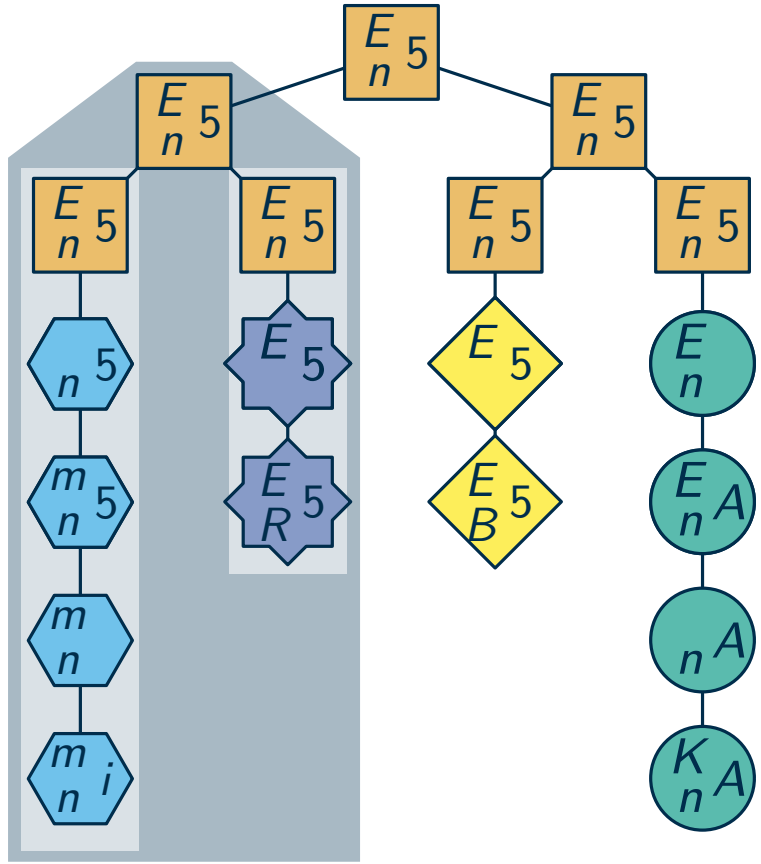
	$\emptyset$	$\{E\}$	$\{5\}$	$\{n\}$	$\{E,5\}$	$\{E,n\}$	$\{n,5\}$	$\{E,n,5\}$
G_1	1	2	2	2	3	$-\infty$	2	$-\infty$
G_2	1	1	1	2	2	$-\infty$	2	$-\infty$
G_3								





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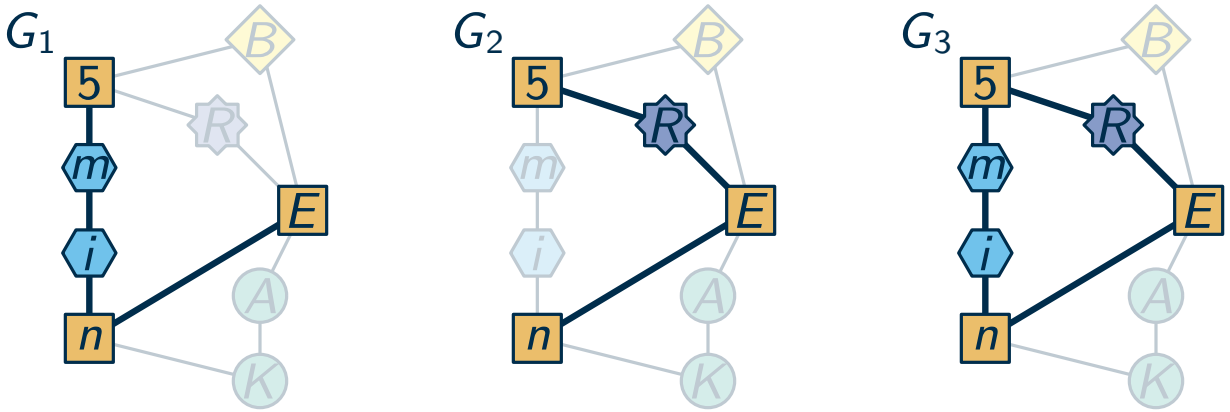
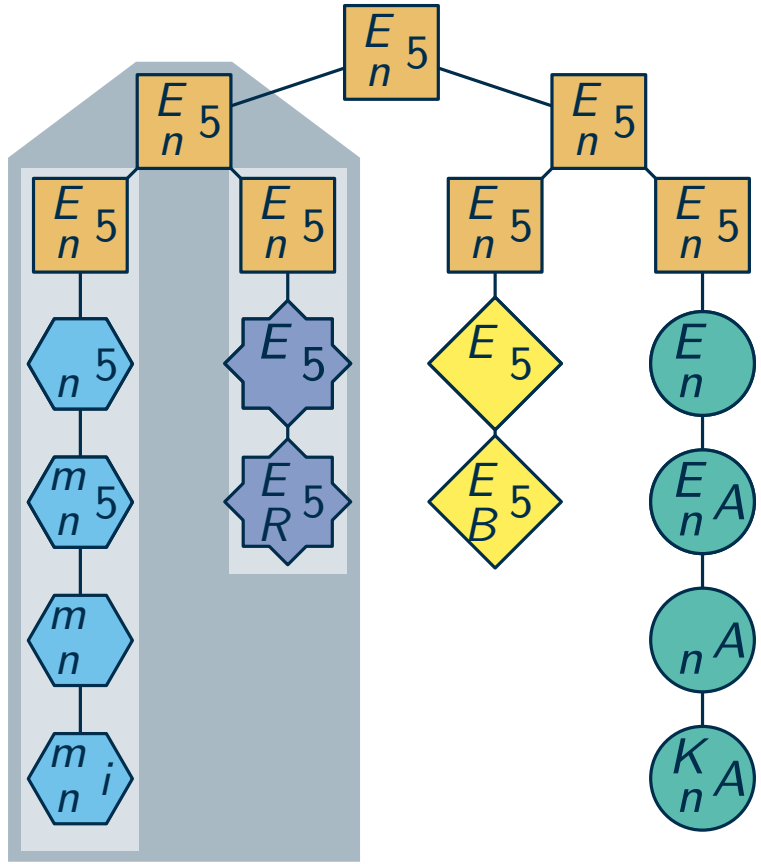
G_2	1	1	1	2	2	$-\infty$	2	$-\infty$
G_3	2							

union the independent sets $\Rightarrow$ sum values





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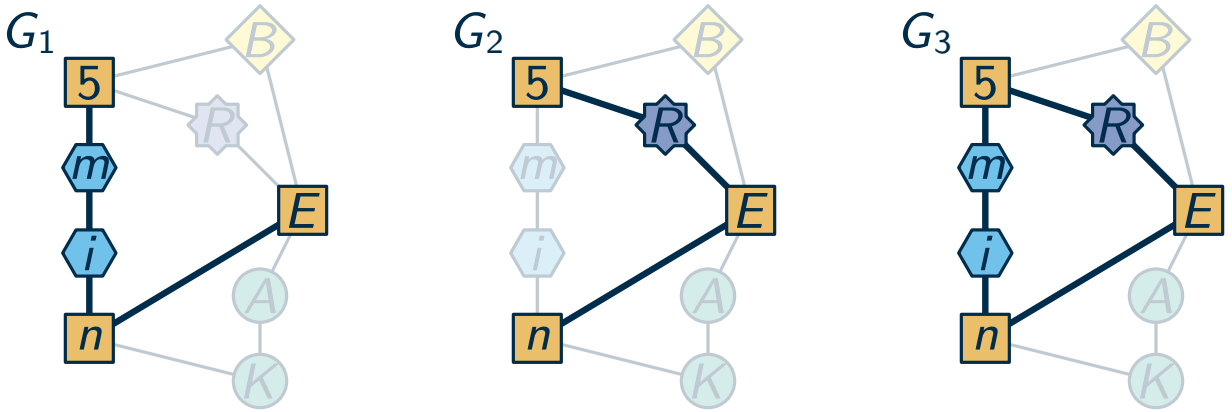
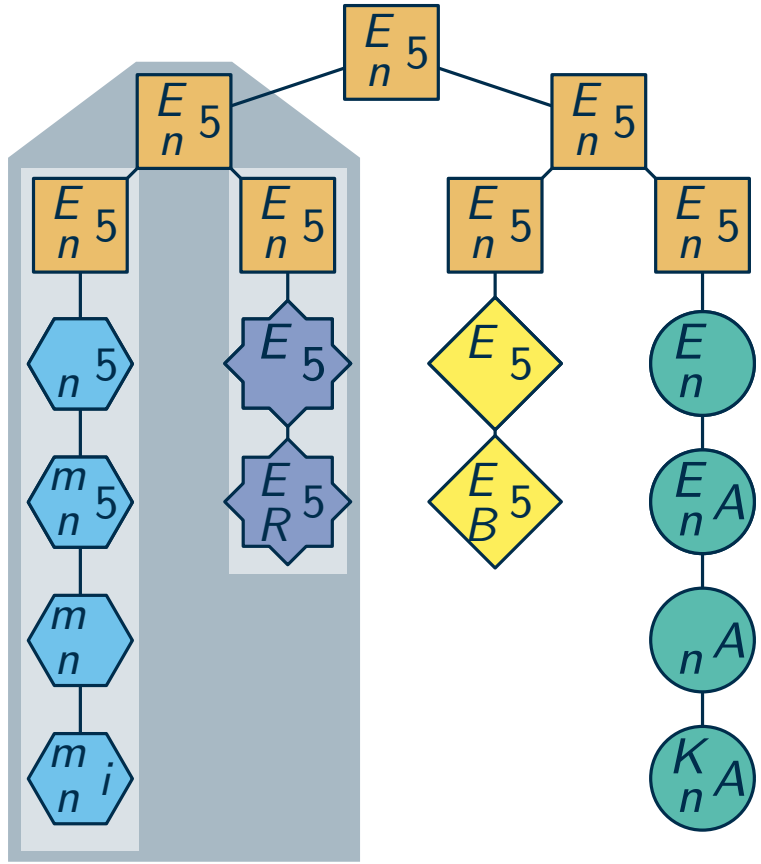
G_3	2	2	union the independent sets $\Rightarrow$ sum values $-\lvert\{E\}\rvert$					





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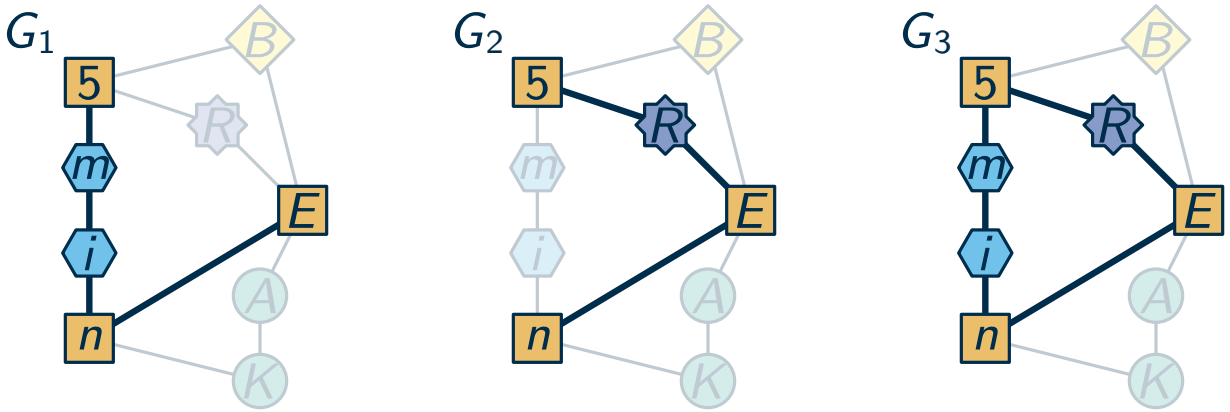

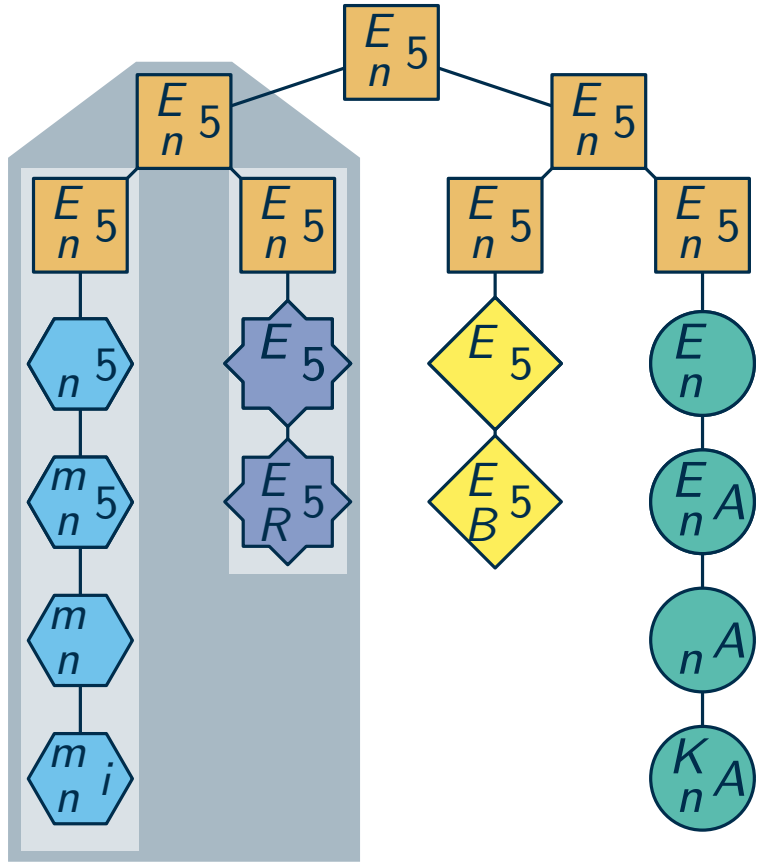

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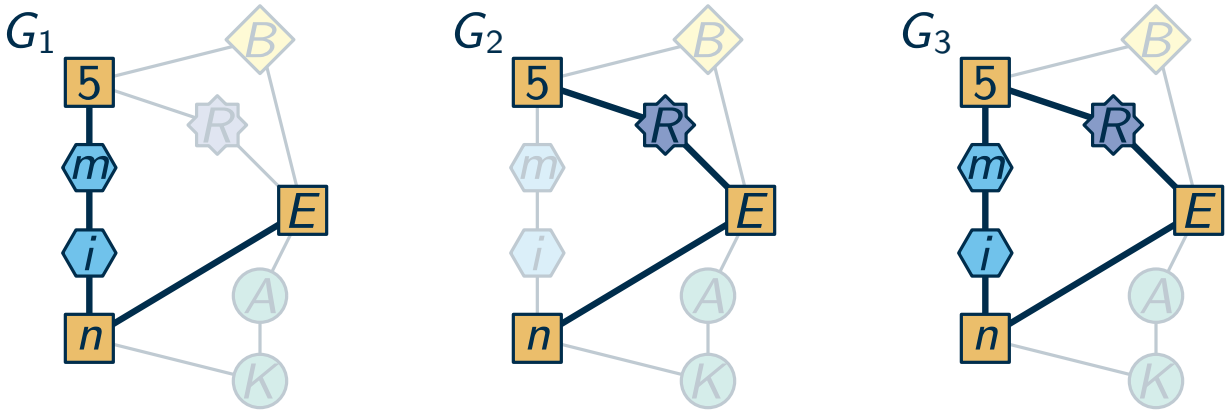

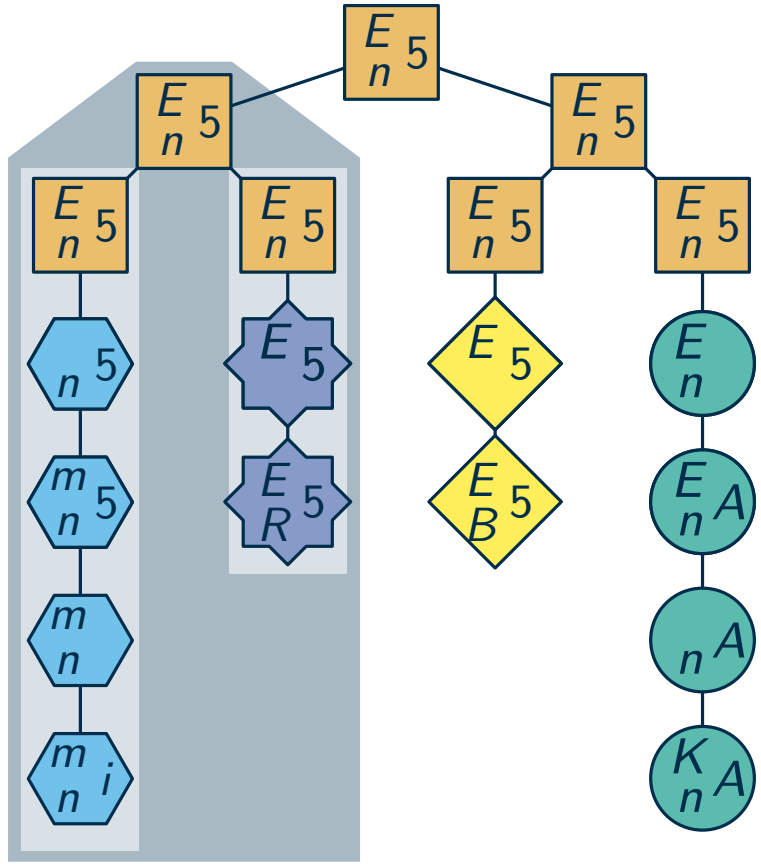

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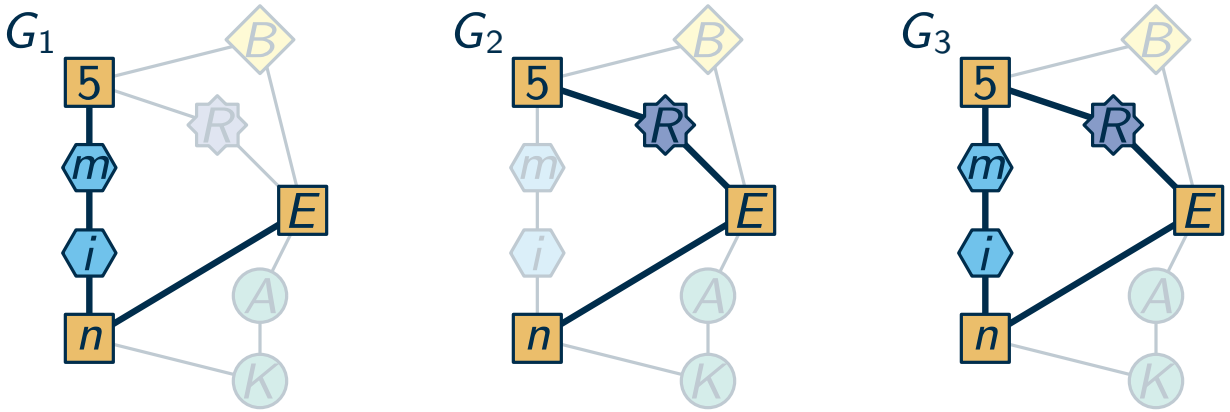

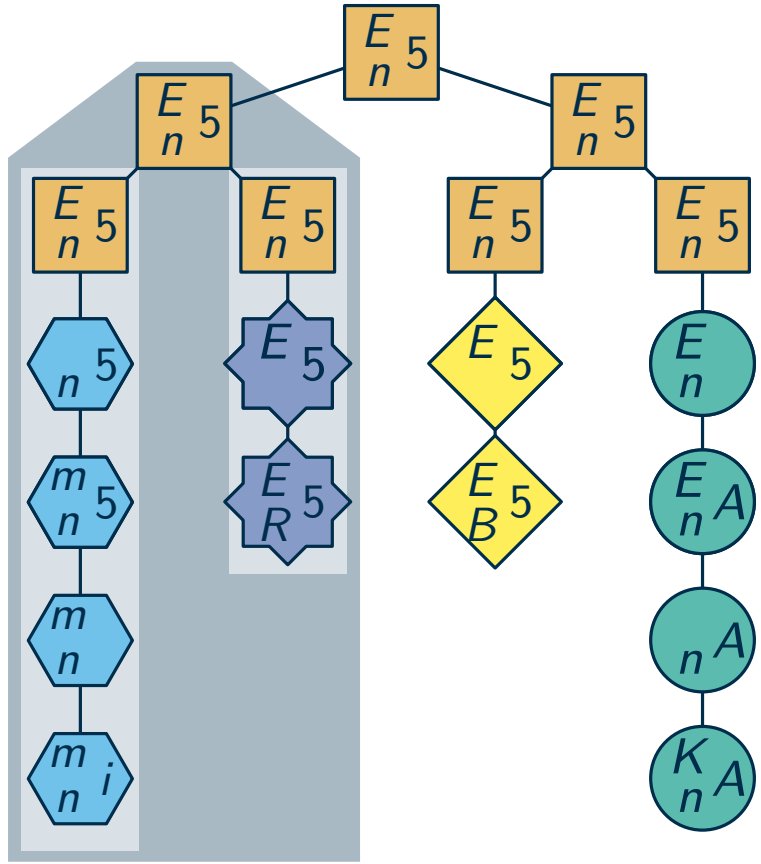

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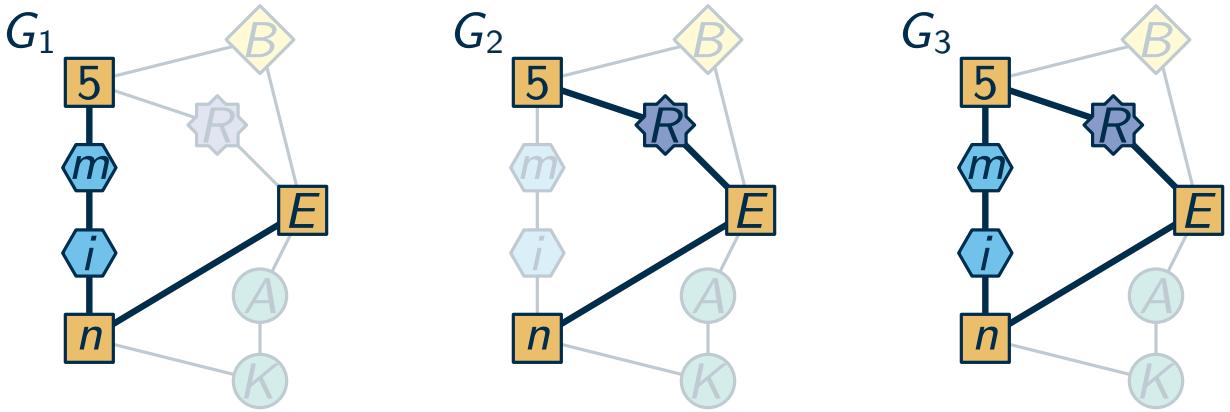

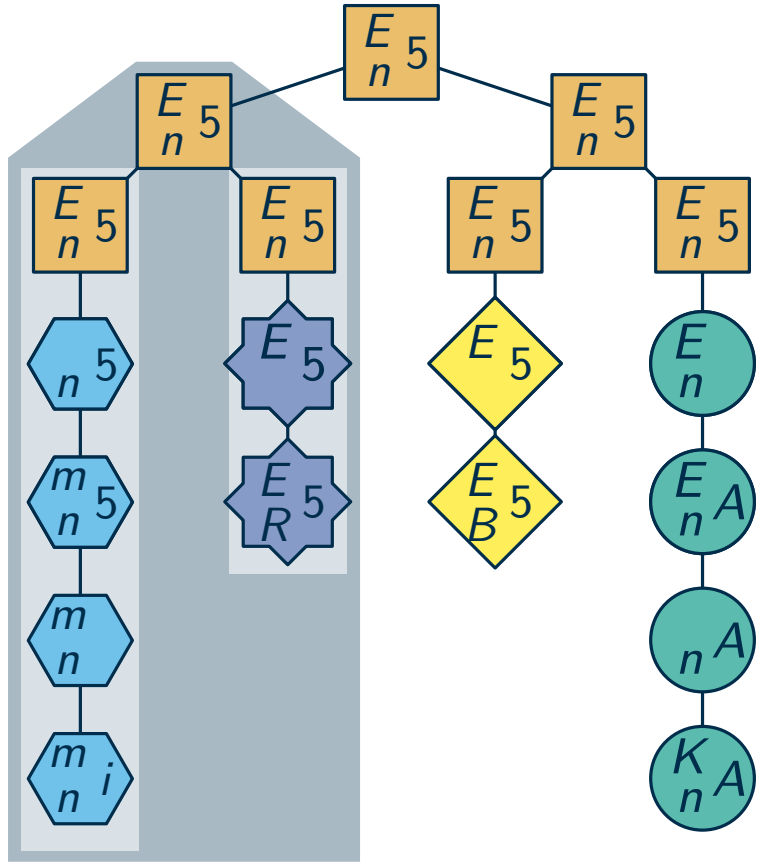

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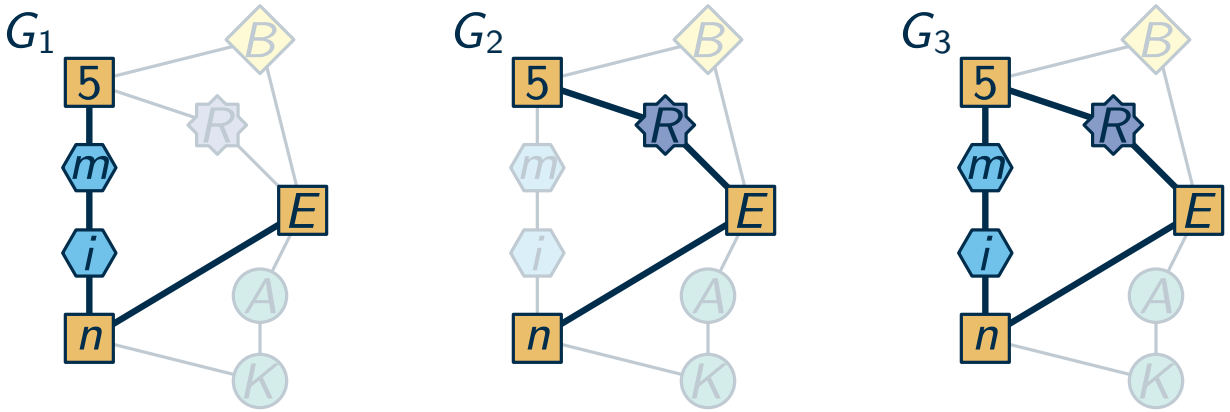
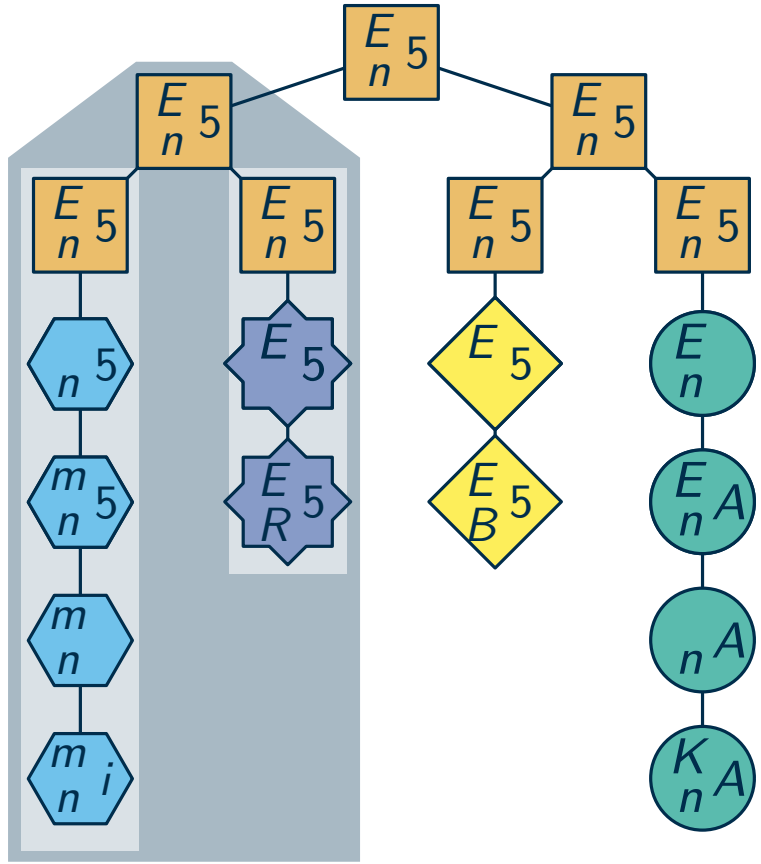
	$\emptyset$	$\{E\}$	$\{5\}$	$\{n\}$	$\{E,5\}$	$\{E,n\}$	$\{n,5\}$	$\{E,n,5\}$
G_1	1	2	2	2	3	$-\infty$	2	$-\infty$
G_2	1	1	1	2	2	$-\infty$	2	$-\infty$
G_3	2	2	2	3	3	$-\infty$	2	





DP over nice tree decompositions

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- vertex set with no edges
- maximize size
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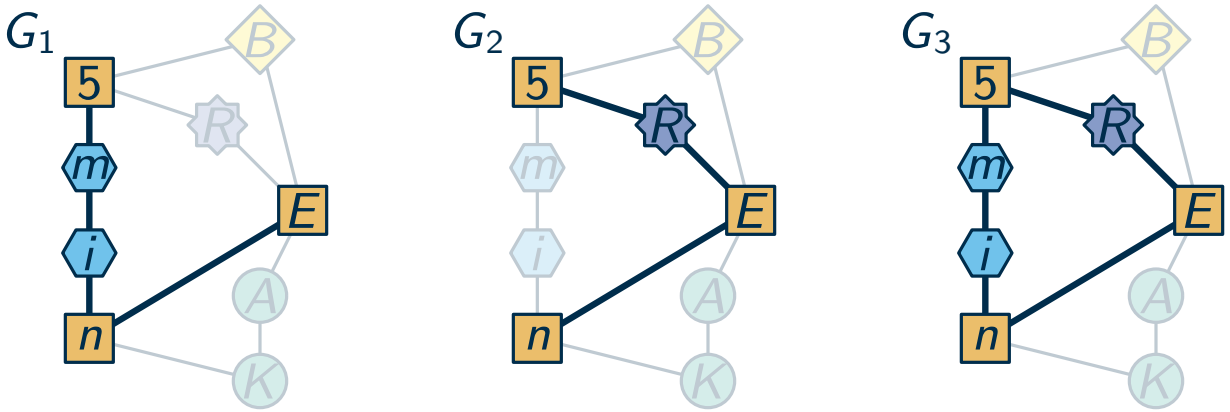

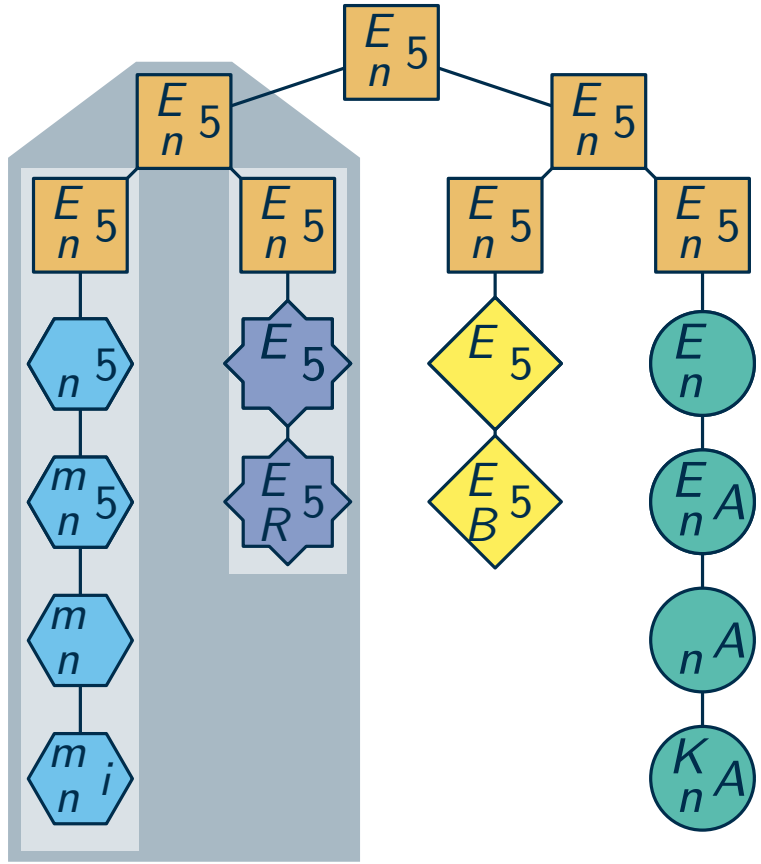

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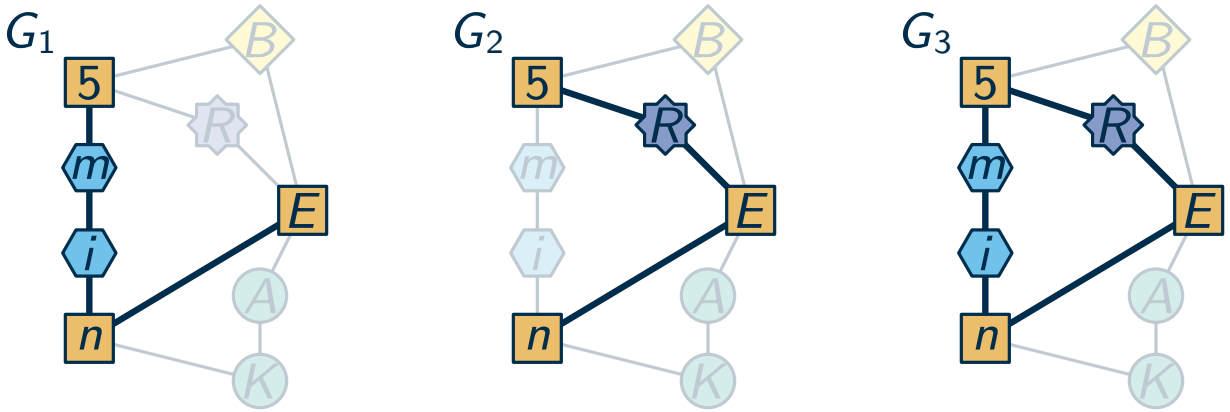

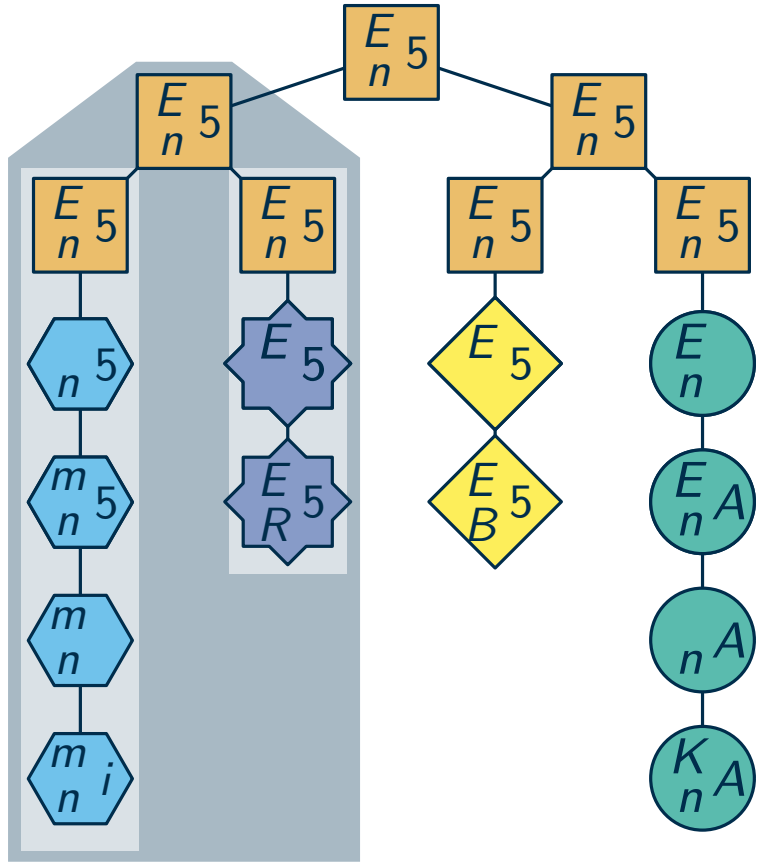

Theorem: INDEPENDENT SET is FPT with respect to treewidth.

KIT

DPs on tree decompositions

Notation and basic properties

- for node x_i: $V_i = X_i \cup \{v \in V \mid v \in X_j \text{ for successor } x_j \text{ of } x_i\}$
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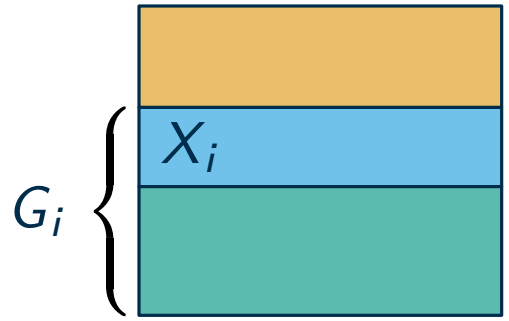

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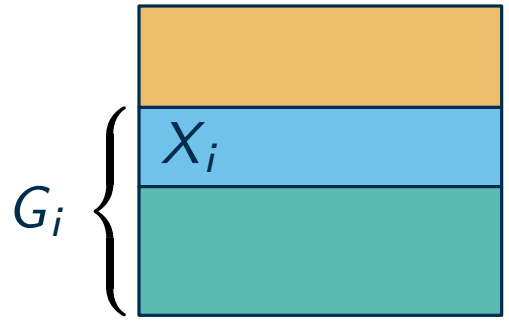

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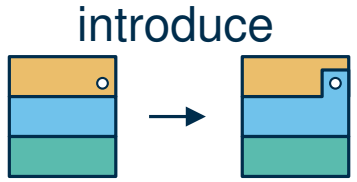

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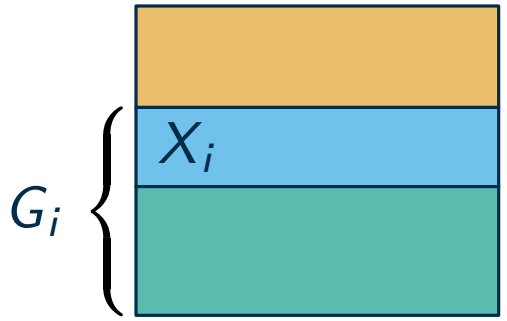

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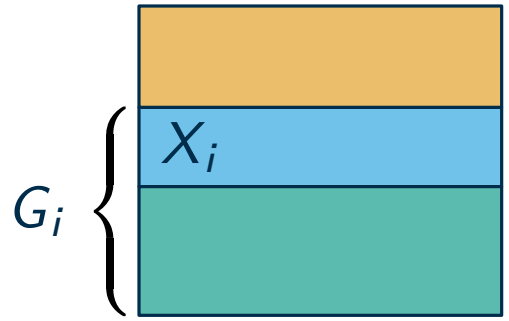

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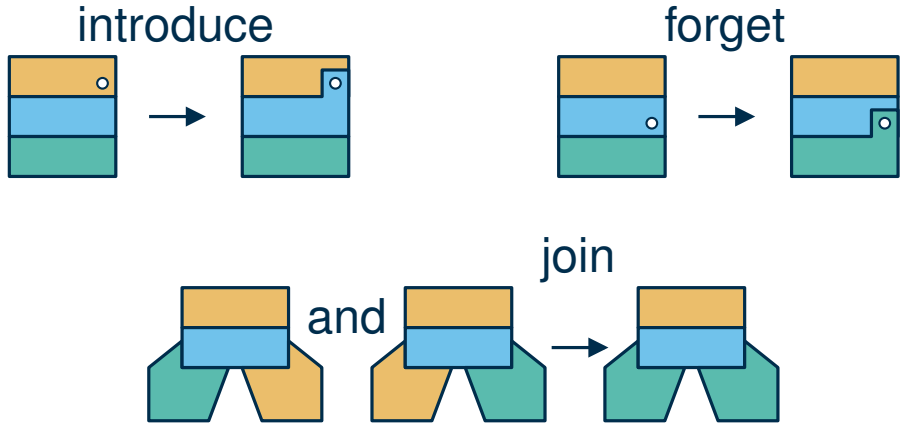

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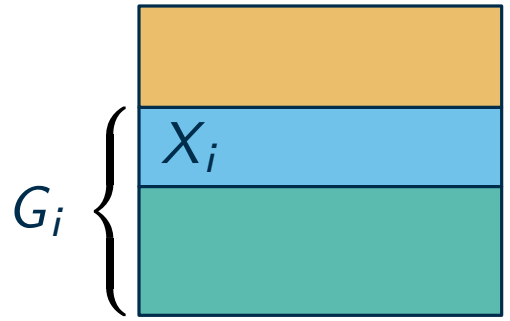

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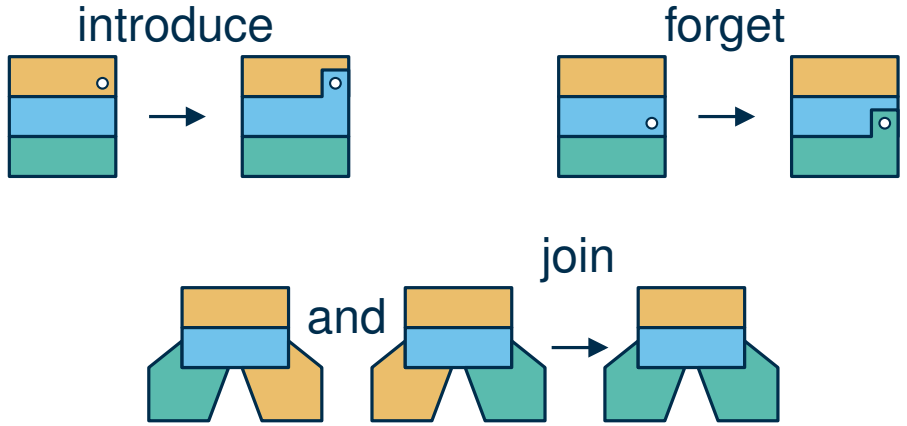

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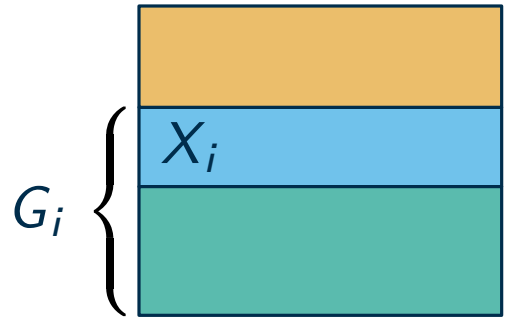

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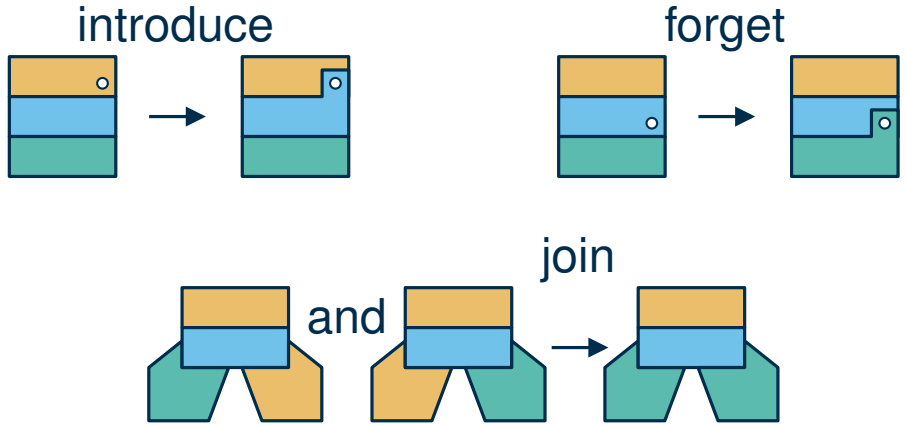
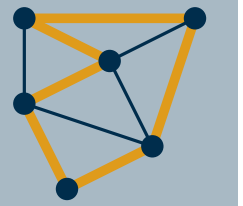


Desired properties for partial solutions on (G_i, X_i)

- for $G_i = G$, the actual solution is included
- in every step: all new partial solutions can be computed based on old ones

DP for Hamiltonian cycle – partial solutions

Problem: HAMILTONIAN CYCLE
Given a graph G. Does G have a cycle that visits all vertices?



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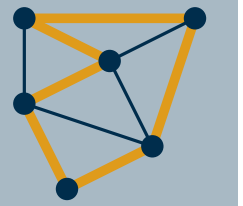

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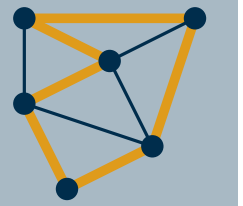

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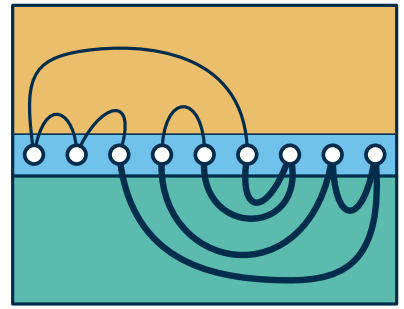

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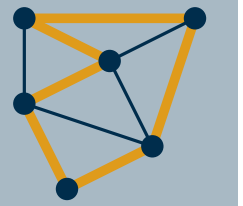

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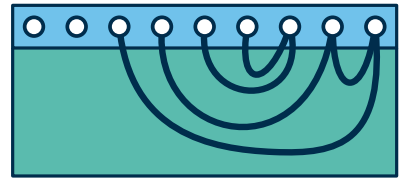

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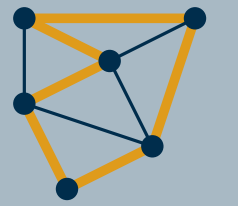

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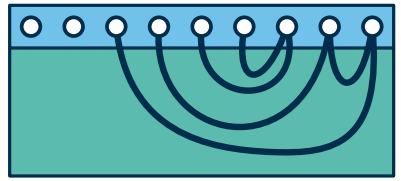

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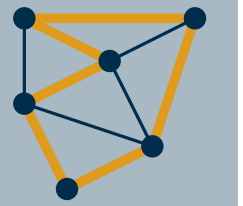

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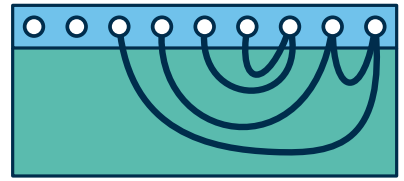

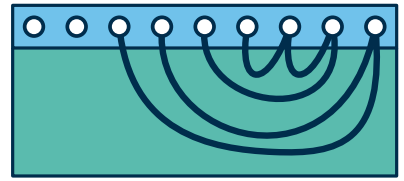

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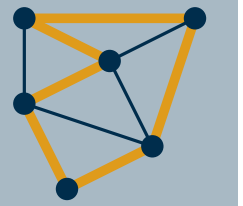

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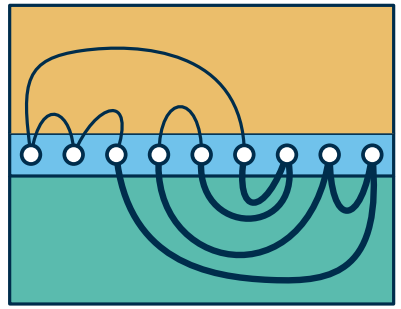

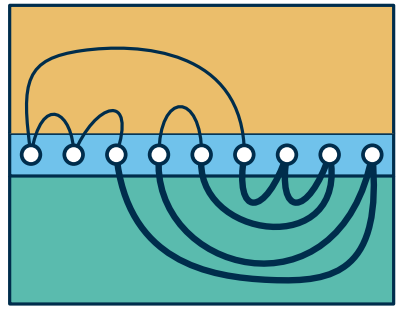

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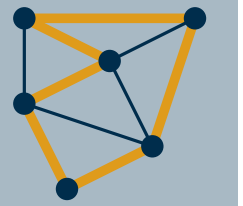

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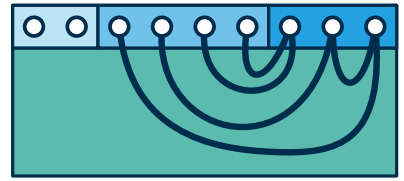

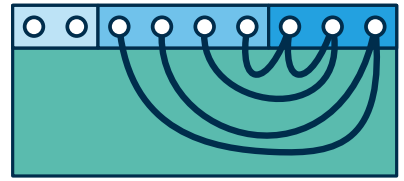

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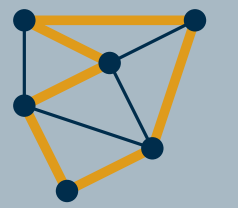

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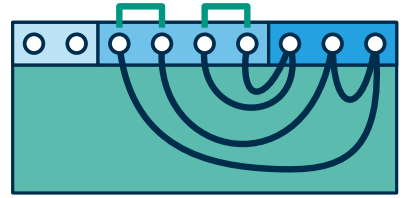

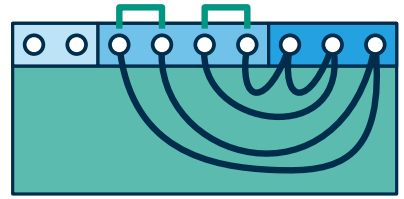

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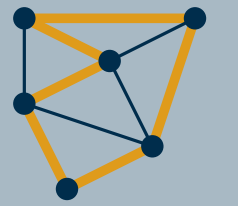

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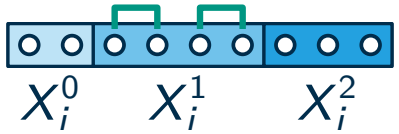
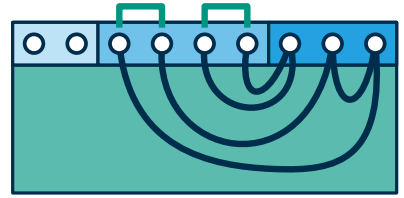
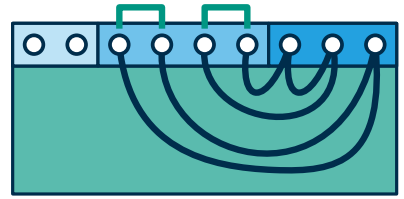
- define partial solution: partition of X_i into $X_i = X_i^0 \cup X_i^1 \cup X_i^2$ and matching on X_i^1

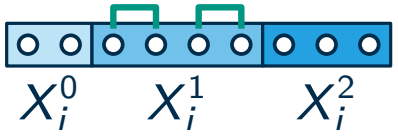






DP for Hamiltonian cycle – forget node

Partial solution:

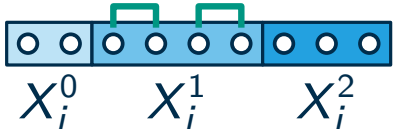


What does the DP do in general?

- process nodes bottom up
- when node is processed: assume all children have been processed

DP for Hamiltonian cycle – forget node

Partial solution:



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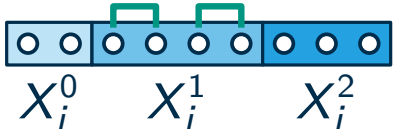

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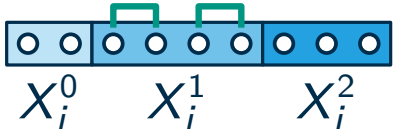

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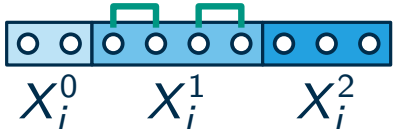

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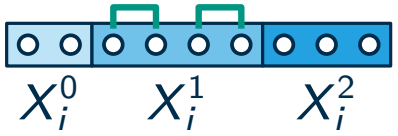

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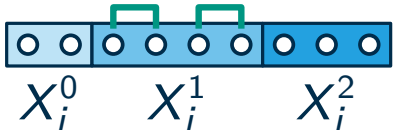

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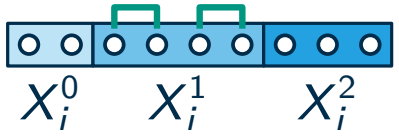
- if $v \notin X_i^2$:
  - v has no chance to be part of the Hamiltonian cycle
  - dead end $\rightarrow$ ignore this partial solution of the child

DP for Hamiltonian cycle – introduce node

Partial solution:



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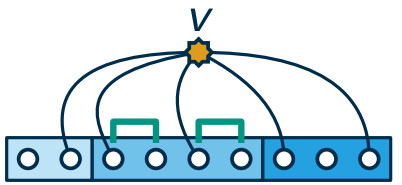
- let v be the introduced vertex and consider its edges to the interface X_i



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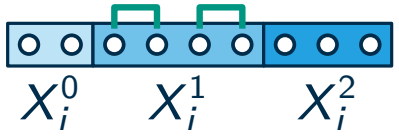

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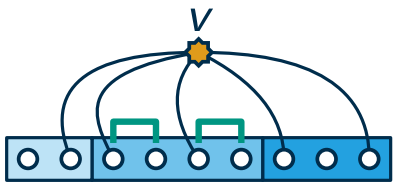

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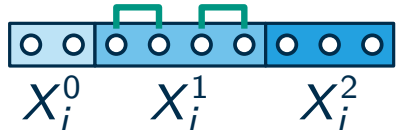

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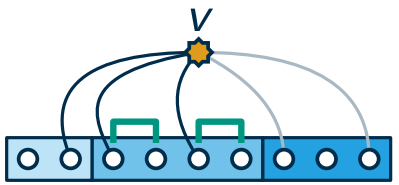

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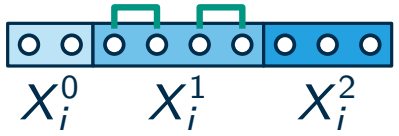

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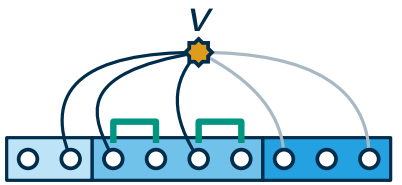

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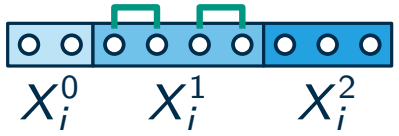

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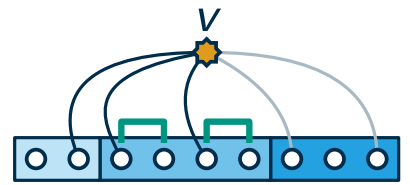

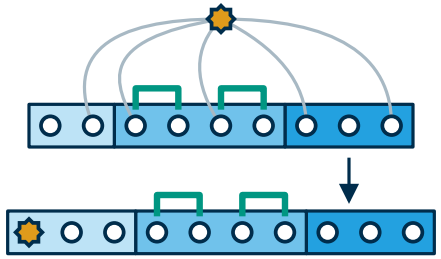

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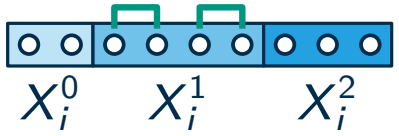

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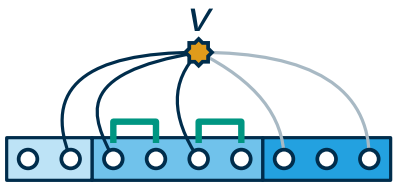

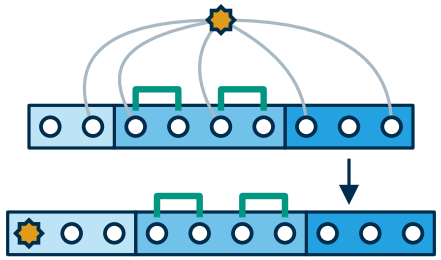

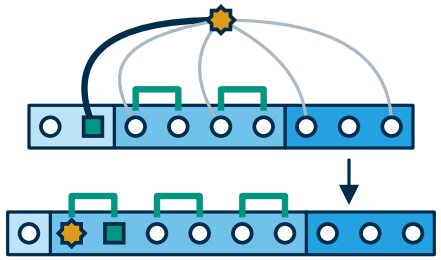

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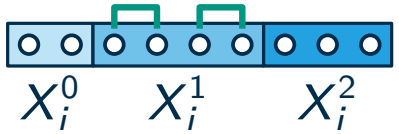

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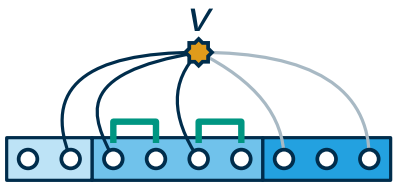

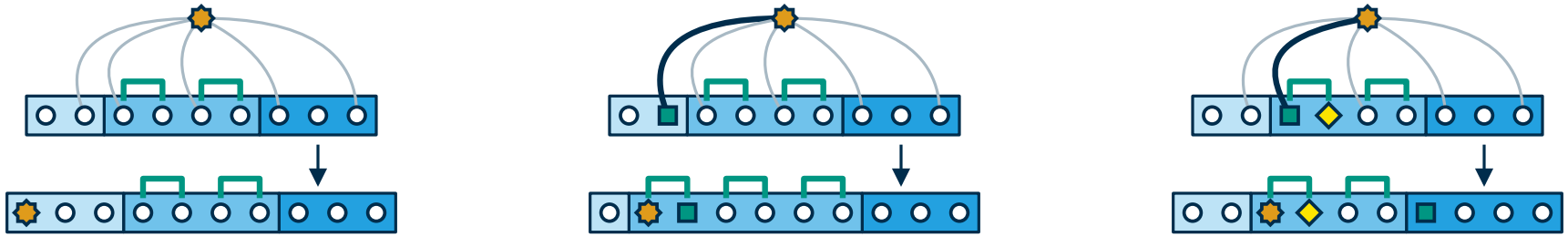

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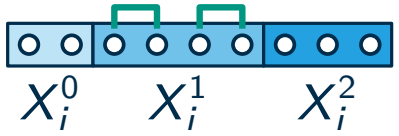

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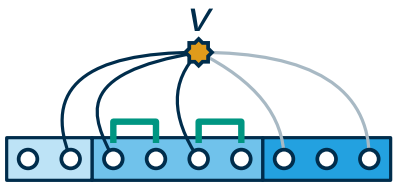

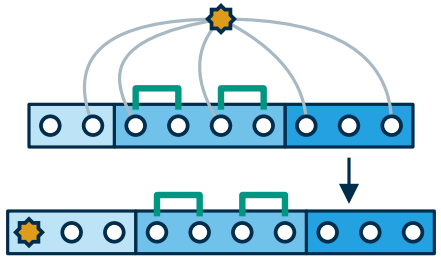

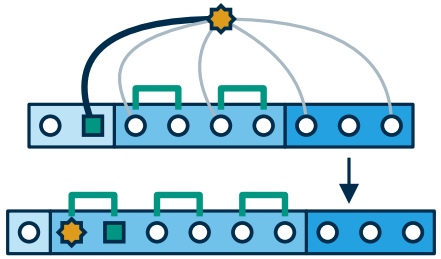

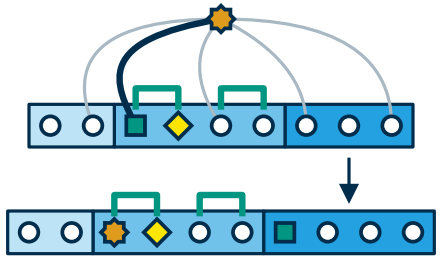

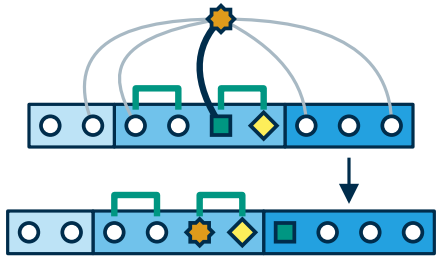

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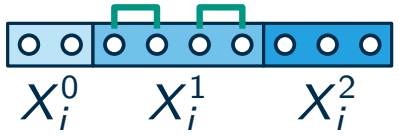

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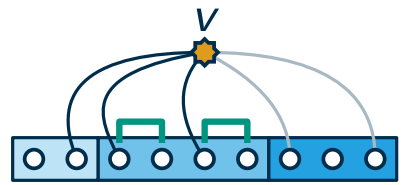

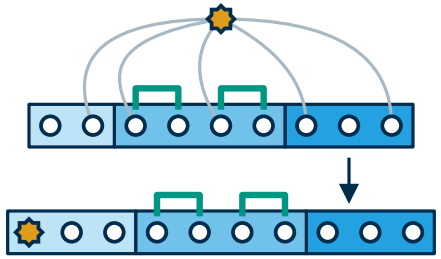

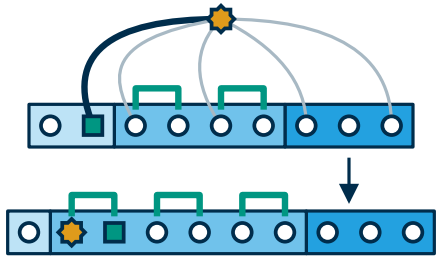

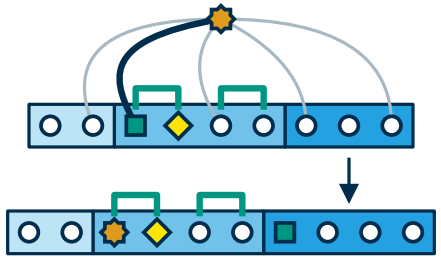

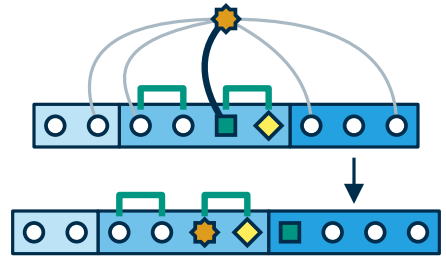

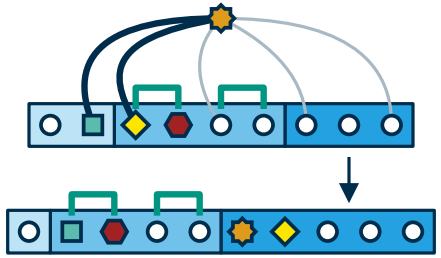

DP for Hamiltonian cycle – introduce node

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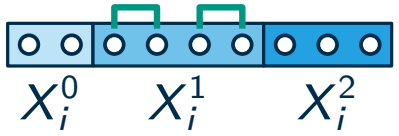

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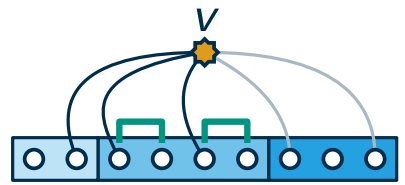

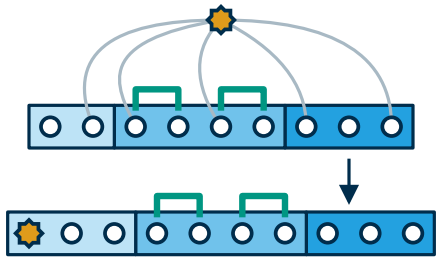

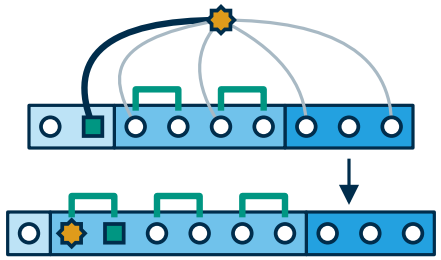

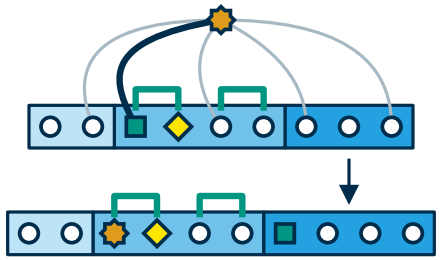

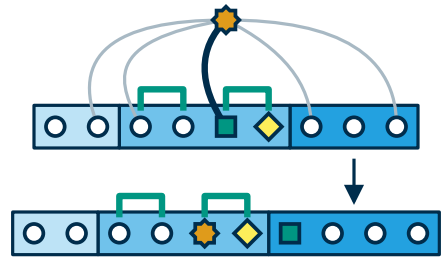

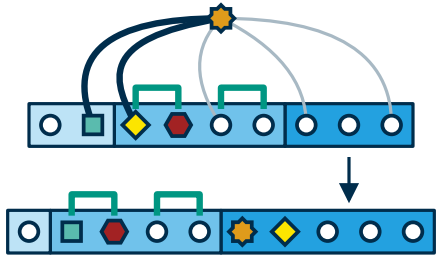

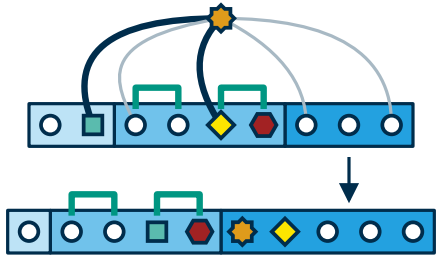

DP for Hamiltonian cycle – introduce node

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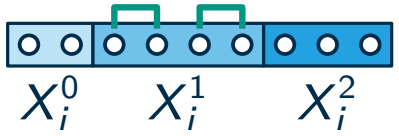

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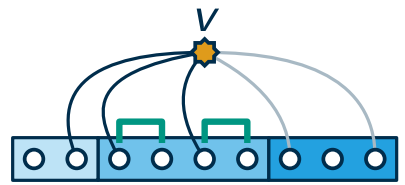
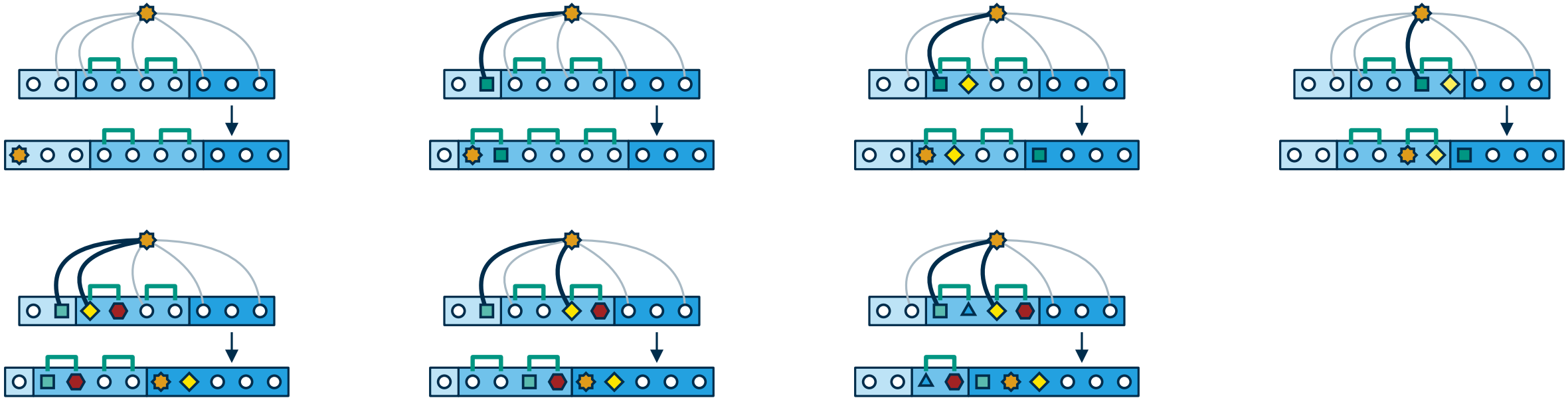
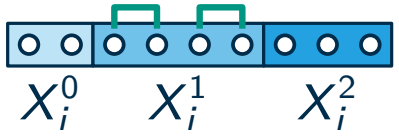




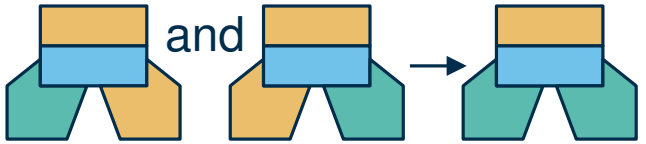
DP for Hamiltonian cycle – join node

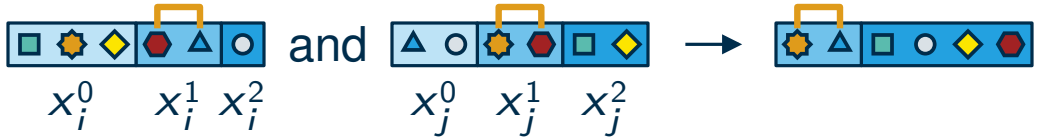
Partial solution:



Computing new partial solutions: join node

- combining two partial solutions $\rightarrow$ union of selected edges





DP for Hamiltonian cycle – join node

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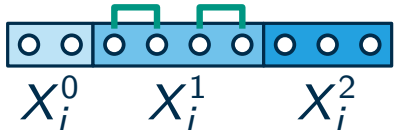

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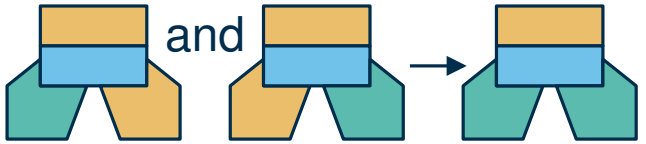

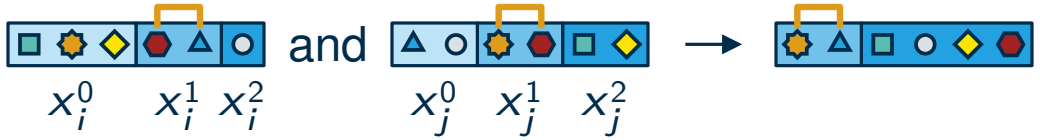

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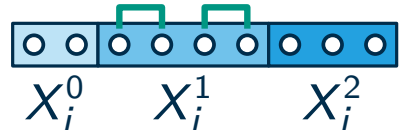

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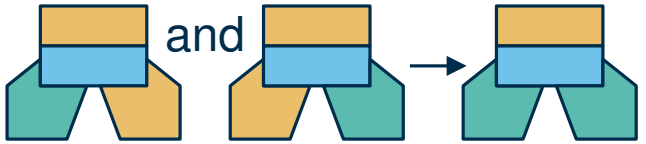

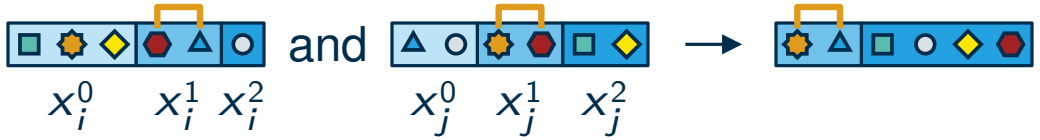

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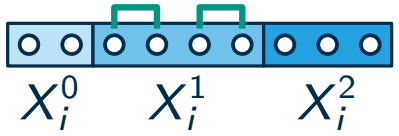

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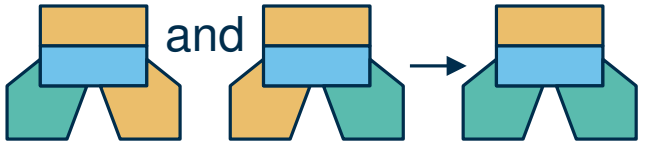

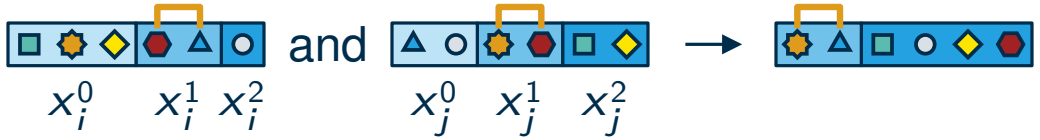

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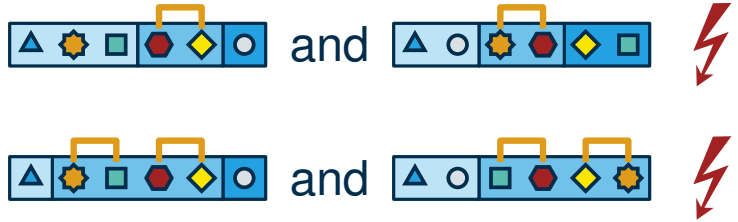

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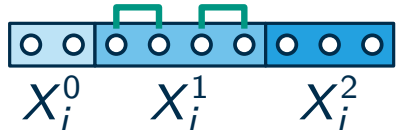

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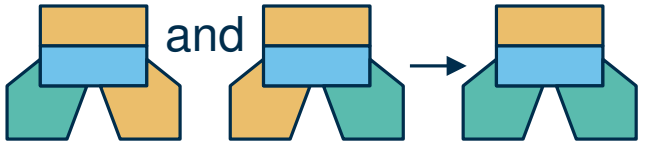

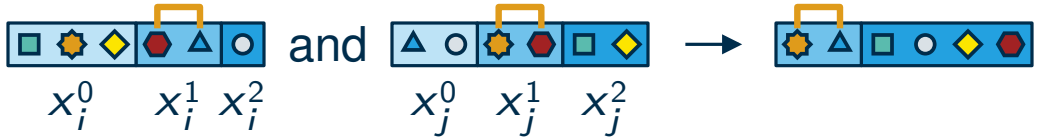

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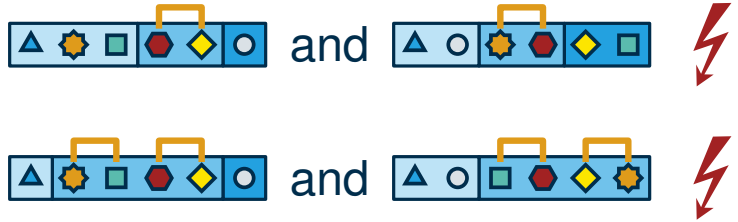

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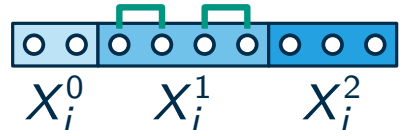

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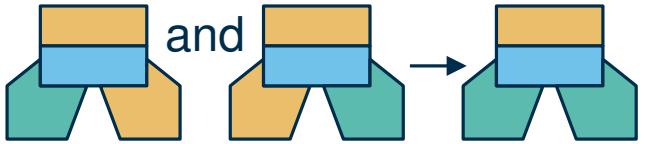

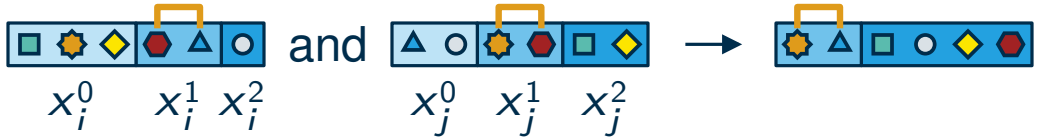

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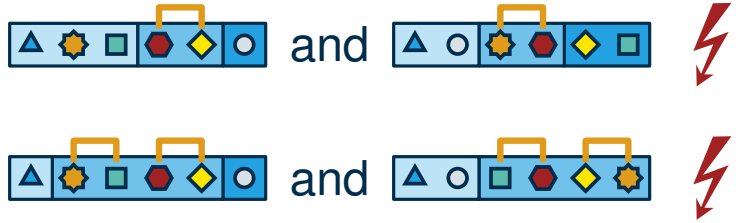
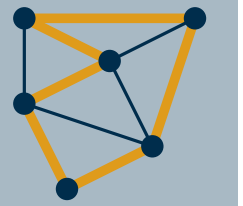


- otherwise:
  - sum the vertex degrees to get the partition
  - compute which pairs of 1-vertices belong to the same path

DP for Hamiltonian cycle

Problem: HAMILTONIAN CYCLE
Given a graph G. Does G have a cycle that visits all vertices?



Goal

- FPT with respect to treewidth
- given: corresponding tree decomposition

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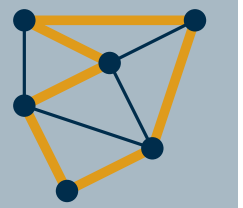

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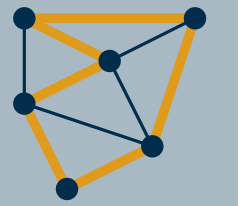

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Theorem
HAMILTONIAN PATH is FPT with respect to the treewidth t. (assuming we know a corresponding composition)

Wrap-UP

Treewidth

- structural graph parameter
- measures similarity to trees
  (in regards to separators)

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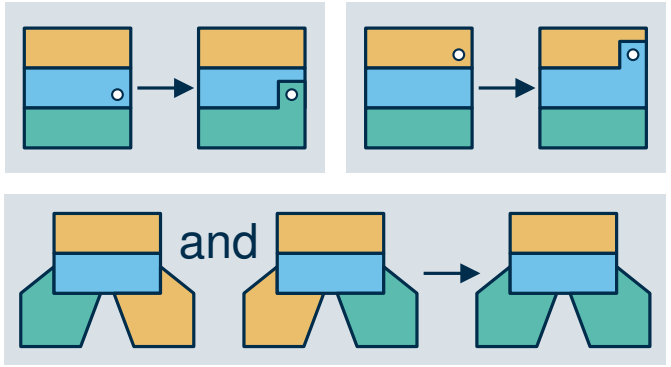

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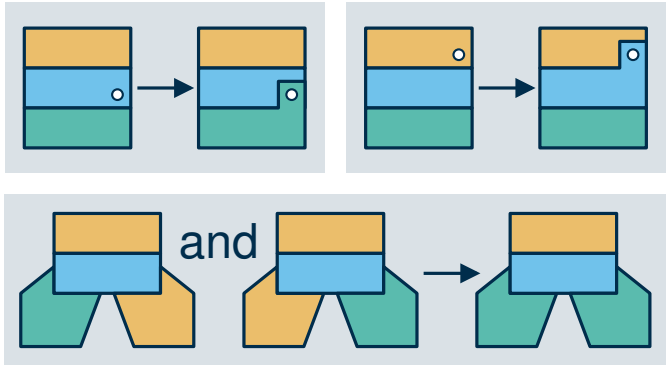

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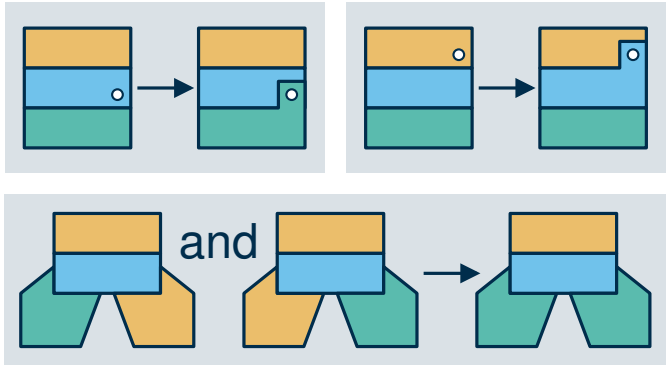

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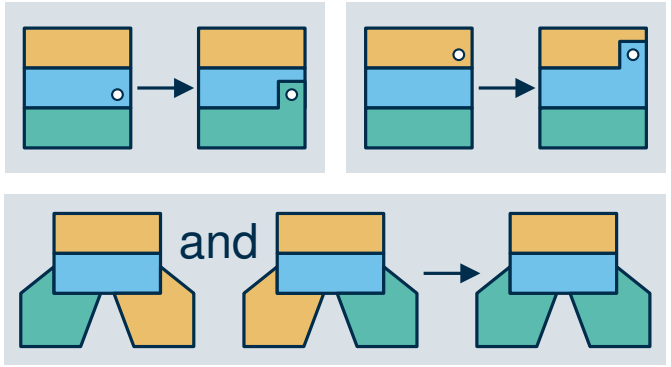

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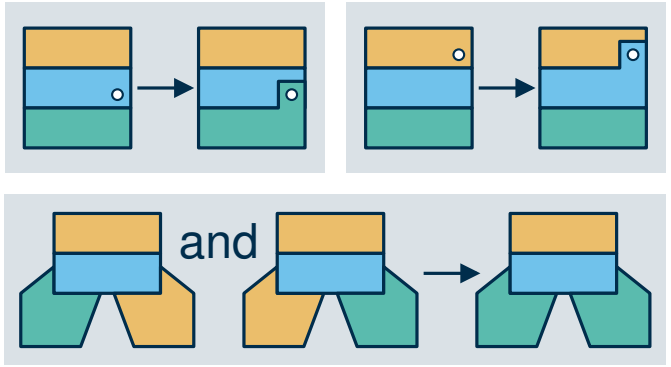

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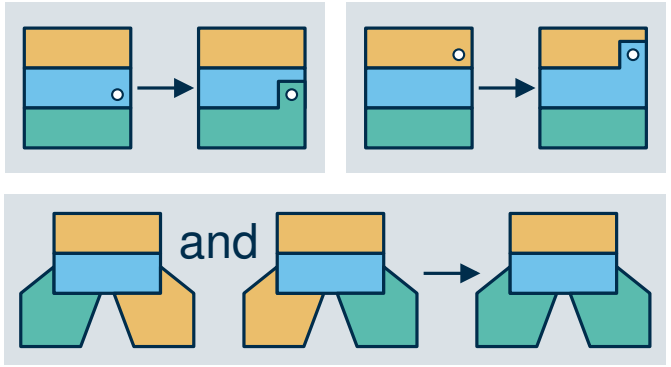

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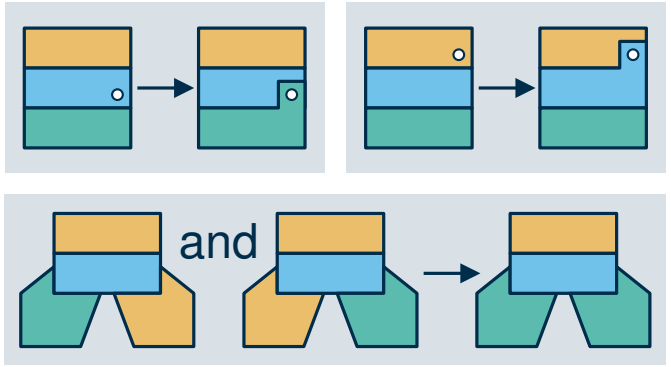

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