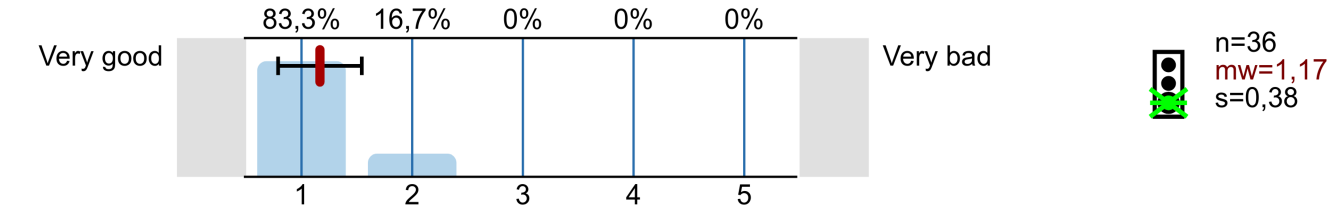
Parameterized Algorithms

Exercise 7 – Sheet 6, *W*-Hierarchy

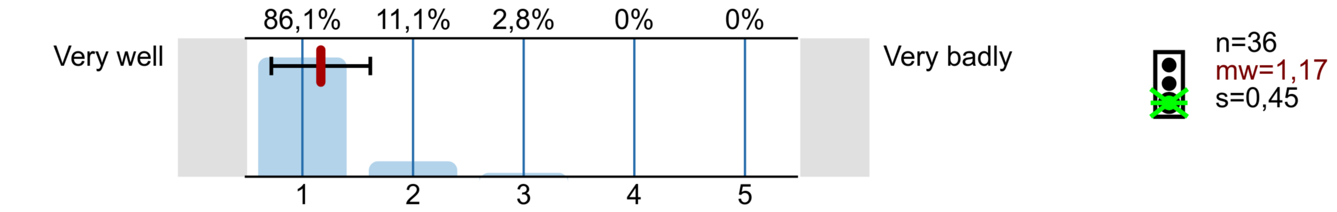
Elly, Jean-Pierre, Wendy

Evaluation Results

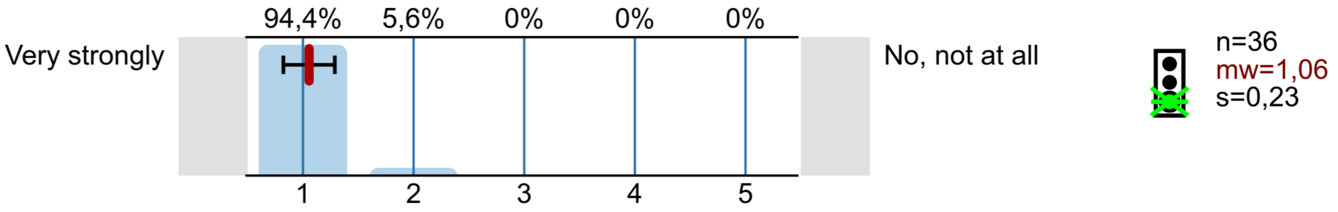
Please rate the course as a whole



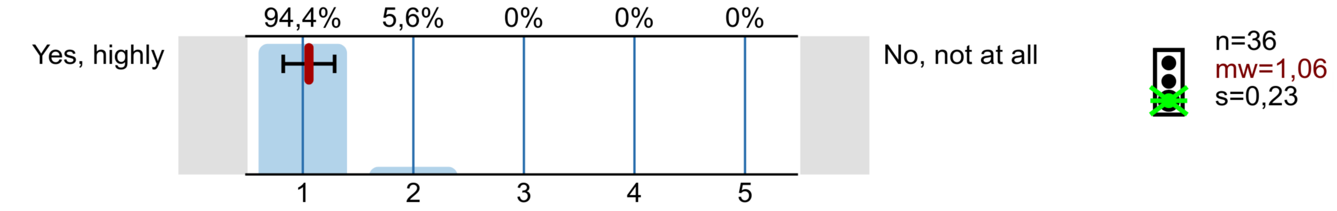
How is the course structured?



Does the lecturer appear dedicated and motivated during the course?

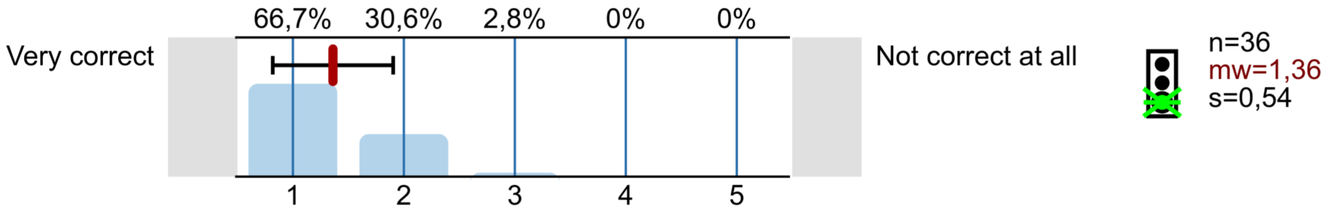


Is the lecturer responsive to questions and concerns of the students?

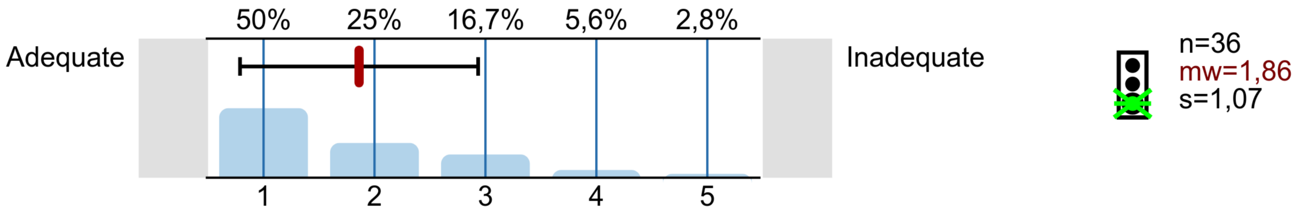


Evaluation Results

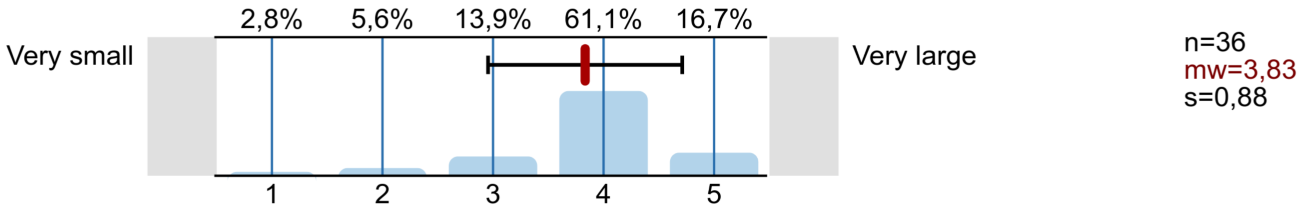
I learn a lot in this course.



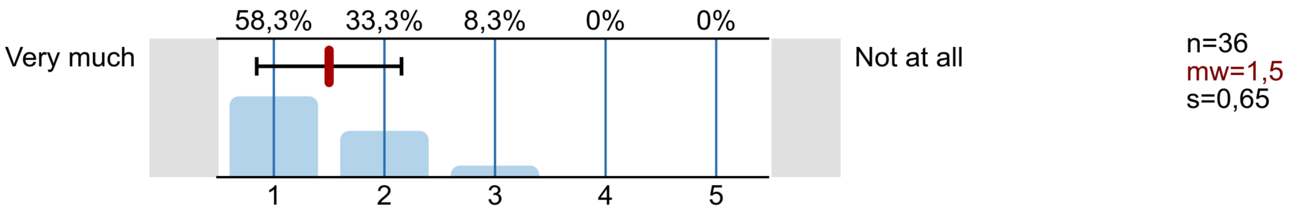
The amount of work required for this course is...



How large is the amount of work for this course?

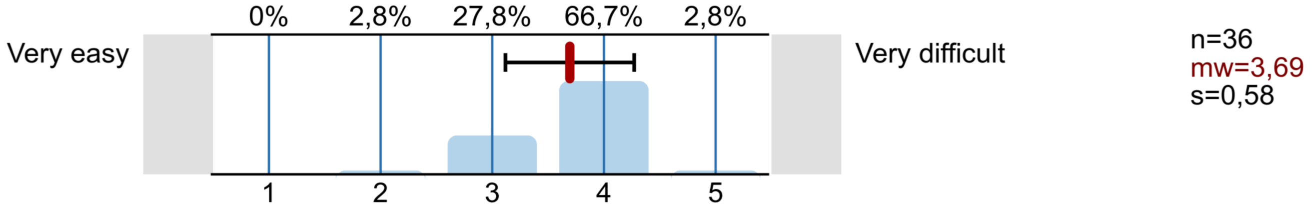


How much do you like attending this course?

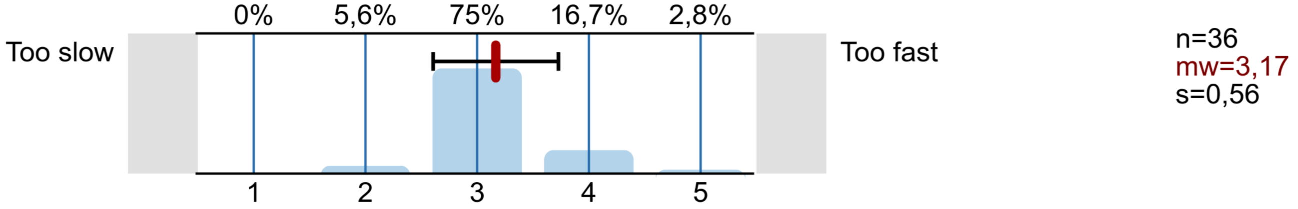


Evaluation Results

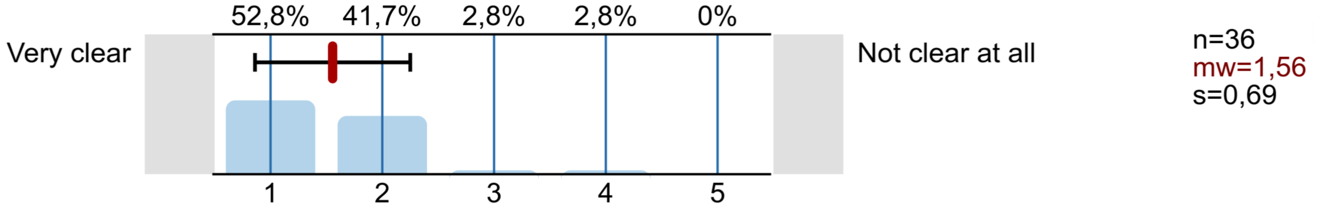
Difficulty of Contents



Speed



Clarity (using helpful examples)



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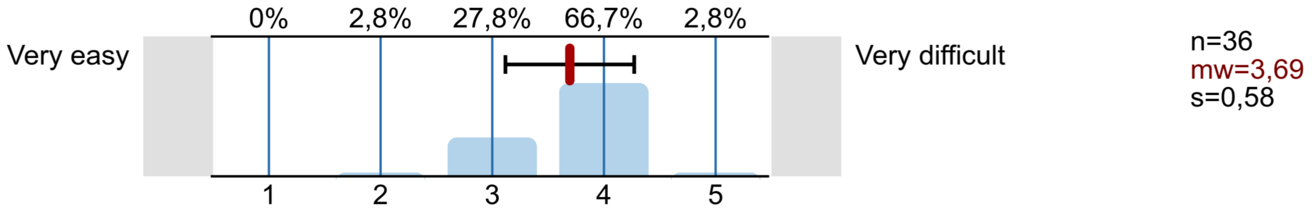

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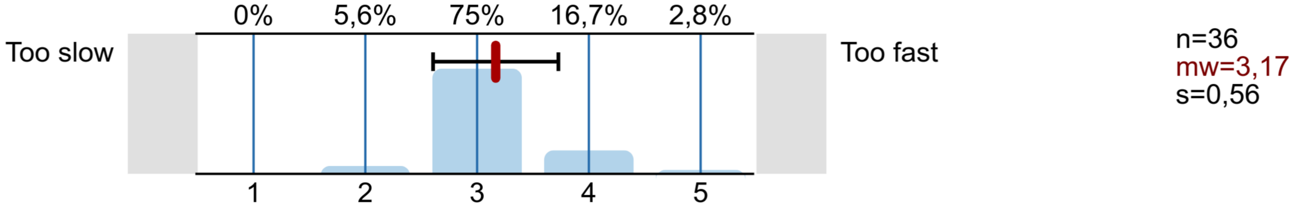

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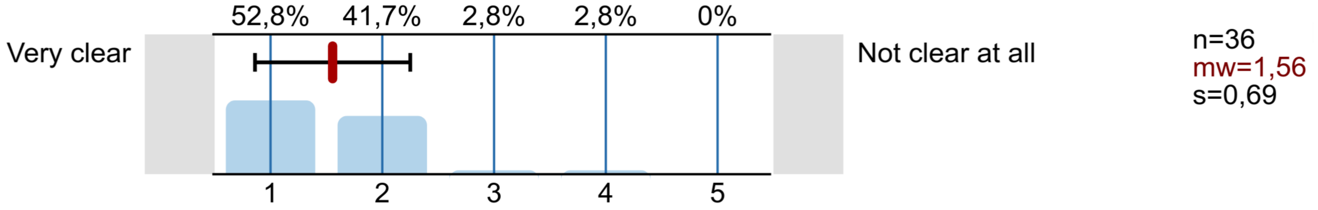

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- sheets are too difficult/time-consuming

Evaluation Results

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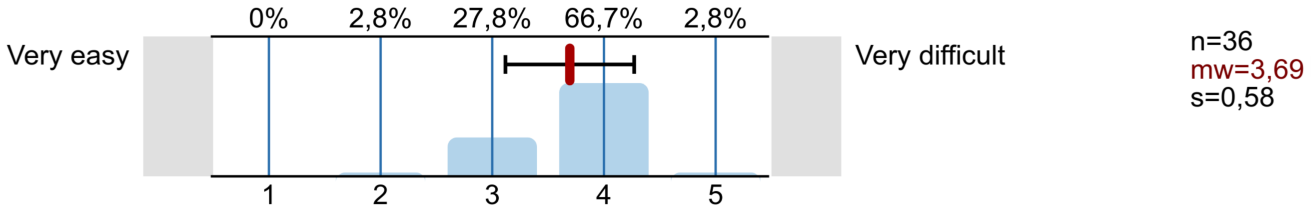

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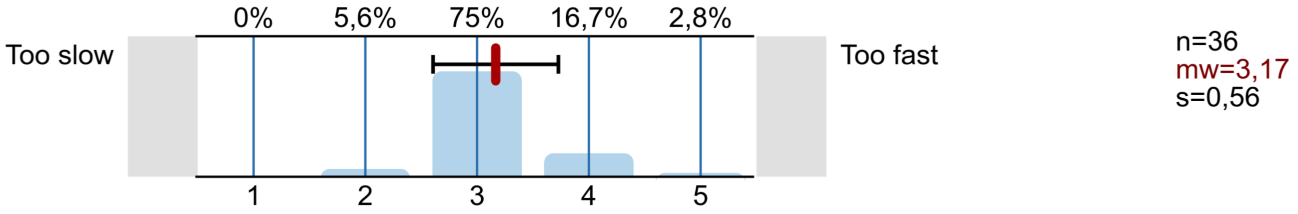

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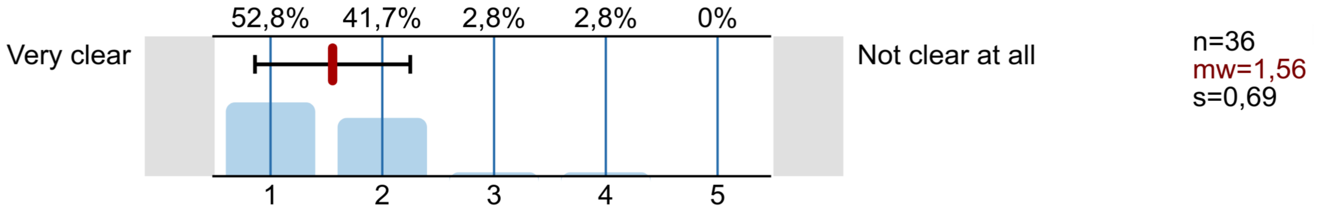

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- problem solving more fun than paper reading

Evaluation Results

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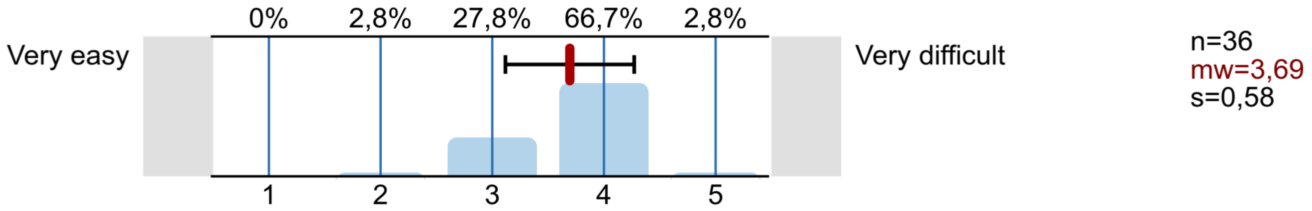

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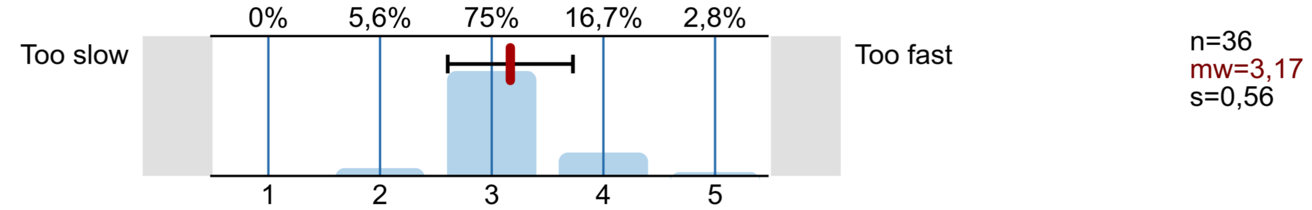

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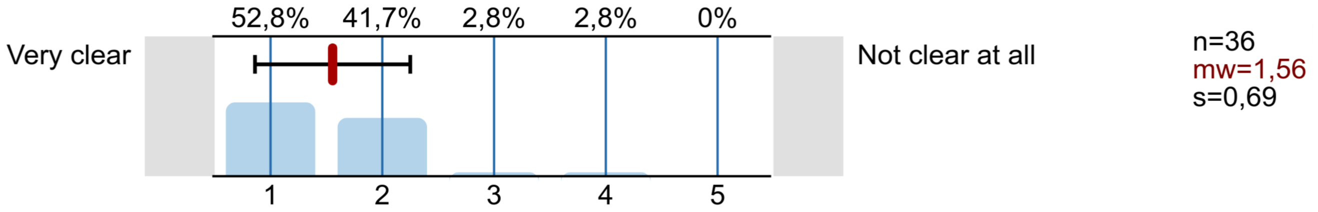

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- implementation tasks

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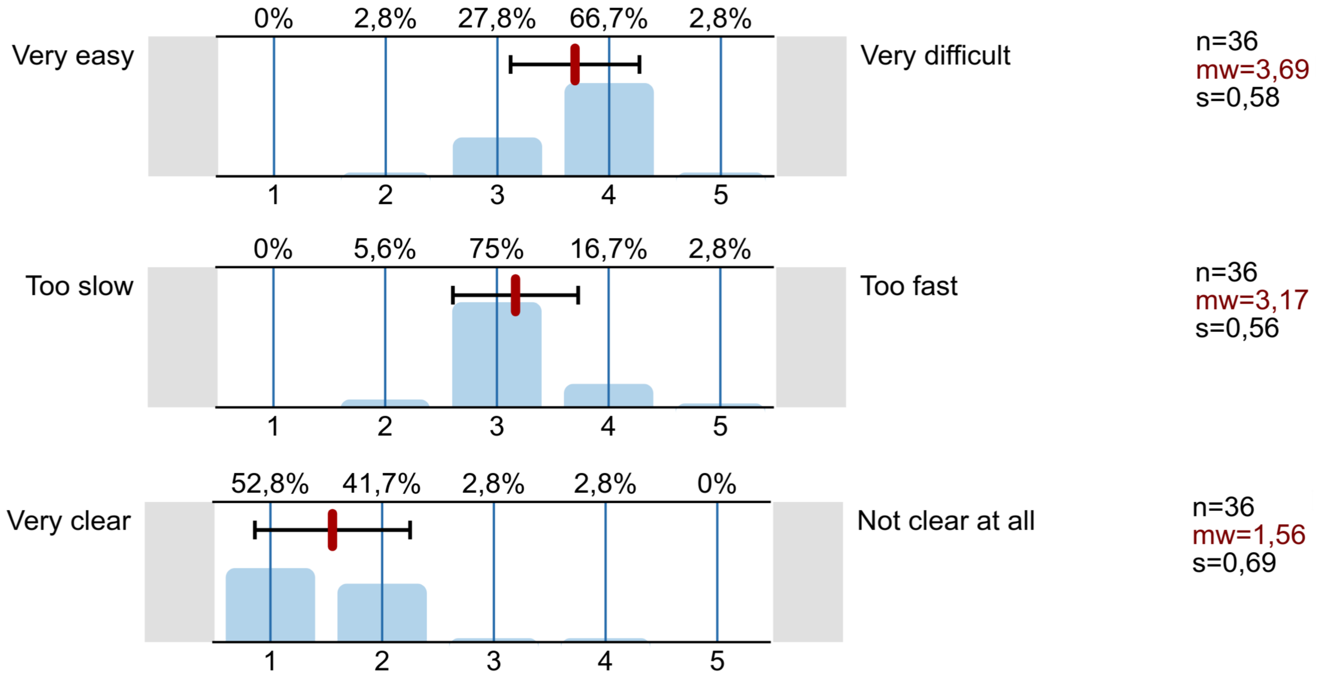

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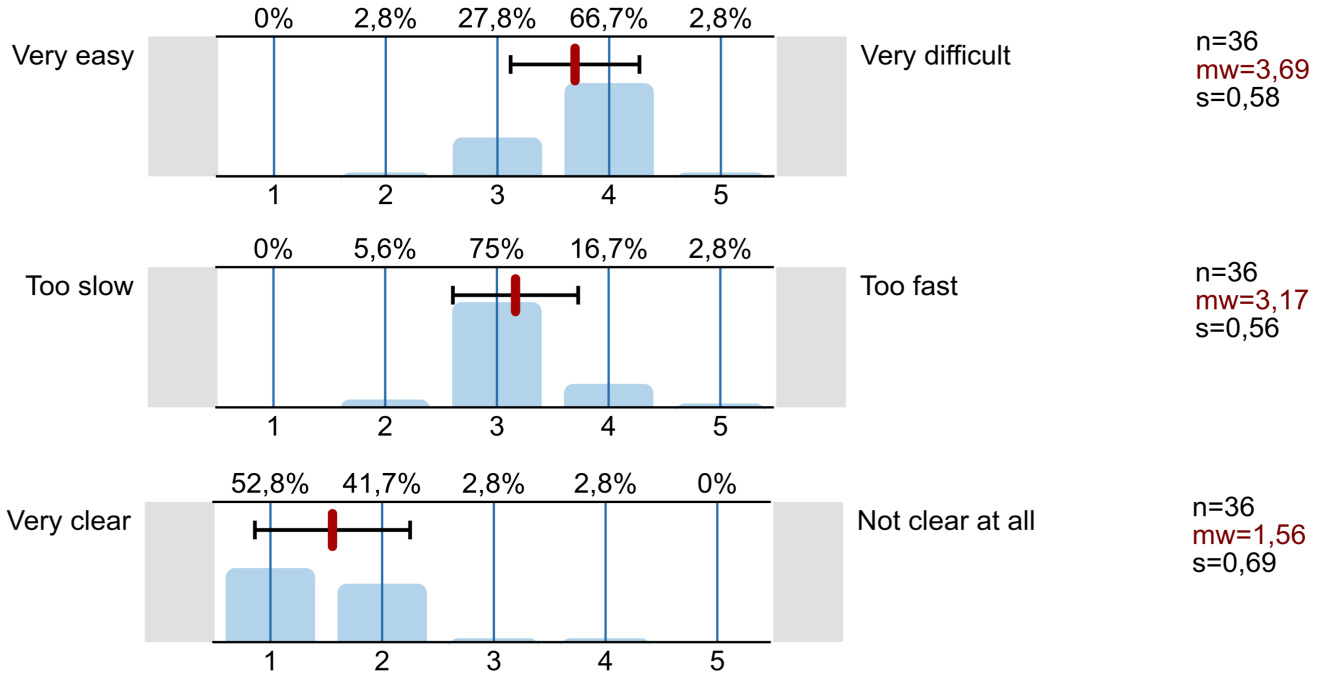

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- implementation tasks
- discussion of old sheet

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- discussion of old sheet too long

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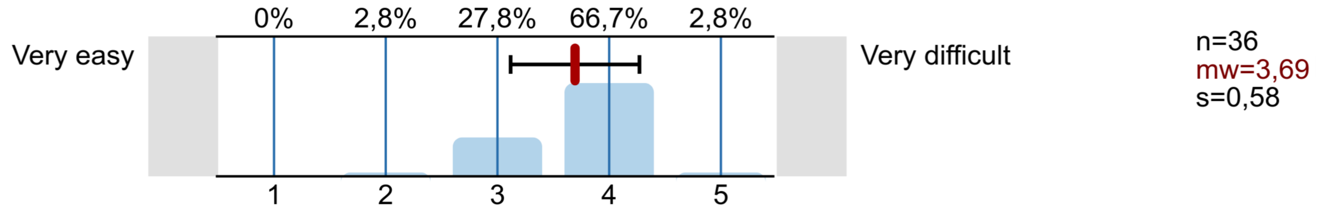

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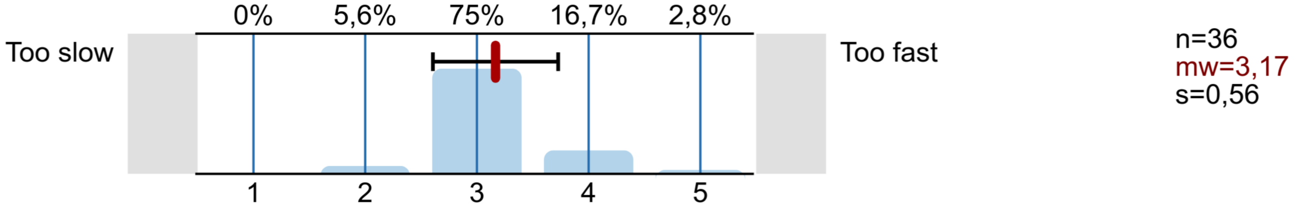

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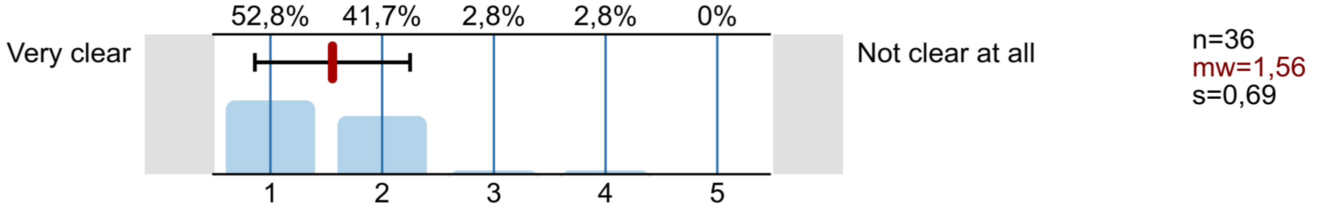

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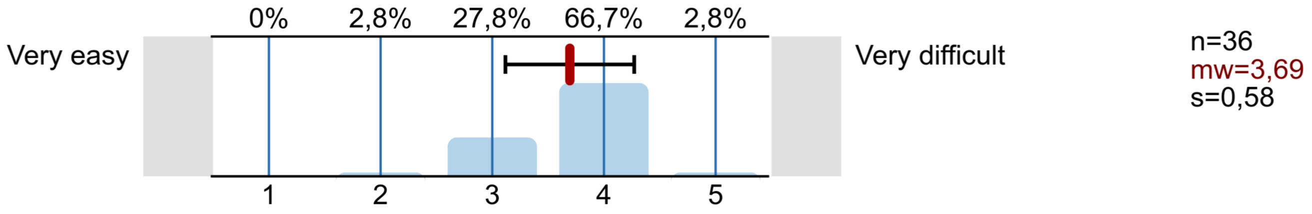

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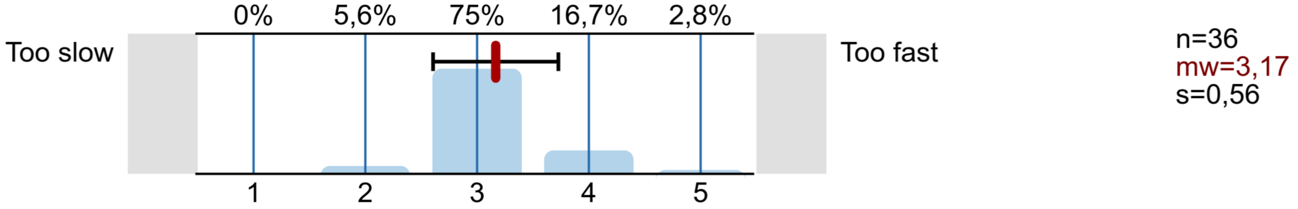

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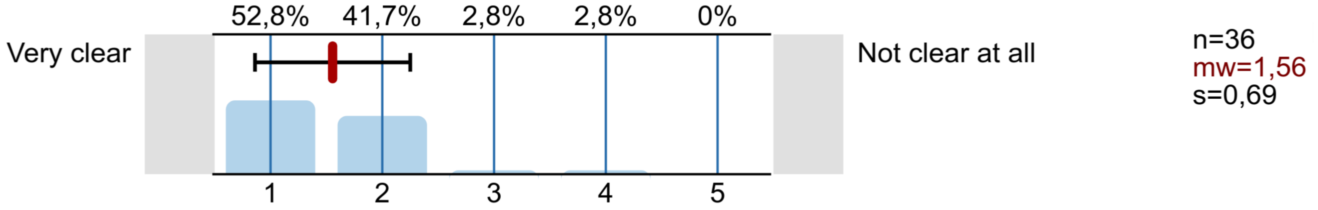

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- implementation tasks
- discussion of old sheet
- good ratio between old and new problems

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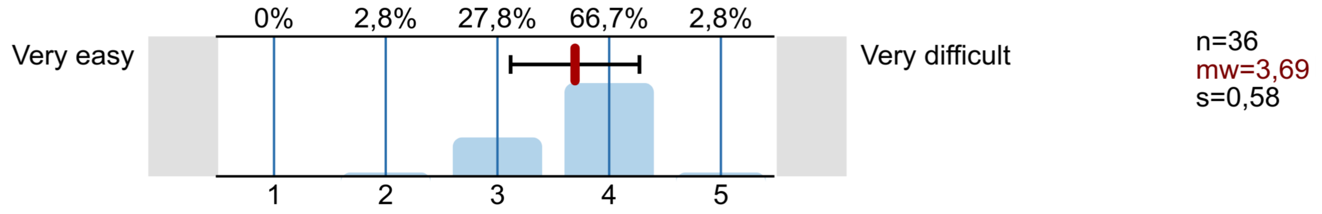

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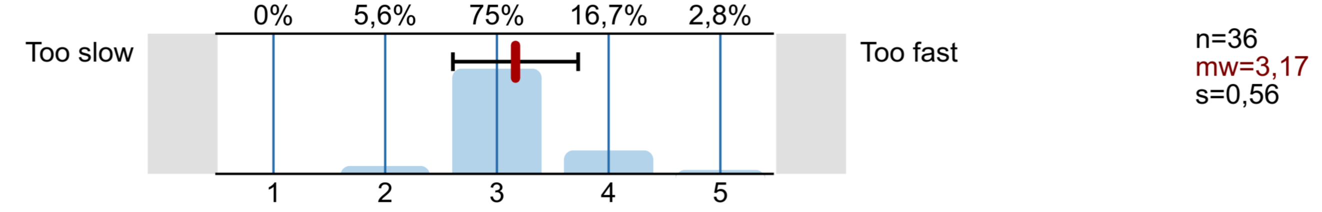
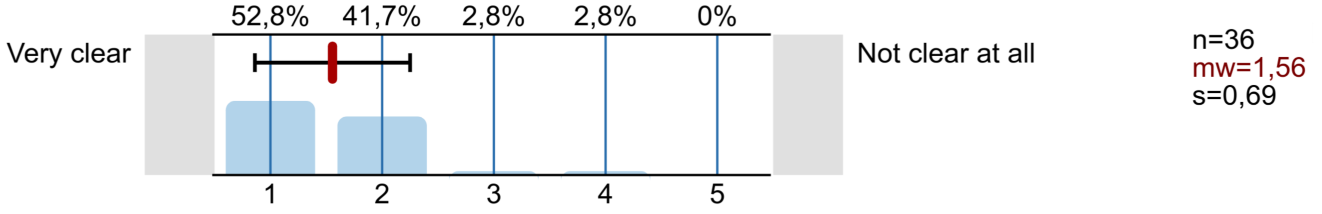


Clarity (using helpful examples)



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- implementation tasks
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- sheets are too difficult/time-consuming
- problem solving more fun than paper reading
- implementation tasks
- discussion of old sheet too long
- too little encouragement to work together

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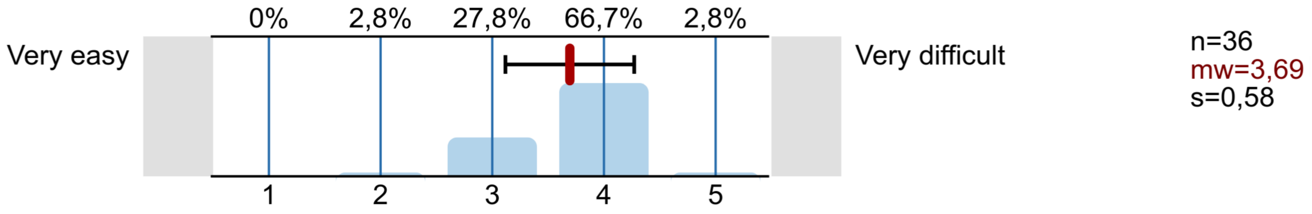

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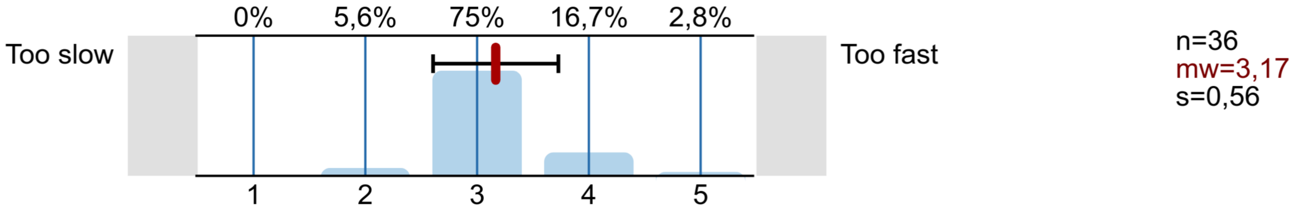
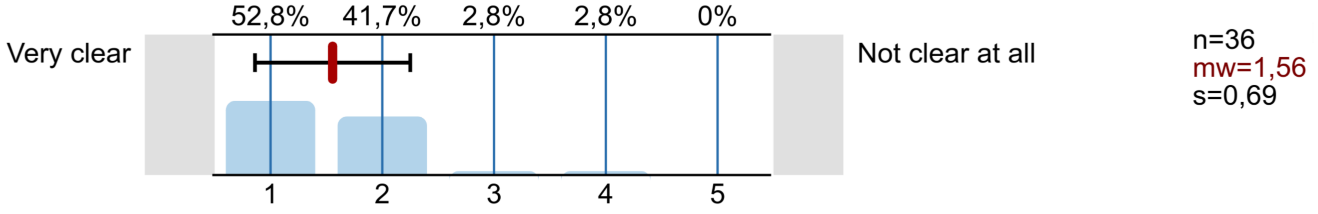


Clarity (using helpful examples)



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- problem solving and personal advice
- implementation tasks
- discussion of old sheet
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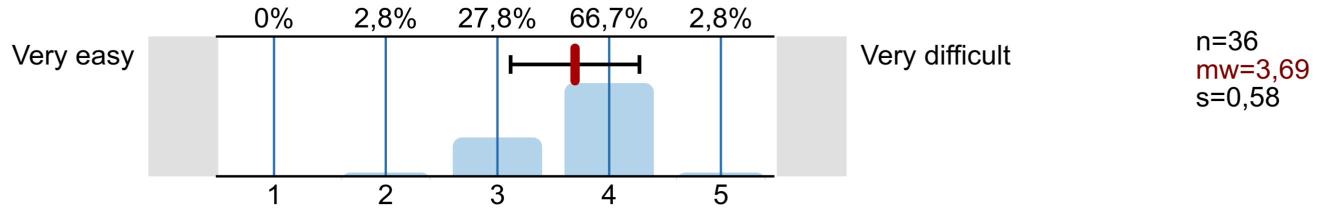

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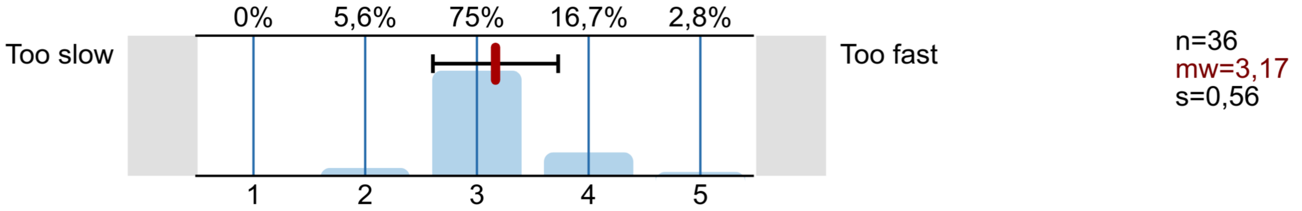

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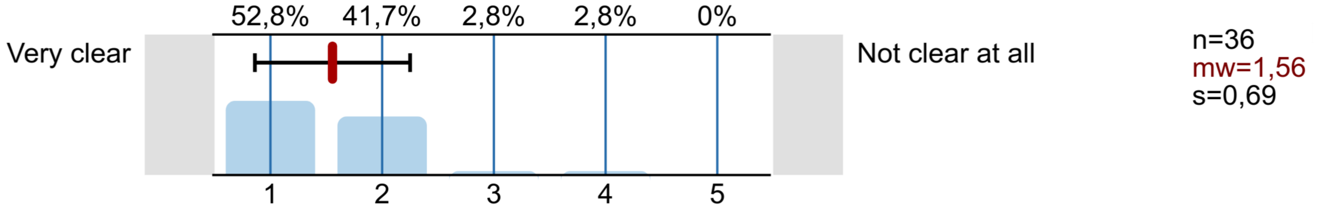

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- problem solving and personal advice
- hints for new sheet
- implementation tasks
- discussion of old sheet
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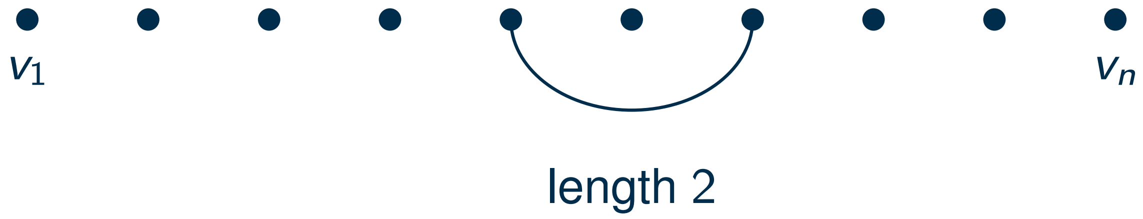
Sheet 6 - Dominating Set on Special Graphs

Given a graph G and an ordering of the vertices $V = \{v_1, \ldots, v_n\}$. The *length* of an edge $\{v_i, v_j\}$ is $|i - j|$. The parameter k is the maximum edge length. Provide an FPT algorithm to find a minimum dominating set.



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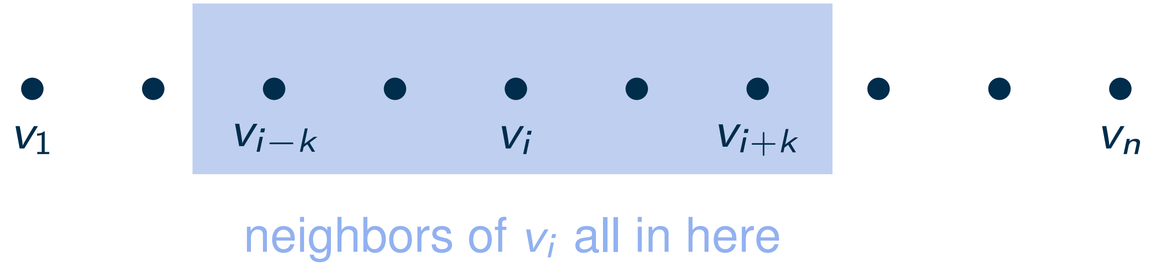

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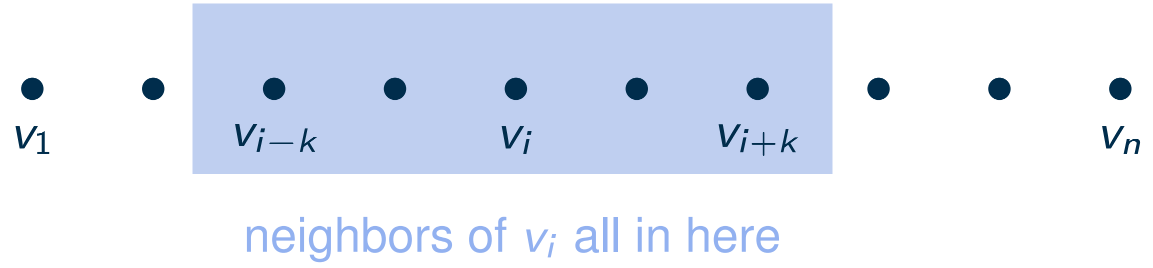

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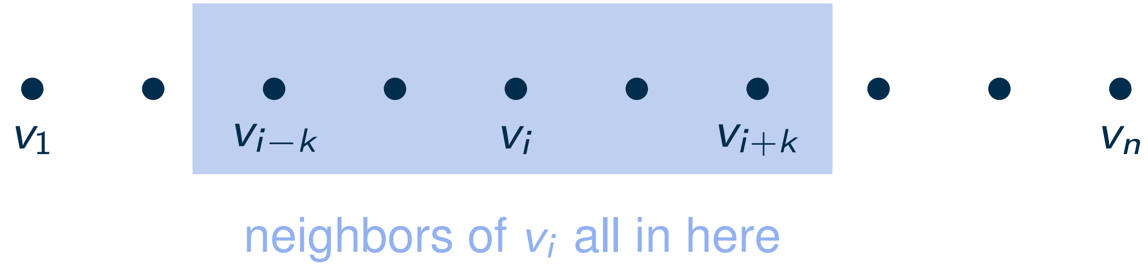

Courcelle?

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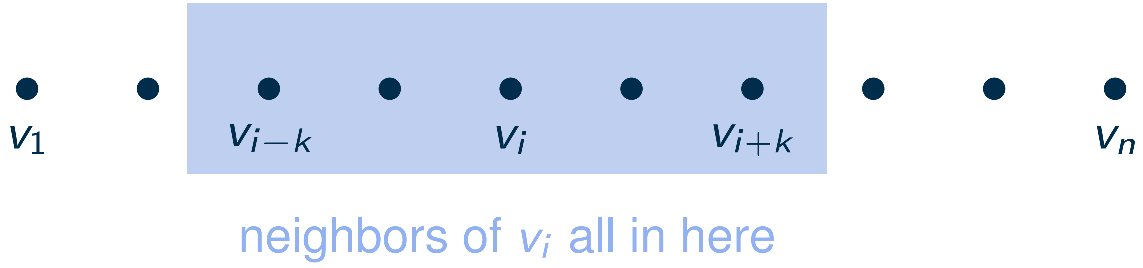

Courcelle?

can be used to show containment in FPT, but only implicitly gives us algorithm...

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- Nice path decomposition of width $k - 1$:

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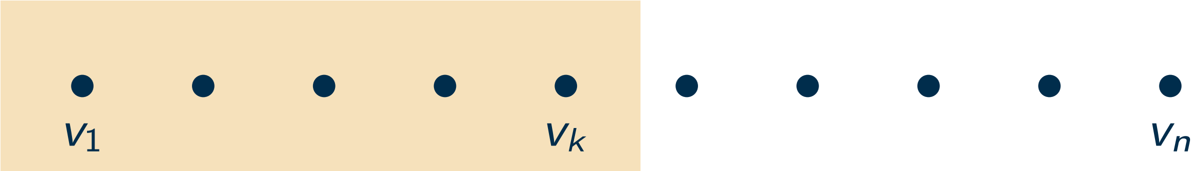

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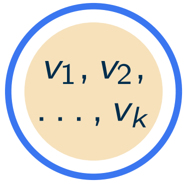

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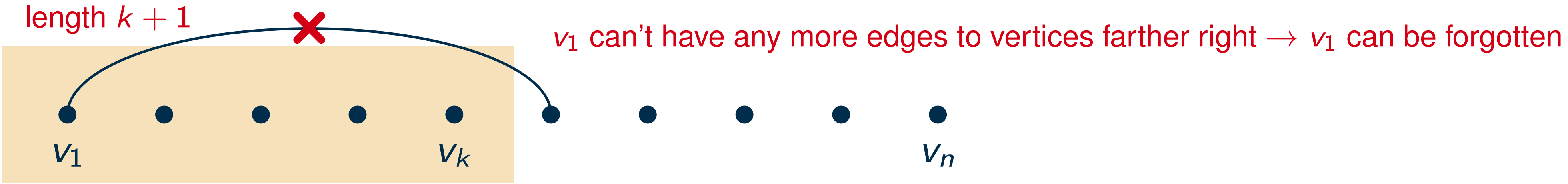

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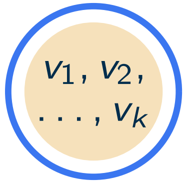

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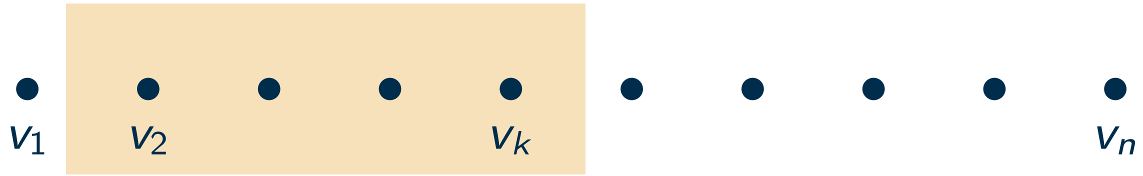

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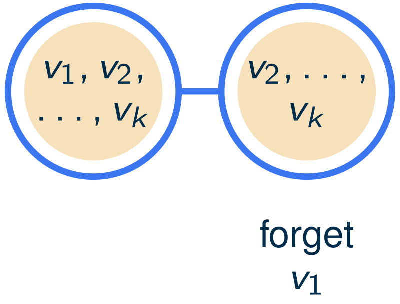

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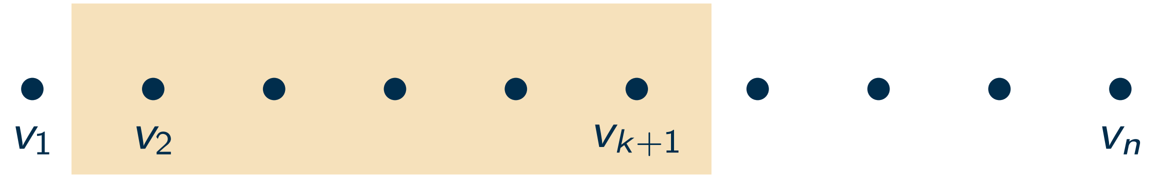

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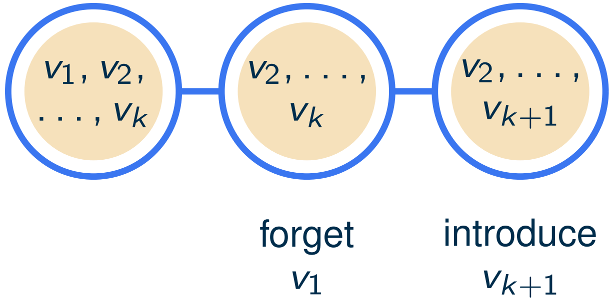

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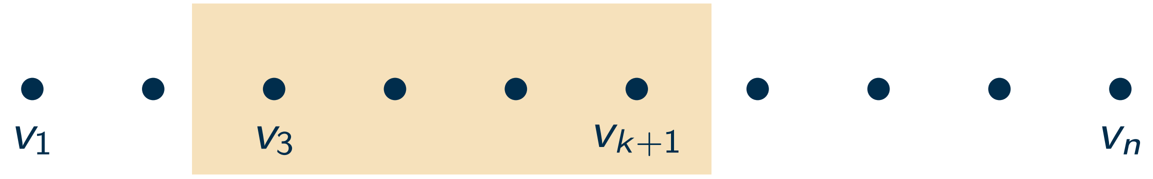

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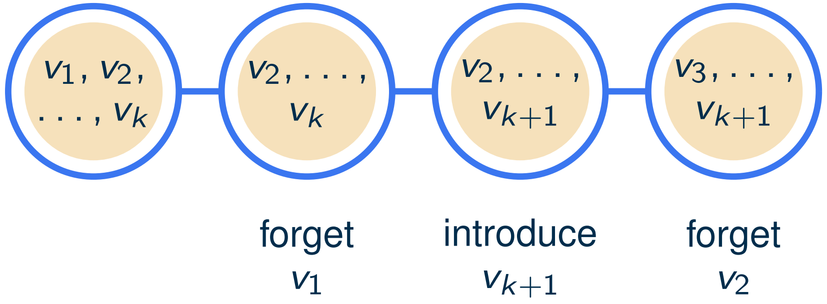

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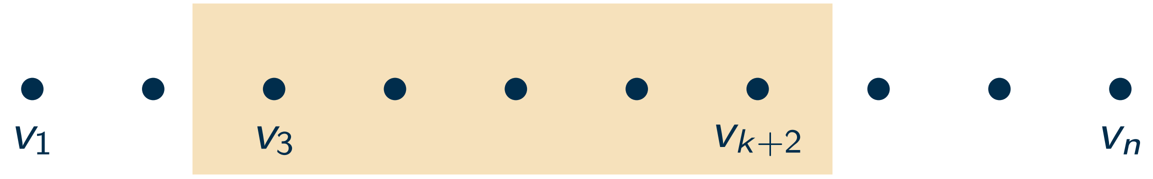

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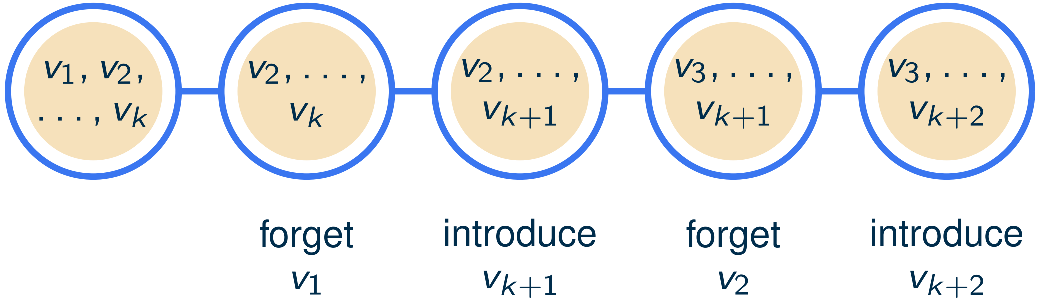

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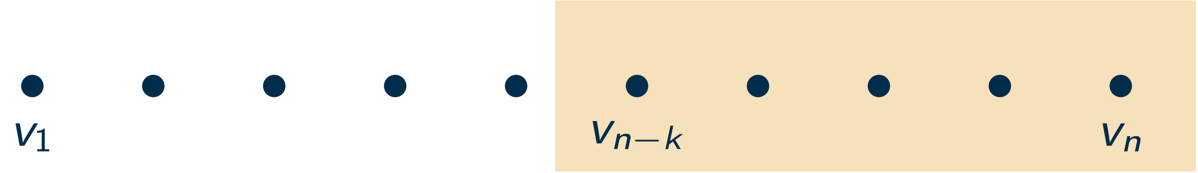

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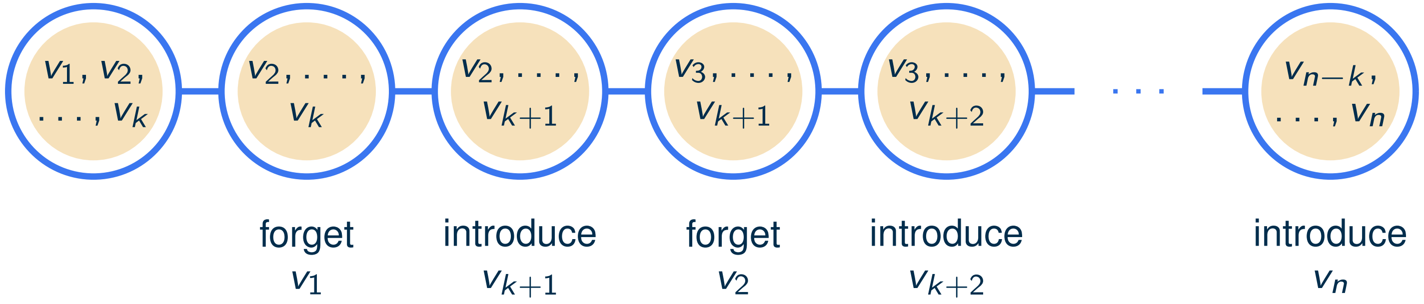

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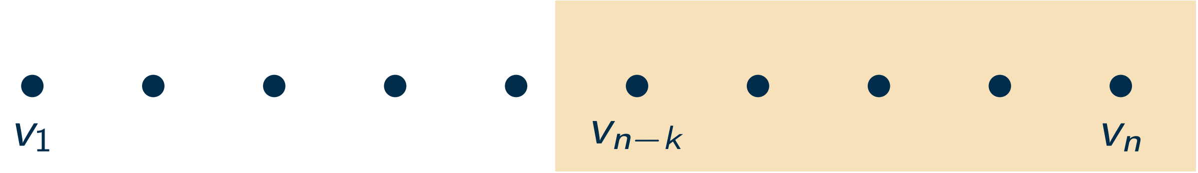
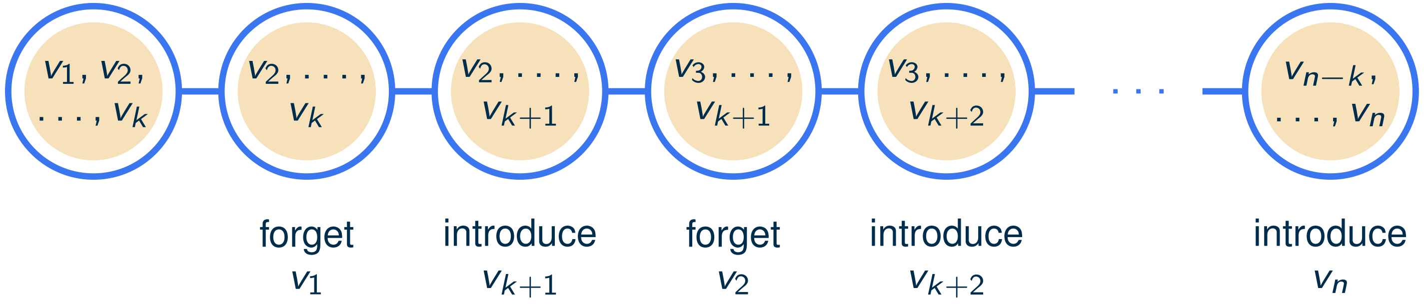


DP algorithm on path decomposition:

- how does a partial solution look like?

- Nice path decomposition of width $k - 1$:



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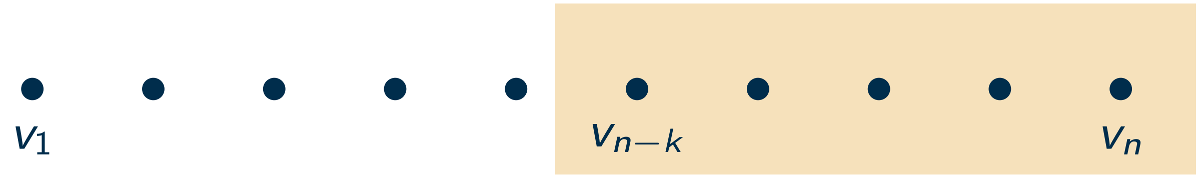

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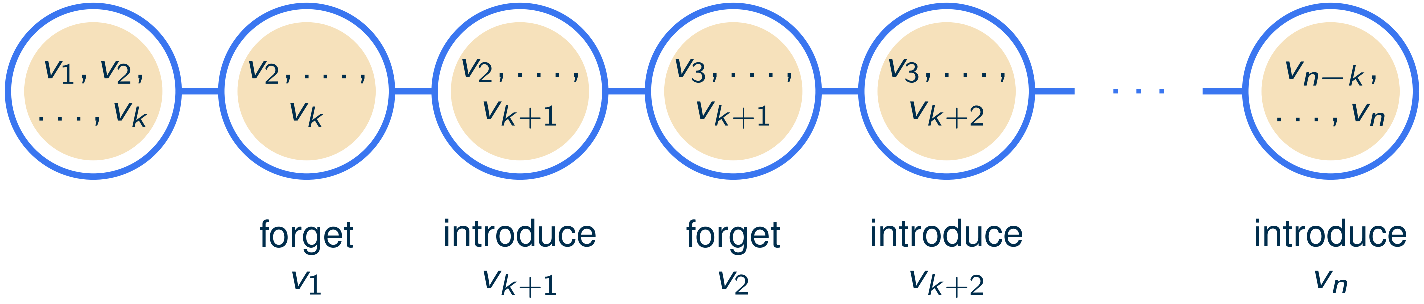

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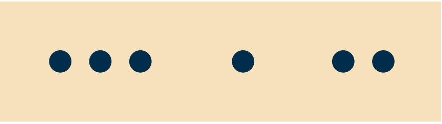

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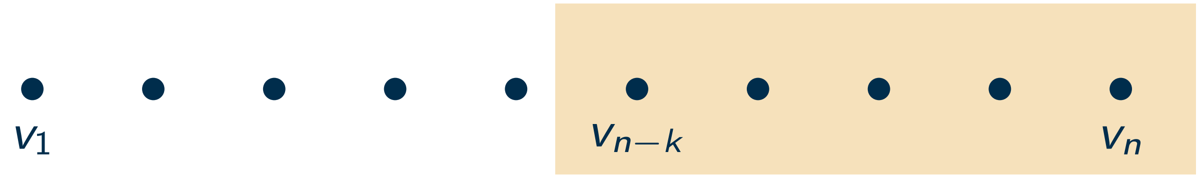

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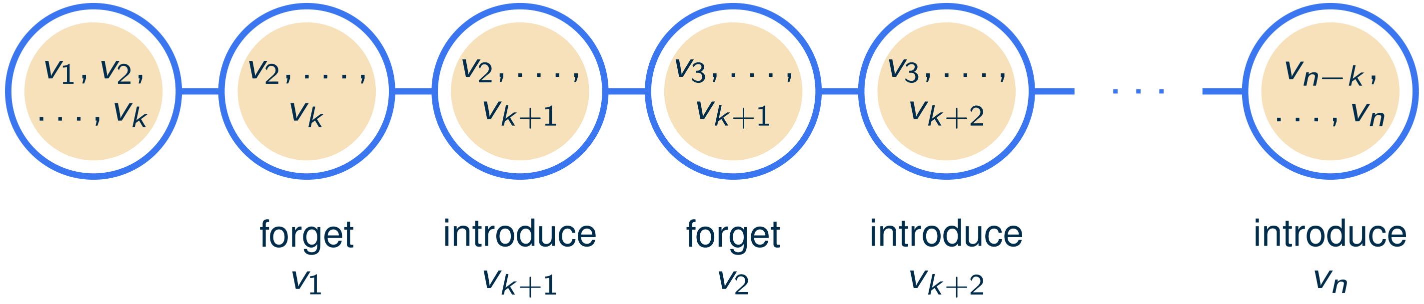

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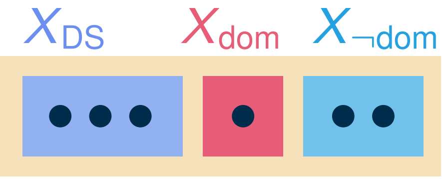

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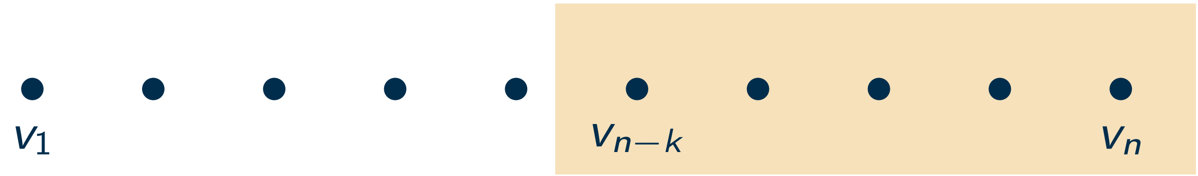

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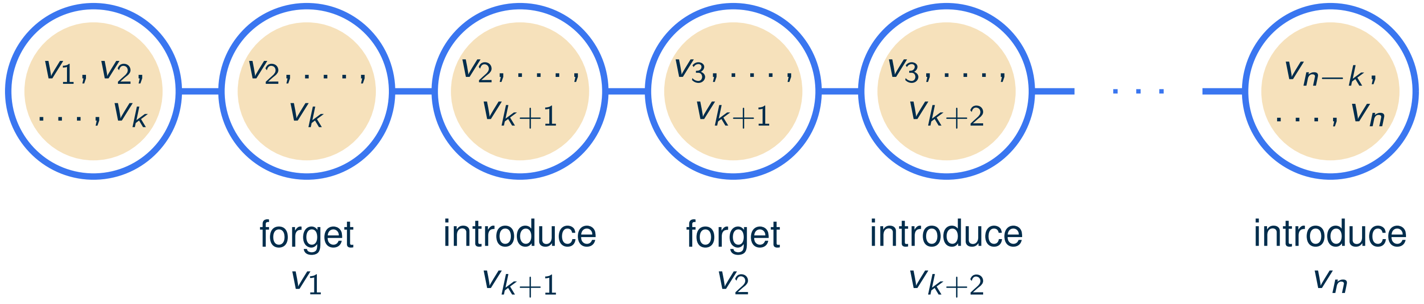

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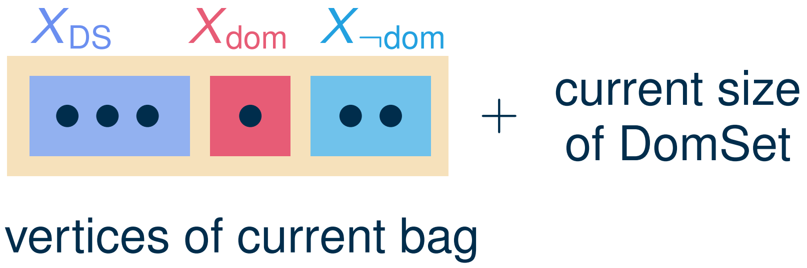

- what do we have to do for introduce nodes and forget nodes?

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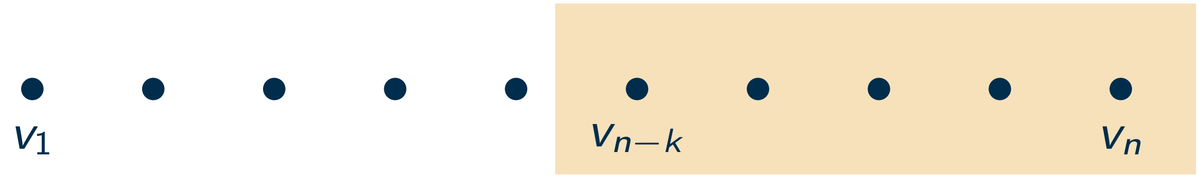

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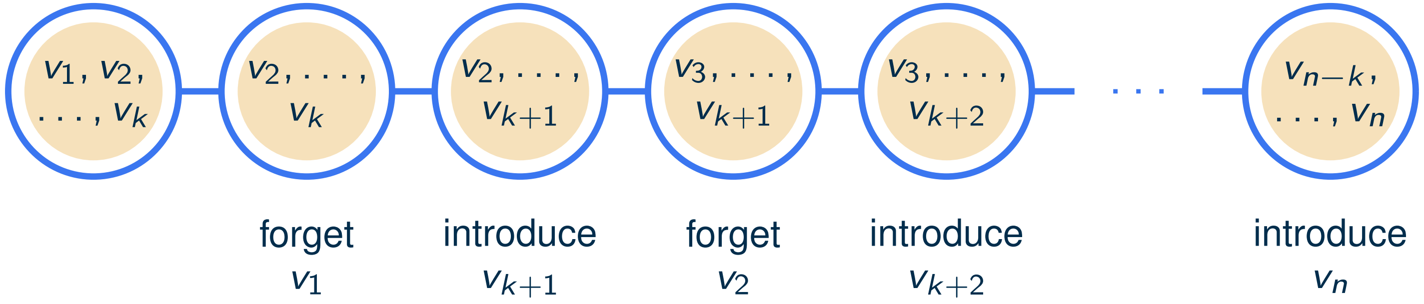

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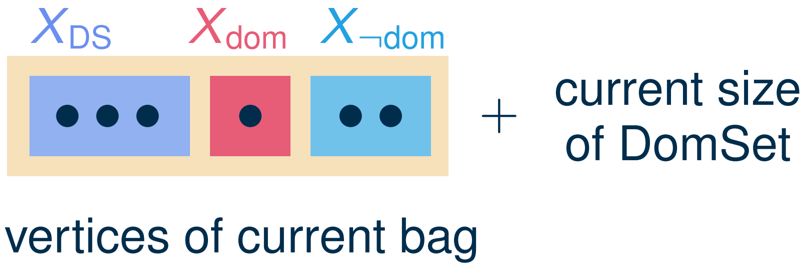

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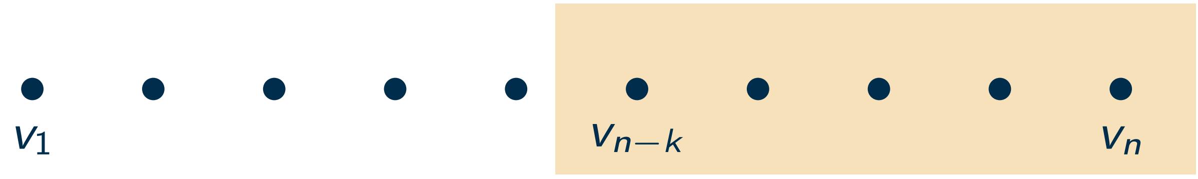

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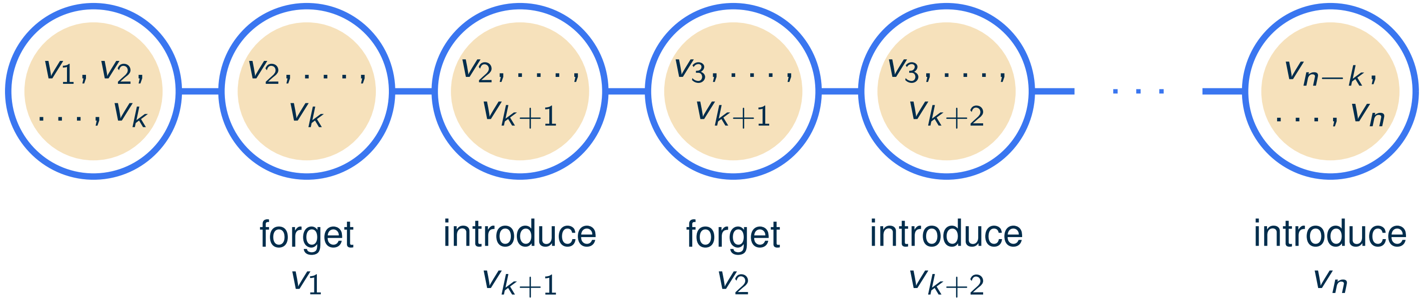

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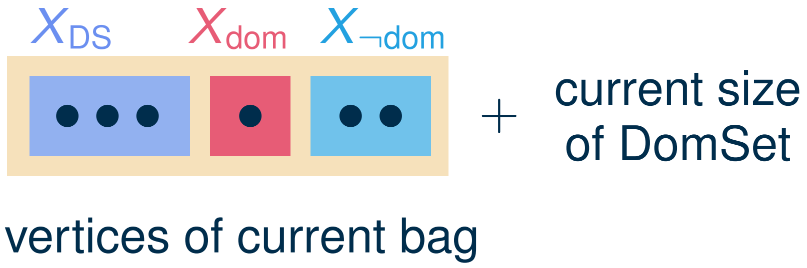

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- runtime?

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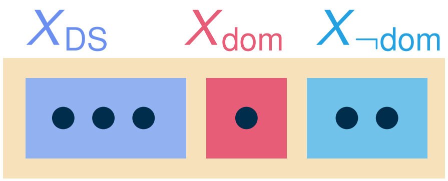

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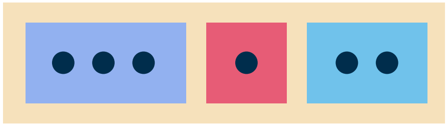

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what else do we need to store?

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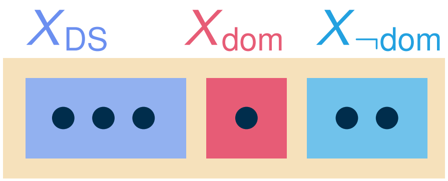

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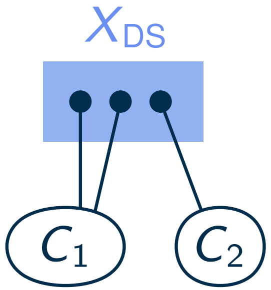

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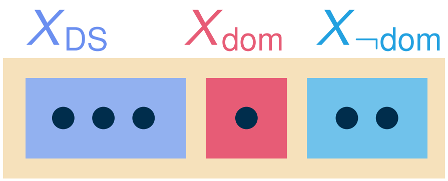

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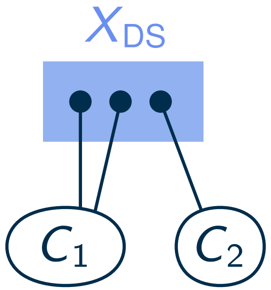

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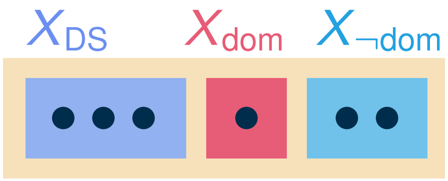

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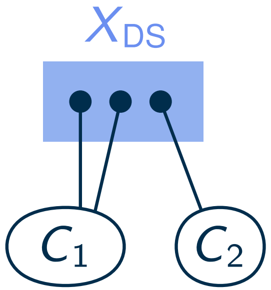

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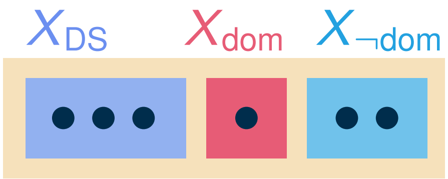

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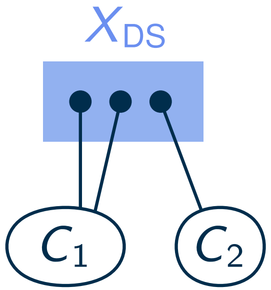

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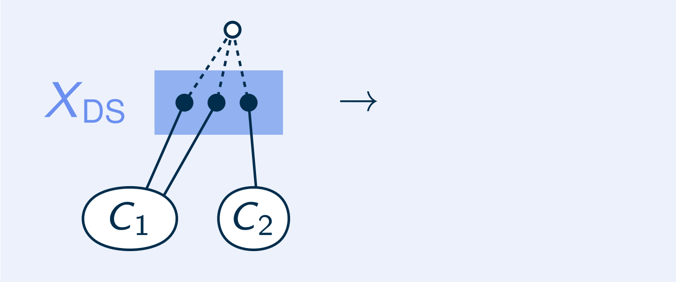

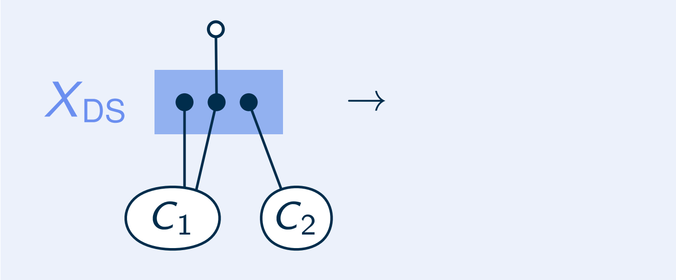

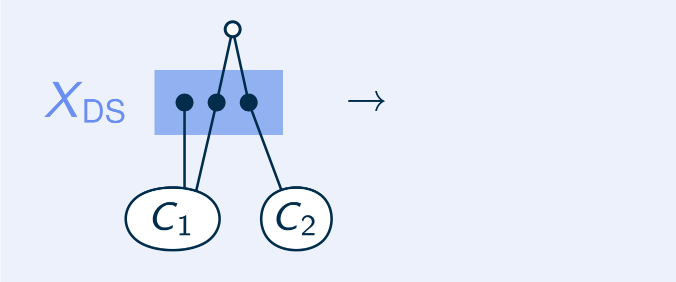

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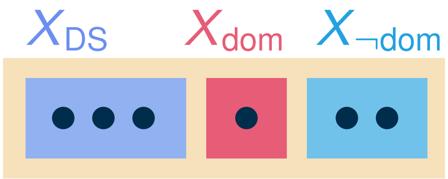

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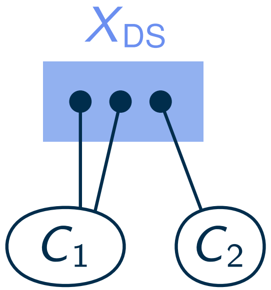

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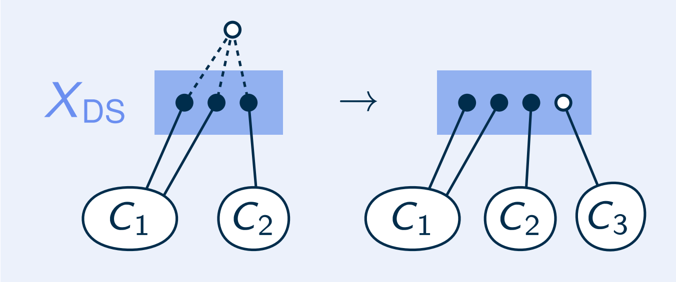

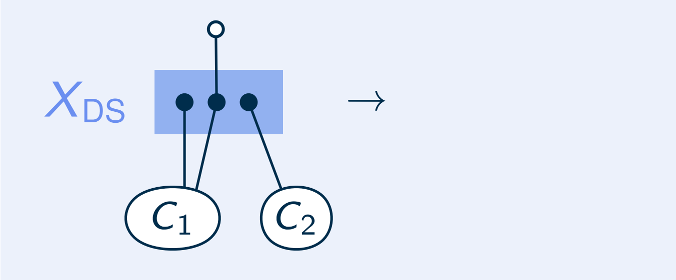

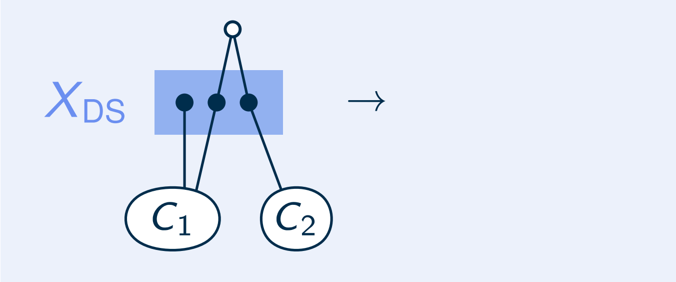

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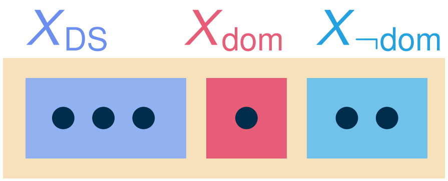

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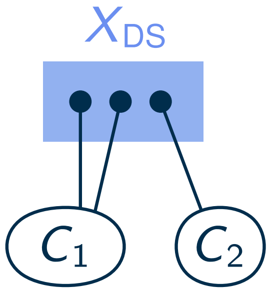

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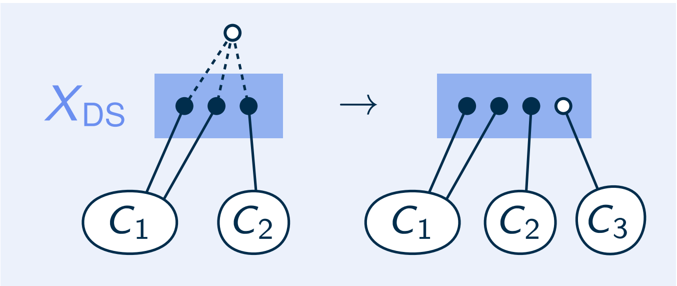

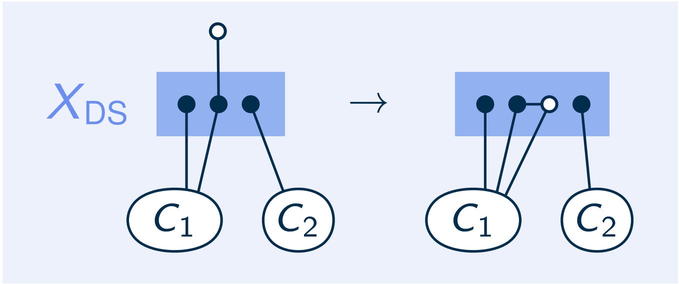

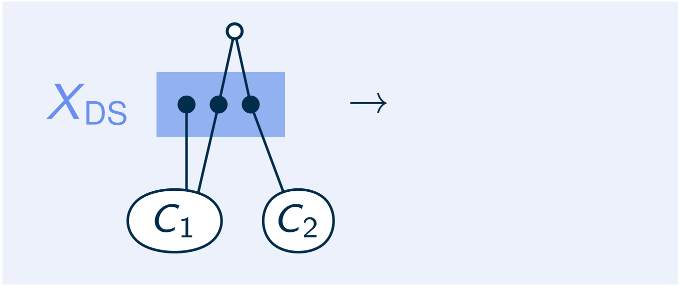

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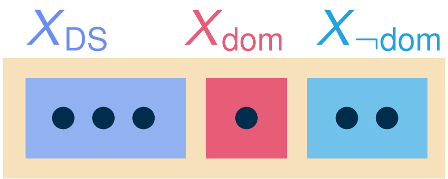

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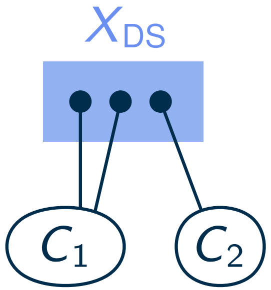

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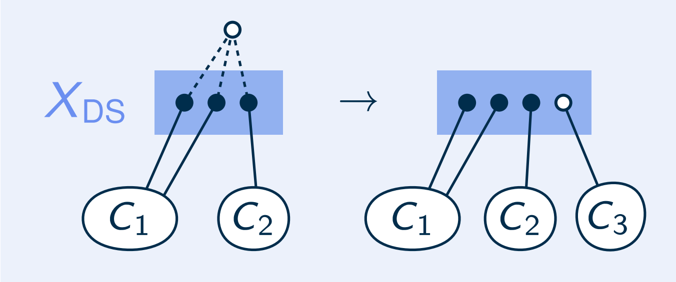

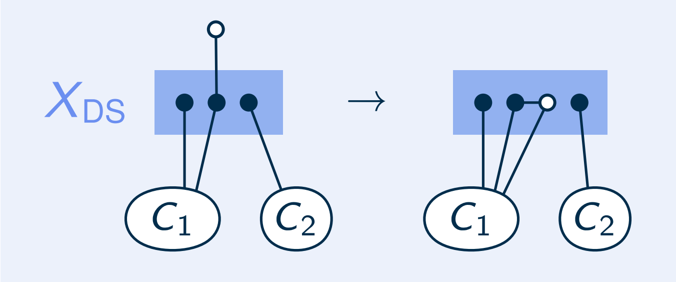

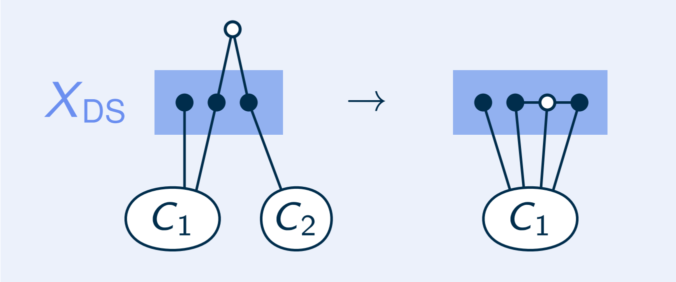

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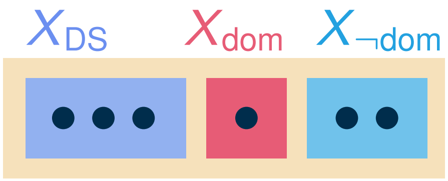

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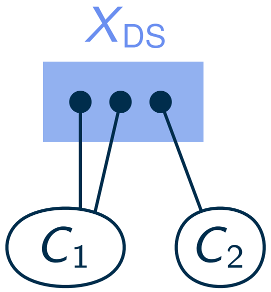

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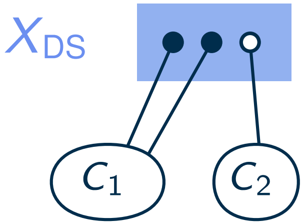

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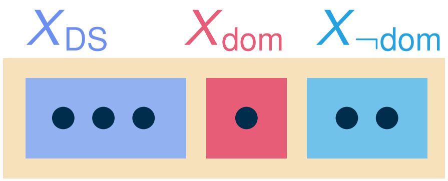

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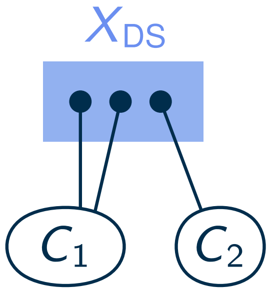

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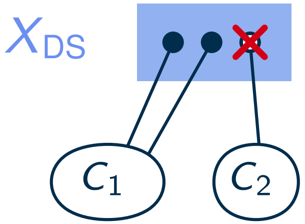

C_2 can not be connected to the rest of the DomSet anymore!

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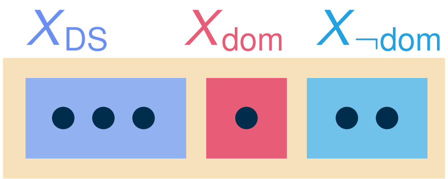

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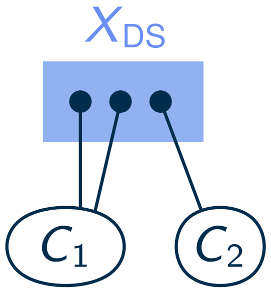

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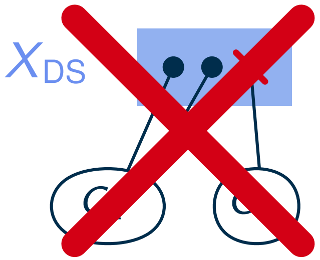


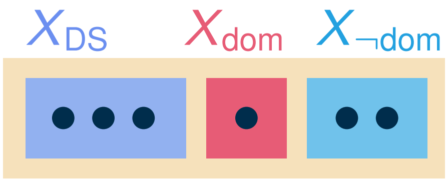
C_2 can not be connected to the rest of the DomSet anymore!

Sheet 6 - Connected Dominating Set

Given a graph G, a parameter k, and a tree decomposition of width k, provide an FPT algorithm to find a minimum connected dominating set. (use dynamic programming on the tree decomposition)

DP algorithm on tree decomposition:

- how does a partial solution look like?



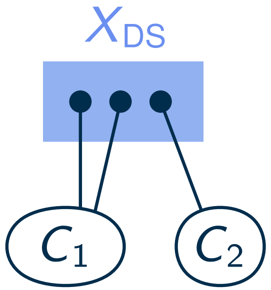
$+$ current size of DomSet

vertices of current bag

$+$

partition of X_{DS} into connected components

connection either in current X_{DS} or over already forgotten DS-vertices



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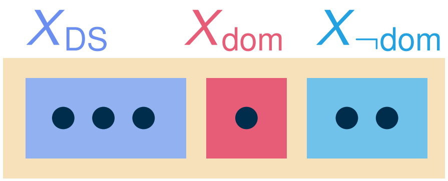

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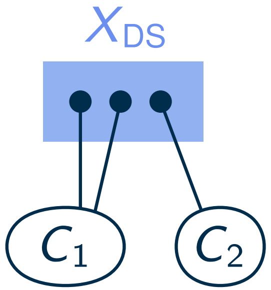

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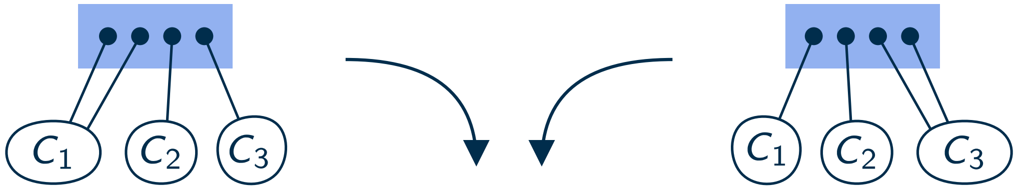

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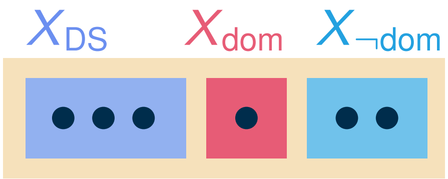

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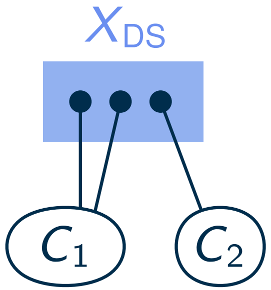

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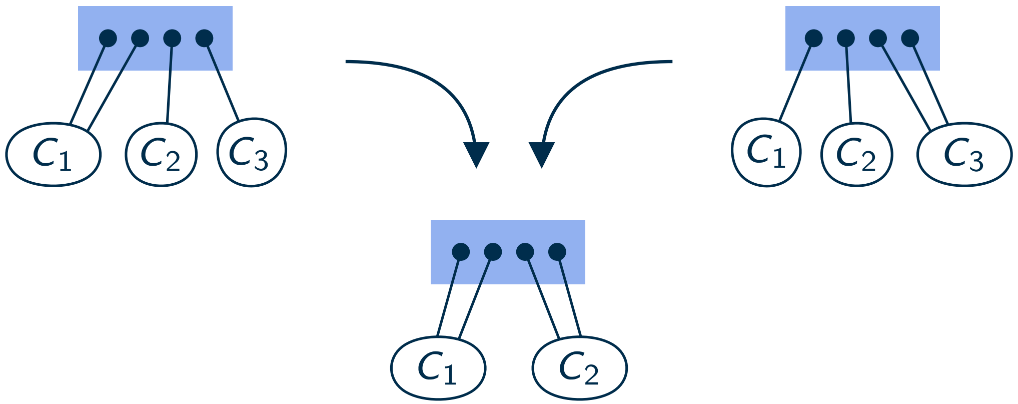

Recap: Parameterized reduction

Reduction from problem $\mathcal{L}$ to problem $\mathcal{L}'$

instance (I, k)
of problem $\mathcal{L}$

$\longrightarrow$

instance (I', k')
of problem $\mathcal{L}'$

with $k' \leq g(k)$

- map each instance (I, k) of $\mathcal{L}$ to instance (I', k') of $\mathcal{L}'$ with $k' \leq g(k)$

Recap: Parameterized reduction

Reduction from problem $\mathcal{L}$ to problem $\mathcal{L}'$

instance (I, k) of problem $\mathcal{L}$ $\Rightarrow$ equivalent instance (I', k') of problem $\mathcal{L}'$ with $k' \leq g(k)$

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Reduction from problem $\mathcal{L}$ to problem $\mathcal{L}'$

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FPT time $f(k) \cdot |I|^{O(1)}$

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(f and g computable)

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- FPT algorithm for $\mathcal{L}'$ $\Rightarrow$ FPT algorithm for $\mathcal{L}$

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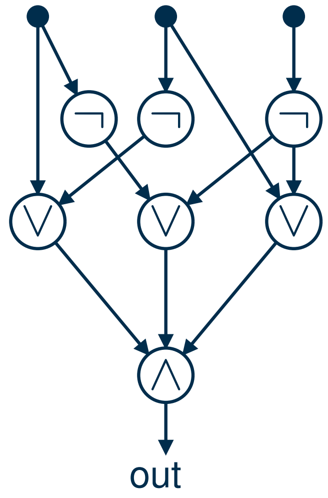
- problem $\mathcal{L}'$ known to be $\in W[t]$ $\Rightarrow$ problem $\mathcal{L} \in W[t]$

Recap: W-Hierarchy

Problem: WEIGHTED CIRCUIT SATISFIABILITY (WCS)
Given a Boolean circuit and a parameter k, is there a satisfying assignment of weight exactly k?



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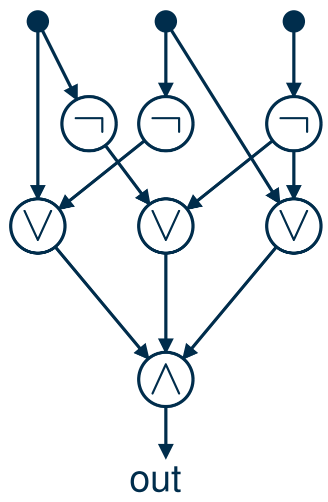
weft: largest number of nodes with in-degree > 2 on a directed path



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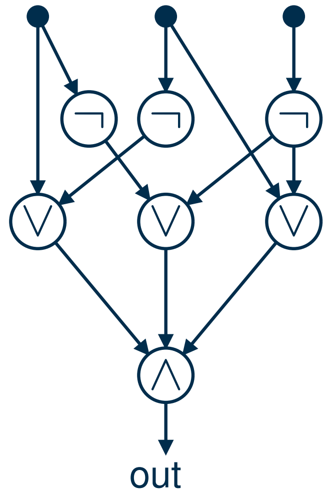

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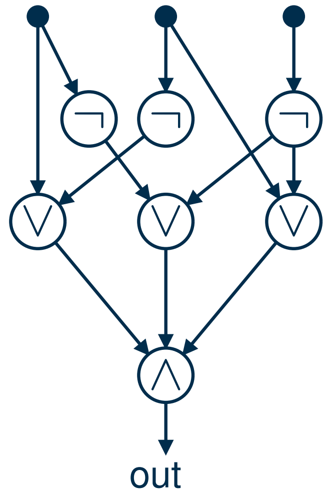

FPT

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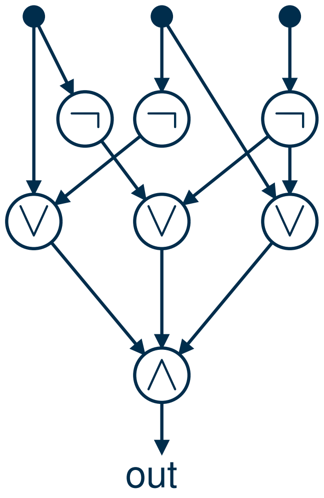

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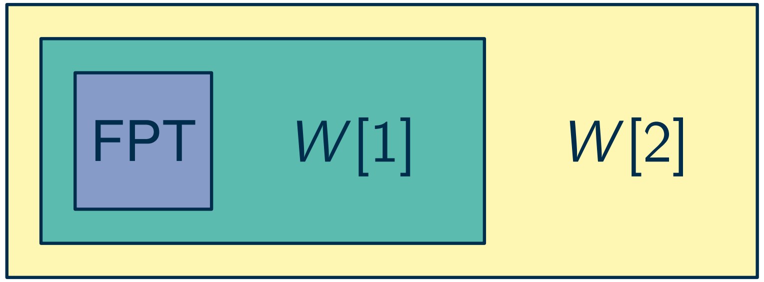

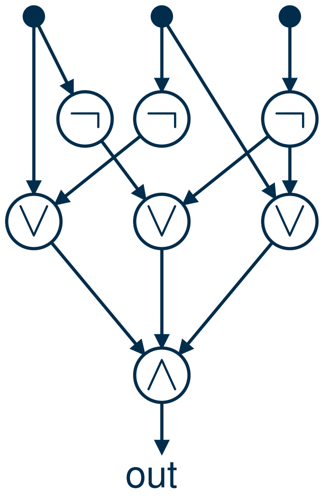

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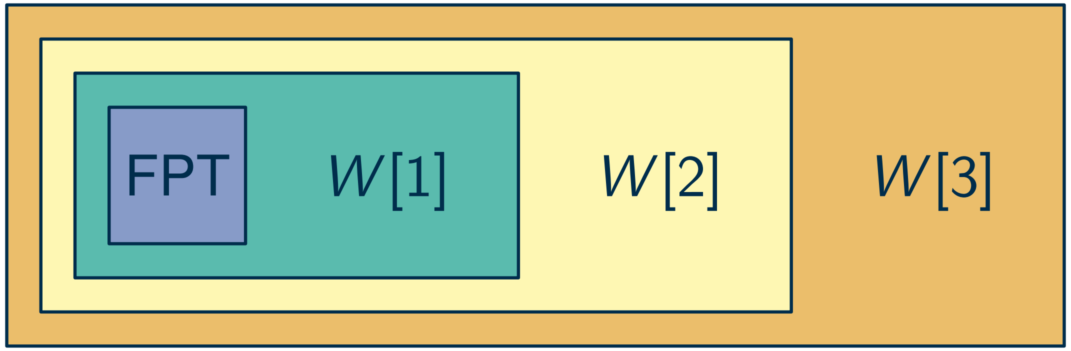

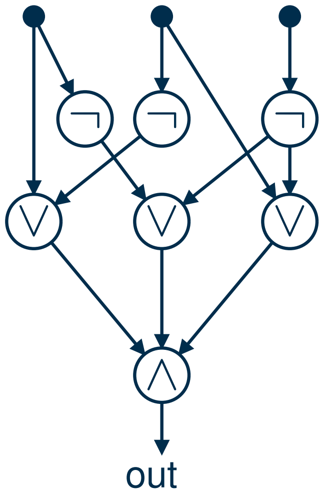

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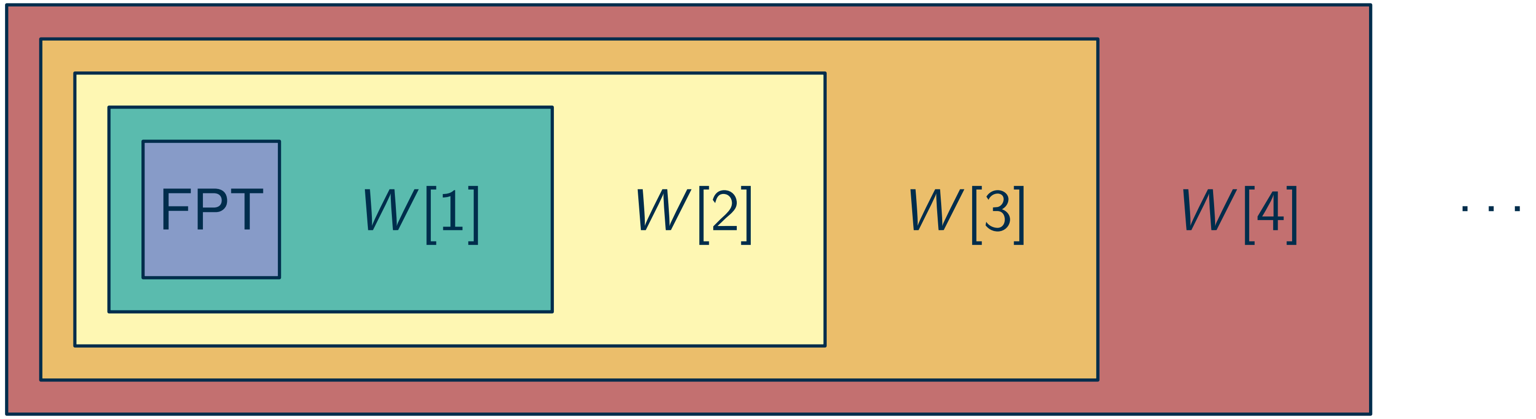

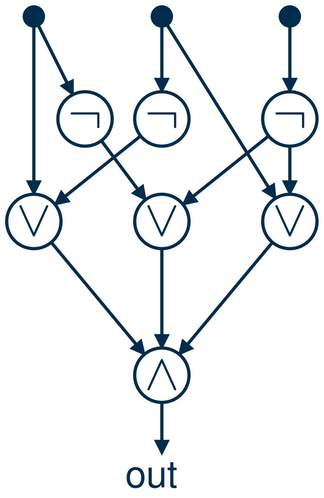

Recap: Showing $W[t]$-hardness

problem for which
I want to show
$W[t]$-hardness

Recap: Showing $W[t]$-hardness

problem known to be $W[t]$-hard

parameterized reduction

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Recap: Showing $W[t]$-hardness

problem known to be $W[t]$-hard → parameterized reduction → problem for which I want to show $W[t]$-hardness

which problem should I use?

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which problem should I use?

$W[1]$-hard problems:

- Indpendent Set
- Clique
- Multicolored Independent Set
- Multicolored Clique
- List Coloring
- Strongly Connected Steiner Subgraph
- ...
- WCS[1] (often not a good choice)

Recap: Showing $W[t]$-hardness

problem known to be $W[t]$-hard

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which problem should I use?

often good idea to try one of these first

$W[1]$-hard problems:

- INDPENDENT SET
- CLIQUE
- MULTICOLORED INDEPENDENT SET
- MULTICOLORED CLIQUE
- LIST COLORING
- STRONGLY CONNECTED STEINER SUBGRAPH
- . . .
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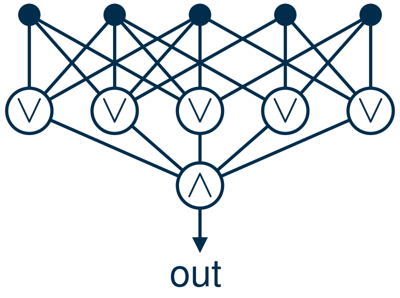
$W[2]$-hard problems:

- Dominating Set
- Hitting Set
- WCS[2] (often not a good choice)
- Weighted Monotone 2-Normalized Satisfiability



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$W[2]$-hard problems:

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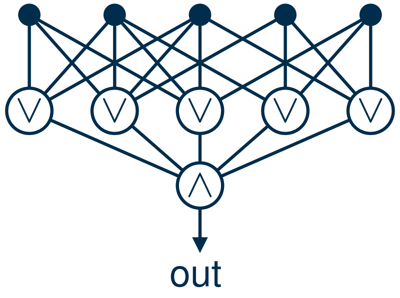


Hierarchical Problems

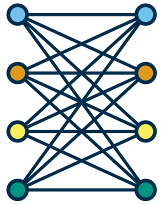
Is there a parameterized reduction from VERTEX COVER to INDEPENDENT SET? If so, provide one. If not, argue why not. ⋆

Hierarchical Problems

Is there a parameterized reduction from Vertex Cover to Independent Set? If so, provide one. If not, argue why not. ⋆

- Show $W[1]$-hardness for:

Multicolored Biclique Given a colored bipartite graph $G = (V, E)$ with k colors, does G contain the complete bipartite graph $K_{k,k}$ as an induced subgraph, where every side of the bipartition is colorful? ⋆⋆


$K_{4,4}$

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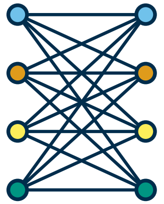

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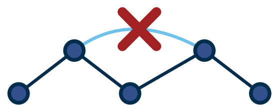
LONG INDUCED PATH Given a graph $G = (V, E)$ and a parameter k, does G contain an induced path of length k? ★★★



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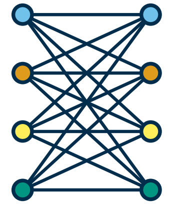

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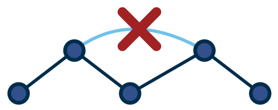
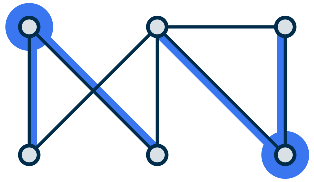


- Determine the parameterized complexity ($W[t]$-completeness for some t) of:

INDEPENDENT DOMINATING SET: Given a graph $G = (V, E)$ and a parameter k, does G contain an independent set S of size k s.t. each vertex in $V \setminus S$ is adjacent to a vertex in S? ★★★



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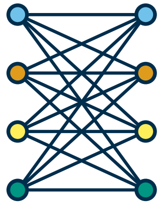

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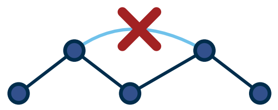

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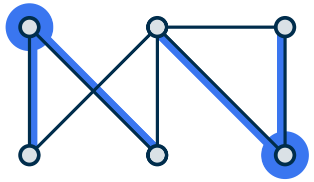
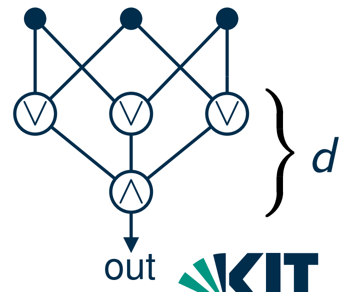


- Find an FPT algorithm for:

Monotone Weft-1 Circuits: Given an instance of WEIGHTED CIRCUIT SATISFIABILITY with weft 1, constant depth d, and without negation nodes, together with a parameter k. Is there a satisfying assignment of weight $\leq k$? ★★★★



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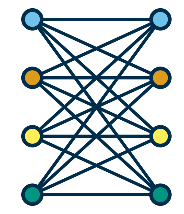

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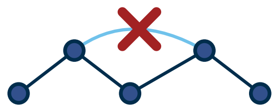

hints on the next slide

- Determine the parameterize

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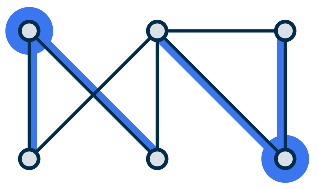

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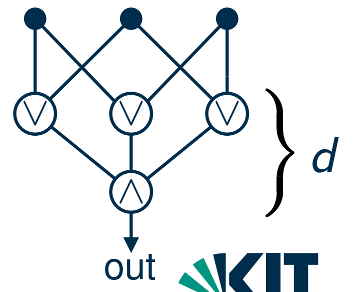

Hierarchical Problems

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- Show $W[1]$-hardness for:

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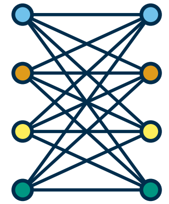
reduce from MULTICOLORED CLIQUE

★★



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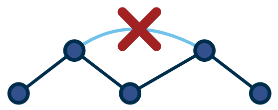
★★★



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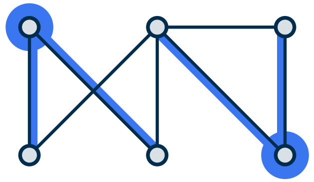

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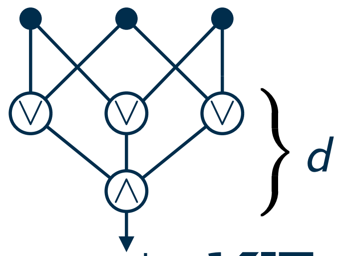
★★★★



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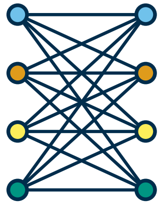
$K_{4,4}$

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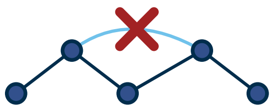
★★★

reduce from MULTICOLORED INDEPENDENT SET



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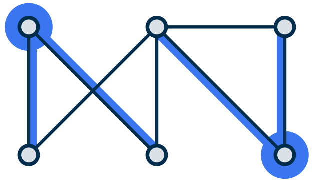

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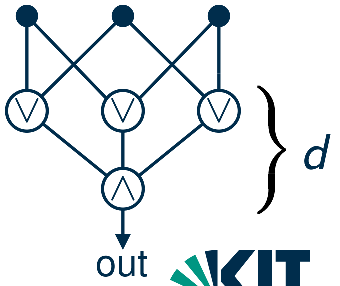

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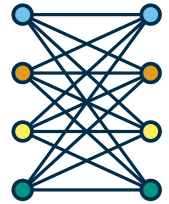
reduce from MULTICOLORED CLIQUE ★★



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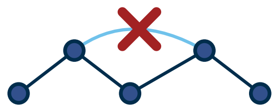
★★★ reduce from MULTICOLORED INDEPENDENT SET



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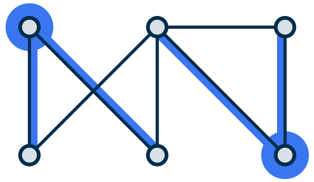
reduce from DOMINATING SET and to WCS[2]



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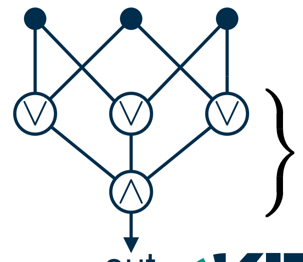

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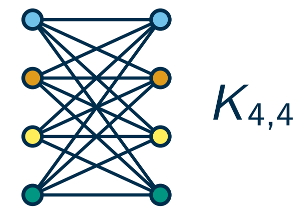

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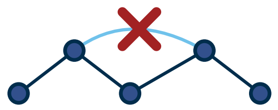

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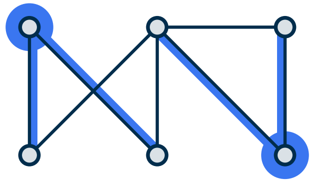

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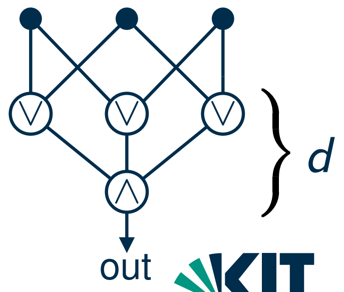
how many high in-degree gates are there?



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reduce from MULTICOLORED CLIQUE

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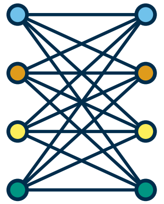

$K_{4,4}$

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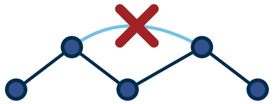
reduce from MULTICOLORED INDEPENDENT SET

★★★



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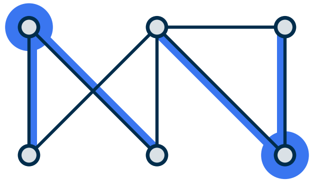
★★★

reduce from DOMINATING SET and to WCS[2]



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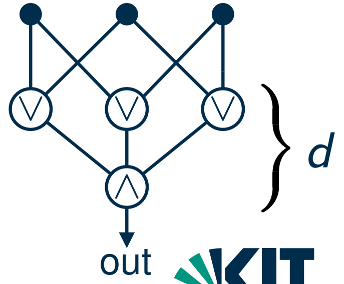
★★★★

how many high in-degree gates are there?

on how many circuit inputs do the parents of the high in-degree gates depend?



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reduce from MULTICOLORED CLIQUE

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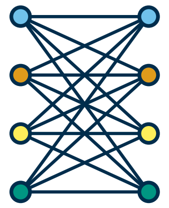

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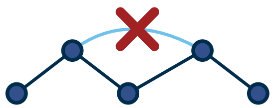

reduce from MULTICOLORED INDEPENDENT SET

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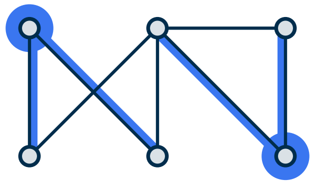

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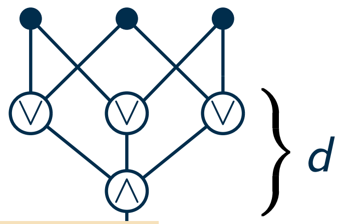


how many high in-degree gates are there?

on how many circuit inputs do the parents of the high in-degree gates depend?

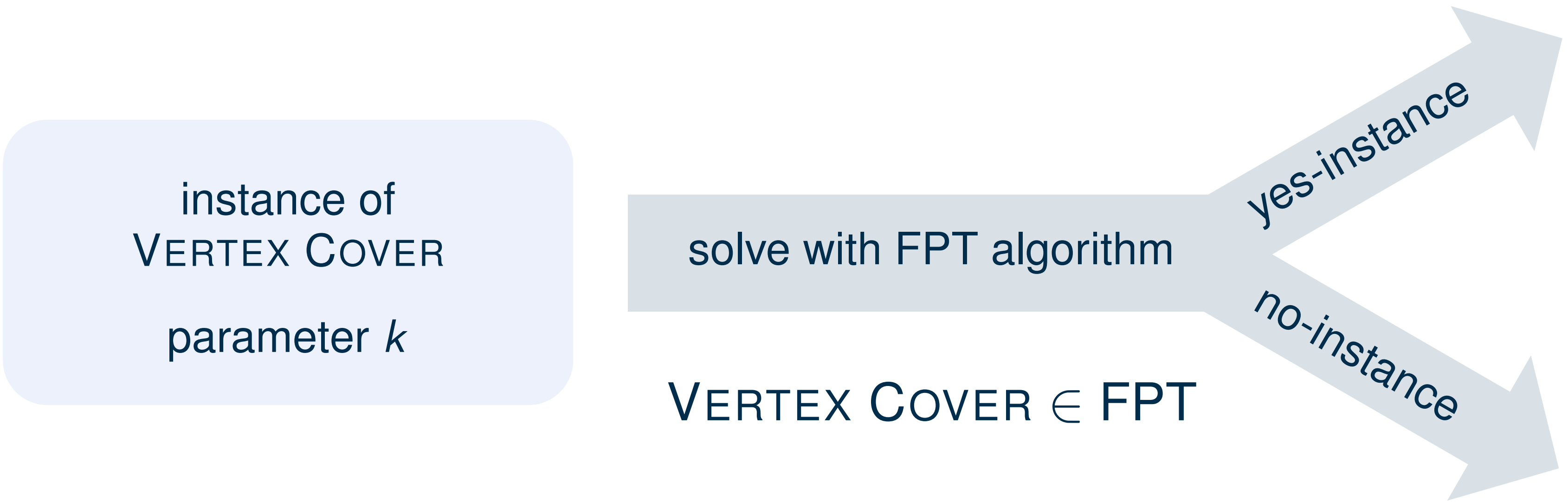
use branching algorithm for AND gates

Vertex Cover → Independent Set

instance of
Vertex Cover

parameter k

VERTEX COVER $\to$ INDEPENDENT SET



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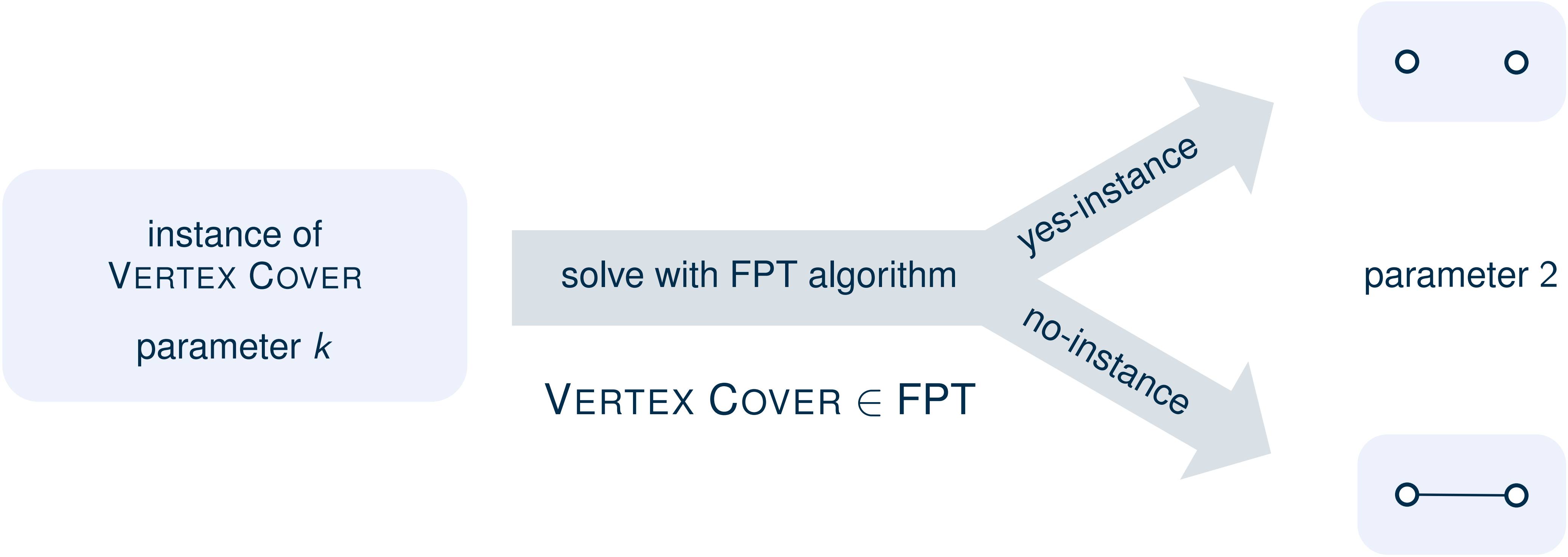


VERTEX COVER → INDEPENDENT SET

parameterized reduction

instance of
VERTEX COVER

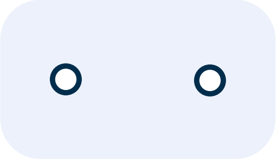
parameter k

solve with FPT algorithm

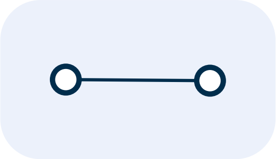
yes-instance

no-instance

VERTEX COVER ∈ FPT



parameter 2



VERTEX COVER → INDEPENDENT SET

instance of
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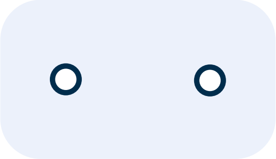
solve with FPT algorithm

VERTEX COVER $\in$ FPT

yes-instance

no-instance



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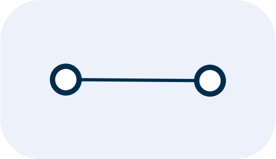
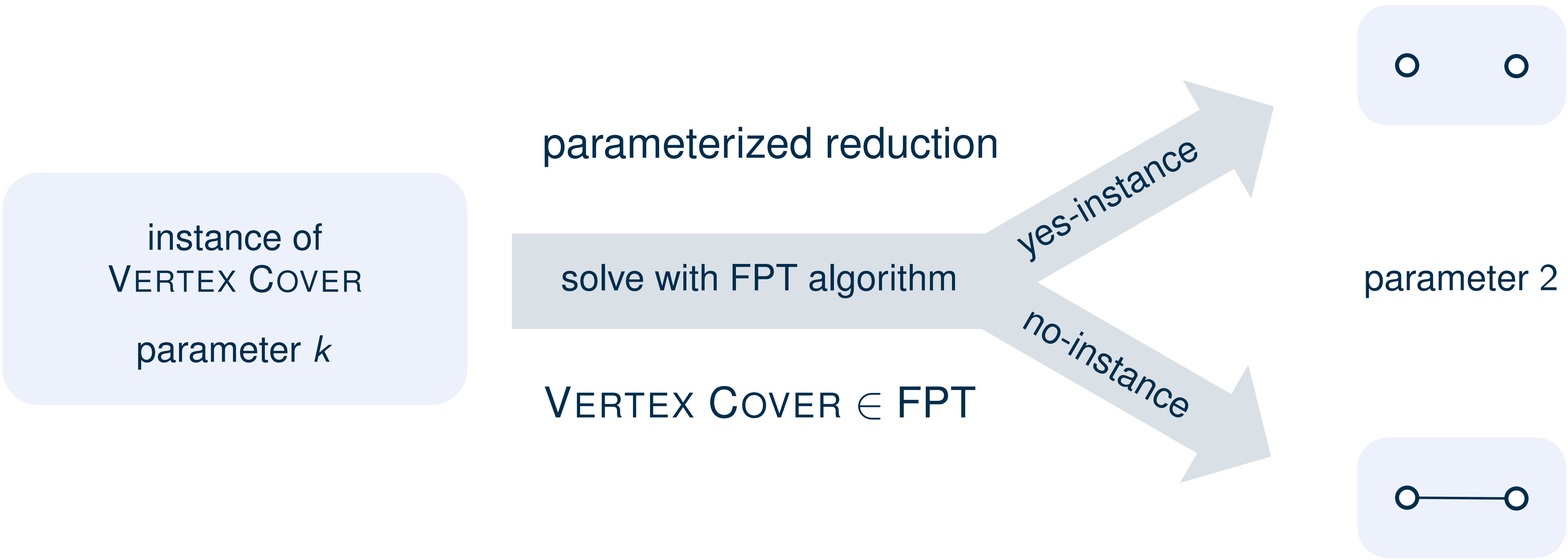


INDPENDENT SET $\in W[1] \implies$ VERTEX COVER $\in W[1]$

Vertex Cover $\rightarrow$ Independent Set

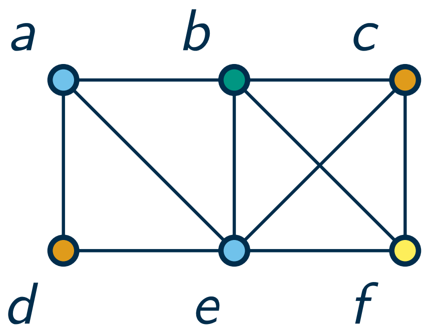


Indpendent Set $\in W[1] \implies$ Vertex Cover $\in W[1]$

repeat analogously for every FPT-problem $\rightarrow$ FPT $\subseteq W[1]$

Multicolored Biclique is $W[1]$-hard

instance of
MULTICOLORED CLIQUE



parameter k

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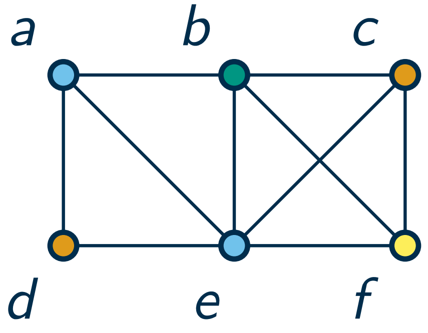

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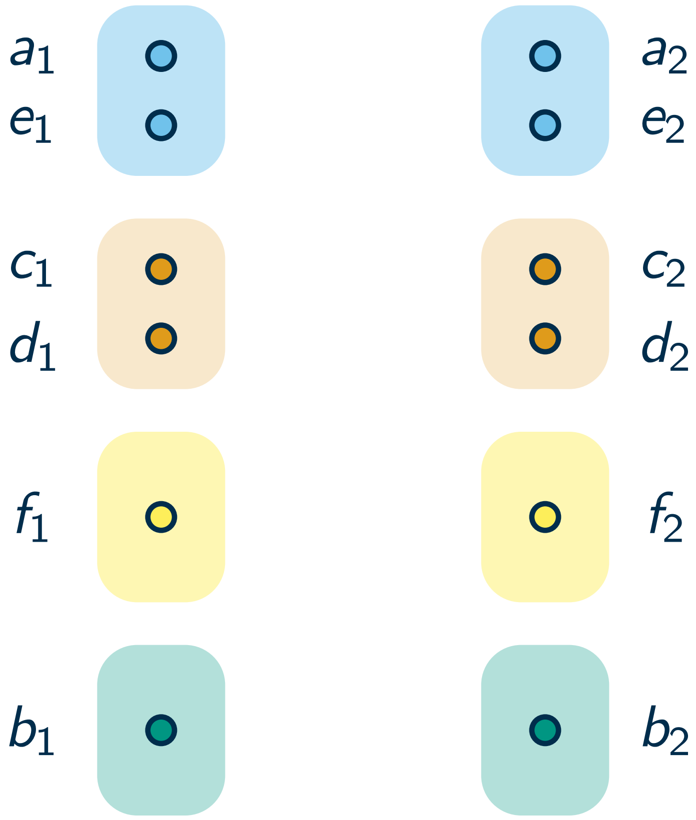
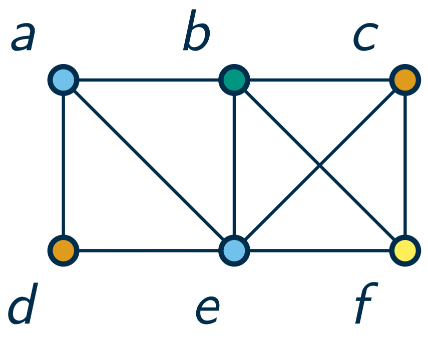


- make two copies for each vertex (left and right)

Multicolored Biclique is $W[1]$-hard

instance of
MULTICOLORED CLIQUE



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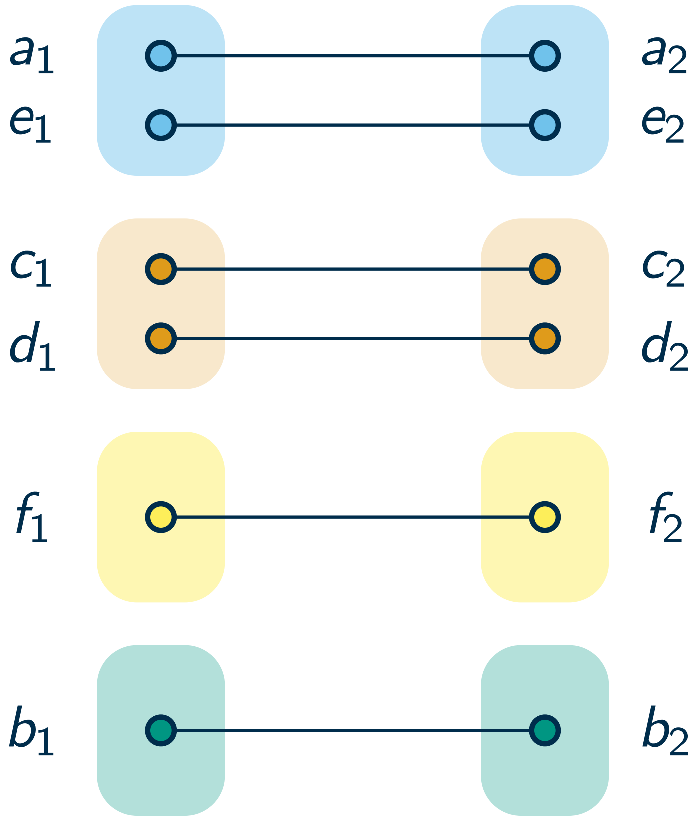
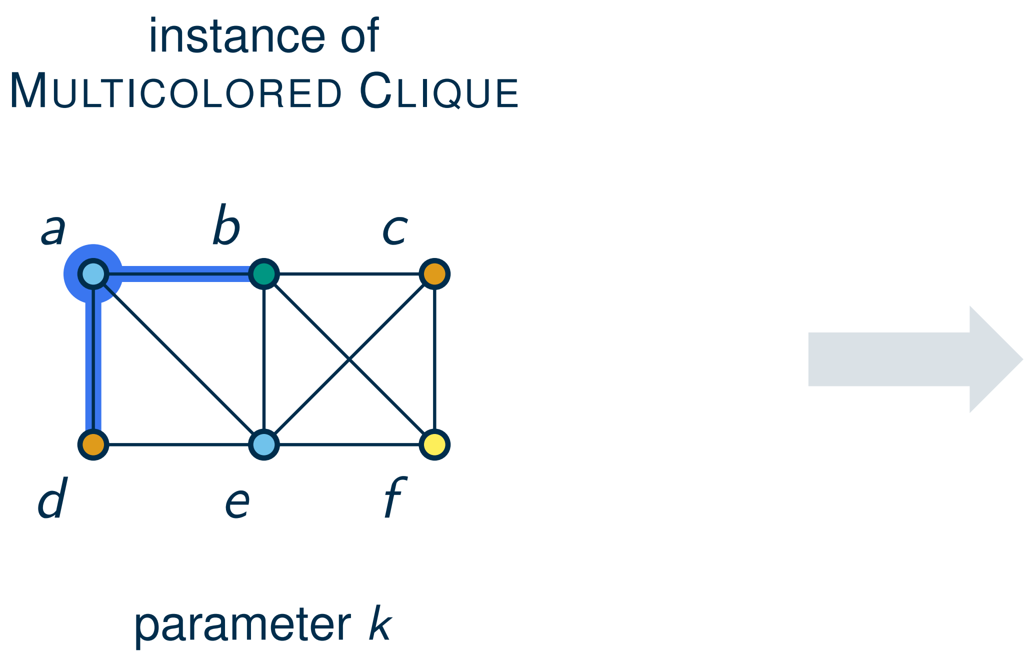
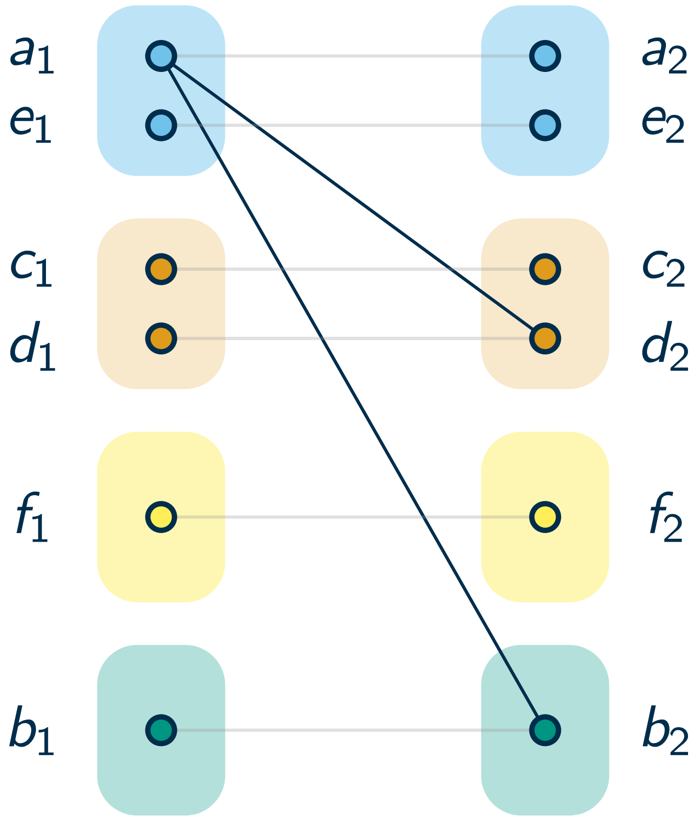


- make two copies for each vertex (left and right)
- connect copies that correspond to same input vertex

Multicolored Biclique is $W[1]$-hard





- make two copies for each vertex (left and right)
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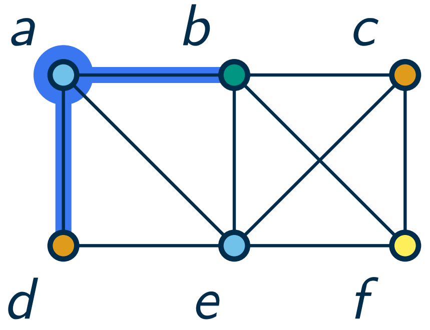
- add edge between v_i and w_j if v and w adjacent in input graph and $i \neq j$, unless same color

Multicolored Biclique is $W[1]$-hard

instance of
Multicolored Clique



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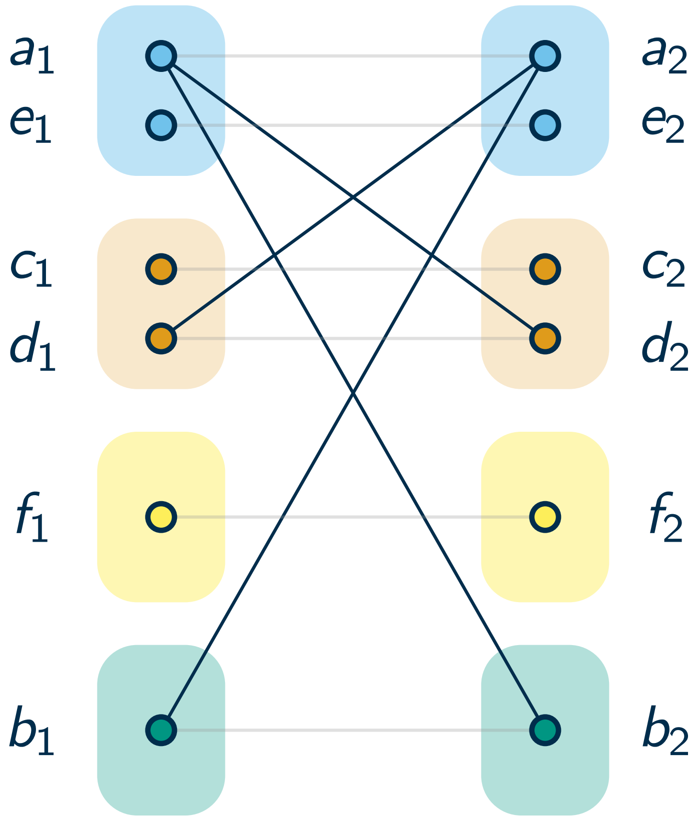

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instance of
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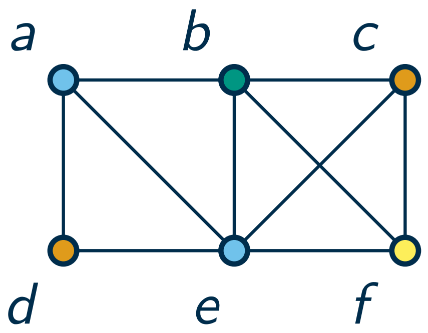
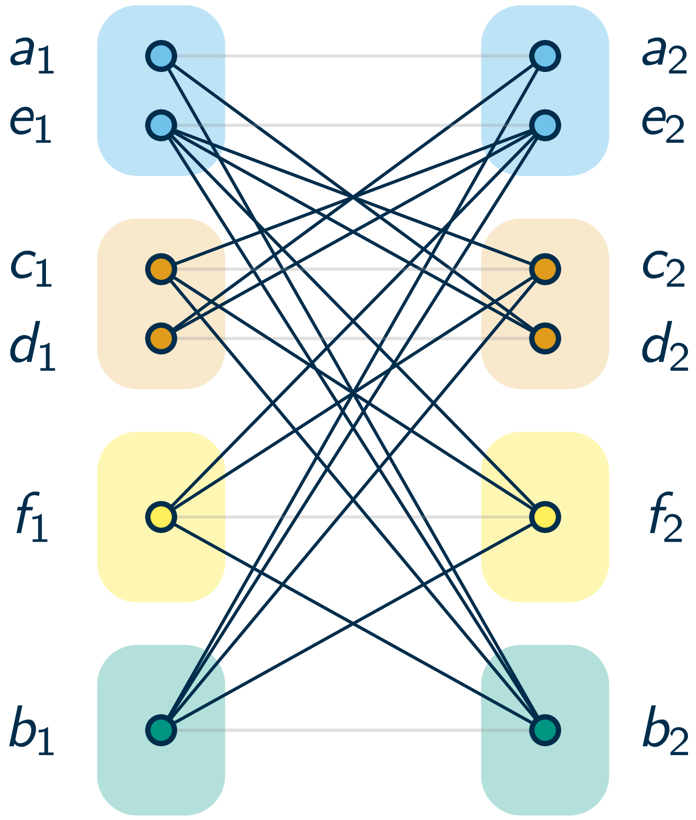


parameter k



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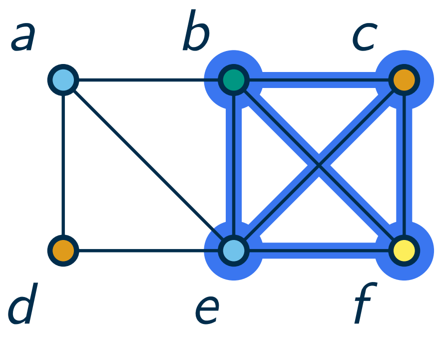
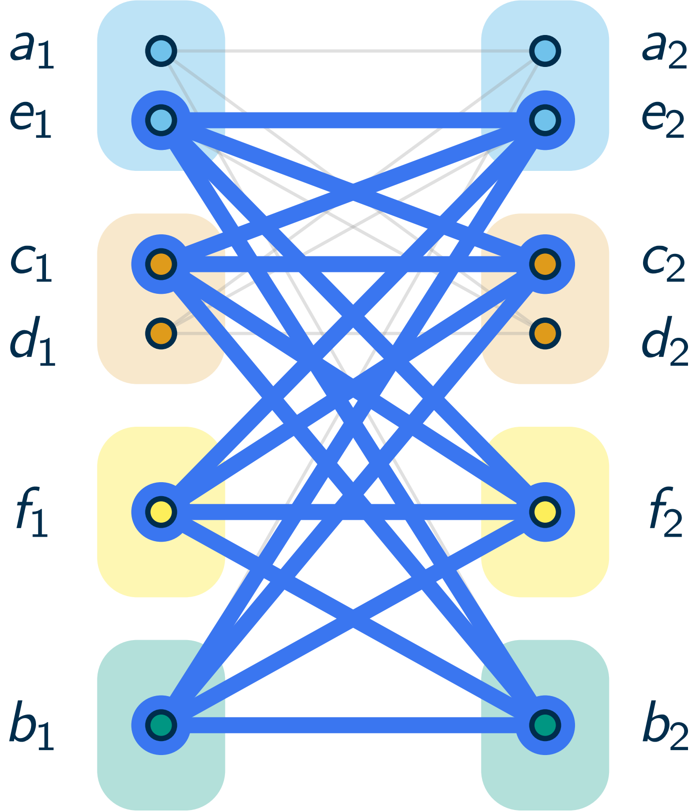


parameter k



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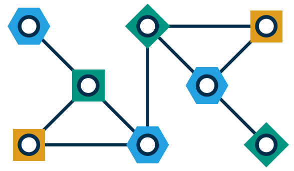
LONG INDUCED PATH IS $W[1]$-HARD

LONG INDUCED PATH Given a graph $G = (V, E)$ and a parameter k, does G contain an induced path of length k?

MULTICOLORED INDEPENDENT SET



parameter k

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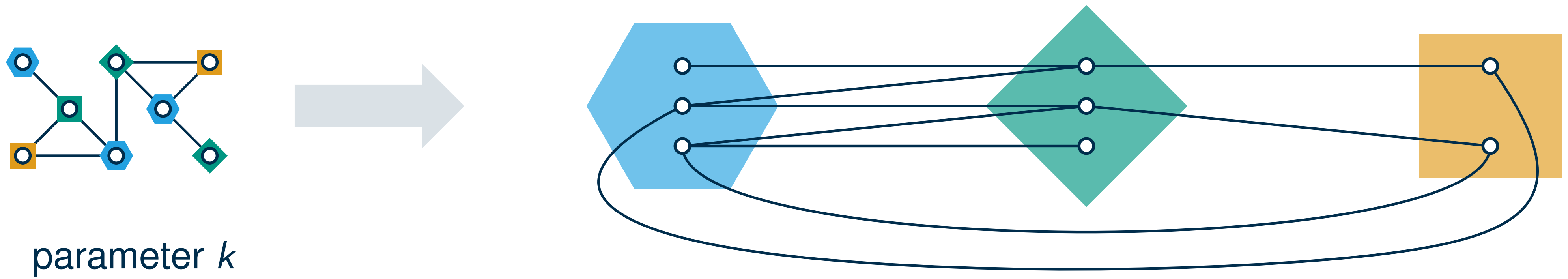

parameter k

- copy over all vertices and edges

Long Induced Path is $W[1]$-hard

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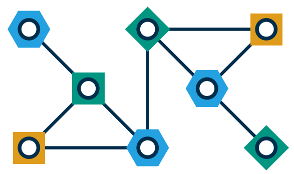
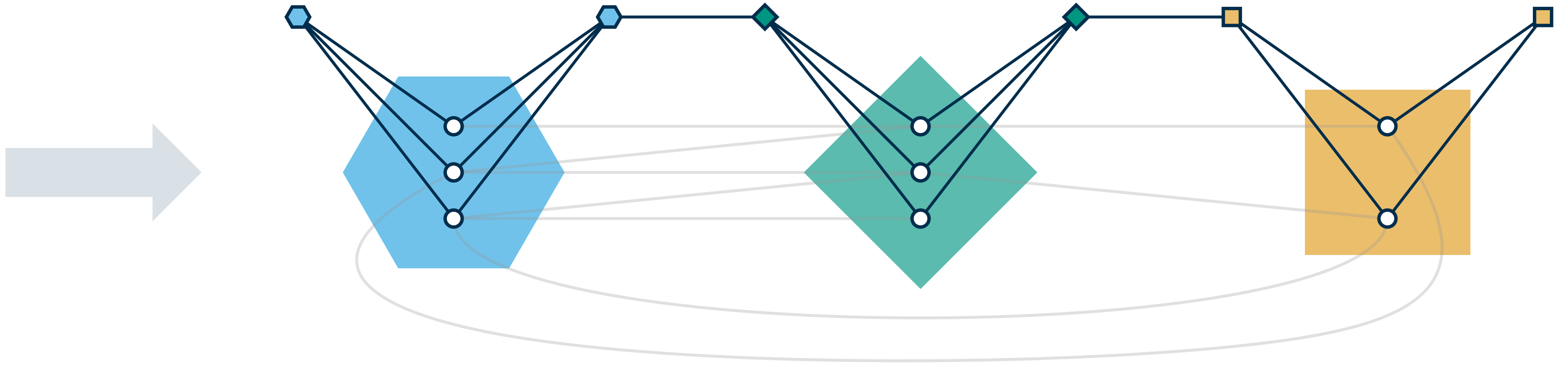


parameter k

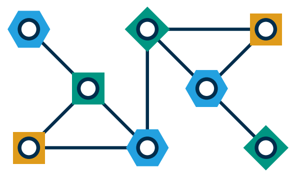


- copy over all vertices and edges
- add two universal vertices for each color class ("entry and exit"), connect exit to next entry

Long Induced Path is $W[1]$-hard

Long Induced Path Given a graph $G = (V, E)$ and a parameter k, does G contain an induced path of length k?

Multicolored Independent Set



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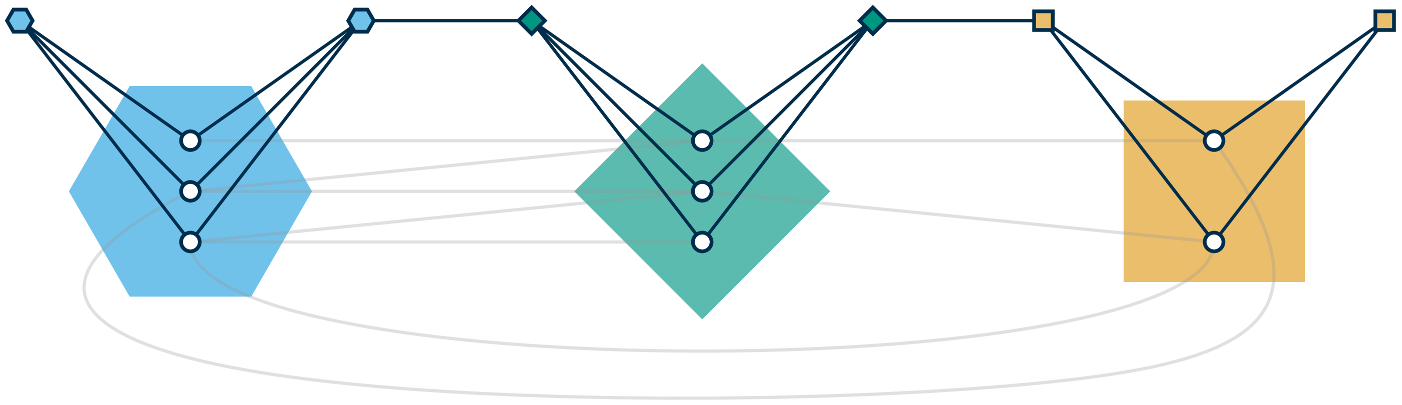

parameter $k' = 3k$

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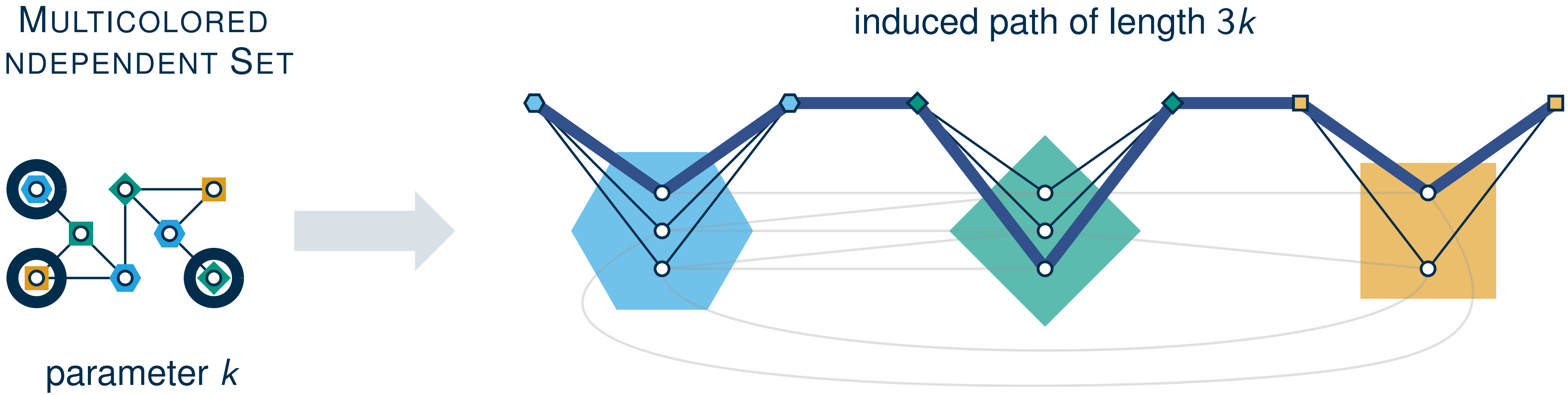

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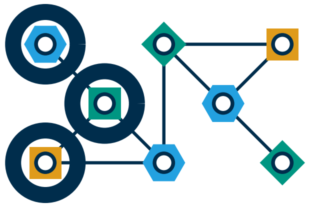

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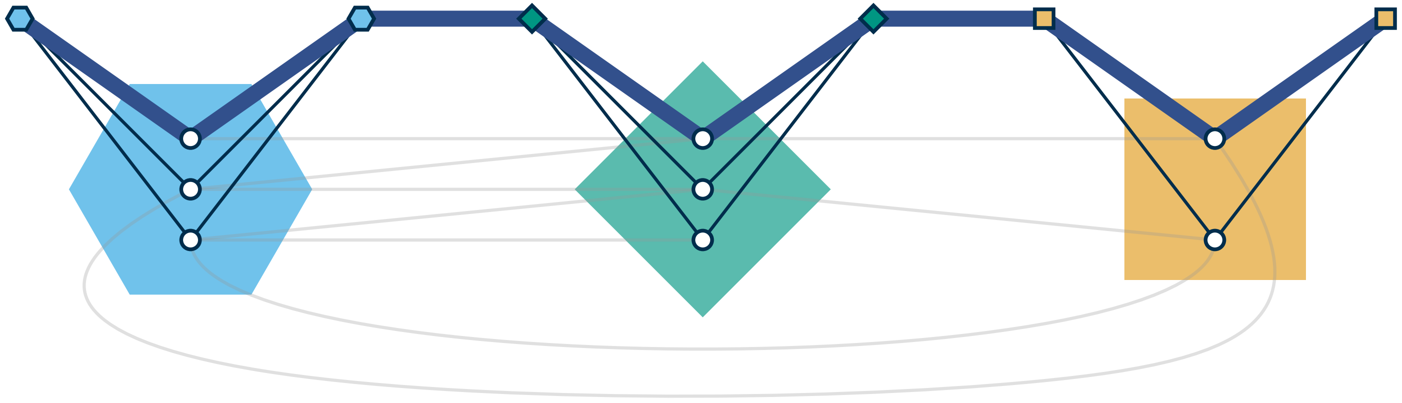

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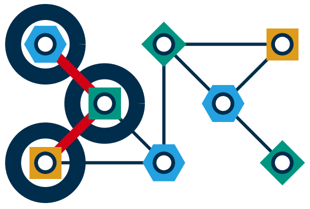
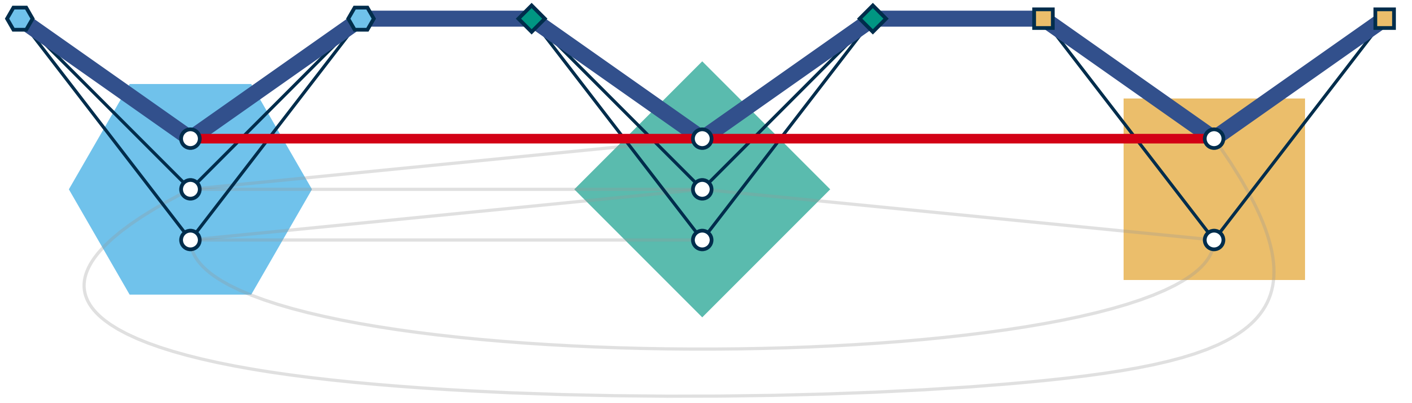


parameter k



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LONG INDUCED PATH IS $W[1]$-HARD

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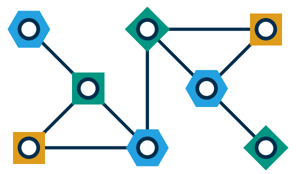
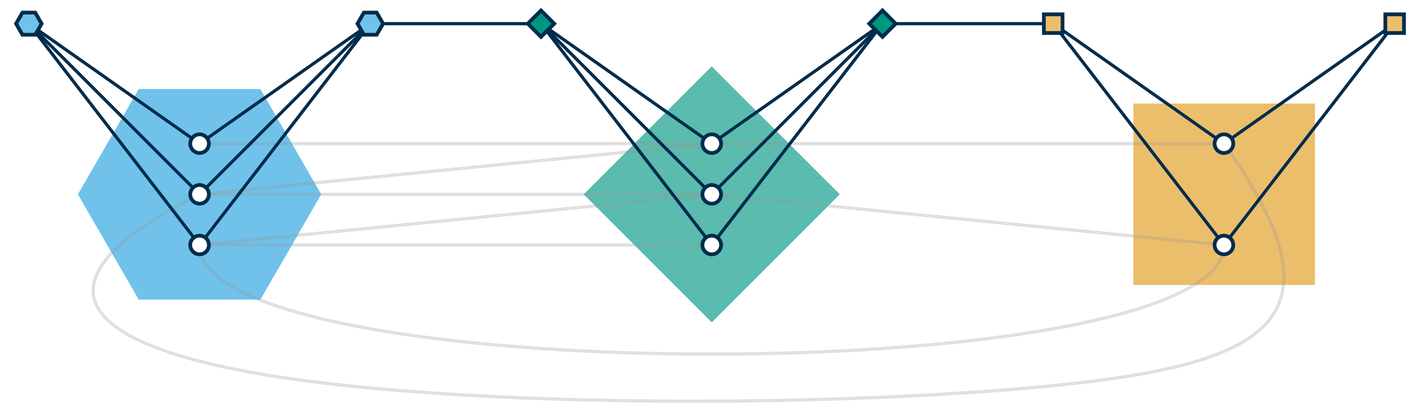
correct?

MULTICOLORED INDEPENDENT SET



parameter k



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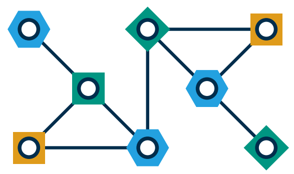
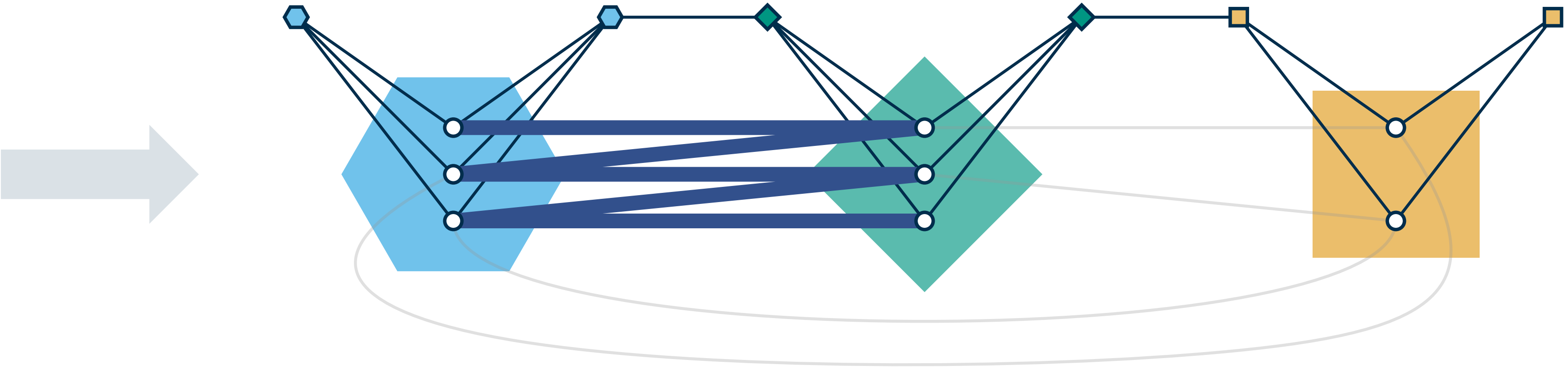


parameter k

there could be long induced path that doesn't use all colors



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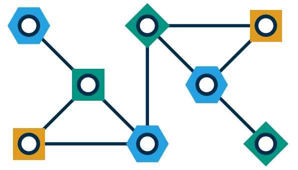
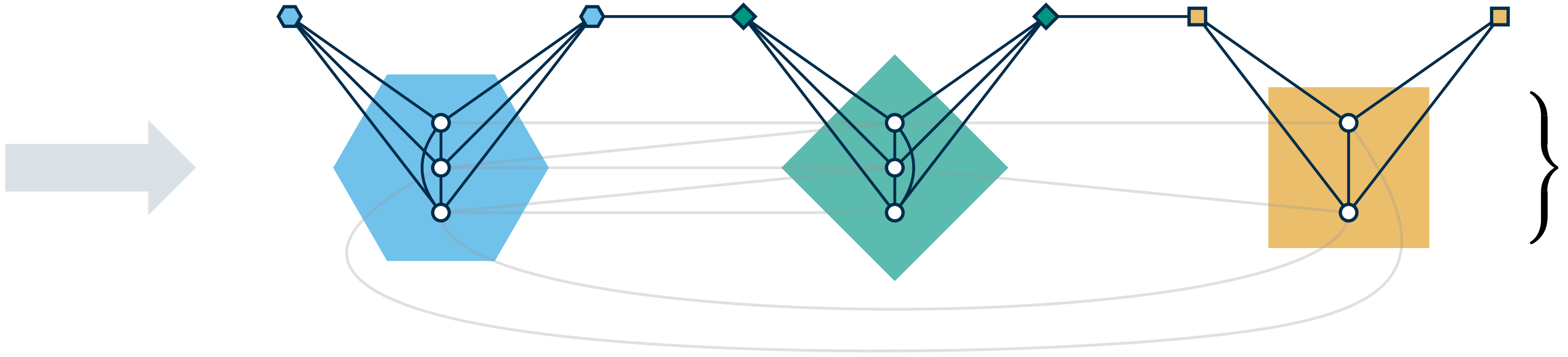


parameter k



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- copy over all vertices and edges
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- make color classes form cliques

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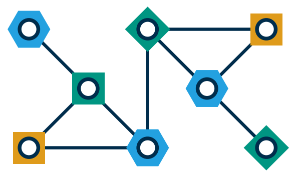

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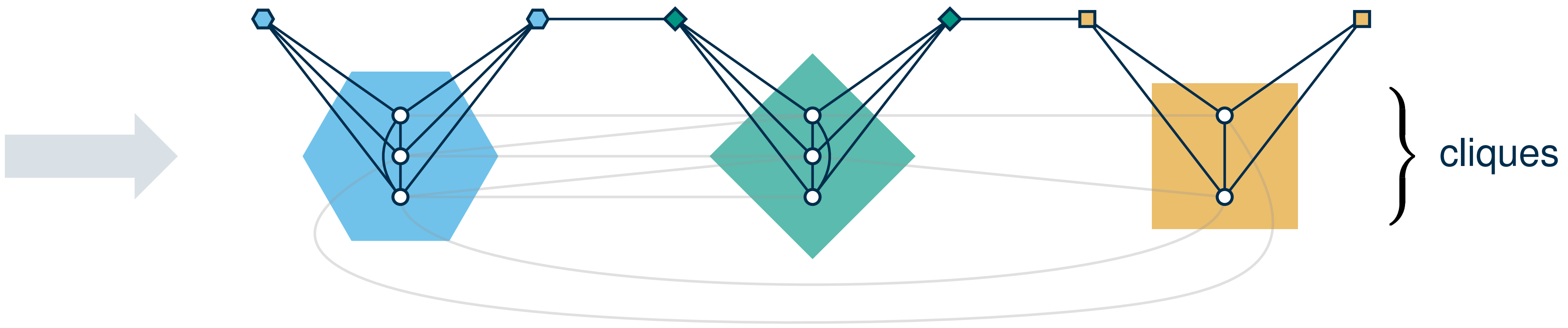

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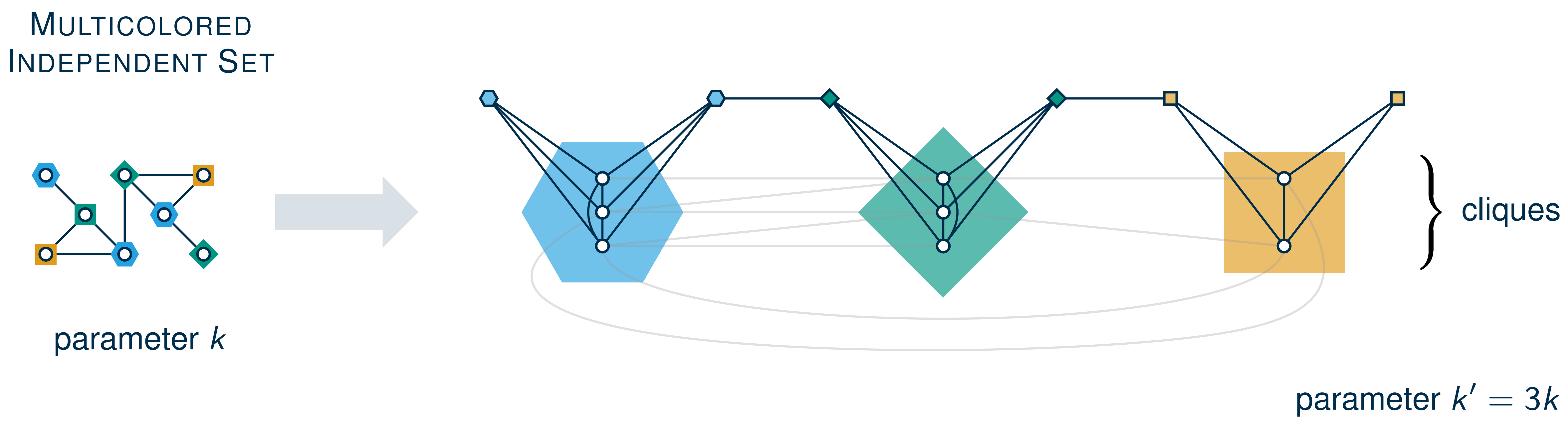

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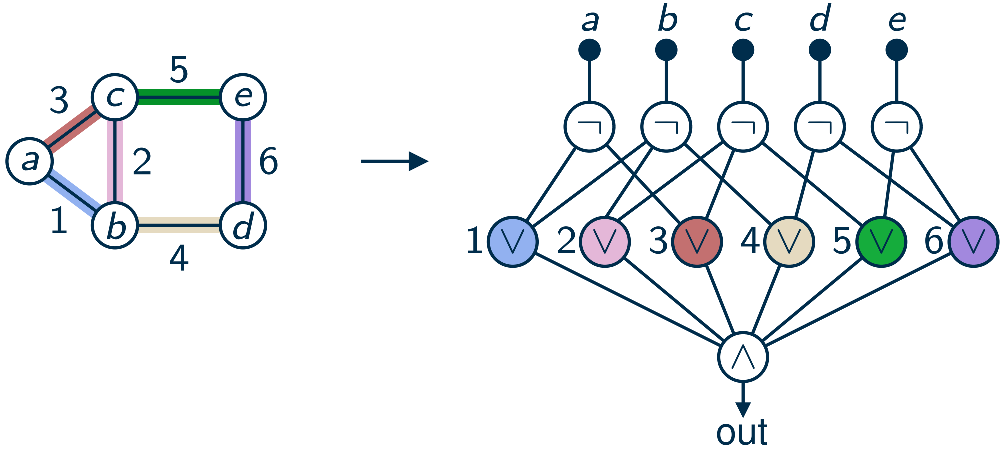
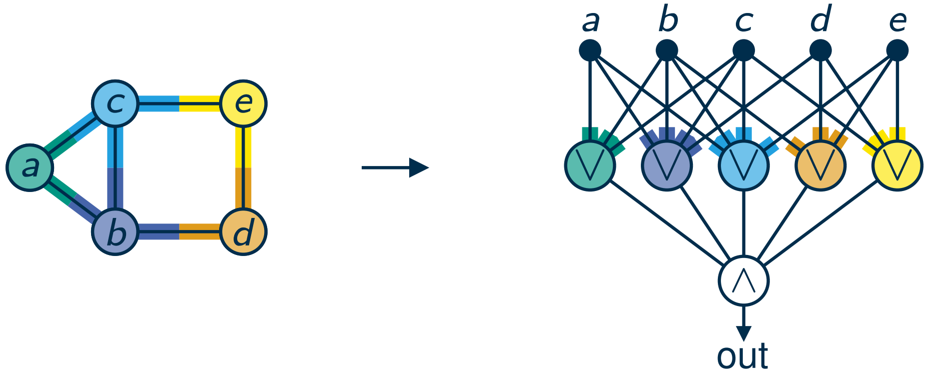
INDEPENDENT DOMINATING SET $\in W[2]$

INDEPENDENT SET → WCS



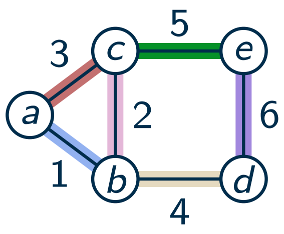
DOMINATING SET → WCS



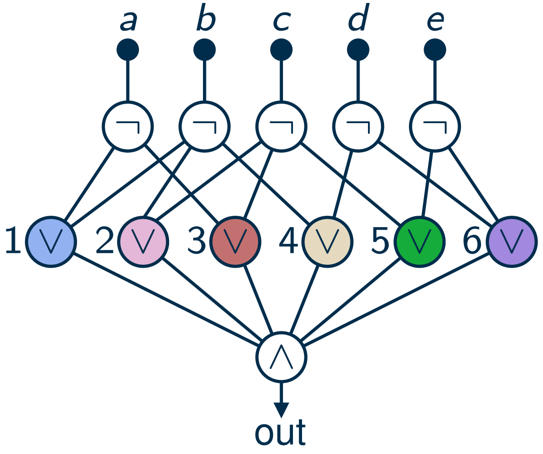
INDEPENDENT DOMINATING SET → WCS

Independent Dominating Set $\in W[2]$

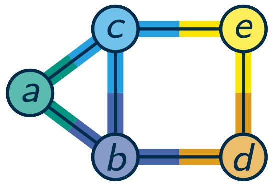
Independent Set → WCS



→



Dominating Set → WCS



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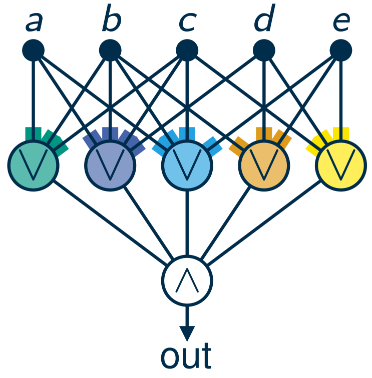

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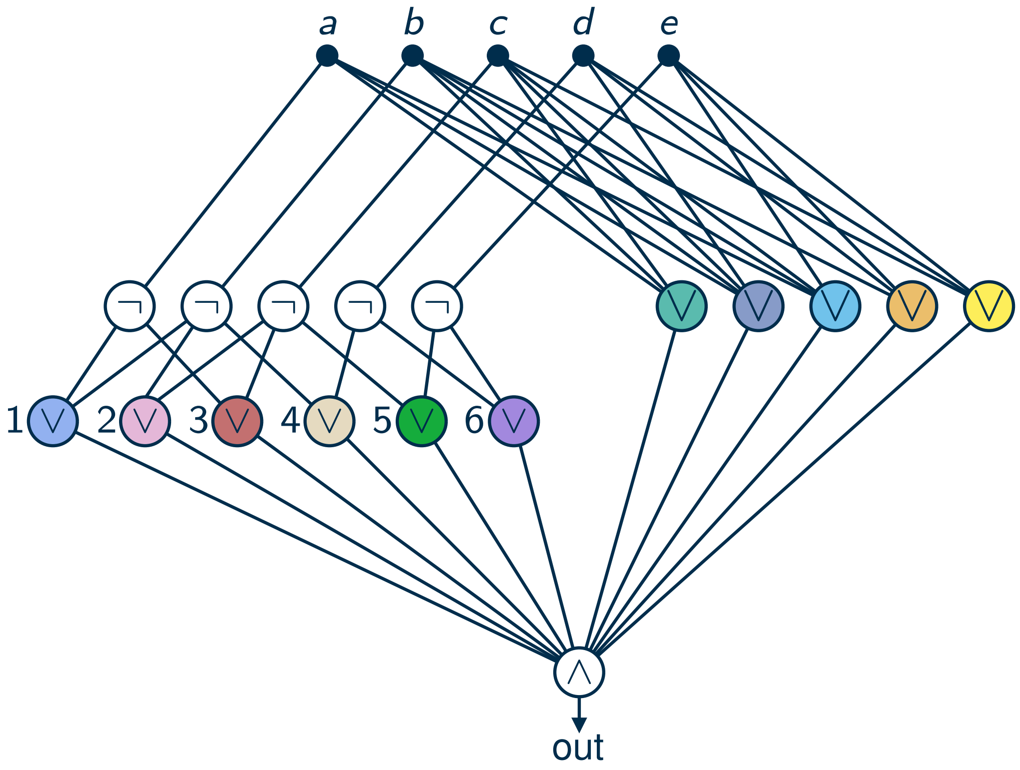
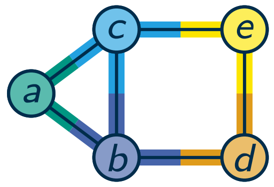


INDEPENDENT DOMINATING SET is $W[2]$-hard

instance of
DOMINATING SET



parameter k

INDEPENDENT DOMINATING SET is $W[2]$-hard

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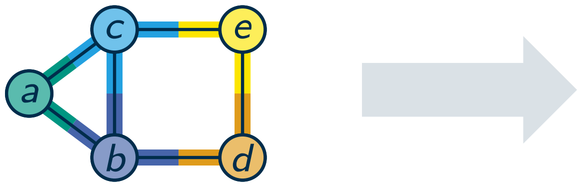
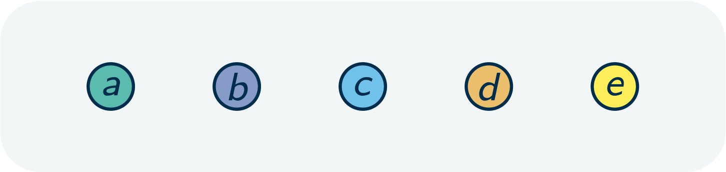


parameter k



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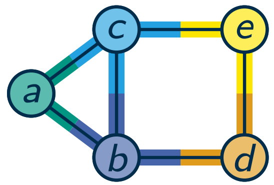


parameter k

possible choices
for vertex 1

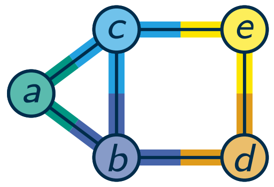
$\dots$

possible choices
for vertex k

parameter $k' = k$

INDEPENDENT DOMINATING SET is $W[2]$-hard

instance of
DOMINATING SET



parameter k

selection gadget

selection gadget

possible choices
for vertex 1

a b c d e

. . .

possible choices
for vertex k

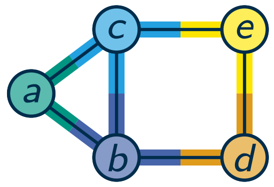
a b c d e

a b c d e

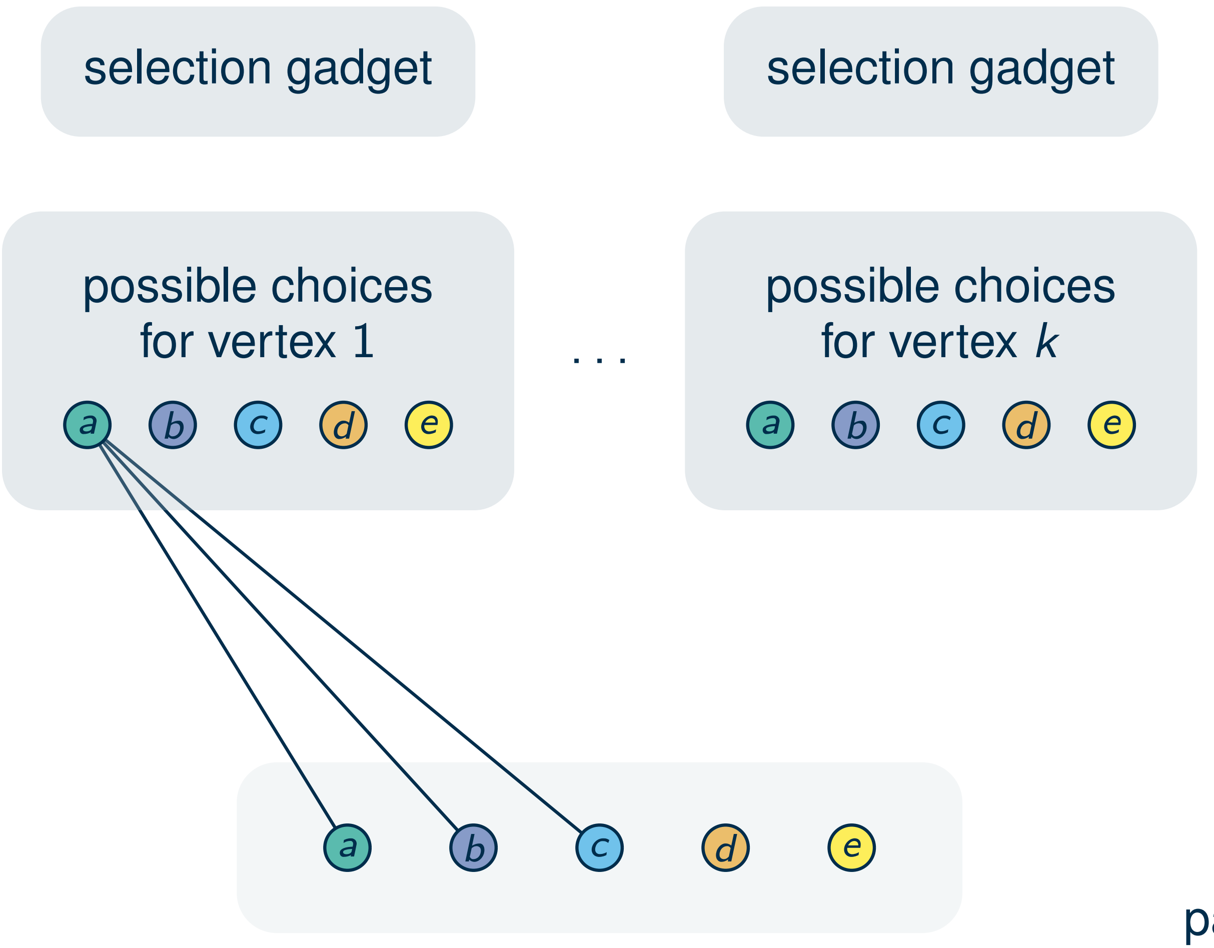
parameter $k' = k$

Independent Dominating Set is $W[2]$-hard

instance of
Dominating Set



parameter k

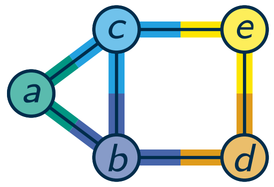


edge if
adjacent or
same vertex

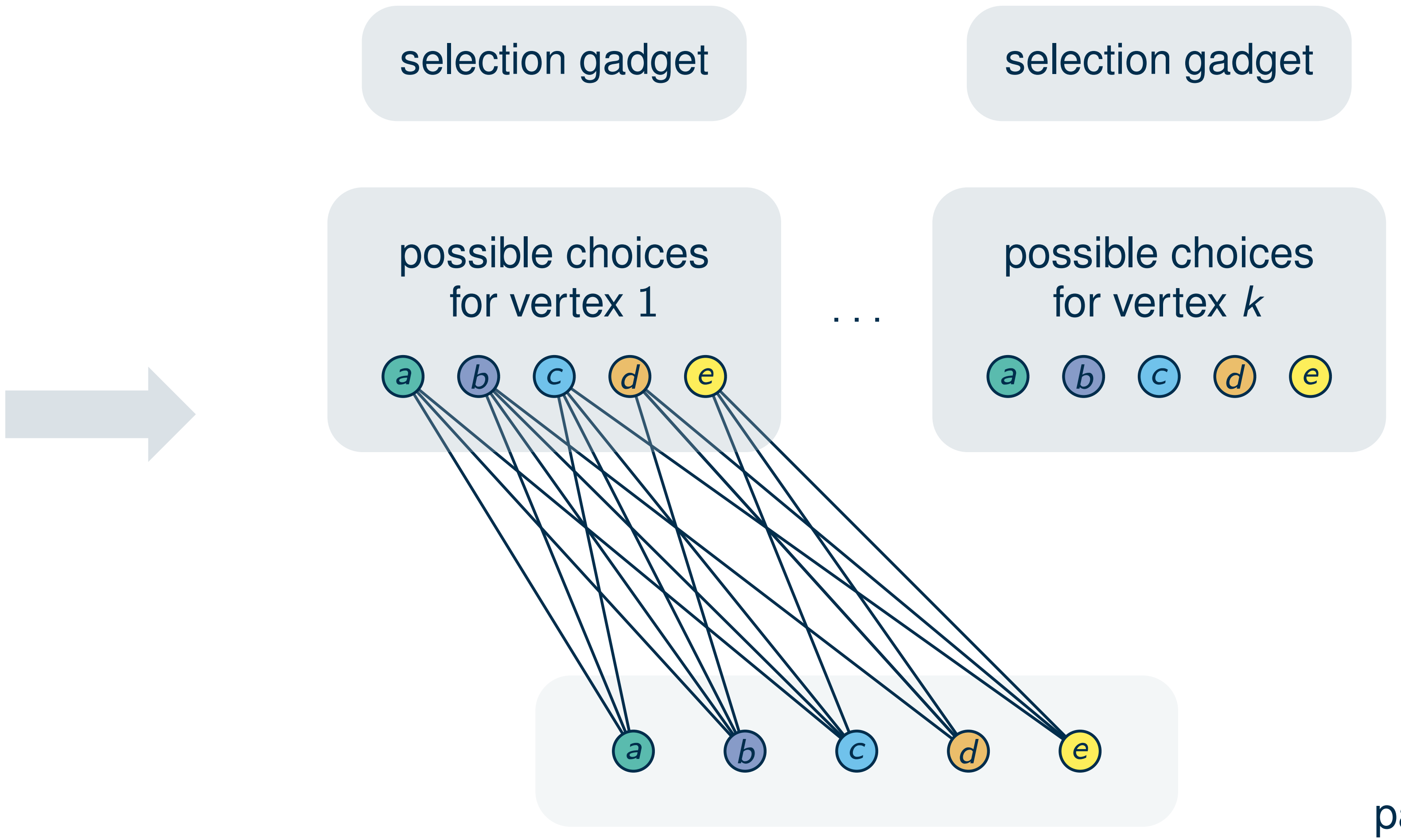
parameter $k' = k$

INDEPENDENT DOMINATING SET is $W[2]$-hard

instance of
DOMINATING SET



parameter k



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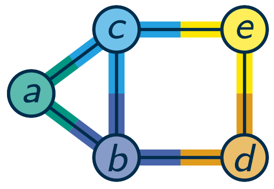

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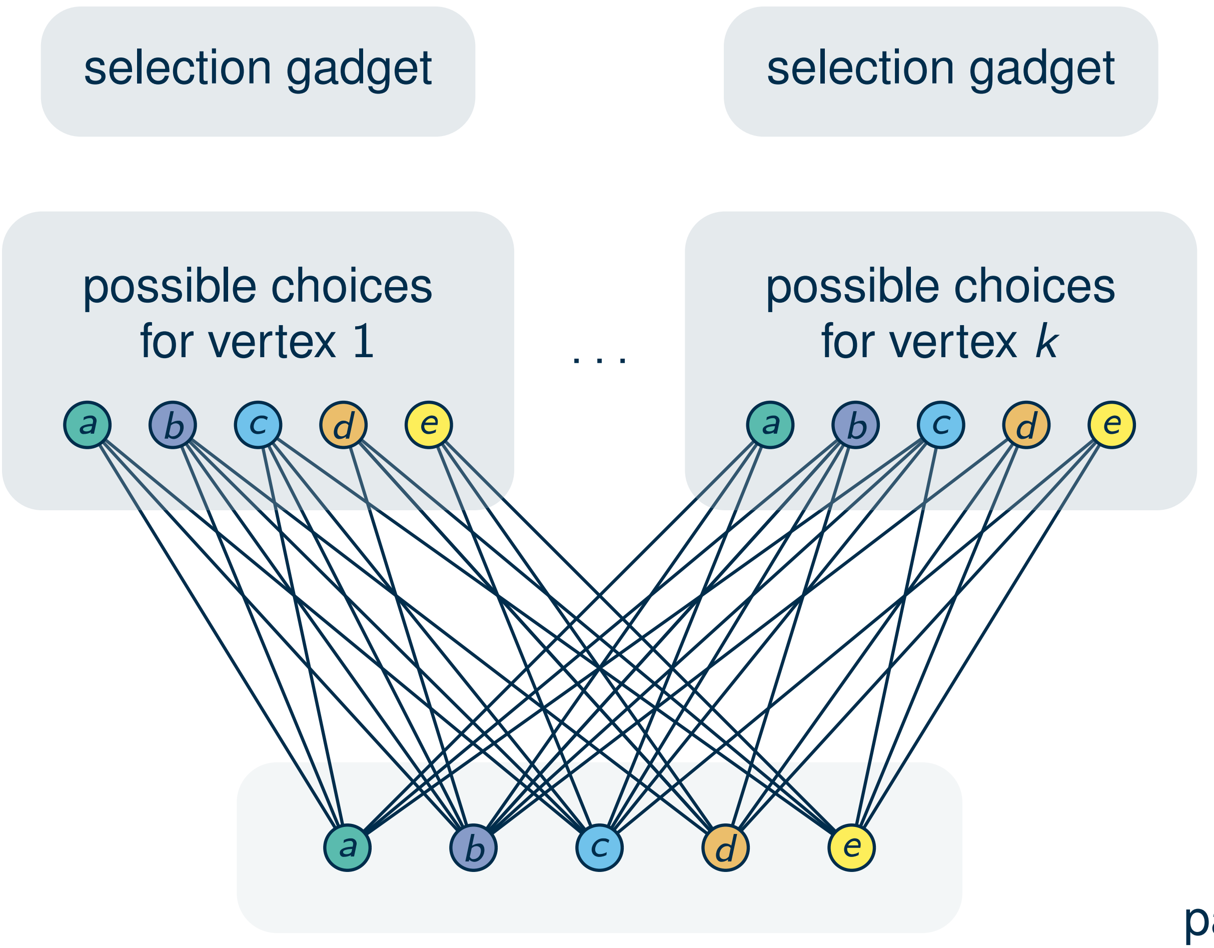

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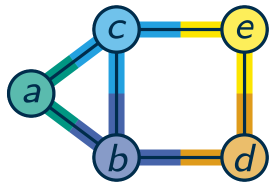

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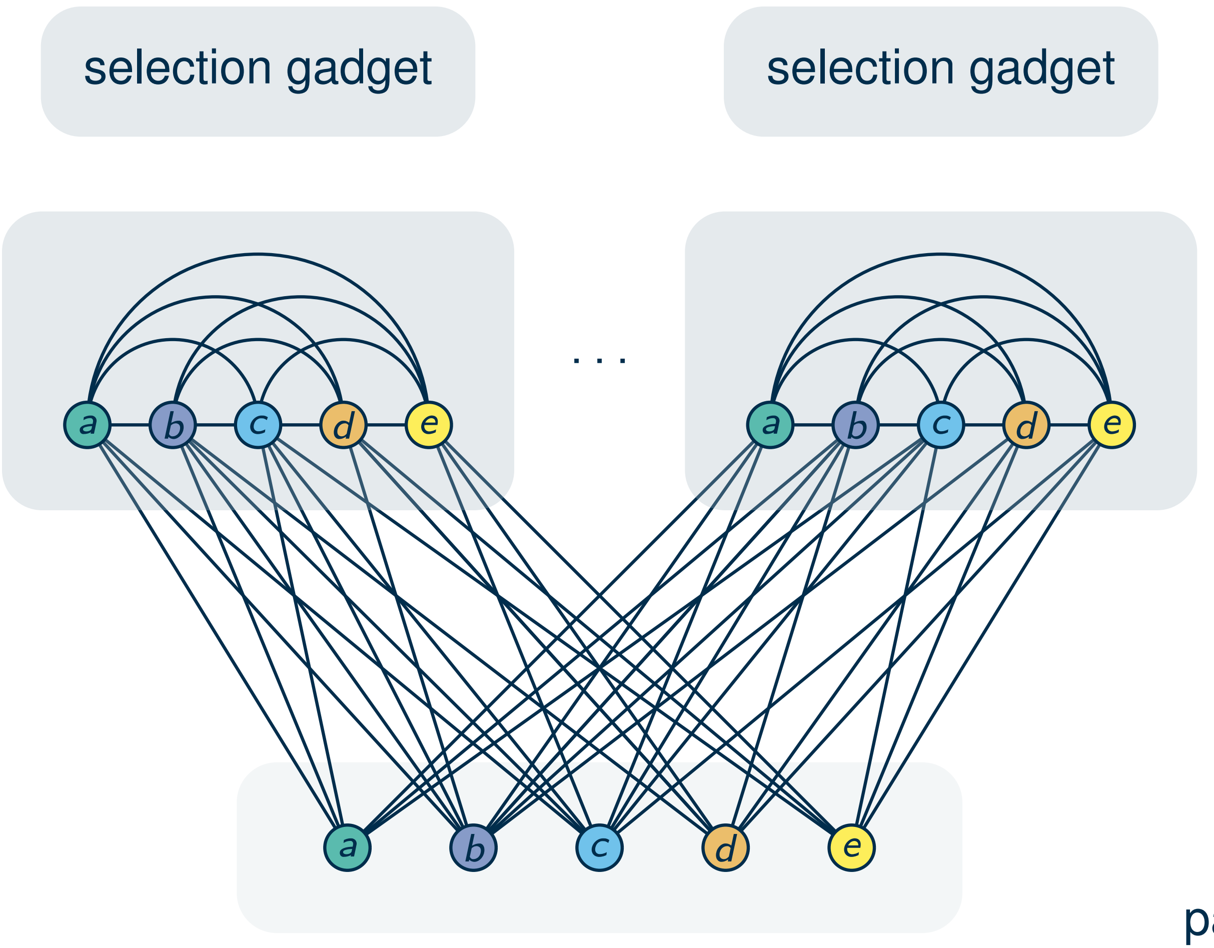

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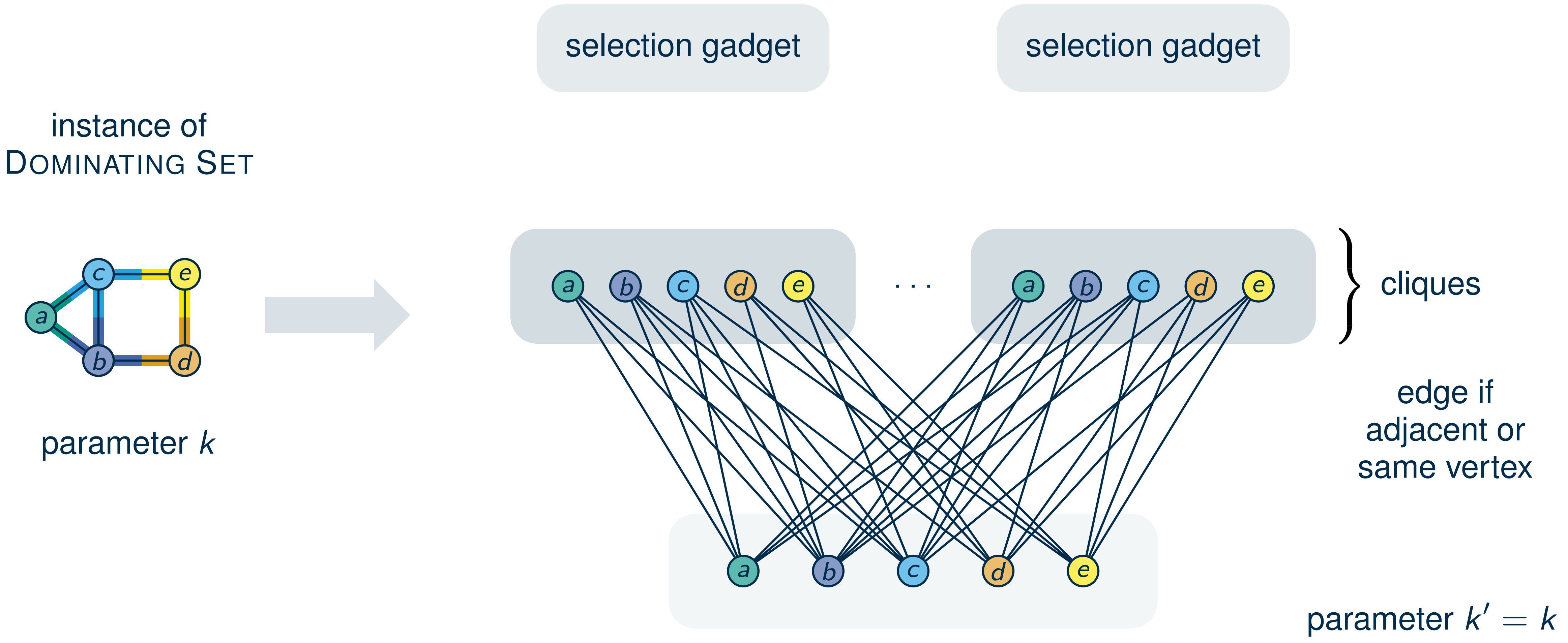

INDEPENDENT DOMINATING SET is $W[2]$-hard

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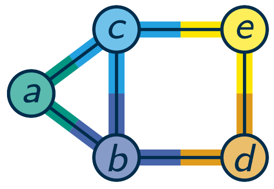
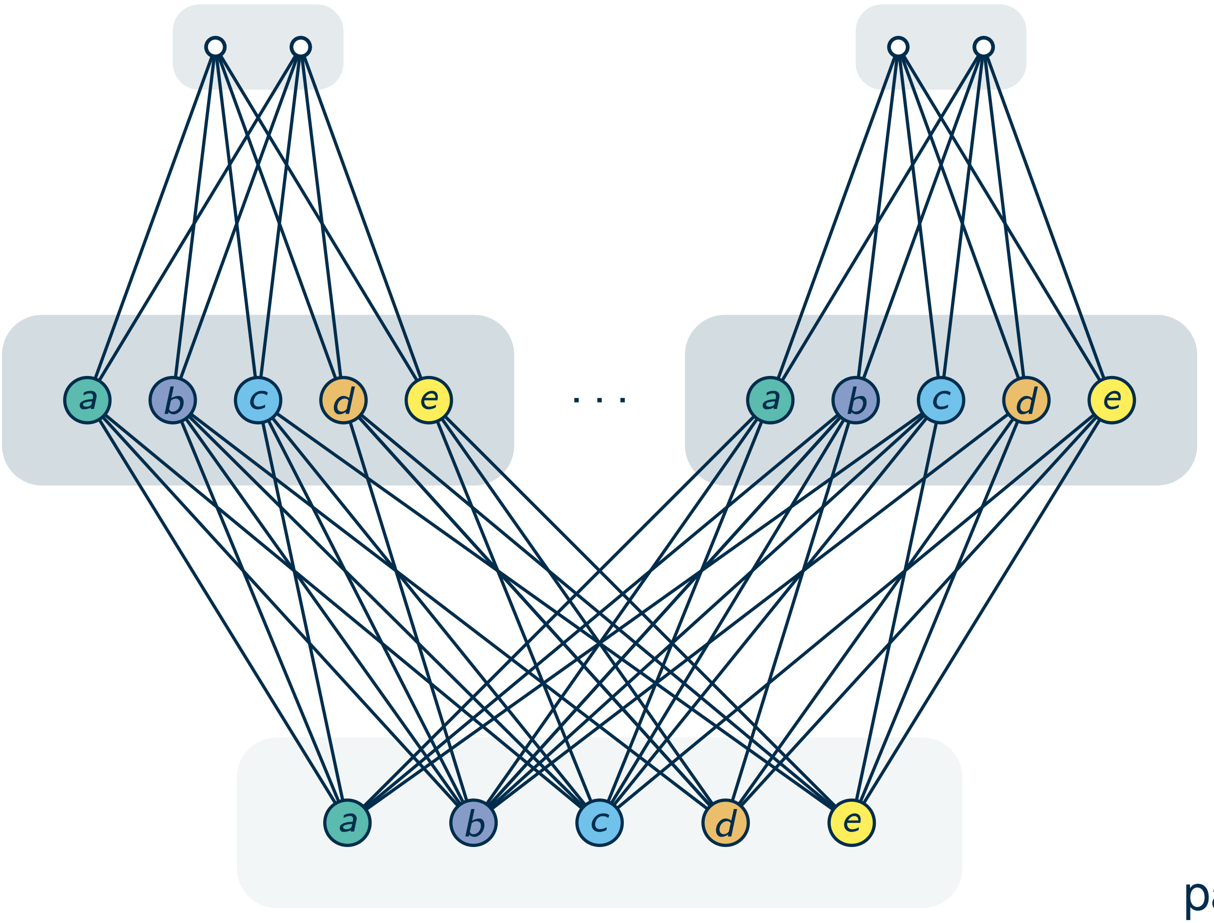


parameter k



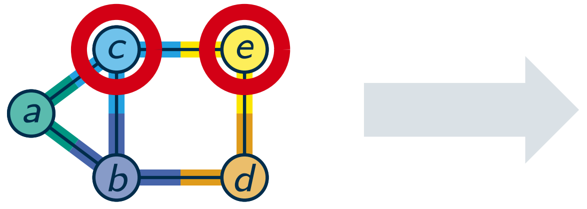
cliques

edge if
adjacent or
same vertex

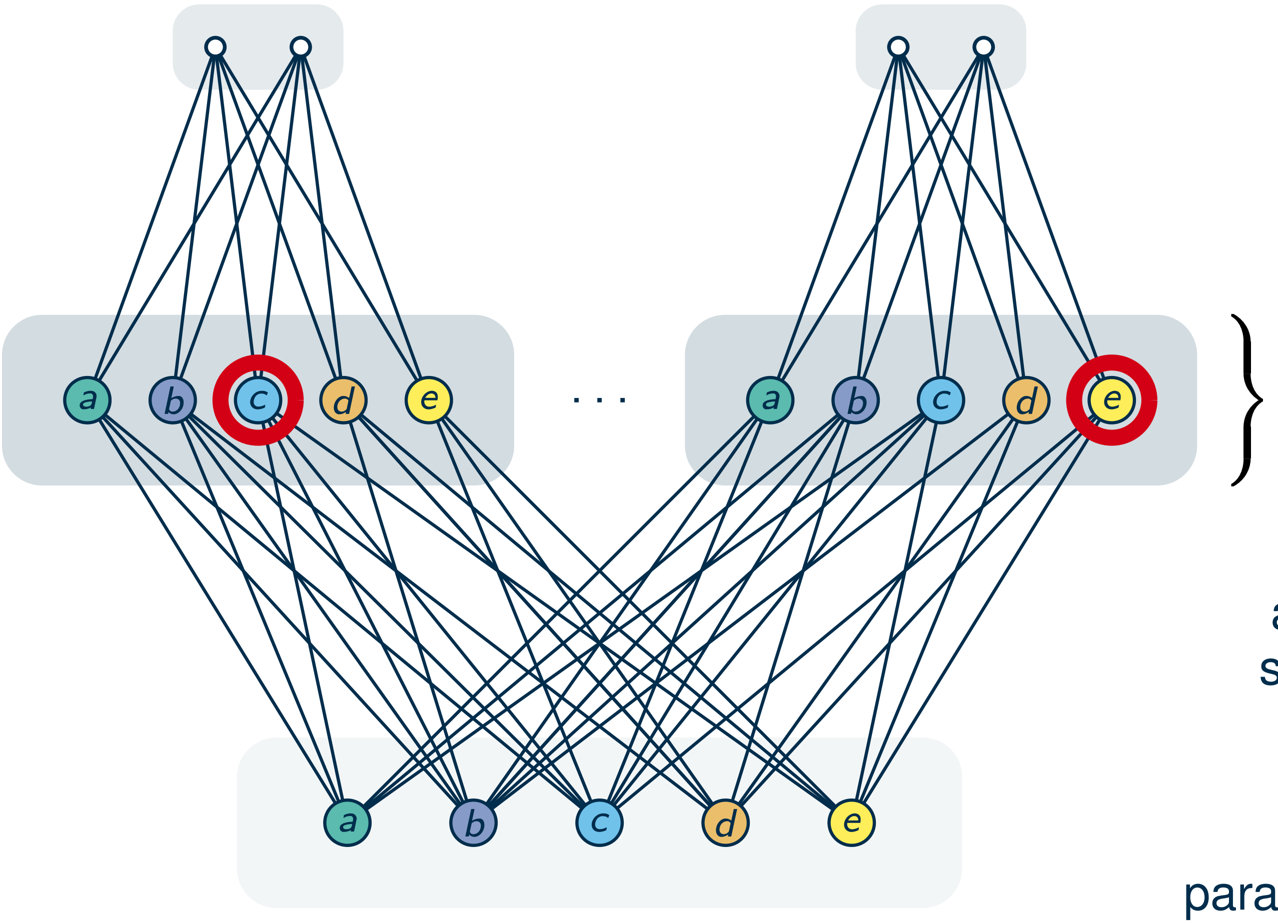
parameter $k' = k$

INDEPENDENT DOMINATING SET is $W[2]$-hard

instance of
DOMINATING SET



parameter k



cliques

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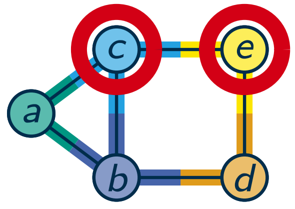
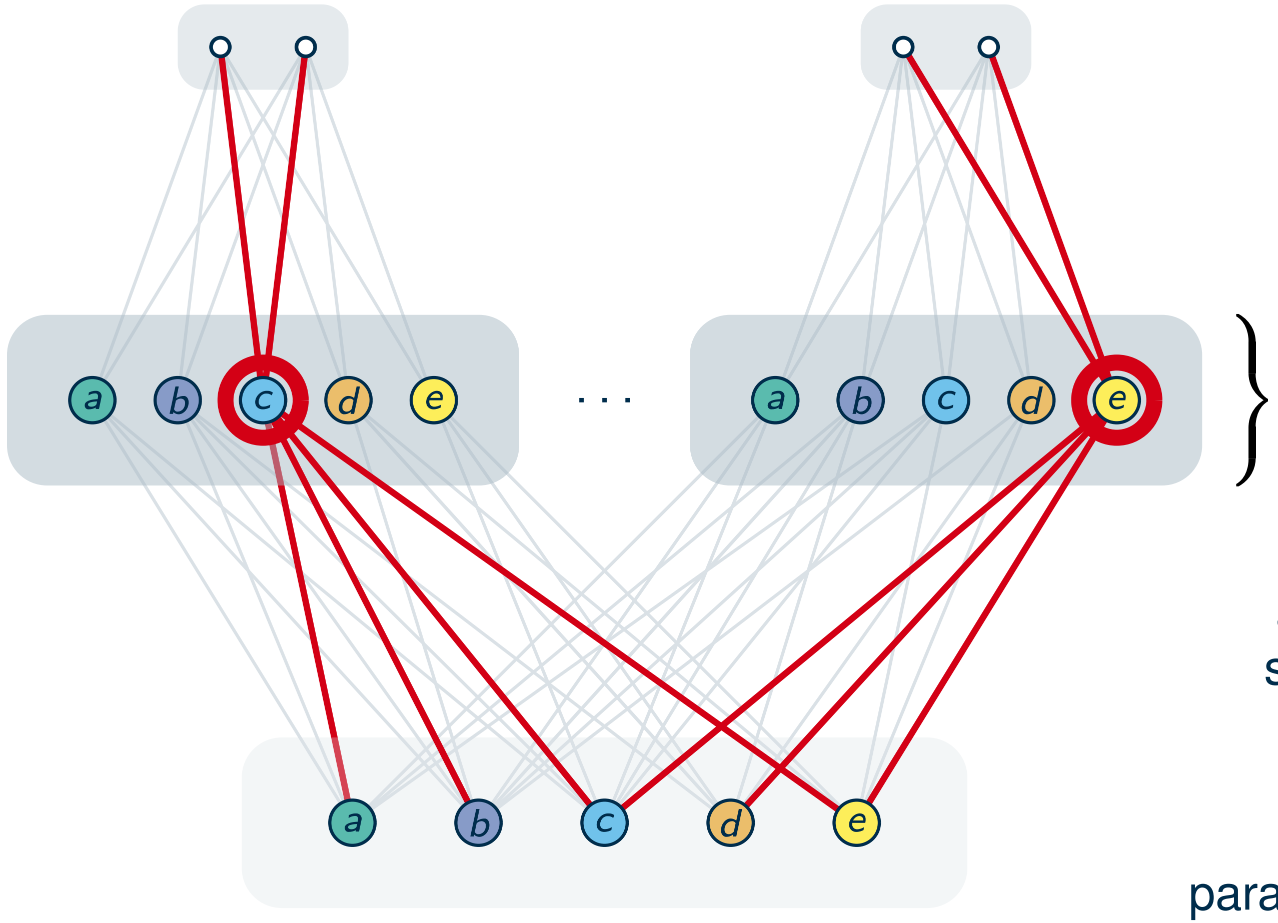


parameter k



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instance of
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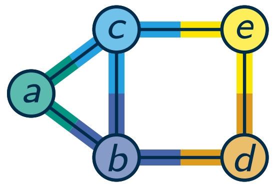
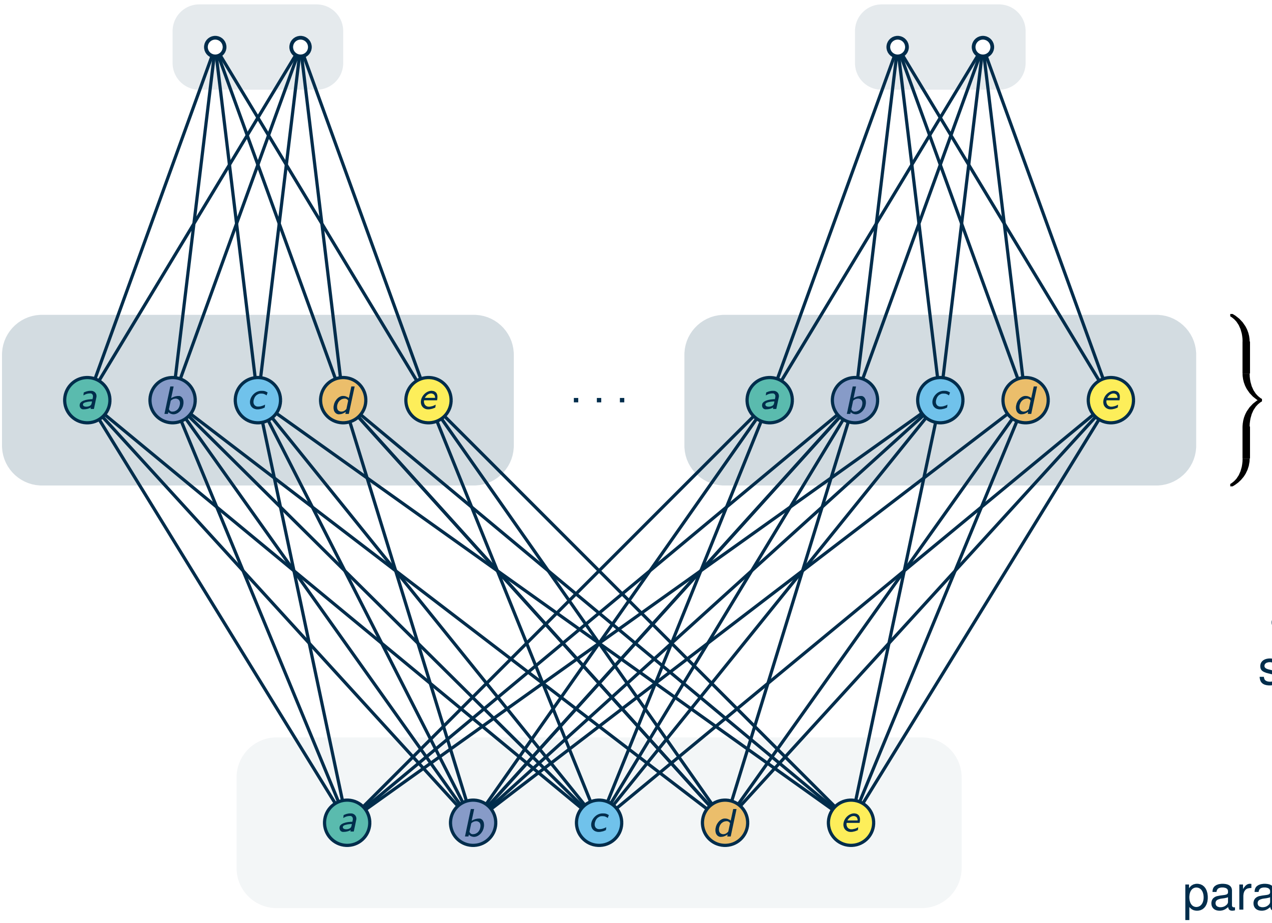


parameter k

correct?



cliques

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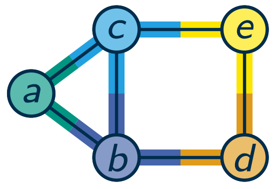
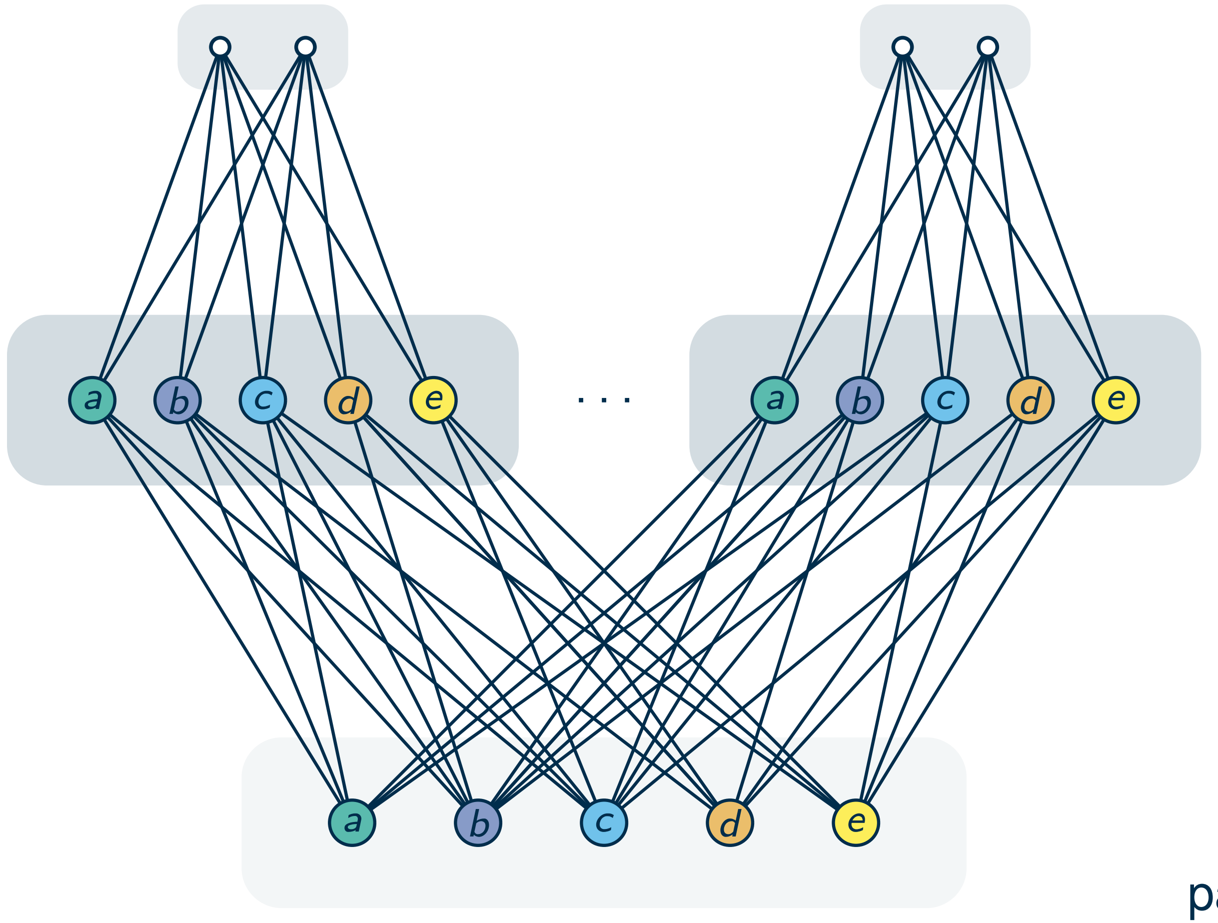


parameter k



cliques

edge if
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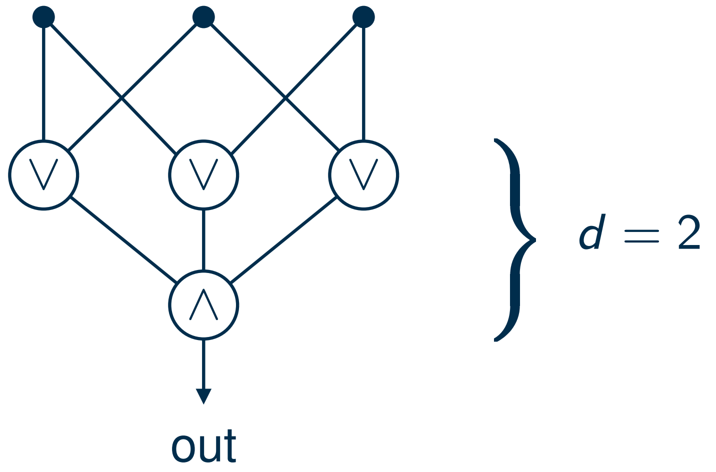
parameter $k' = k$

correct? ✓ (if $n \geq k$ in input graph)

Monotone Weft-1 Circuits

Given an instance of WEIGHTED CIRCUIT SATISFIABILITY with weft 1, constant depth d, and without negation nodes, together with a parameter k. Is there a satisfying assignment of weight $\leq k$?



weft (of circuit): largest number of nodes with in-degree > 2 on a directed path

weight (of assignment): number of 1s for input nodes

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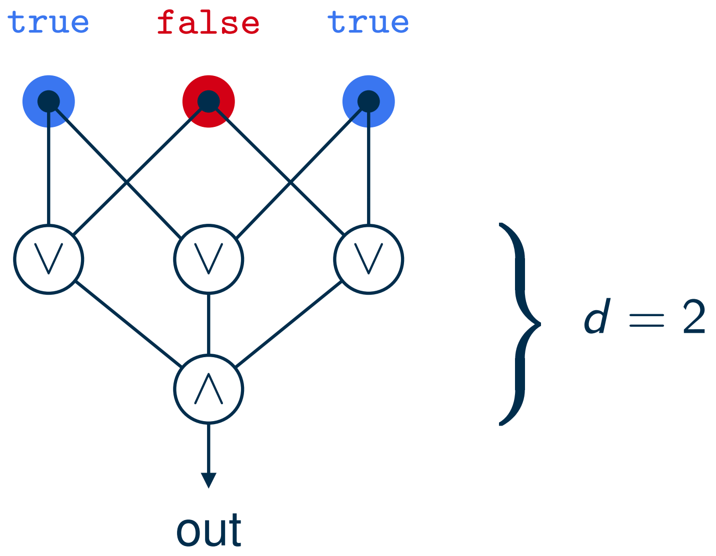

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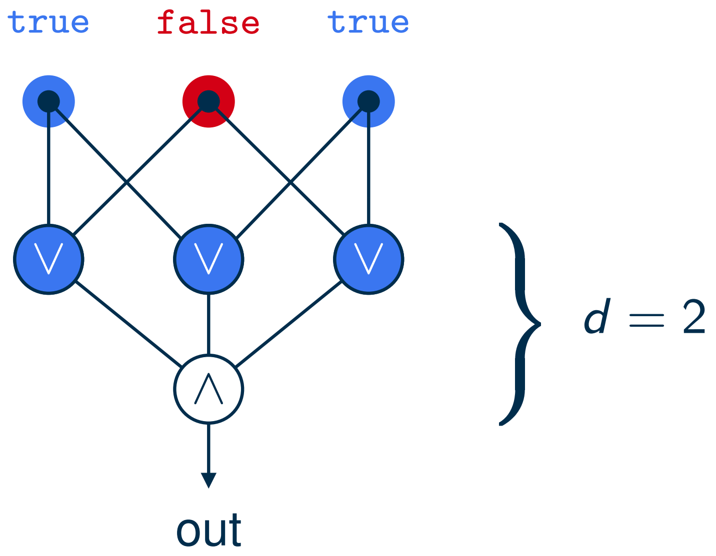

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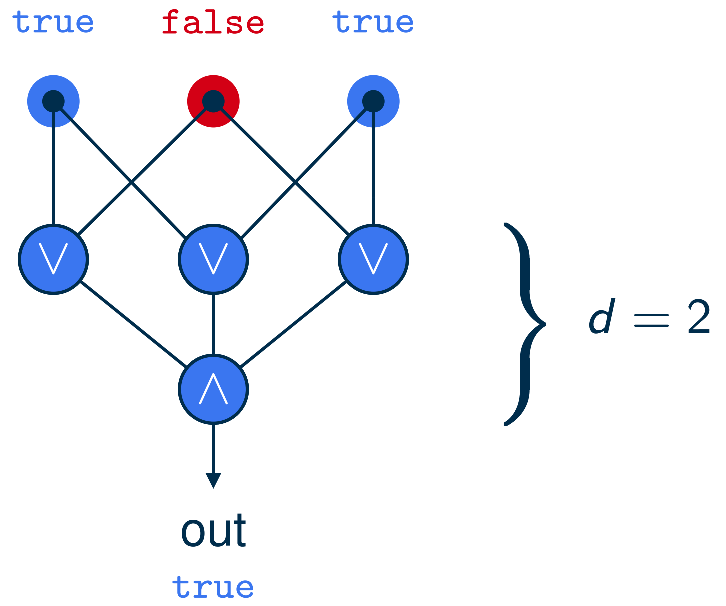

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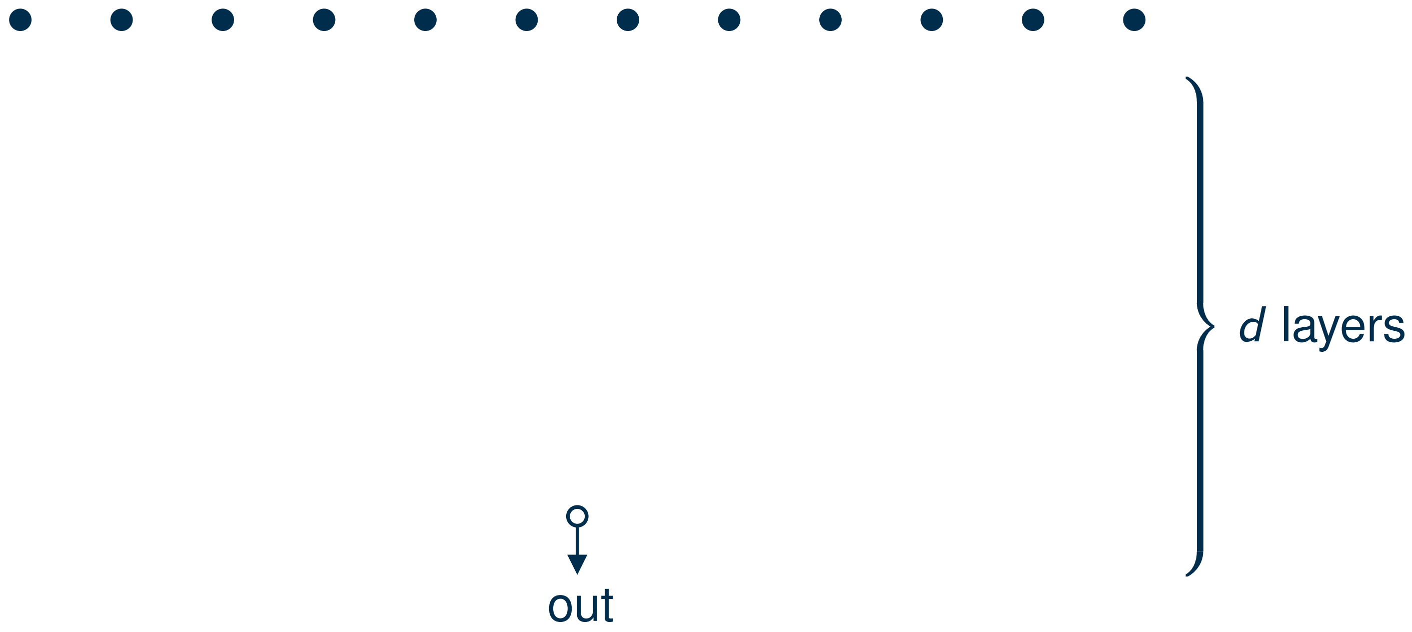

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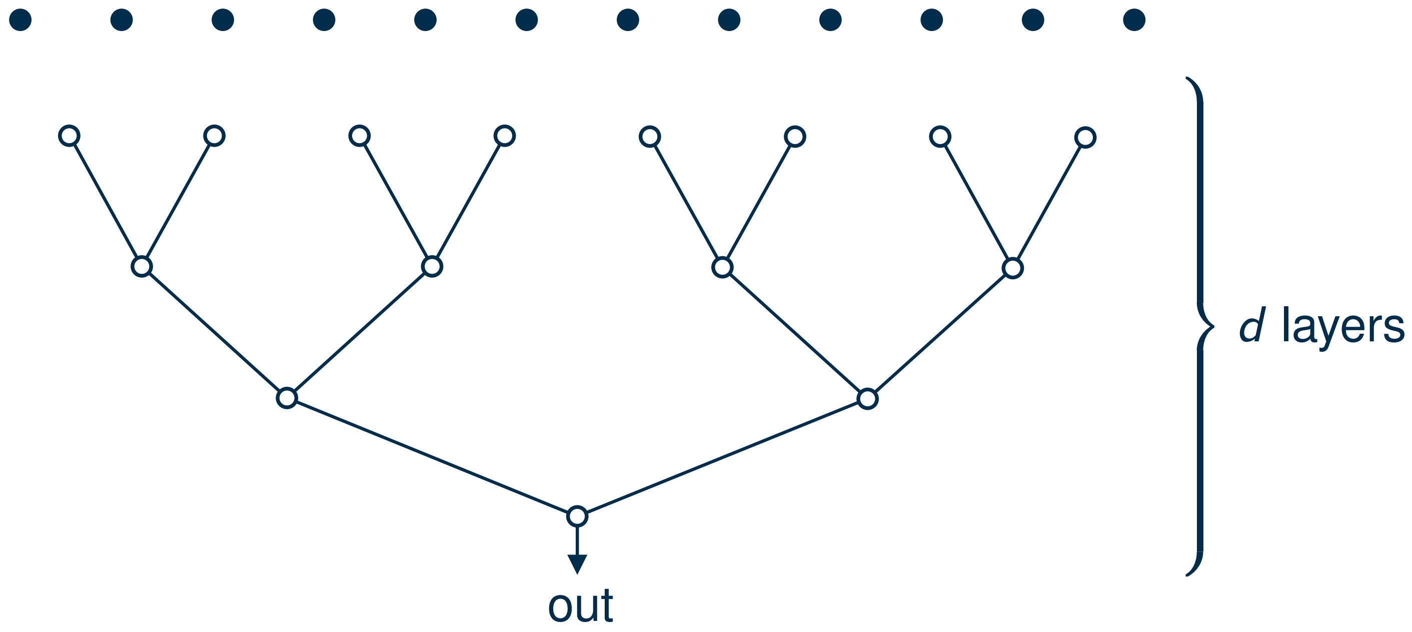

Monotone Weft-1 Circuits

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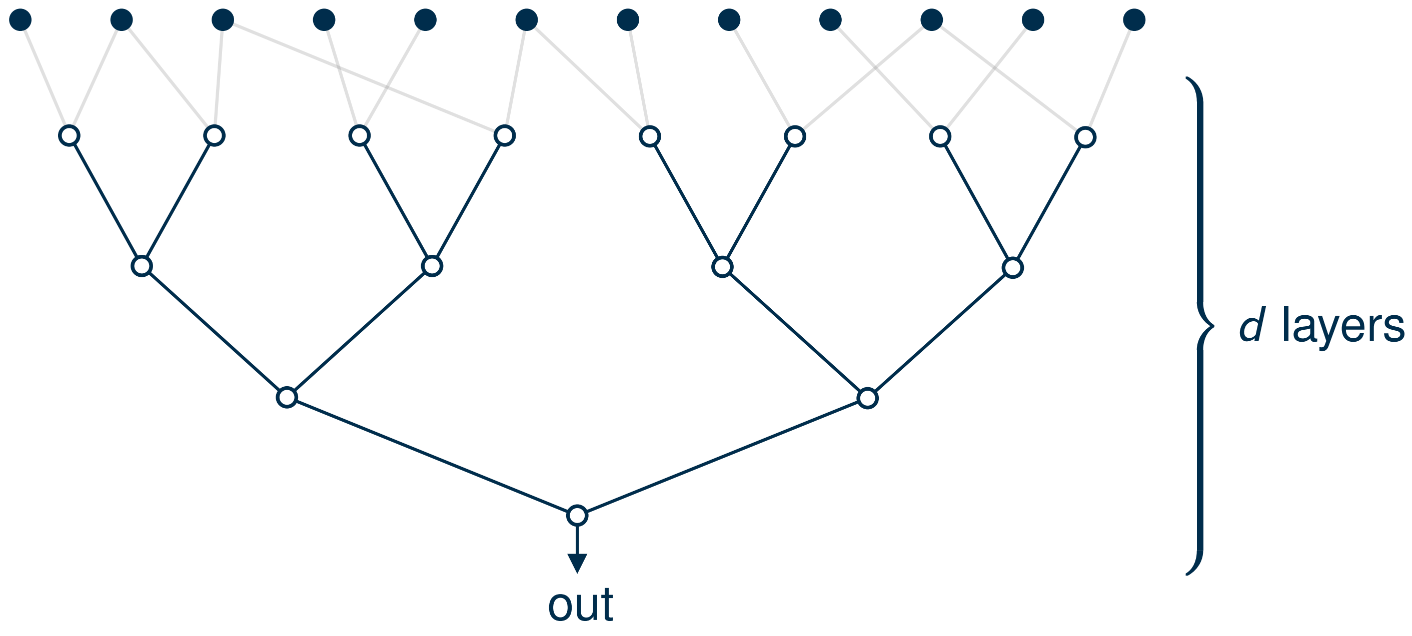

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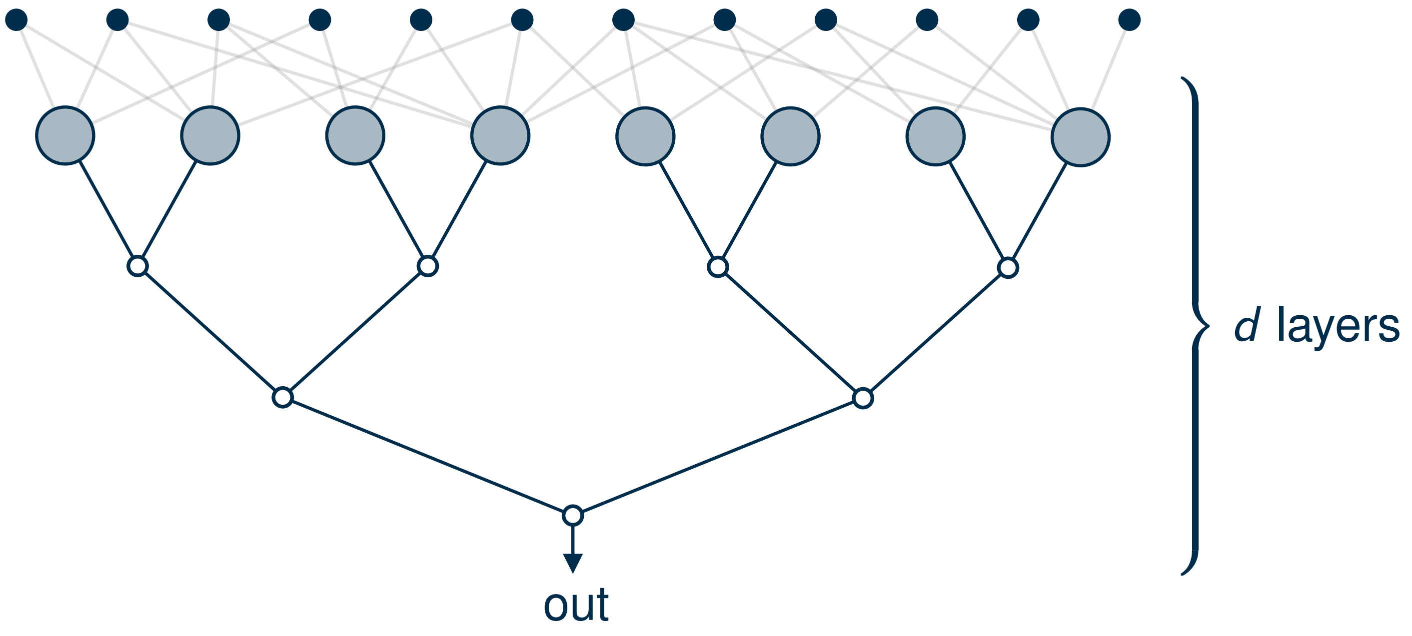
2^{d-1} large gates



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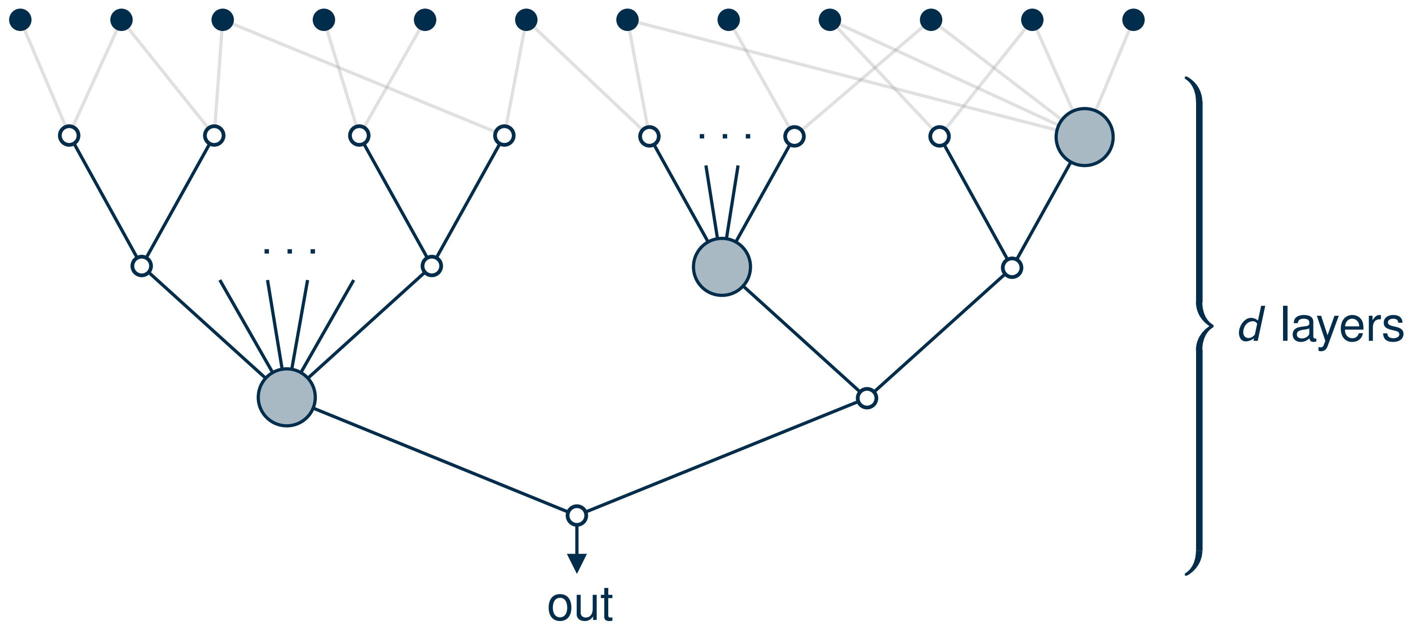

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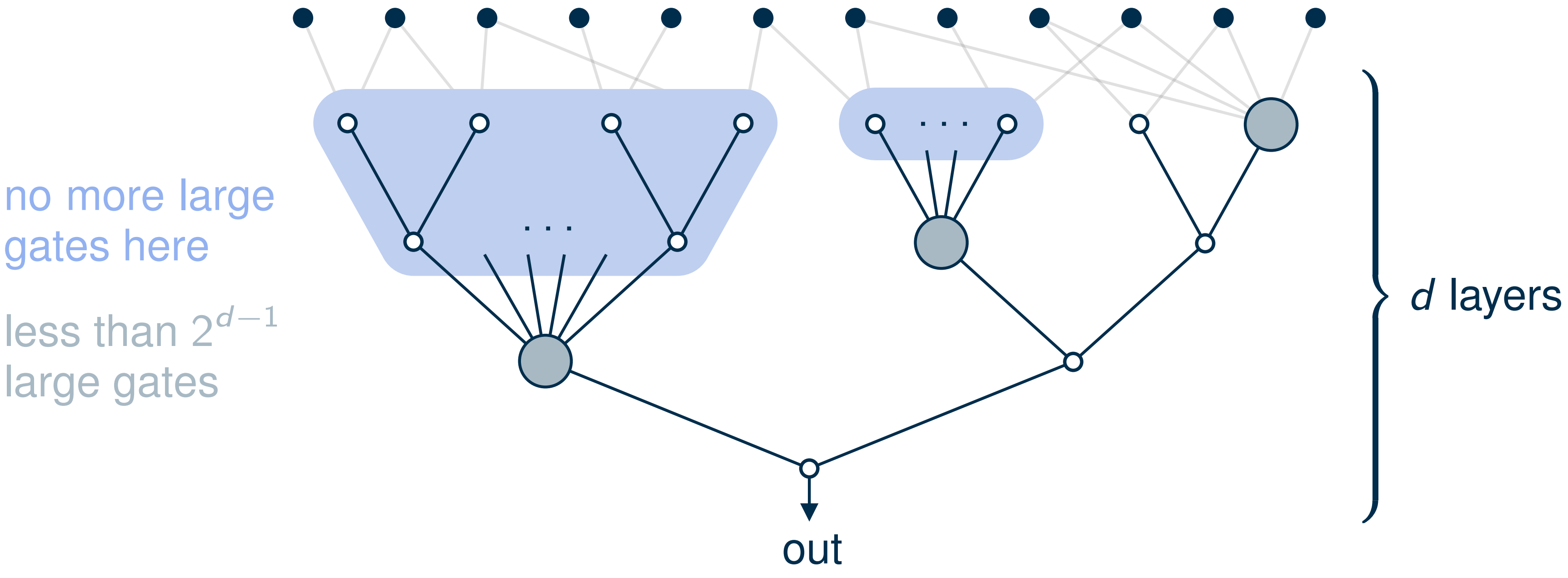

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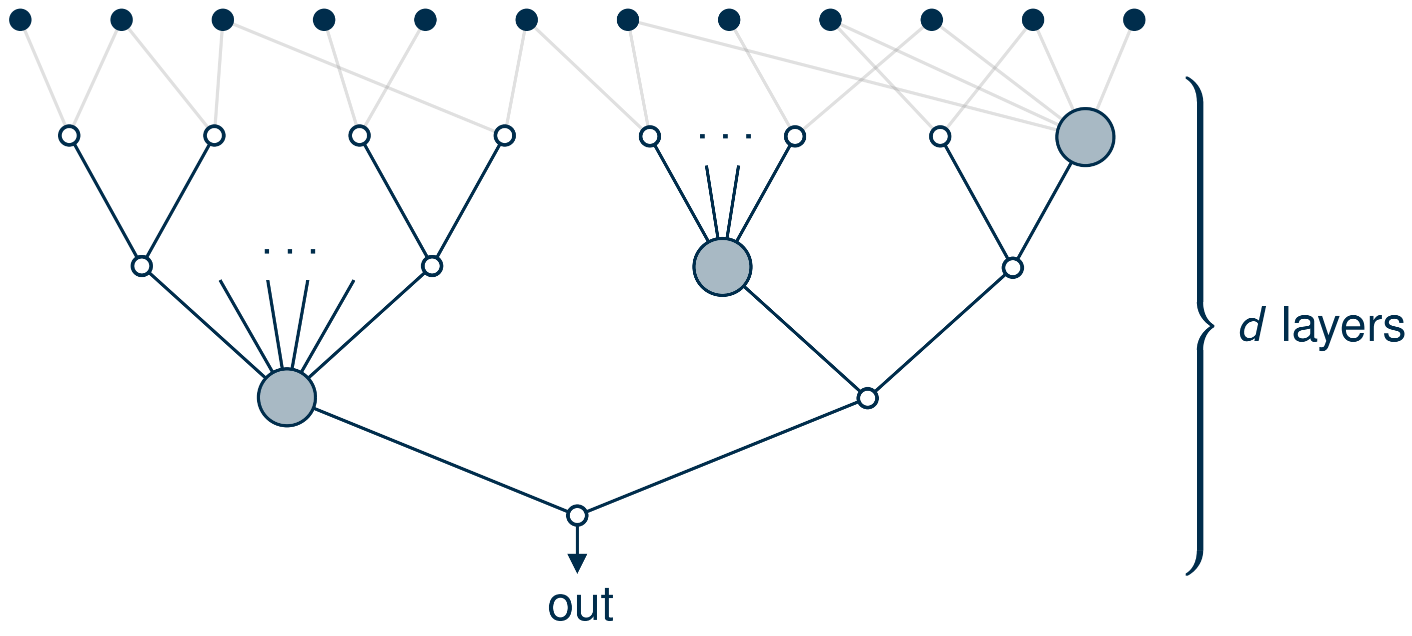

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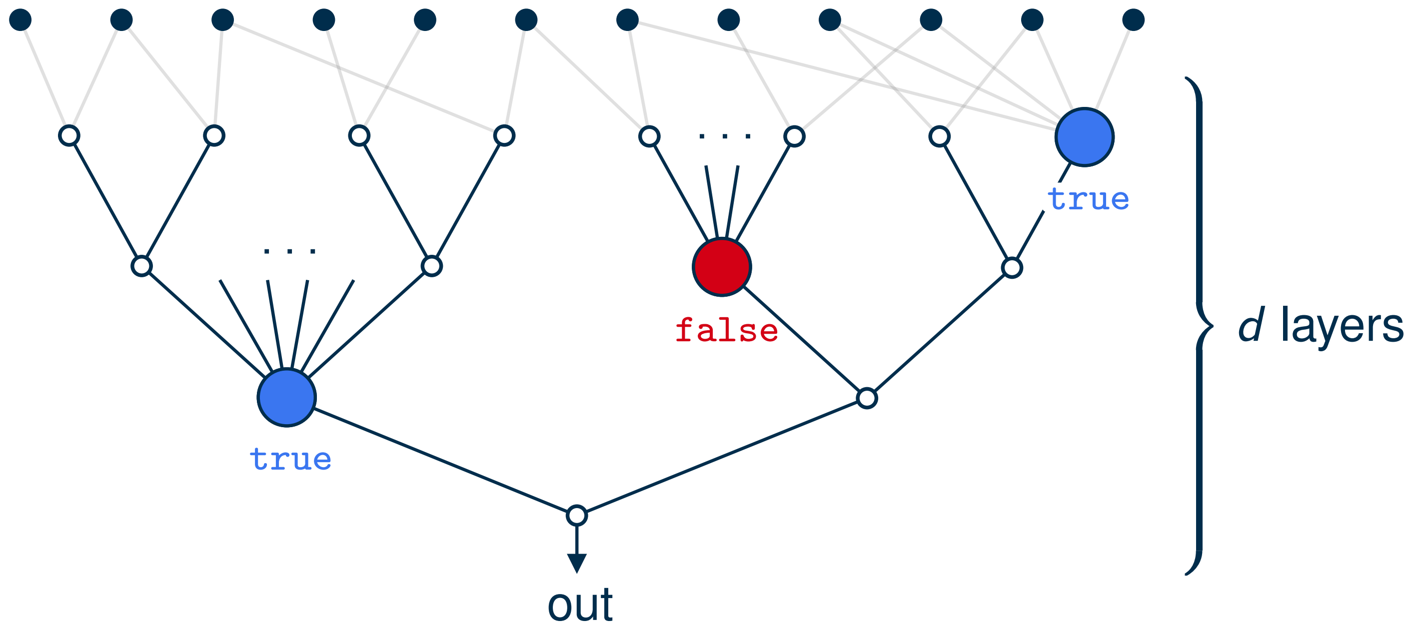

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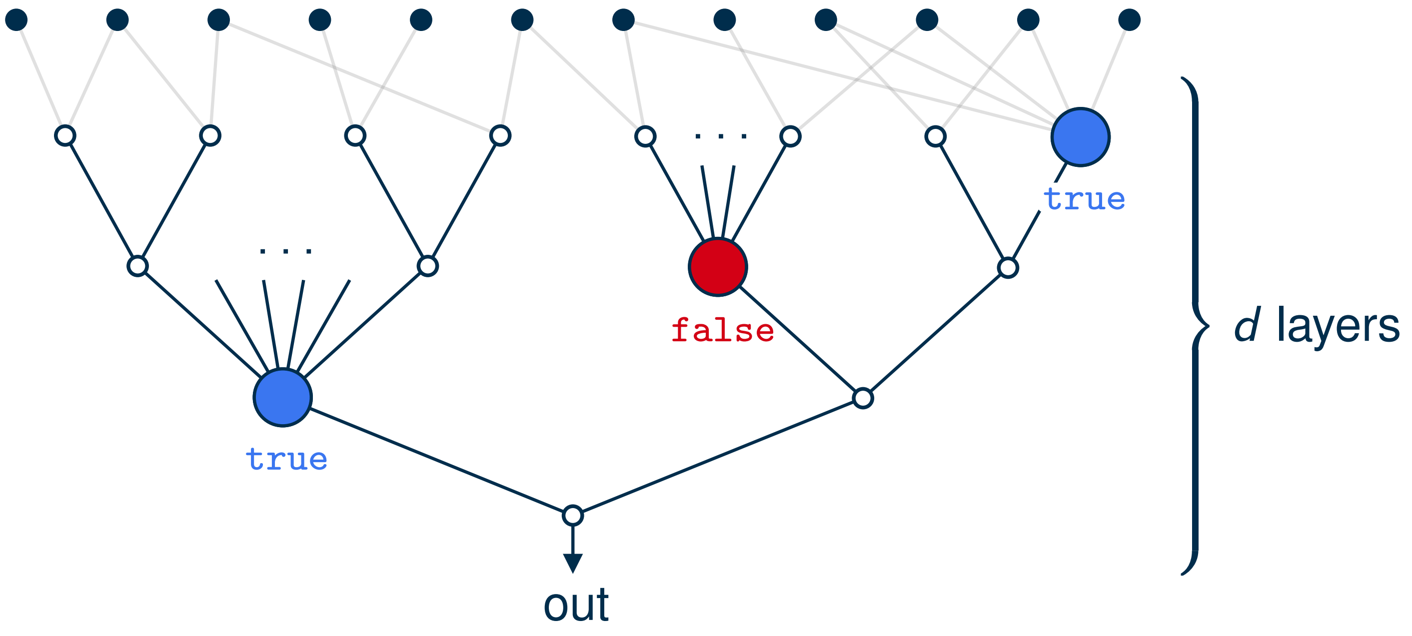

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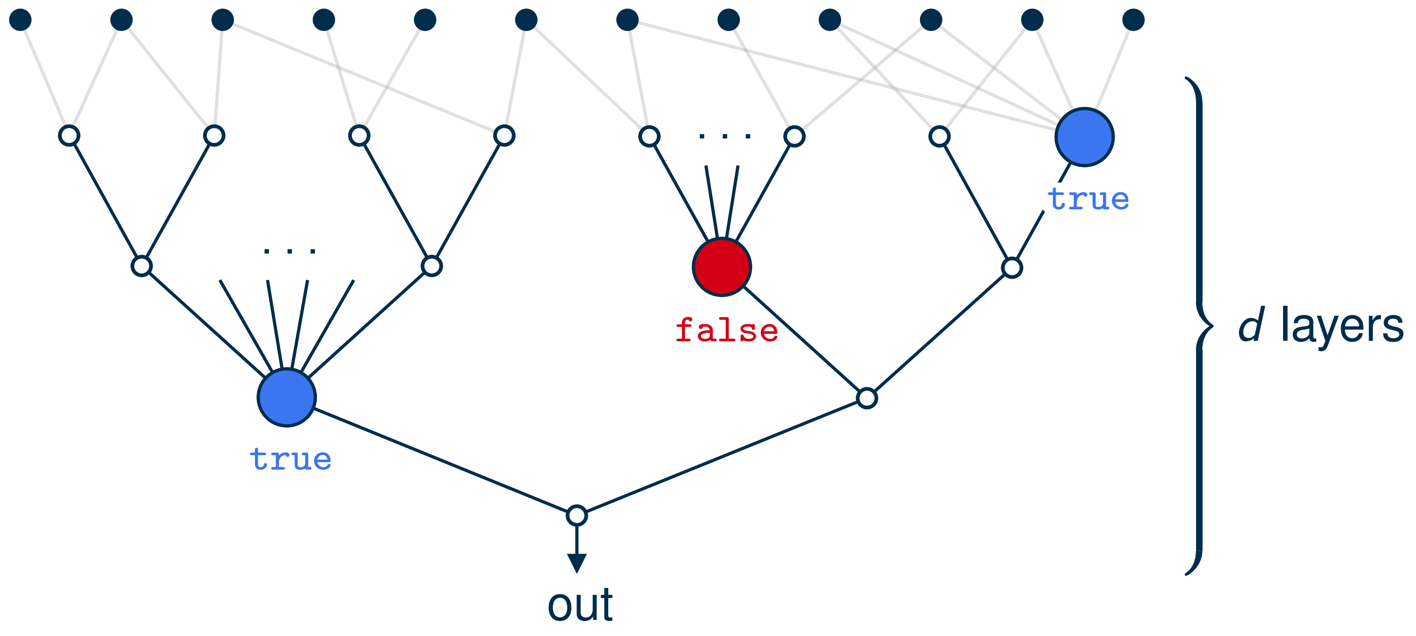

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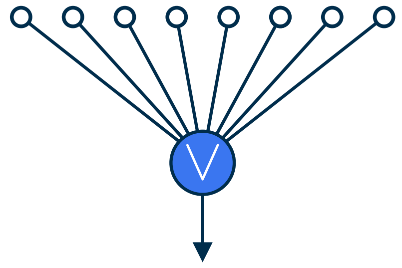

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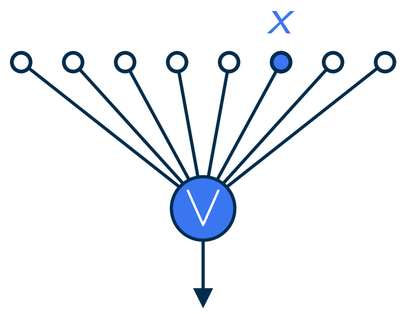

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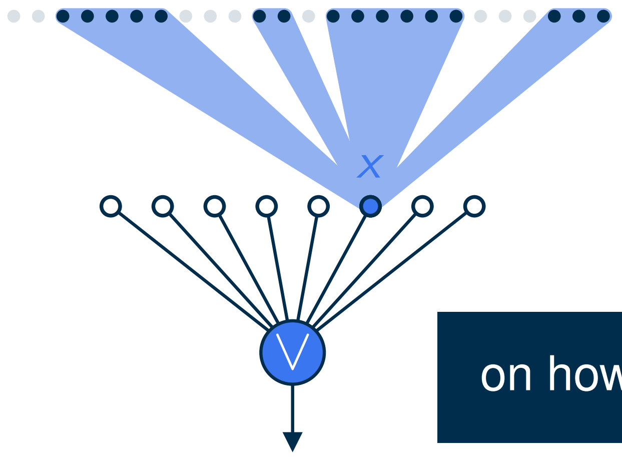

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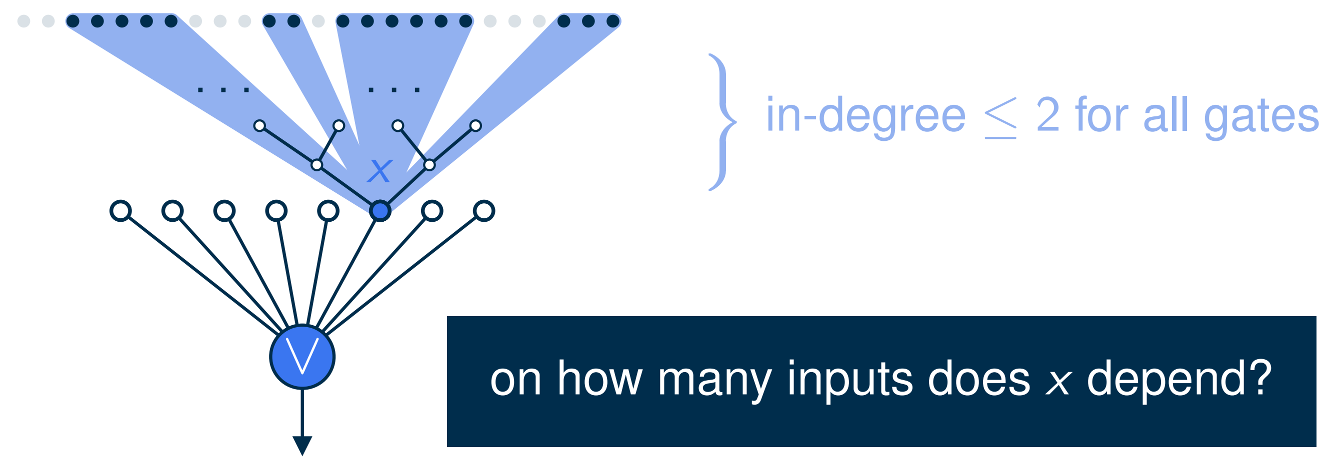

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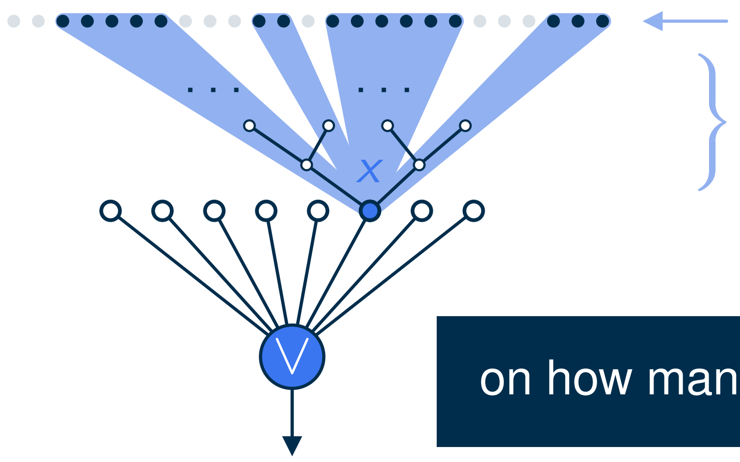

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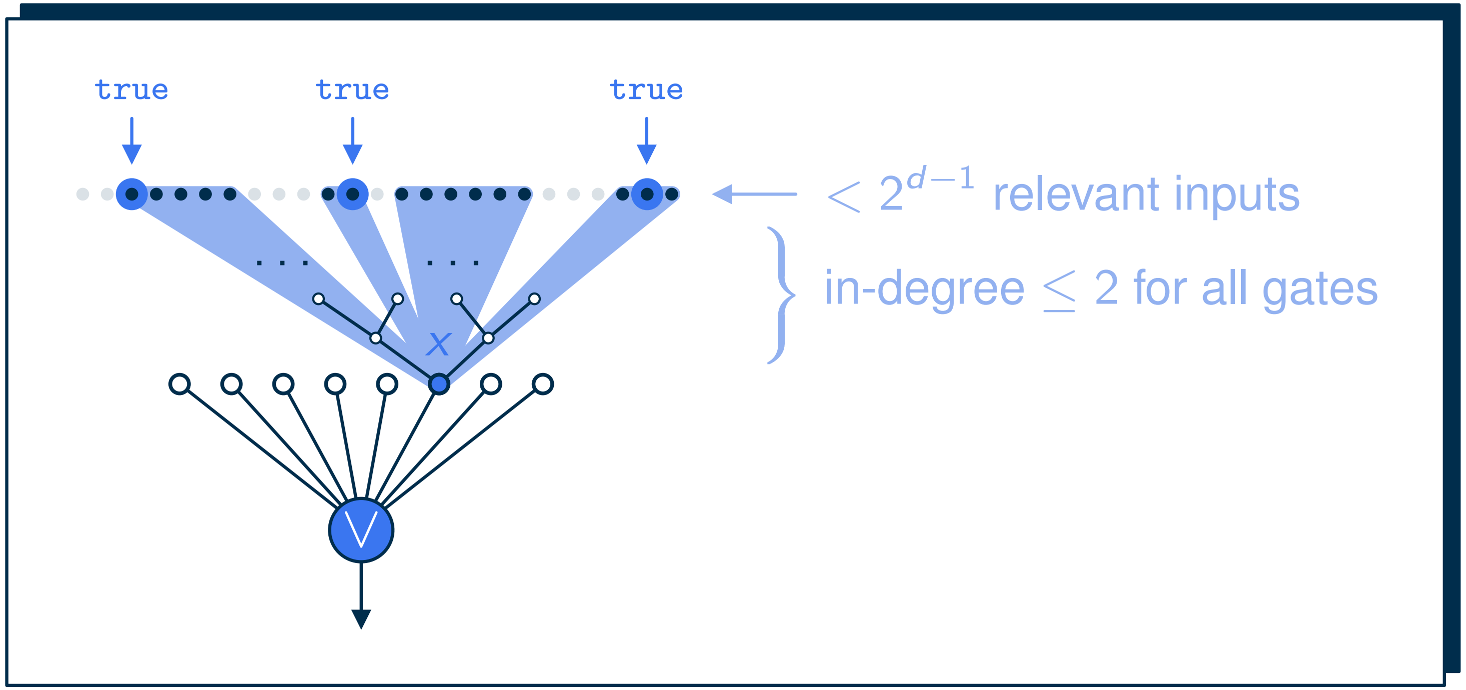

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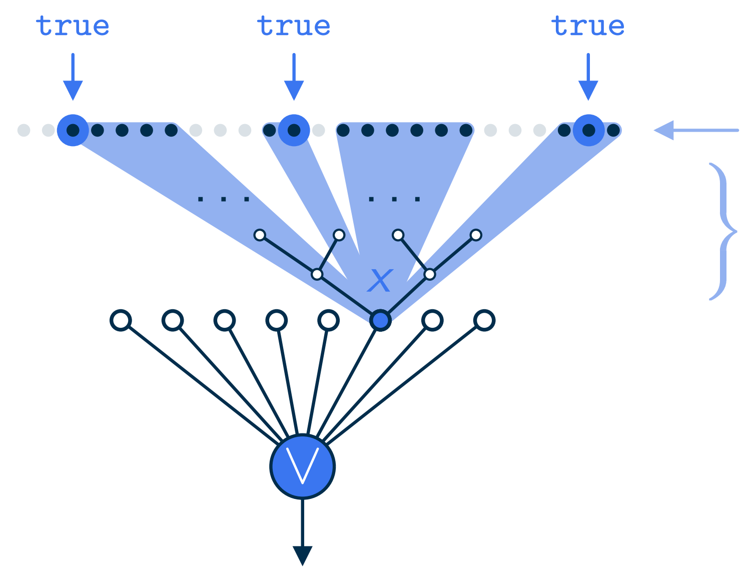

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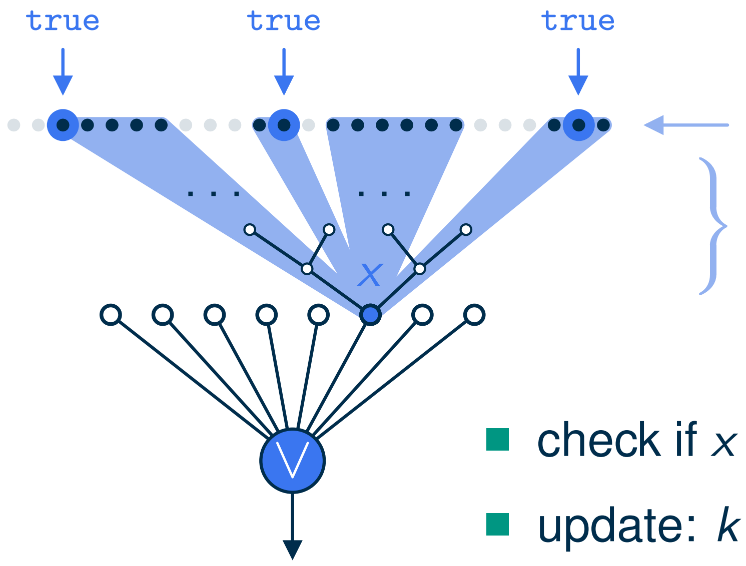

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- check if x satisfied (if not: next guess)
- update: $k \leftarrow k - \#$ true inputs

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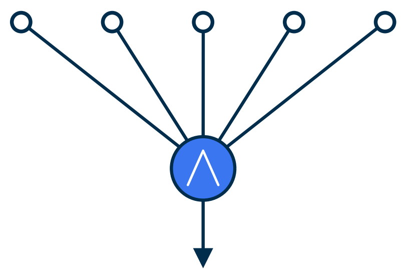

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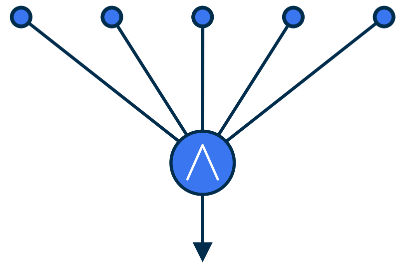

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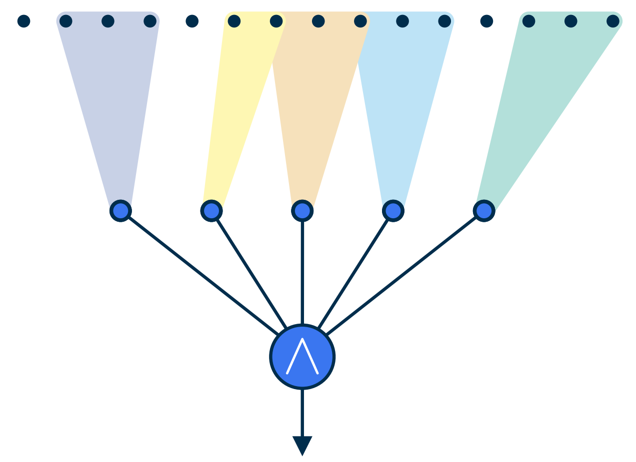

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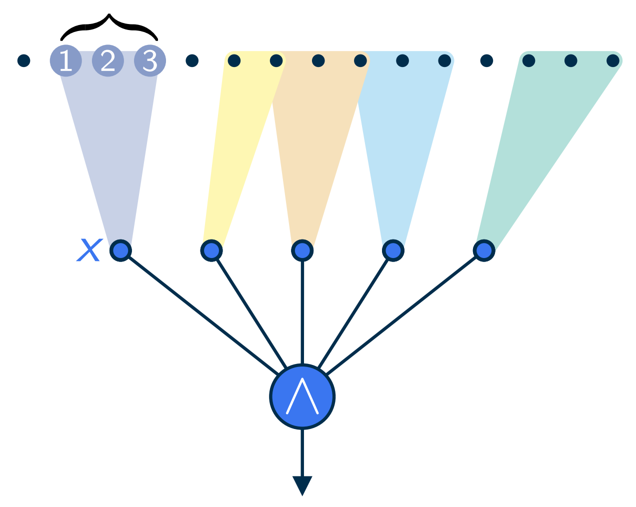

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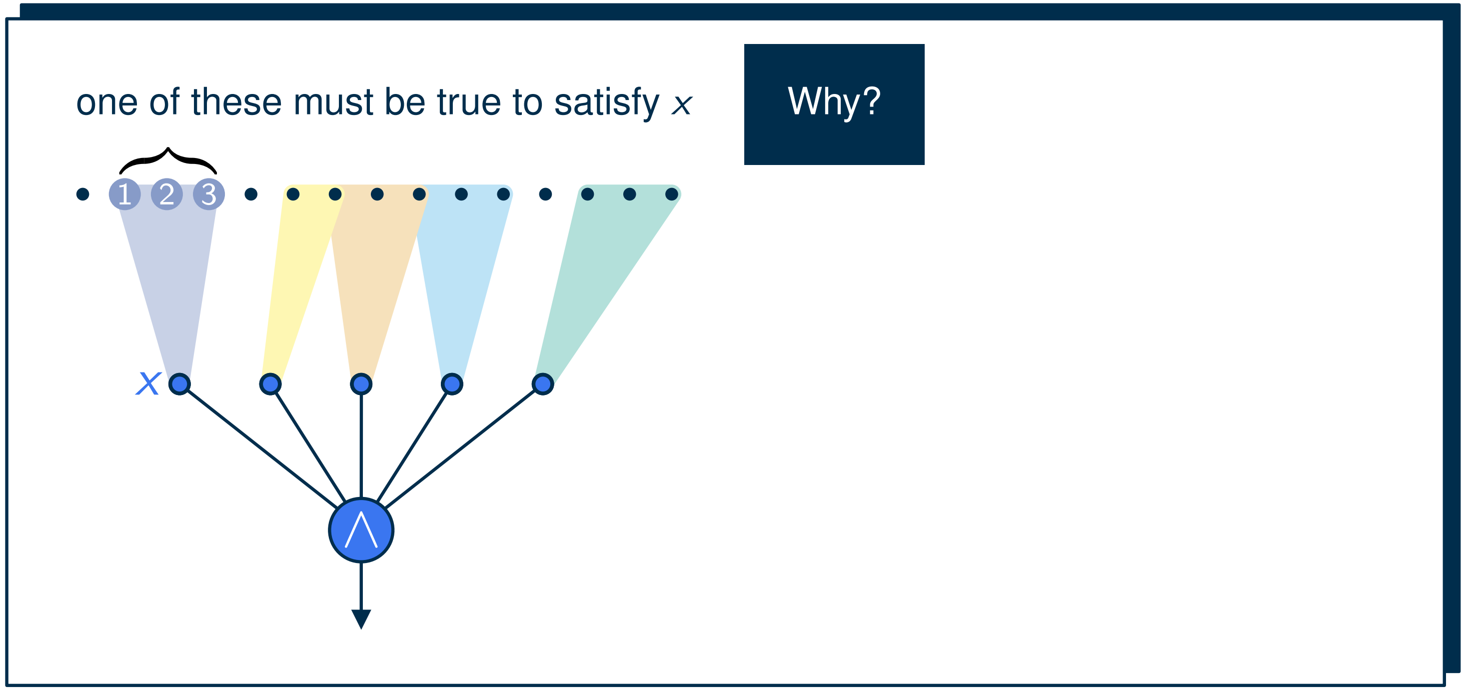

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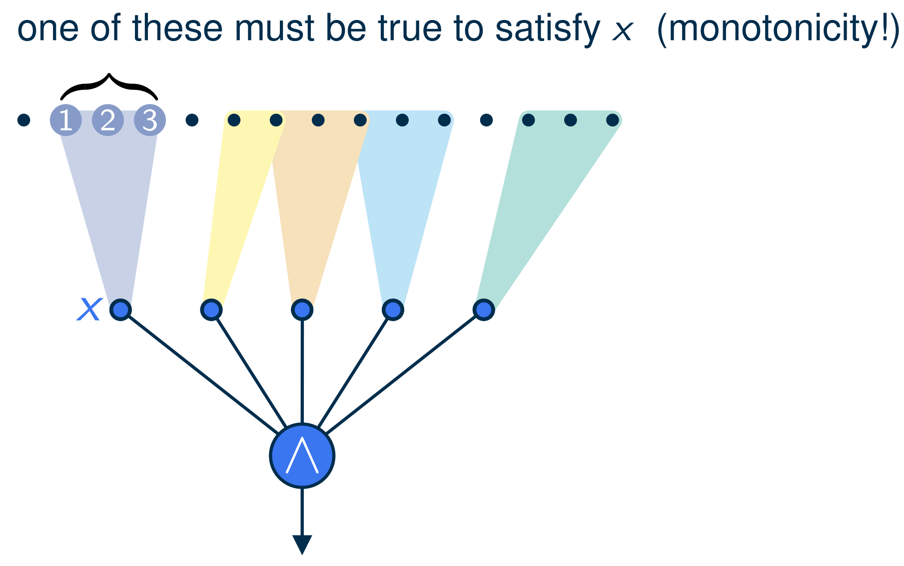

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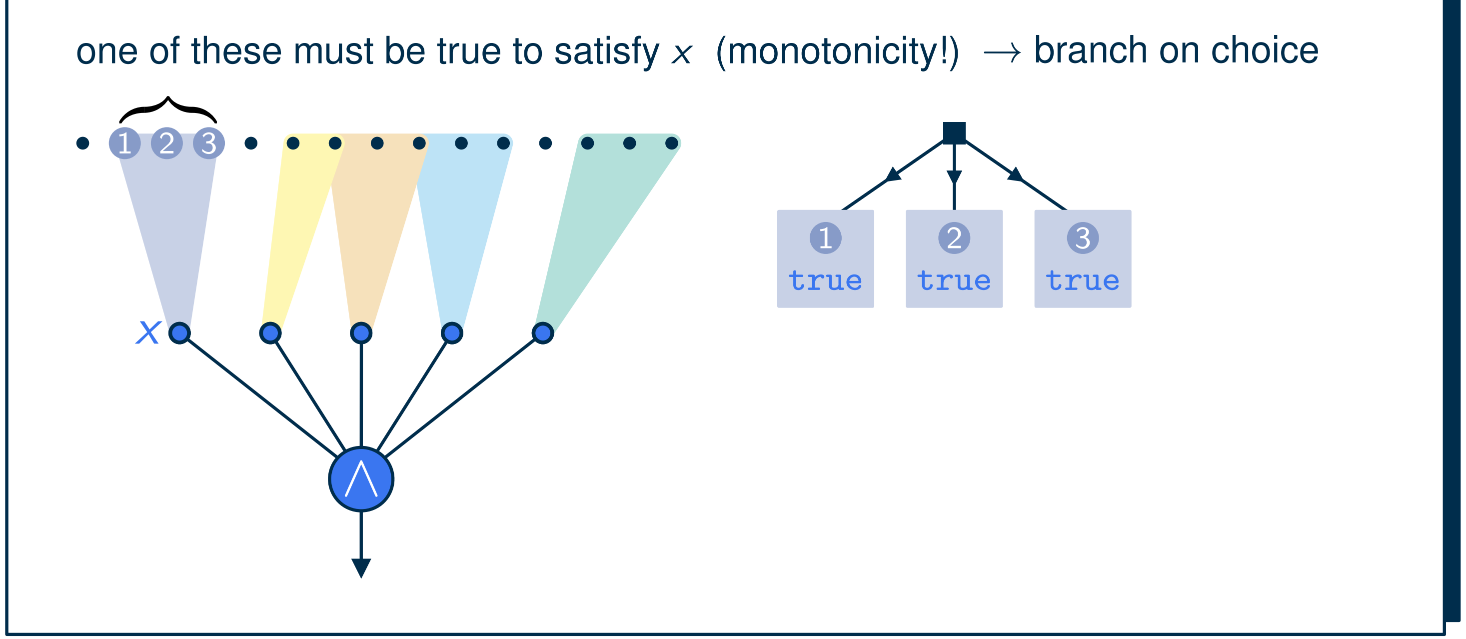

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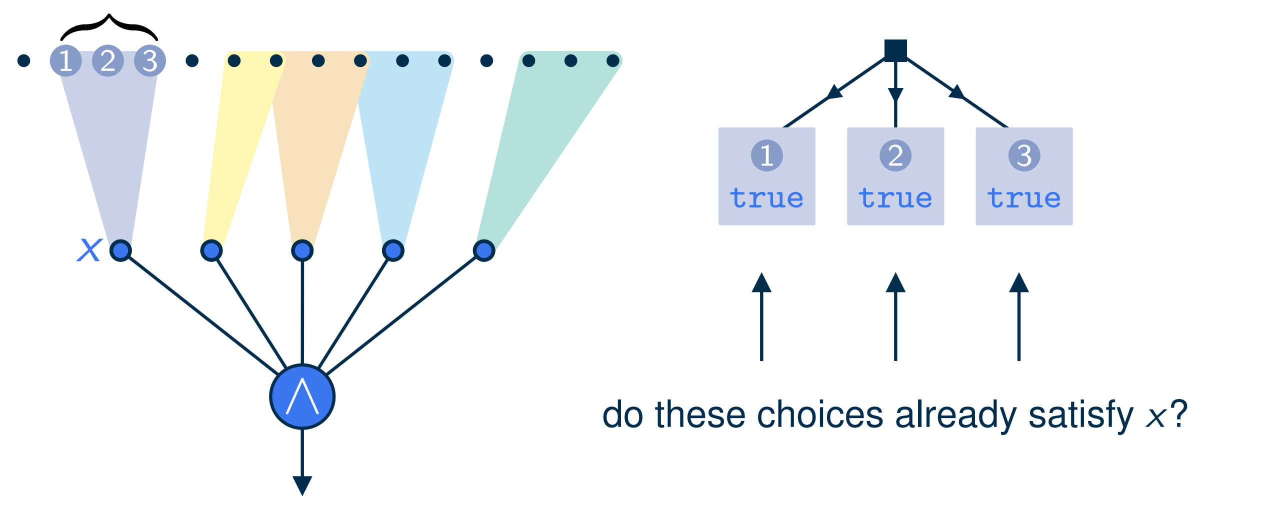

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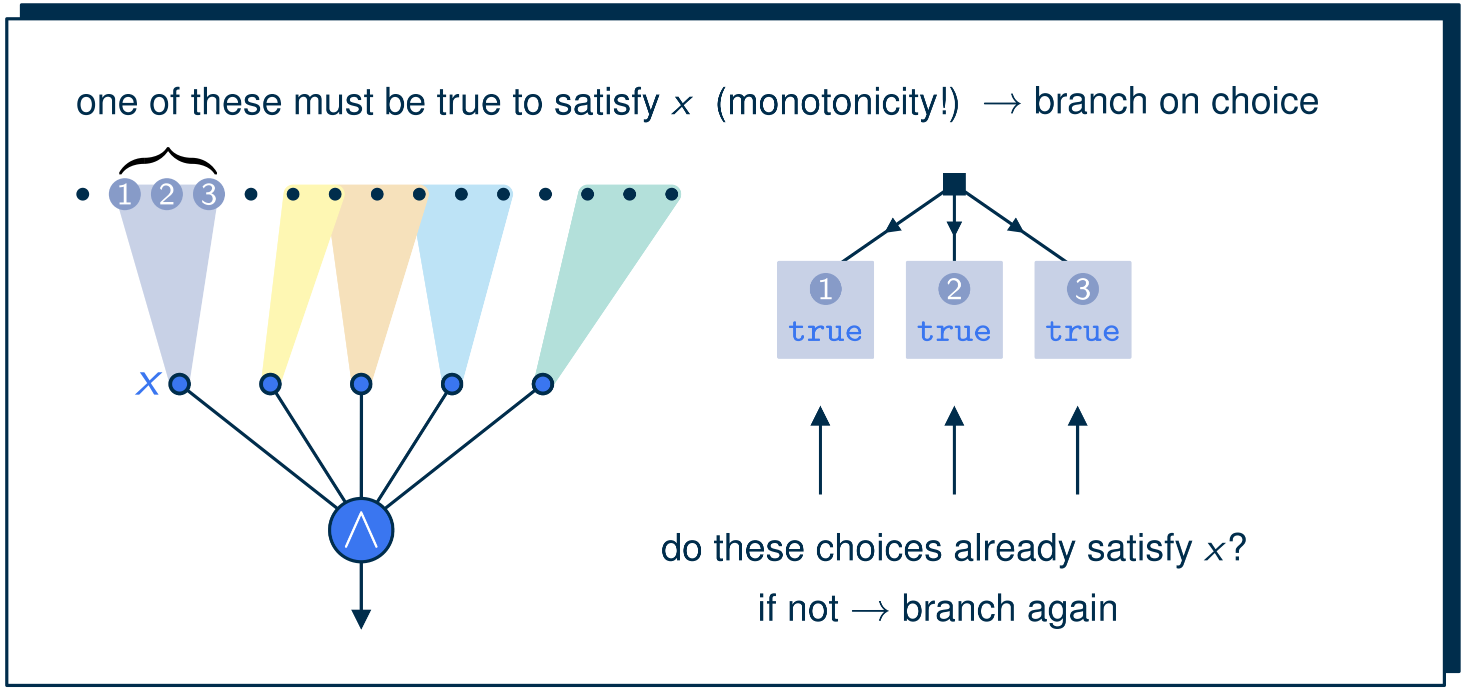

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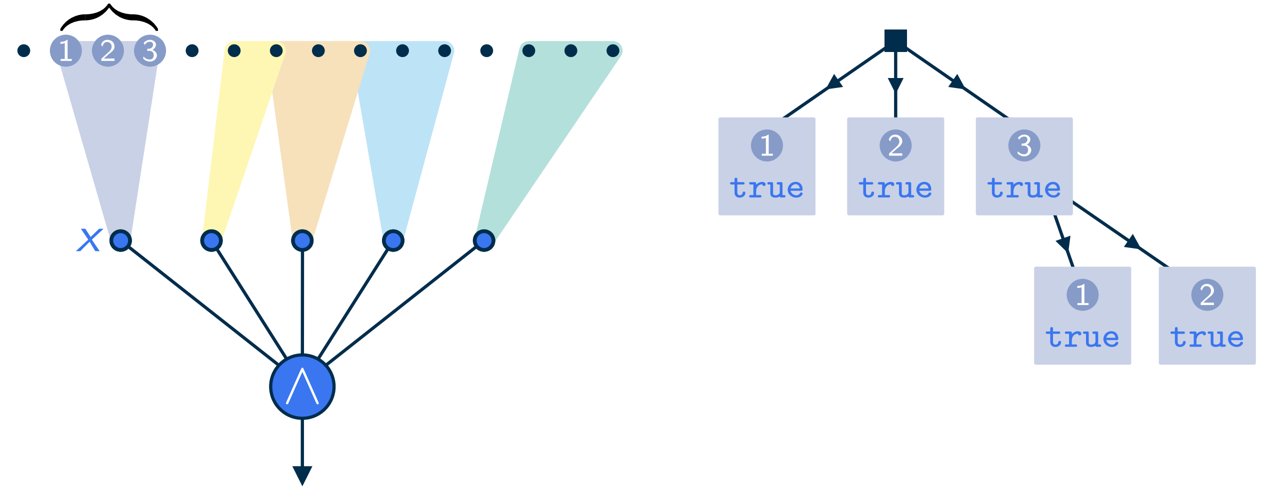
one of these must be true to satisfy x (monotonicity!) $\rightarrow$ branch on choice



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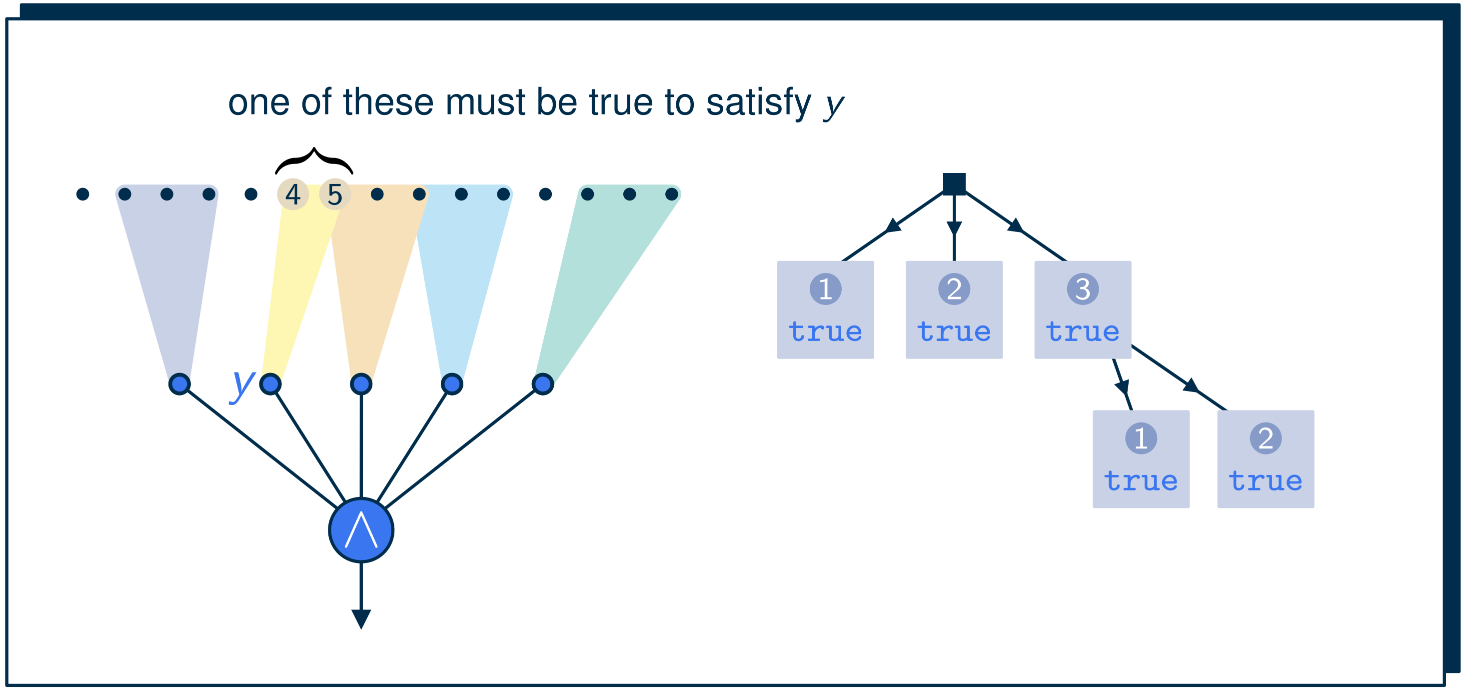

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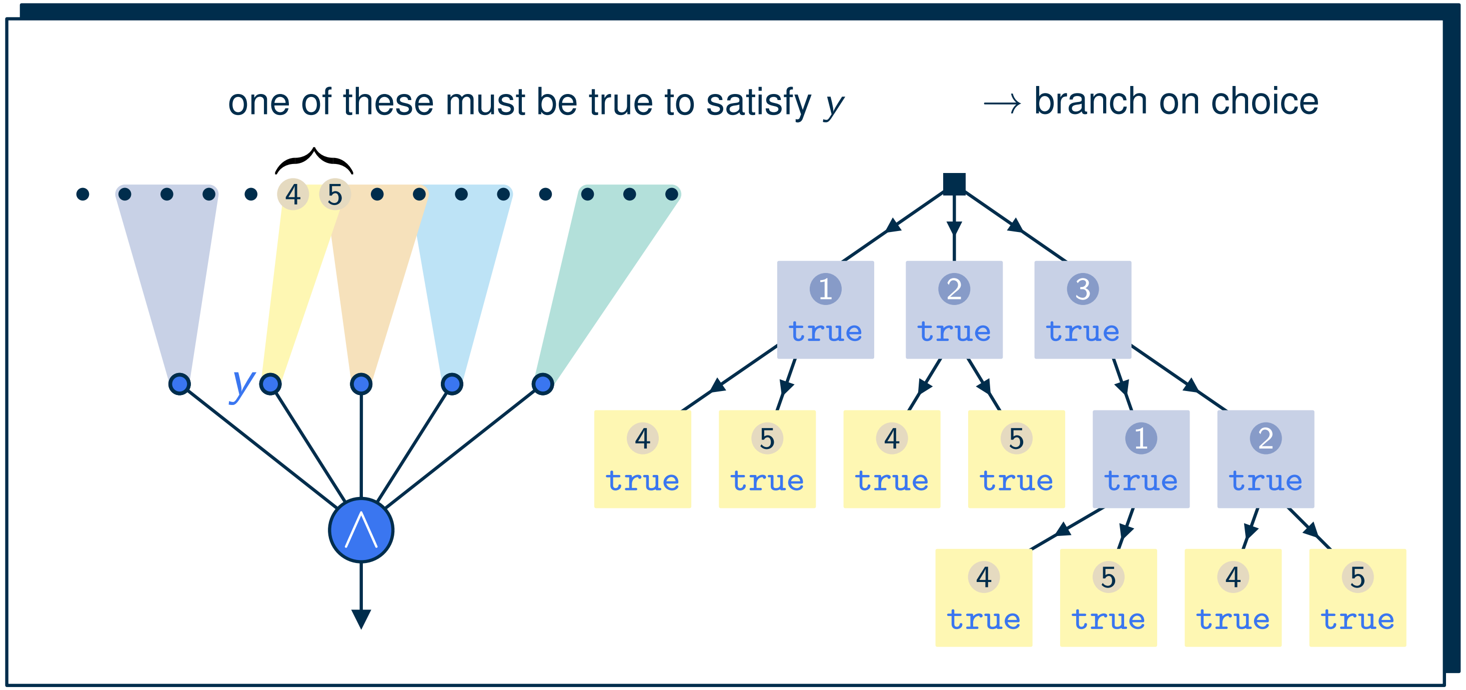

"guess" = actually, we iterate over all possibilities, but for now we fix one and describe how to solve it

Monotone Weft-1 Circuits

Given an instance of WEIGHTED CIRCUIT SATISFIABILITY with weft 1, constant depth d, and without negation nodes, together with a parameter k. Is there a satisfying assignment of weight $\leq k$?

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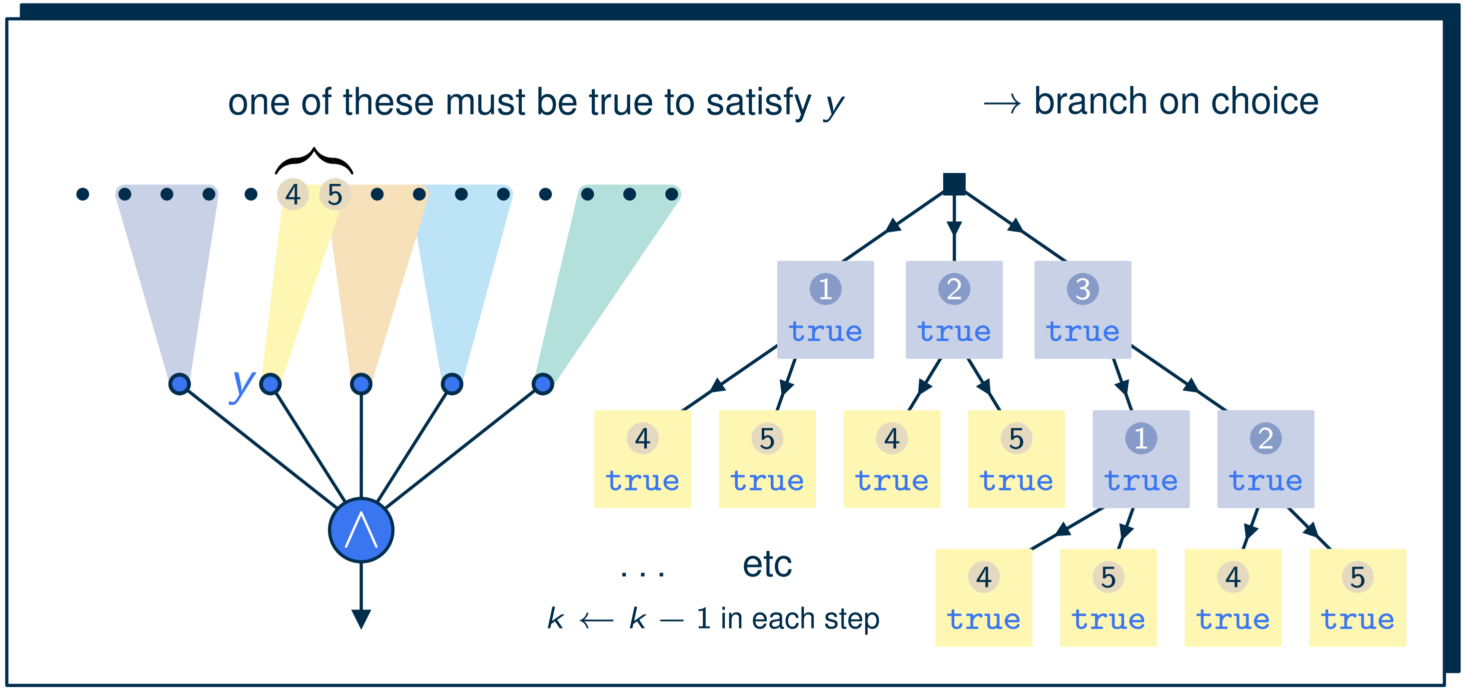

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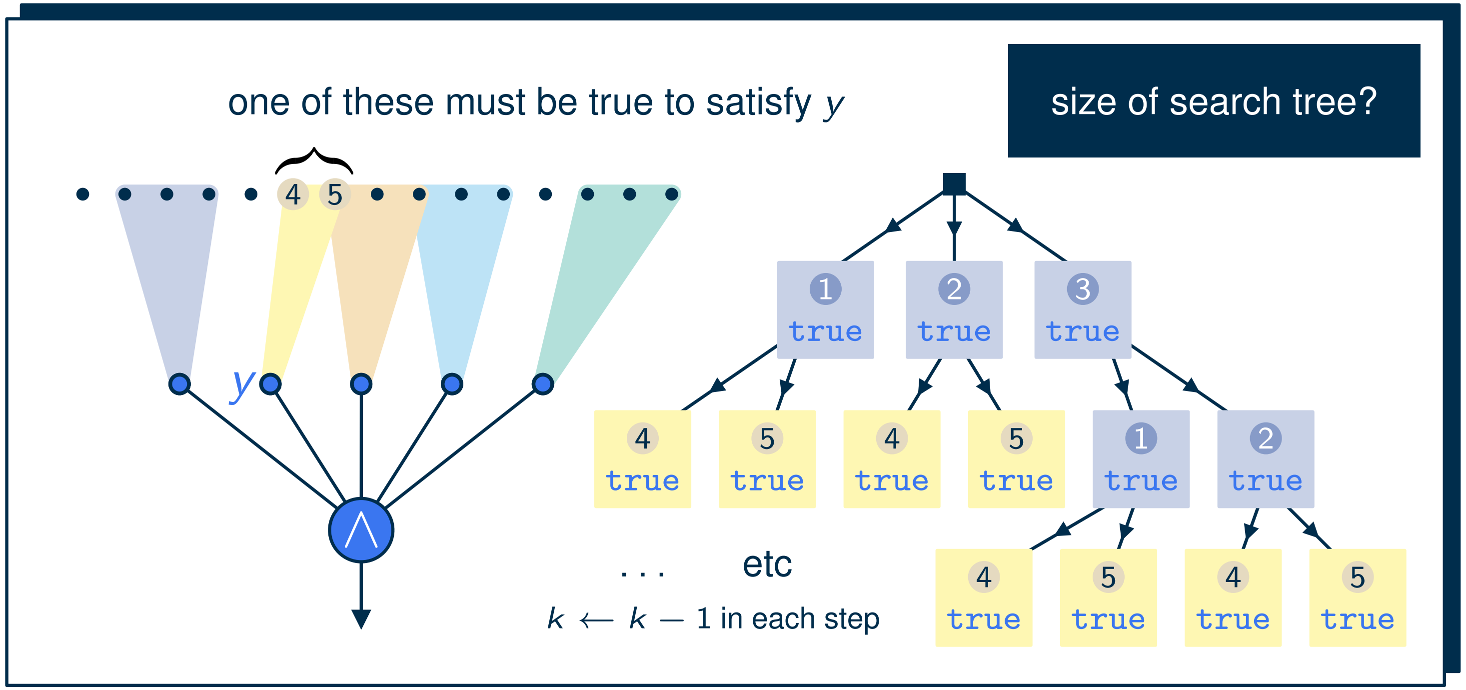

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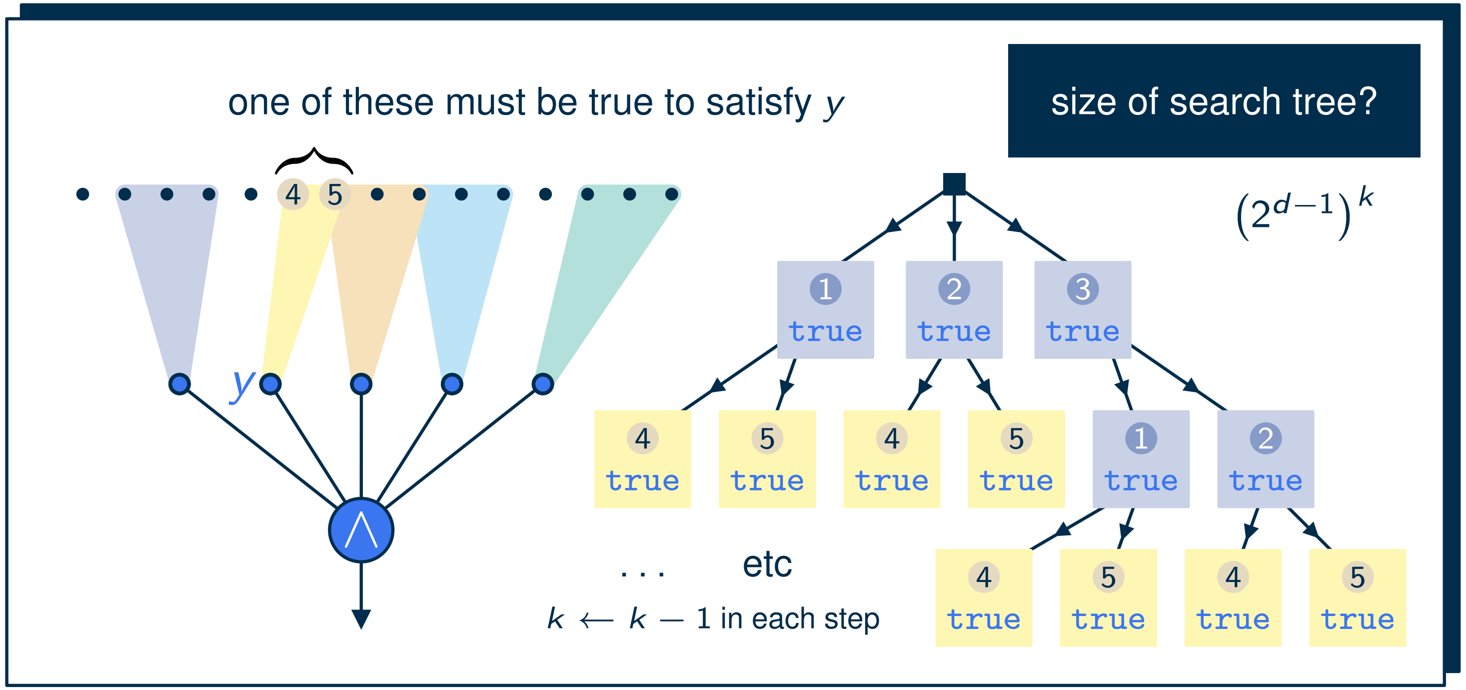

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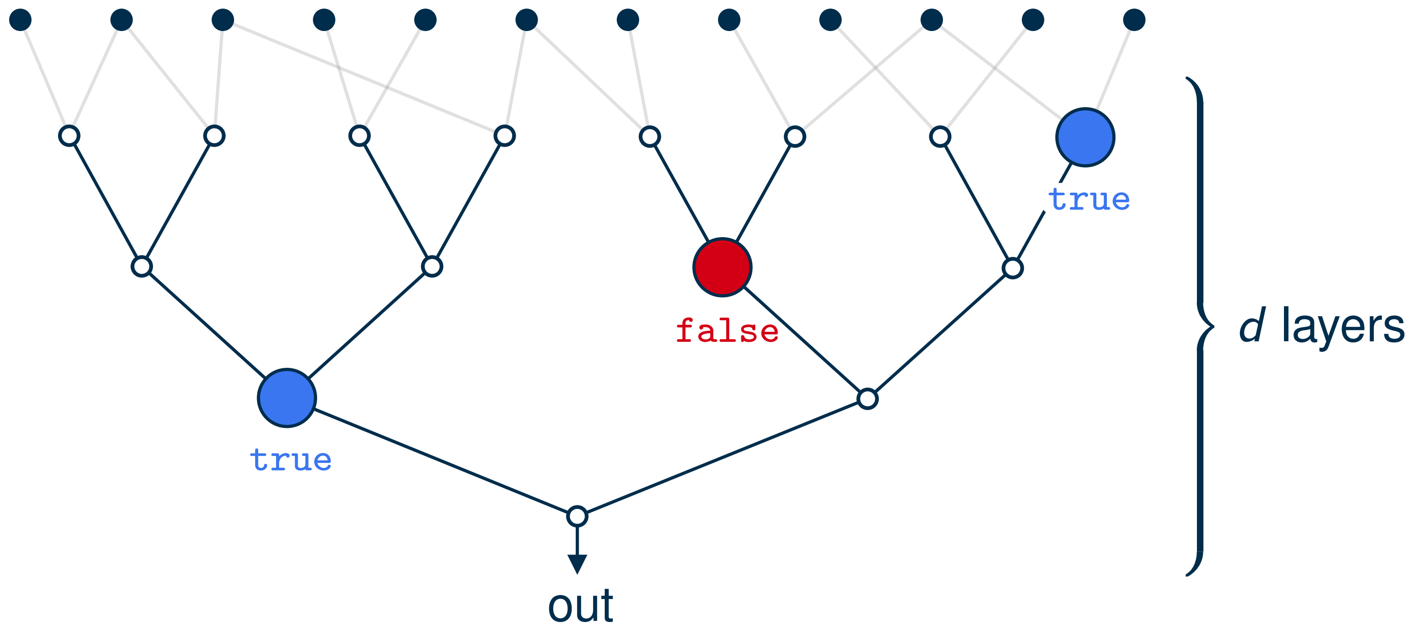

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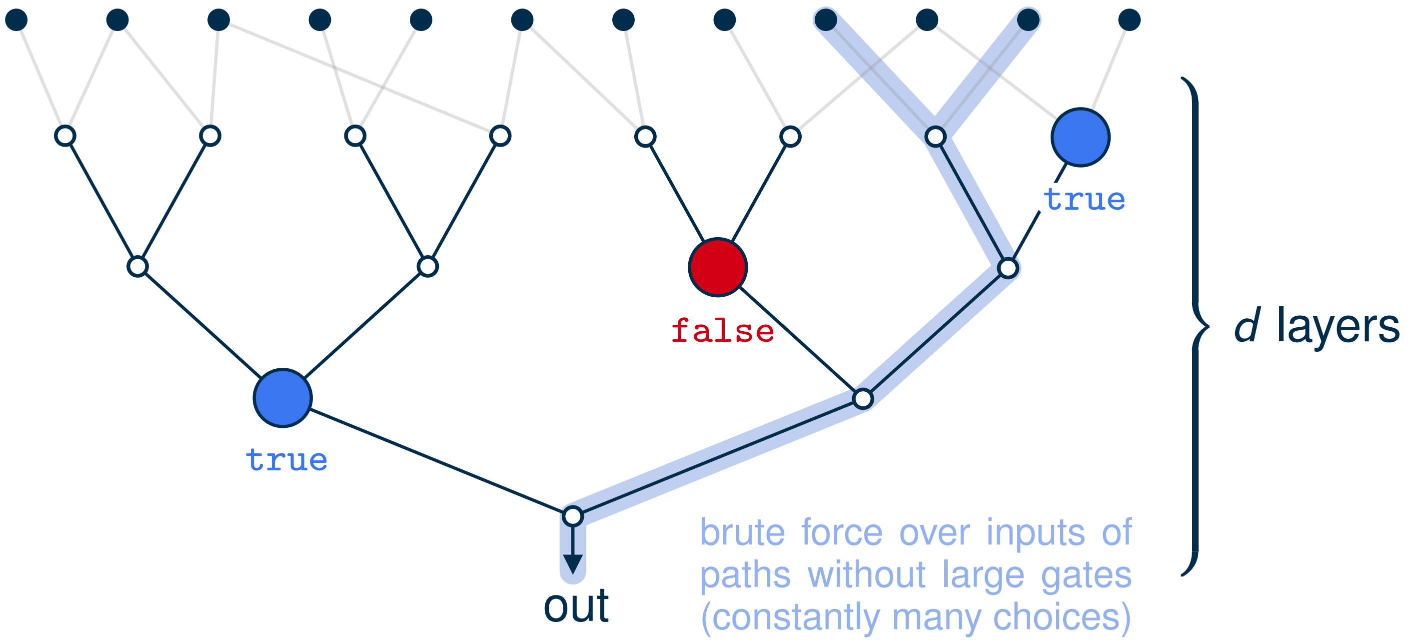

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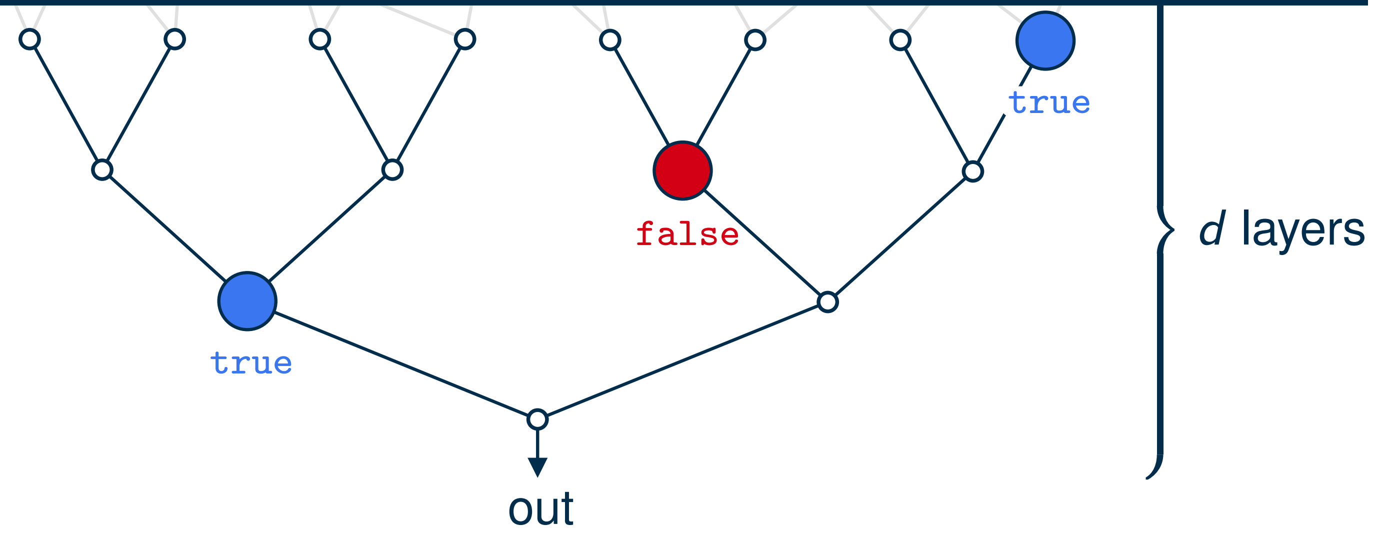

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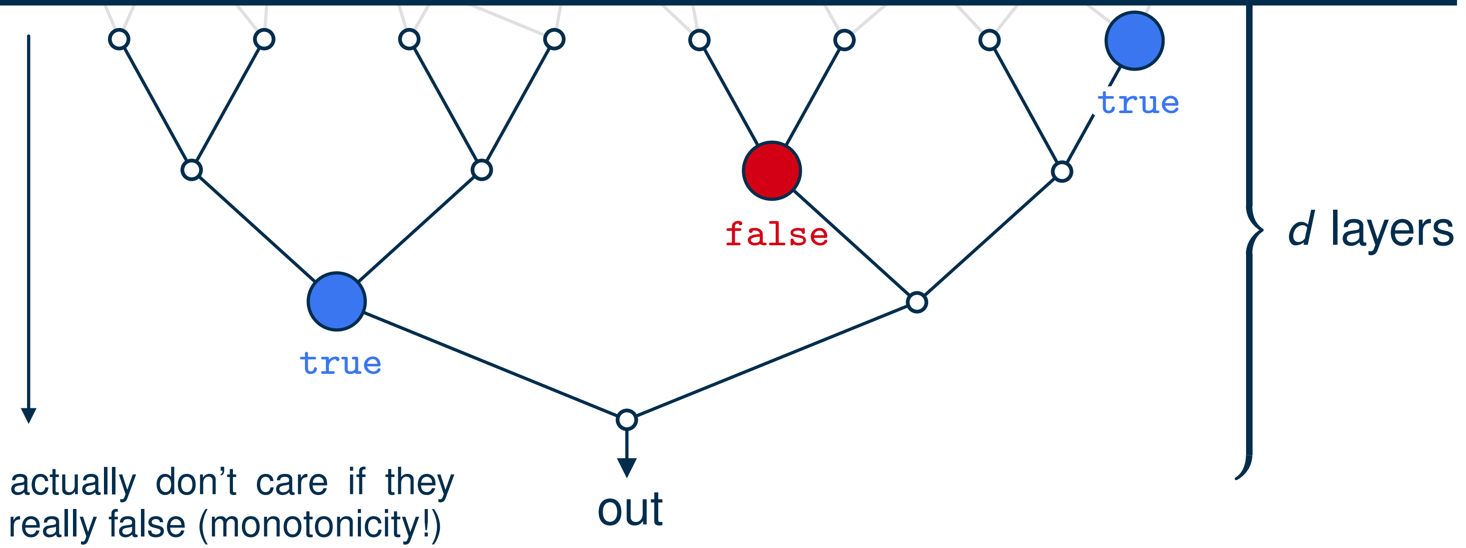

we actually don't care if they are really false (monotonicity!)

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- check if global output is true with chosen assignment
  (if not: next iteration)