Parameterized Algorithms

Exercise 5 – Sheet 4 & 5, Kernel Lower Bounds

Elly, Jean-Pierre, Wendy

Sheet 4 – Makespan Scheduling

Given m machines, n jobs with processing times $p_1, p_2, \ldots, p_n \in \mathbb{N}$ and maximum processing time k. Is there an assignments of jobs to machines s.t. no machine requires more than k time to process its jobs?

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A parameterized problem with parameter k that can be formulated as an ILP with $f(k)$ many variables is in FPT.

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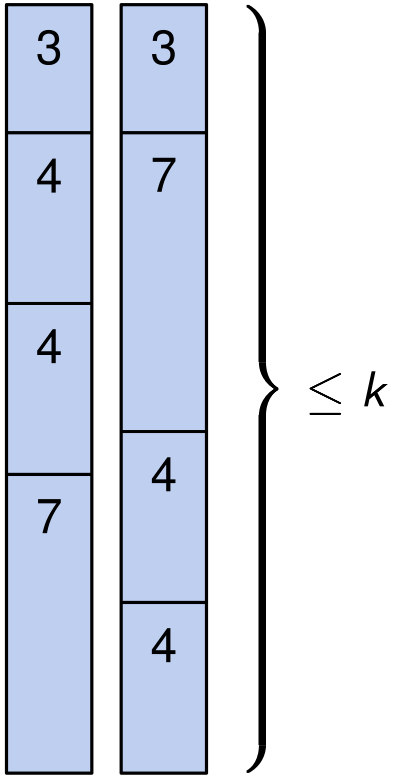

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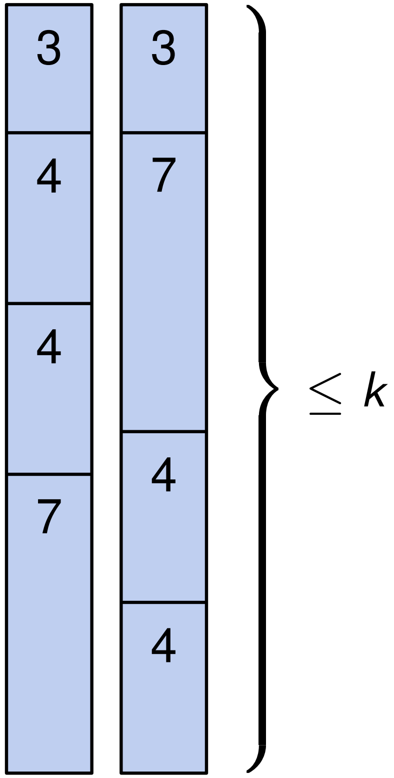

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Constants:

- c_ℓ: number of jobs of length ℓ
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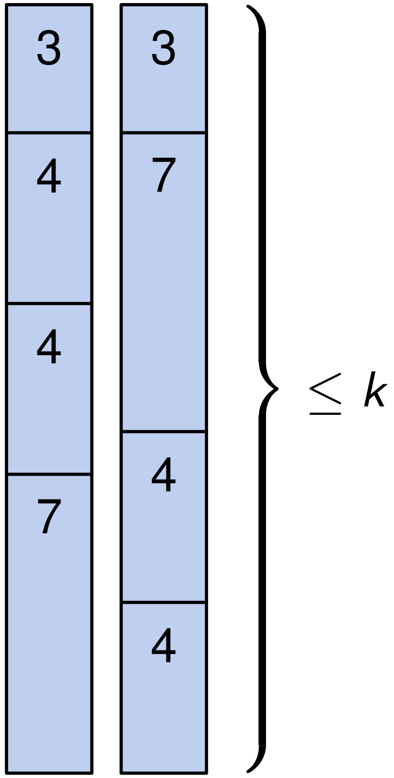

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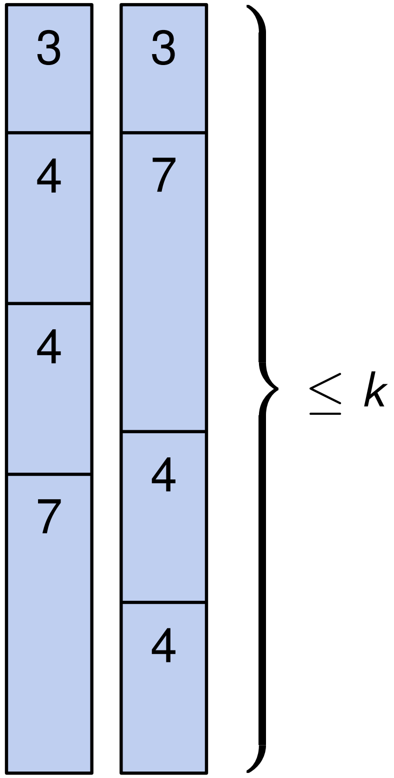

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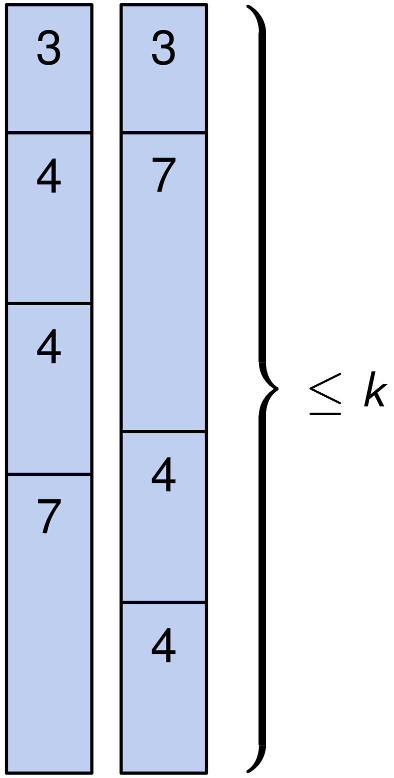

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$$\sum_{s \in S} d_{s,\ell} \cdot x_s = c_\ell \quad \text{for all } \ell \in \{1, \ldots, k\}$$

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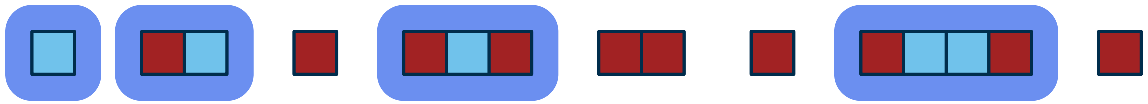

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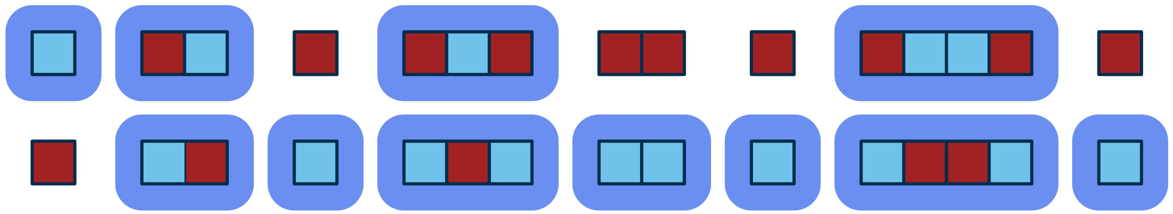

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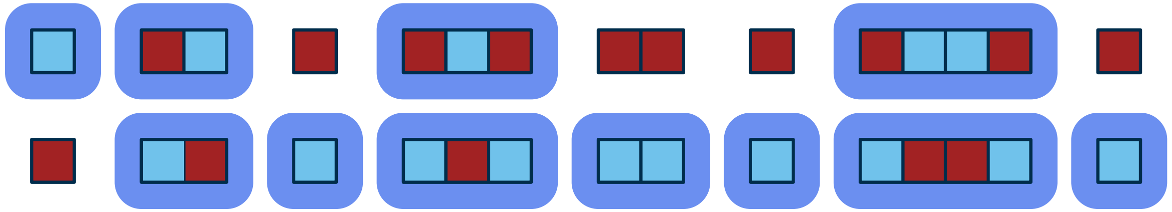

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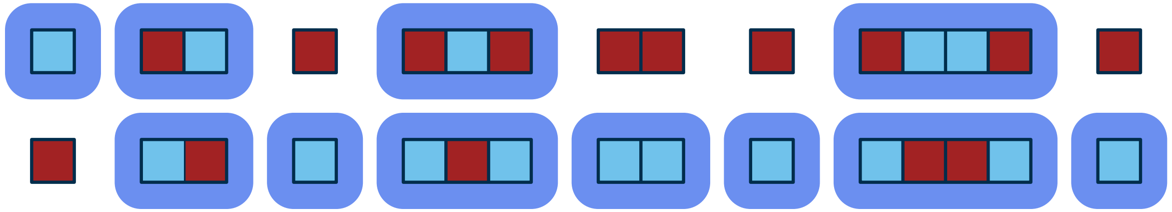

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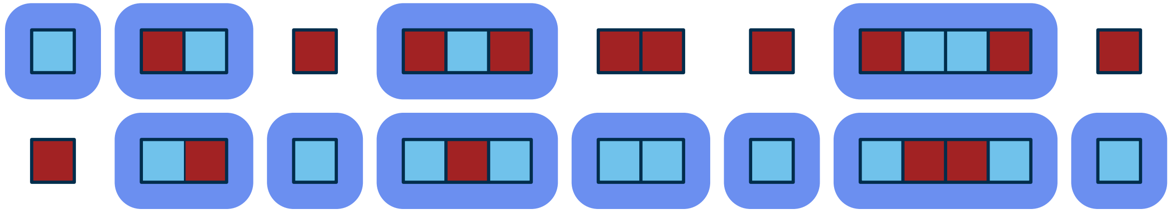

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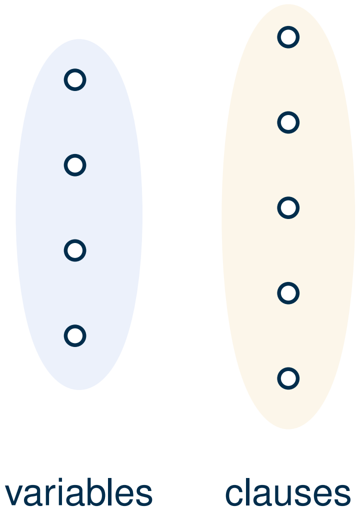

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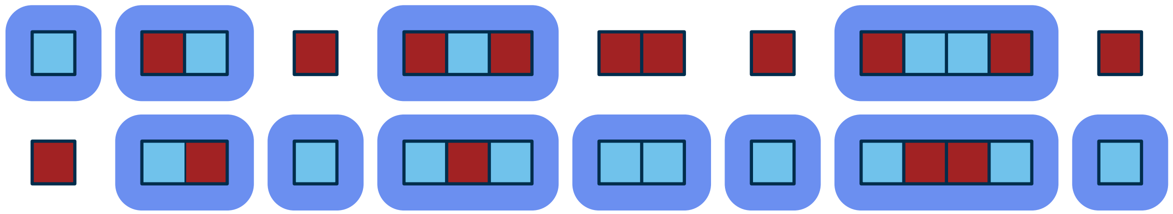

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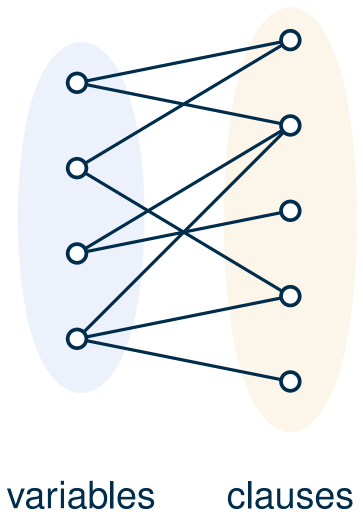

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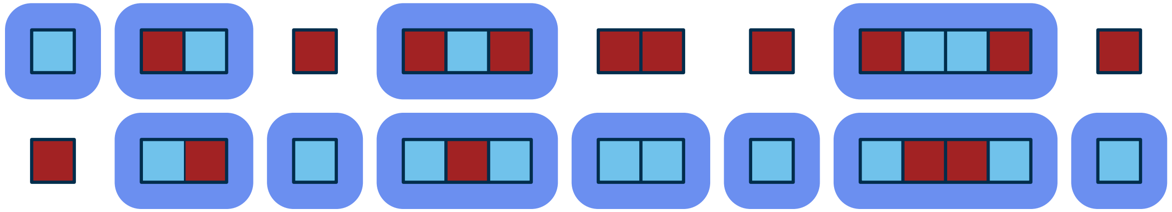

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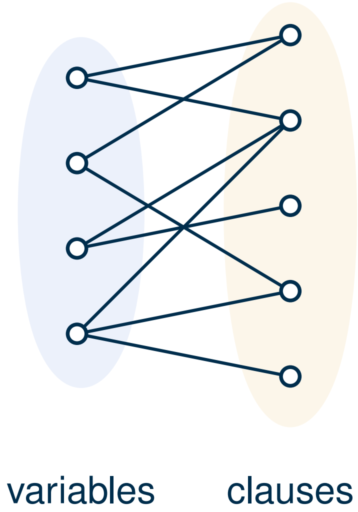

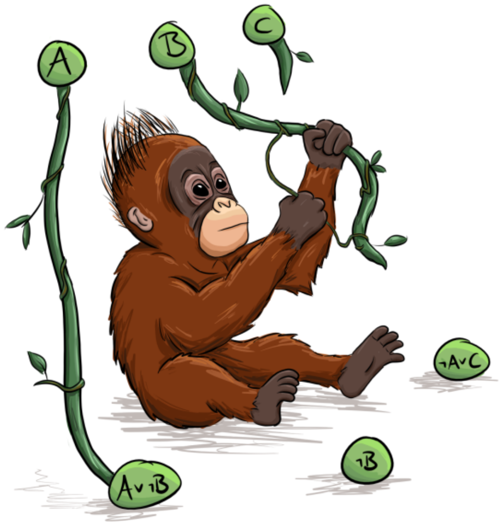

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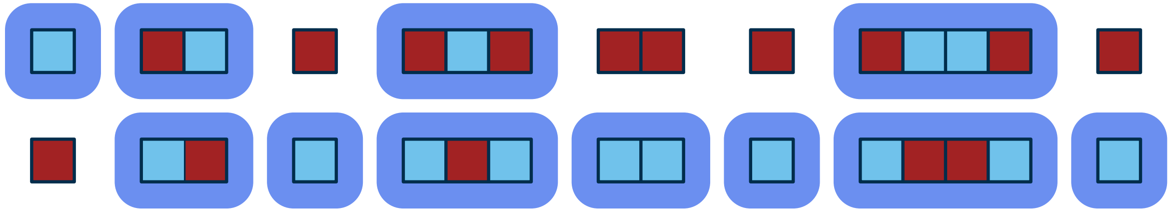

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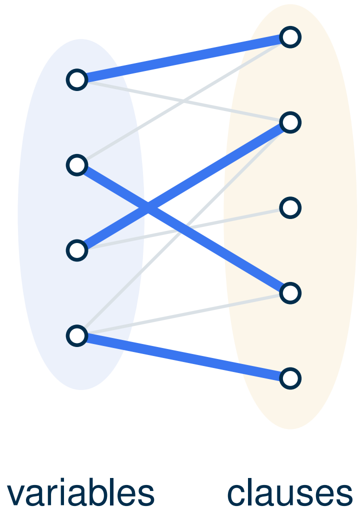

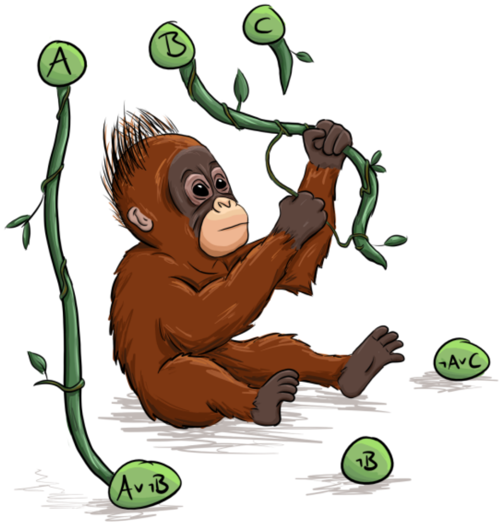

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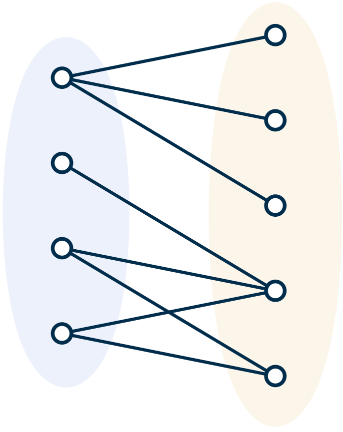

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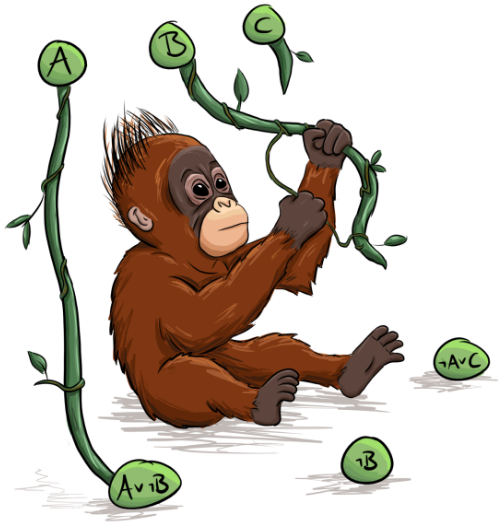

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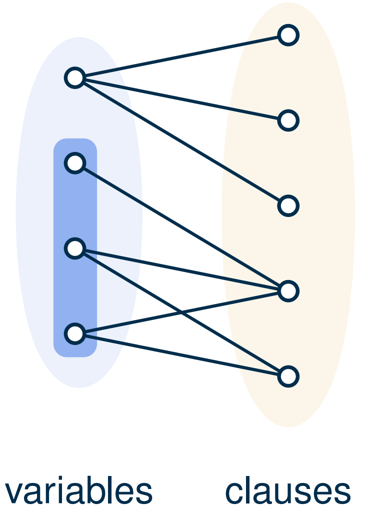

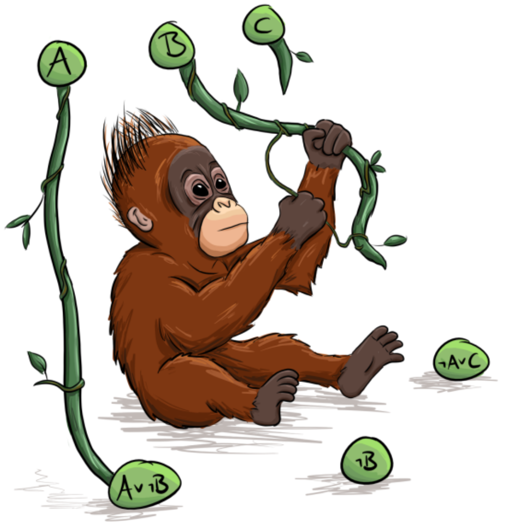

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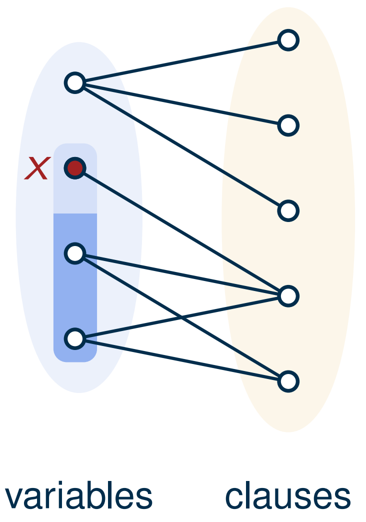
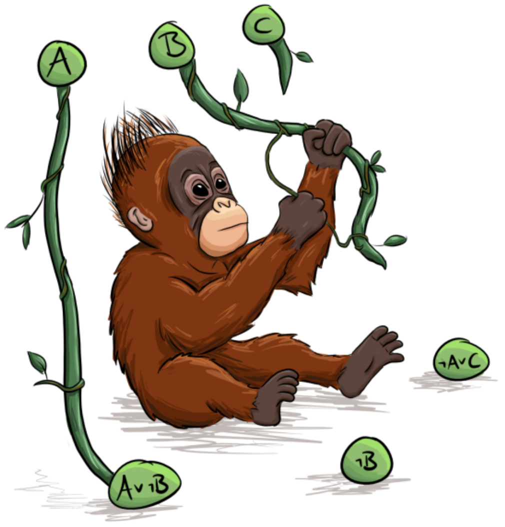
minimality of X $\Longrightarrow$ for $x \in X$ all subsets $X' \subseteq X \setminus x$ satisfy $|X'| \leq |N(X')|$





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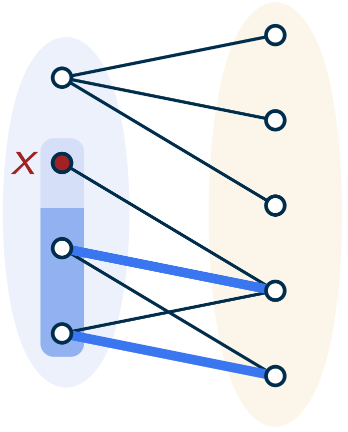

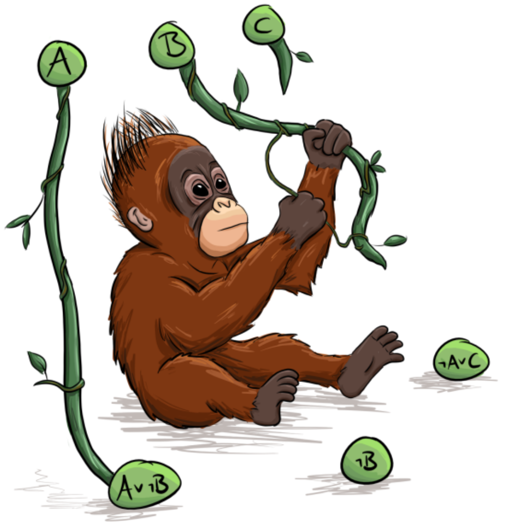

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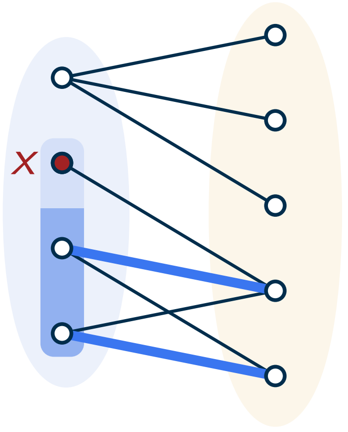
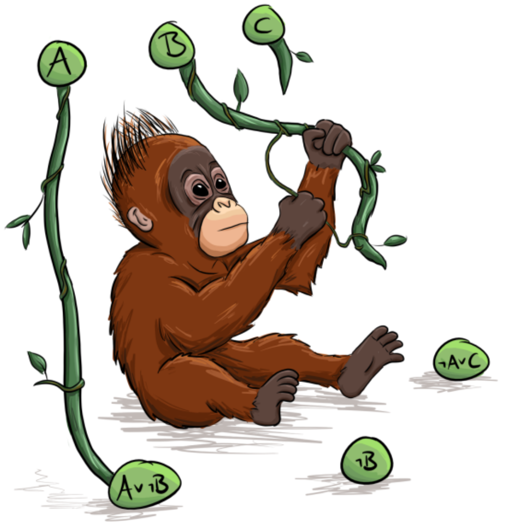
- delete X and $N(X)$, reduce parameter by $|N(X)|$





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Part (b) Find FPT Algo

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- consider number X of satisfied clauses in uniformly distributed variable assignment

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- let $u :=$ #unary clauses, $g :=$ #larger clauses
- after rule 1: $u < \frac{m}{2} + k$ (or yes instance)
- consider number X of satisfied clauses in uniformly distributed variable assignment

Probabilistic method: $E[X] \geq k \Rightarrow Pr[X \geq k] > 0$

$E[X]$

Sheet 4 – MAX SAT ABOVE $\frac{m}{2}$

Given formula φ in CNF with n variables and m clauses, can we assign the variables s.t. at least $\frac{m}{2} + k$ clauses are satisfied?

Part (b) Find FPT Algo

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$$E[X] = E\left[\sum_{i=1}^{m} X_i\right]$$

X_i : is clause i satisfied?

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$$E[X] = E\left[\sum_{i=1}^{m} X_i\right] = \sum_{i=1}^{m} E\left[X_i\right] \geq \frac{u}{2} + \frac{3}{4}g$$

$P(\text{“unary clause is satisfied”}) = \frac{1}{2}$

$P(\text{“larger clause is satisfied”}) \geq \frac{3}{4}$

Sheet 4 – Max Sat Above $\frac{m}{2}$

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↑ $u + g = m$

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$\Rightarrow \frac{m}{2} + \frac{g}{4}$ clauses are satisfiable

(↑ $u + g = m$)

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$\Rightarrow \frac{m}{2} + \frac{g}{4}$ clauses are satisfiable

$u + g = m$

- if $\frac{g}{4} \geq k$: yes instance; else: $g < 4k$

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$\uparrow$ $u + g = m$

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- if $\frac{g}{4} \geq k$: yes instance; else: $g < 4k$
- $m = u + g < \frac{m}{2} + k + 4k = \frac{m}{2} + 5k \Rightarrow m < 10k$

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($u + g = m$)

- if $\frac{g}{4} \geq k$: yes instance; else: $g < 4k$
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- new question: are $\frac{m}{2} + k < 6k$ clauses satisfiable?

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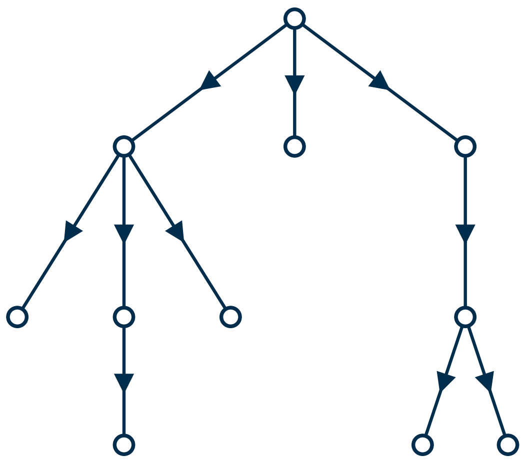
- $m = u + g < \frac{m}{2} + k + 4k = \frac{m}{2} + 5k \Rightarrow m < 10k$
- new question: are $\frac{m}{2} + k < 6k$ clauses satisfiable? $\Rightarrow$ use part (a)

Sheet 4 – WEIGHTED VERTEX COVER ON TREES

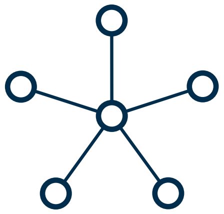
Given a vertex-weighted undirected tree T. Formulate a dynamic program that finds the minimum weight of a vertex cover in T.

- finding the recursion formulas worked generally well
- don't forget the stuff around it:
  - root the tree at arbitrary vertex
    (otherwise there are no parents or children)
  - traverse the tree bottom-up (BFS)
  - argue runtime $\mathcal{O}(n)$



Sheet 5 – Color Coding

5-Stars:



Given a graph G and parameter k, does G contain at least k vertex-disjoint induced 5-stars?

Sheet 5 – Color Coding

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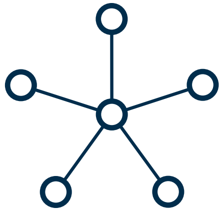

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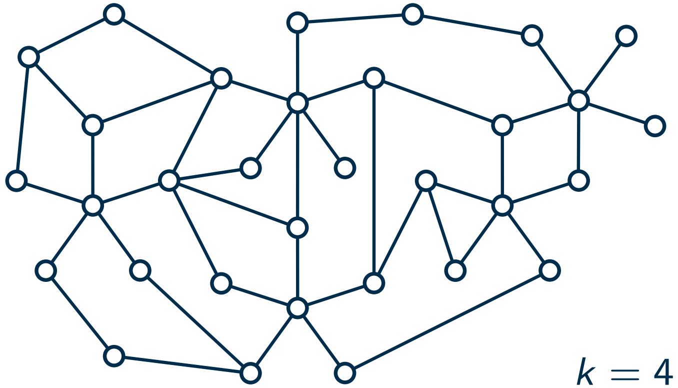

$k = 4$

Sheet 5 – Color Coding

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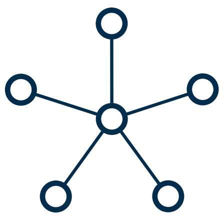

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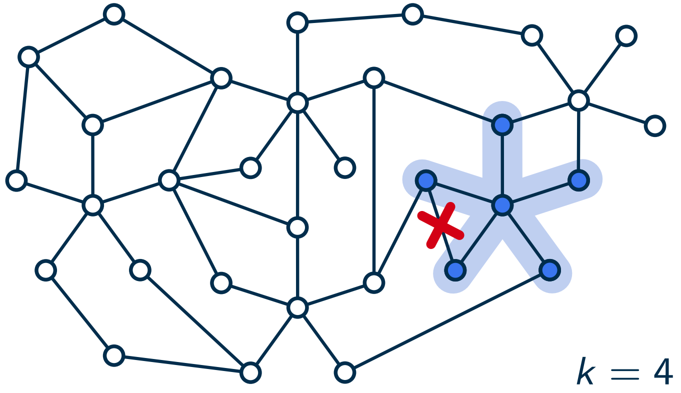

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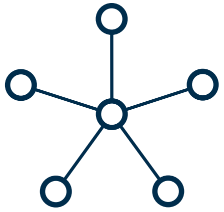

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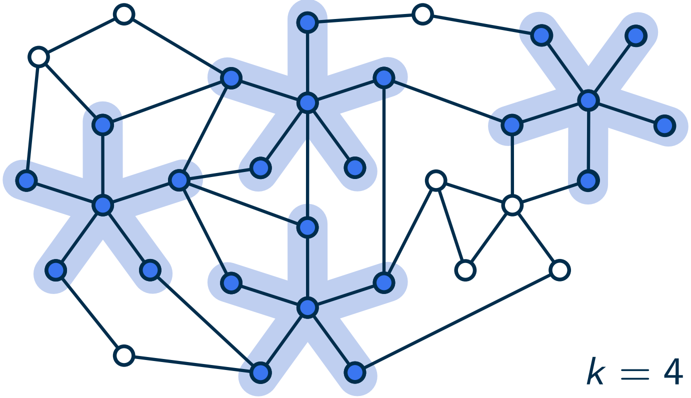

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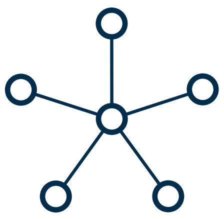

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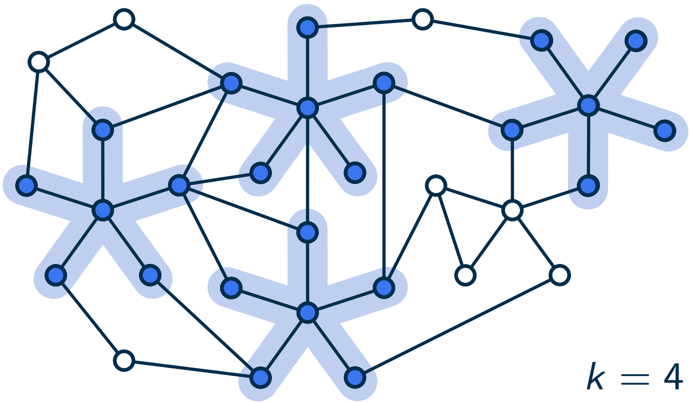

How does a colorful version of the problem look like?

Sheet 5 – Color Coding

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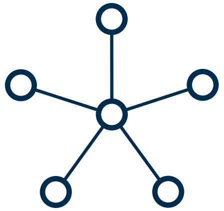

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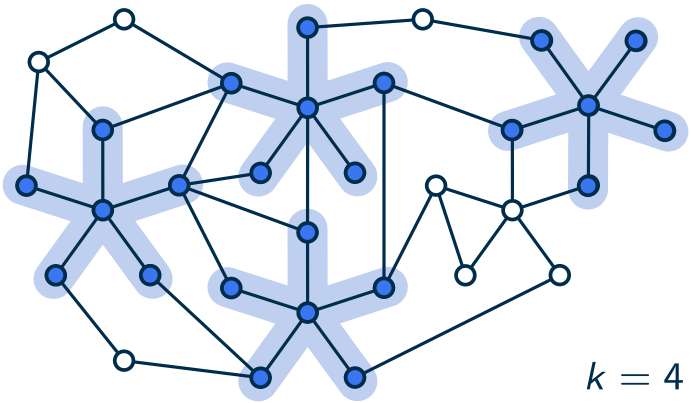

$k = 4$

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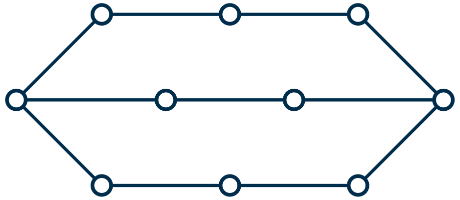
Longest Cycle:

Given a graph G and parameter k, does G contain a cycle of length at least k?



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Sheet 5 – Color Coding

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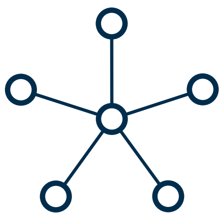

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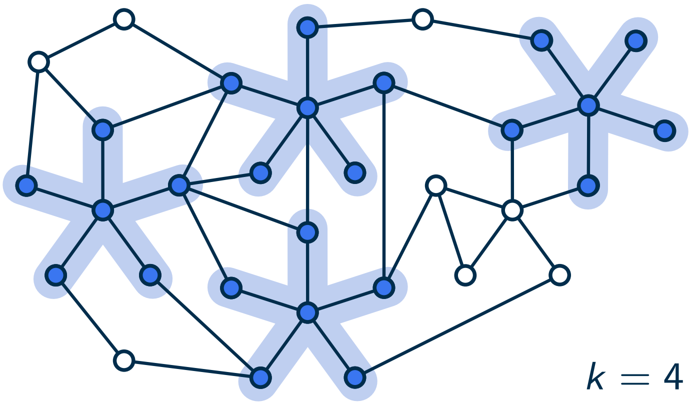

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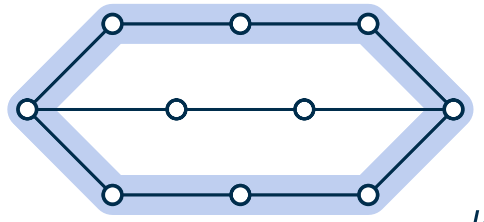

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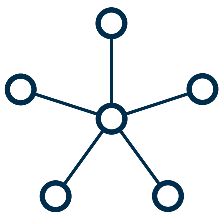

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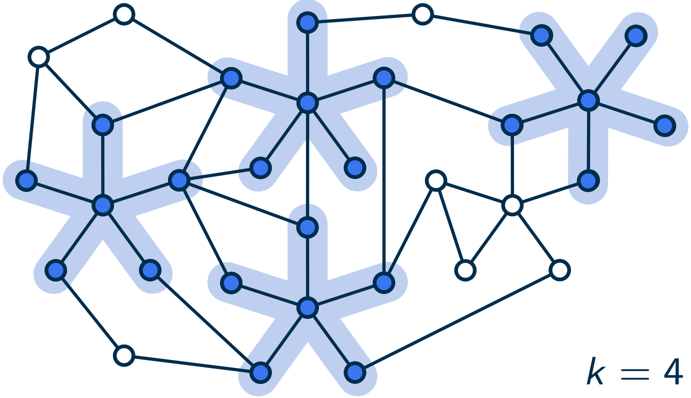

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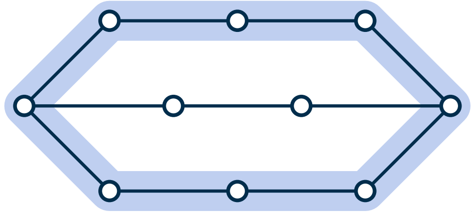

How can we solve the problem for *exactly* k?

Hint 1: Xvh froru frglqj dojr vlplodu wr wkh rqh iru orqjhvw sdwk wr vroyh hAdfw sureohp

Hint 2: Wklqn derxw hgjh frqwudfwlrq (krz grhv wklv fkdqjh wkh ohqjwk ri fBfohv?)

Sheet 5 - Lower Bounds for Kernels

Show that the following problems do not admit polynomial kernels (when parameterized by the respective k), unless $\mathrm{NP} \subseteq \mathrm{coNP/\,poly}$

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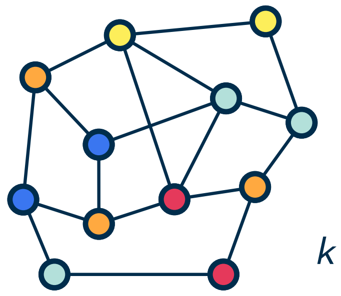
Given a graph G with a vertex coloring with k colors, does G have a colorful connected subgraph of size exactly k?



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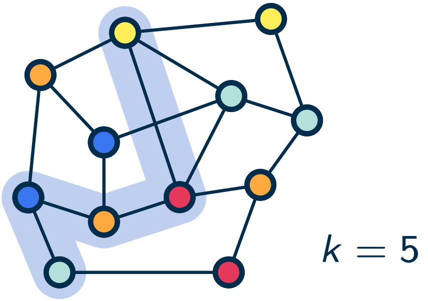

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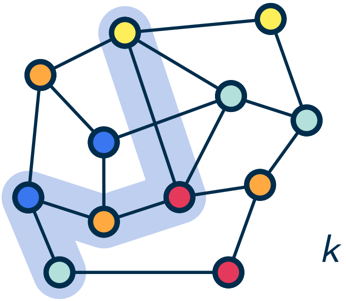

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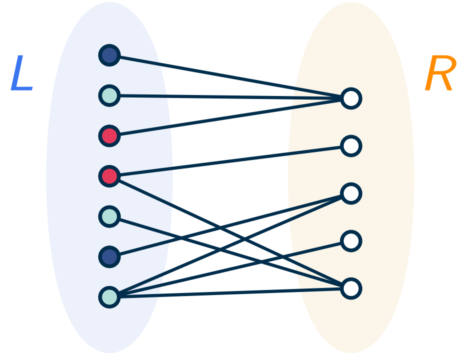
Colorful Left-Right Dominating Set:

Given a bipartite graph G with sides L and R and a coloring of L into ℓ colors.



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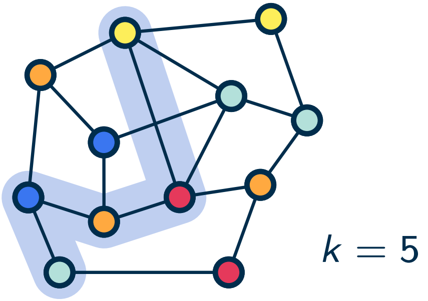
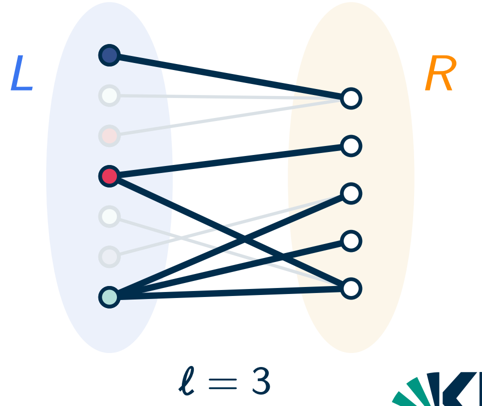


Colorful Left-Right Dominating Set: (parameter: $k = |R| + \ell$)

Given a bipartite graph G with sides L and R and a coloring of L into ℓ colors. Is there a colorful set $X \subseteq L$ of size exactly ℓ that dominates all vertices in R?

this is basically SET COVER



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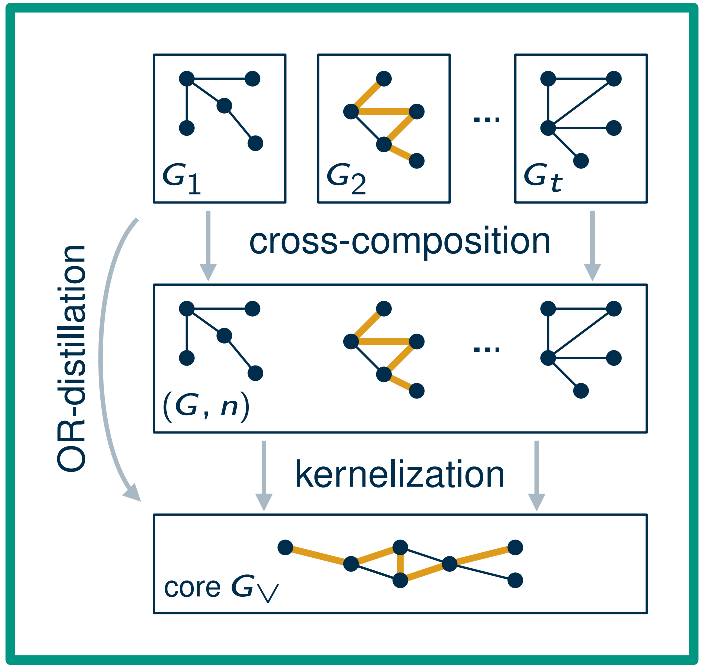

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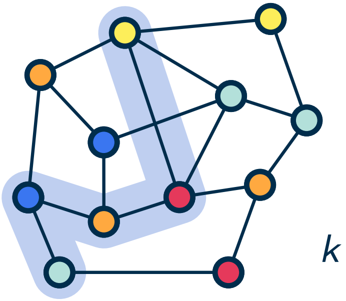

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Theorem (Reminder): If there is an OR-distillation of L into R, then $L \in$ coNP/poly

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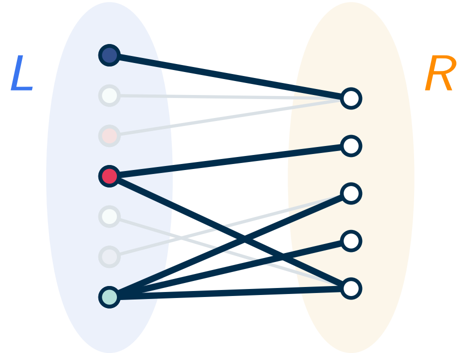

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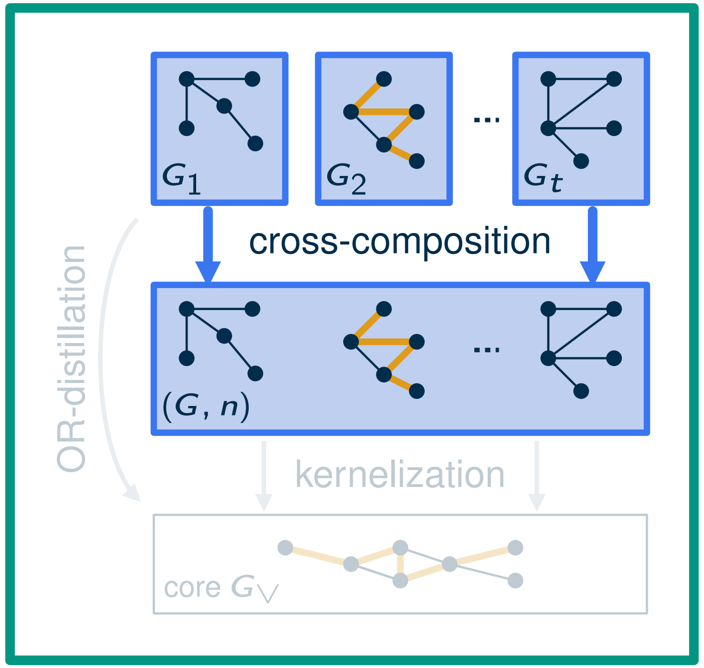

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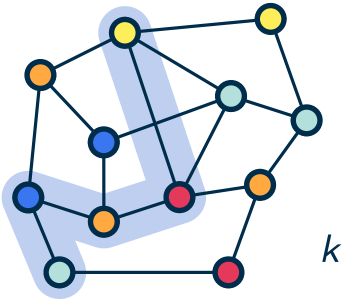

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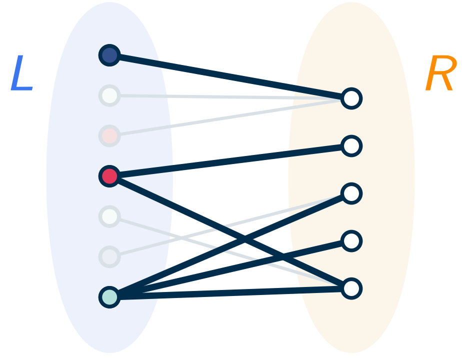

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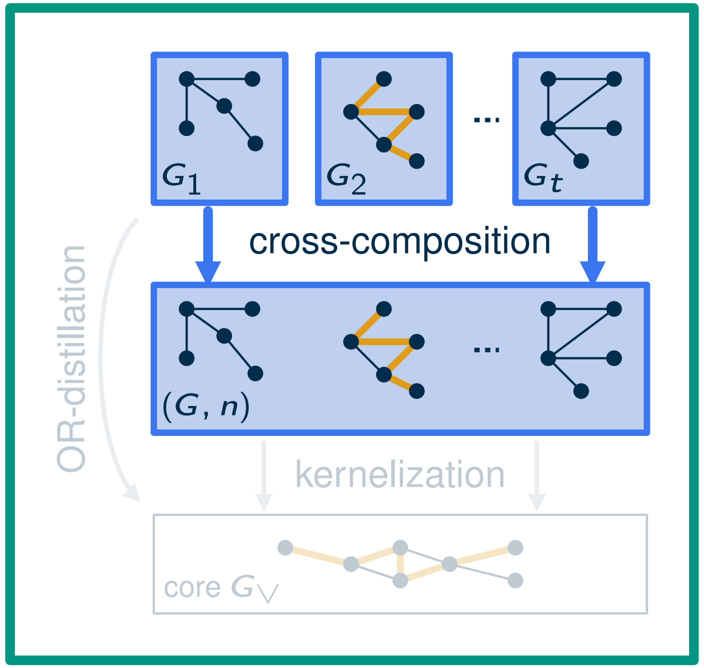

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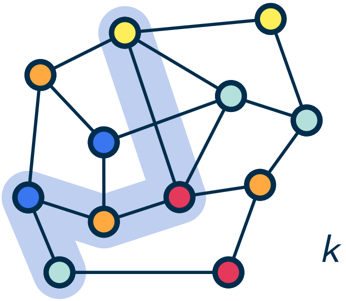
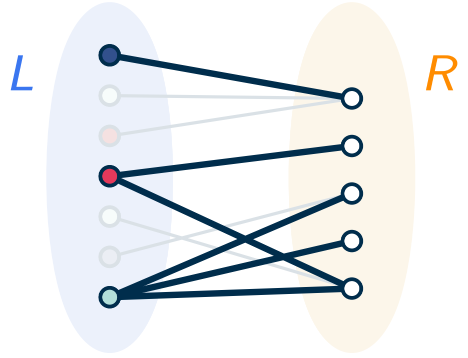


$k = 5$

Colorful Left-Right Dominating Set: (parameter: $k = |R| + \ell$)

Given a bipartite graph G with sides L and R and a coloring of L into ℓ colors. Is there a colorful set $X \subseteq L$ of size exactly ℓ that dominates all vertices in R?

this is basically SET COVER

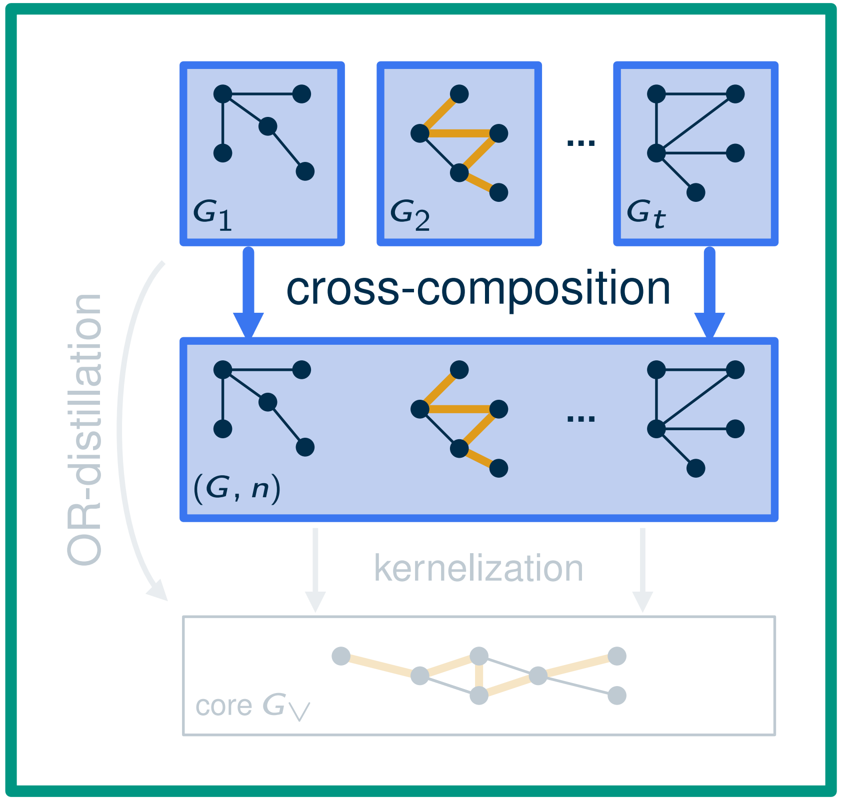


$\ell = 3$

Hint 1: gr qrw gxsolfdwh wkh uljkw vlghv ri wkh elsduwlwlrqv.

Hint 2: Wkh qxpehu ri yhuwlfhv lq Brxu lqvwdqfh vhohfwru vkrxog ghshqg erwk rq wkh qxpehu ri gljlwv lq wkh elqduB uhsuhvhqwdwlrq ri w dqg wkh qxpehu ri froruv.

Recap: Cross Composition



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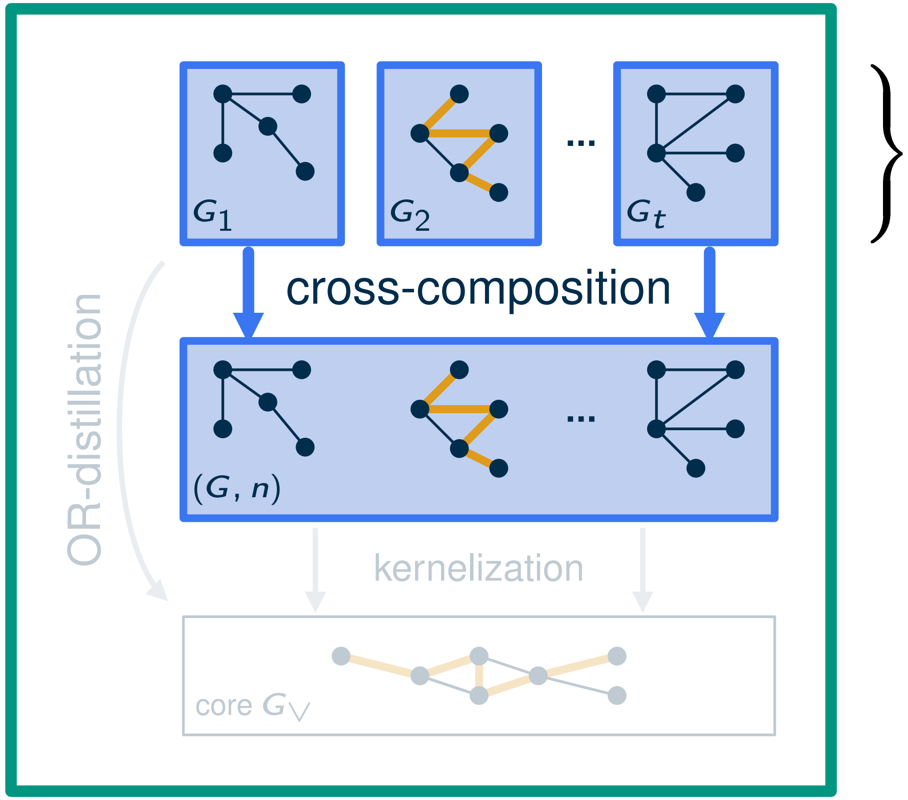

t many "similar" instances x_1, x_2, . . ., x_t of problem P

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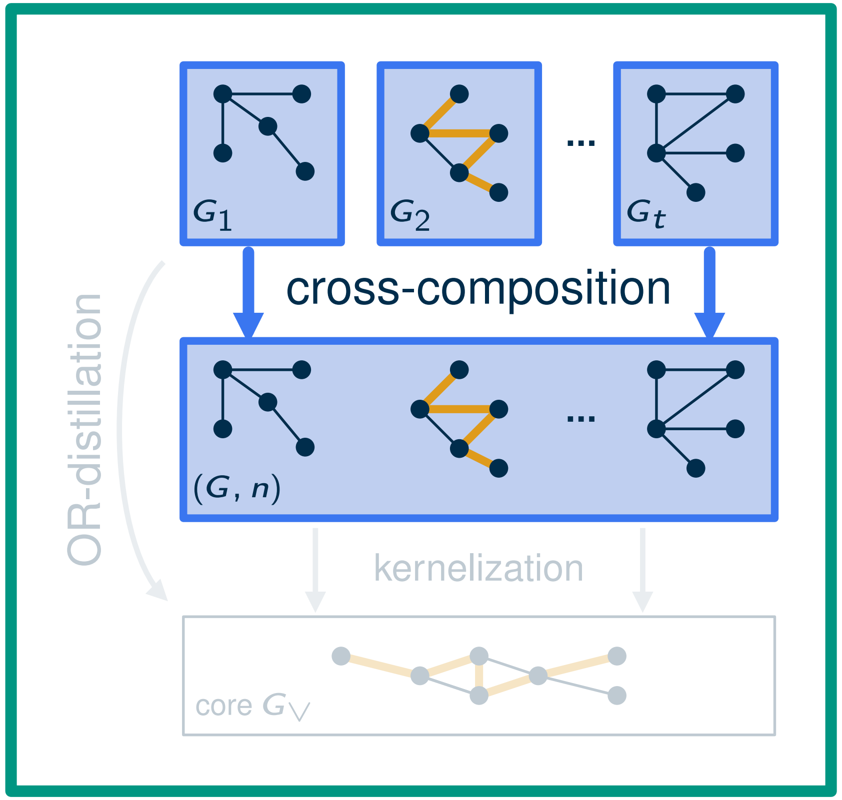

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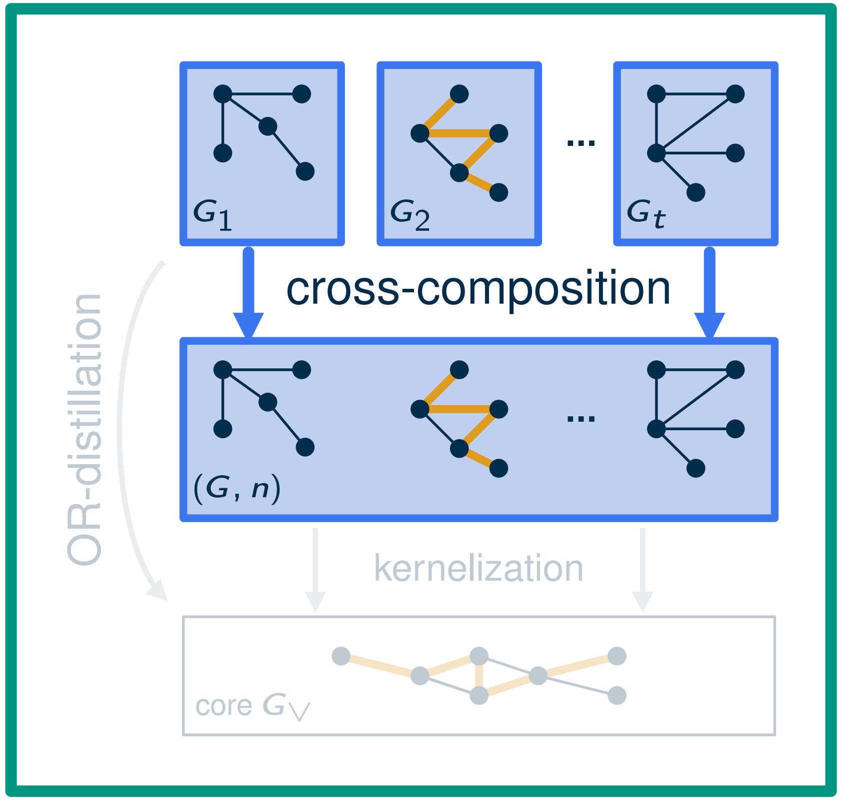

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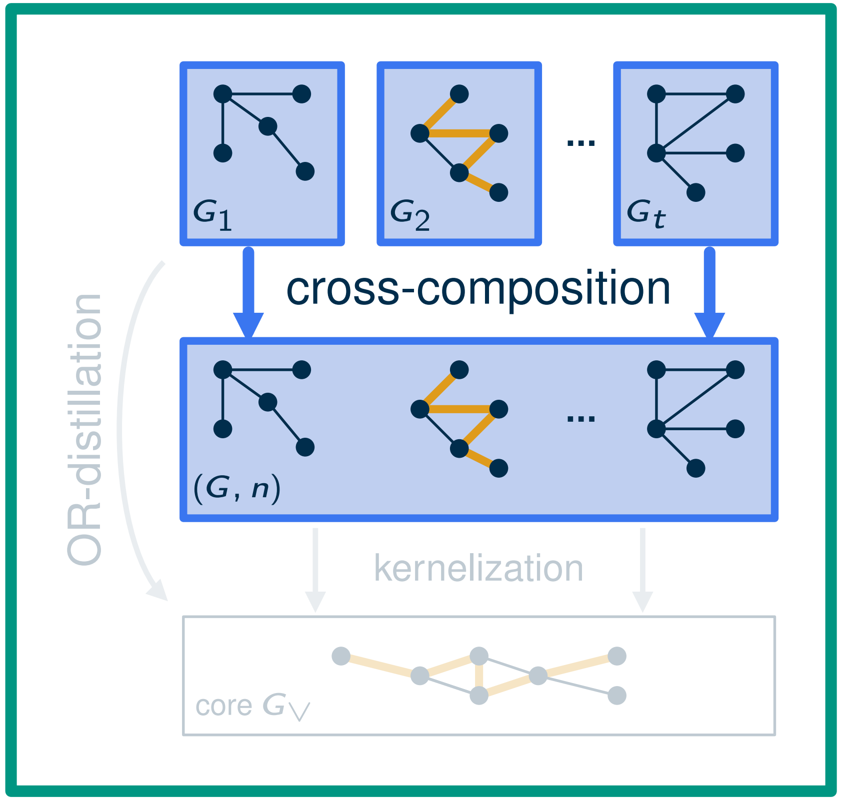

equivalent under some existing equivalence relation

- only polynomially many equivalence classes
- equivalence can be checked in polynomial time

does not have to be parameterized

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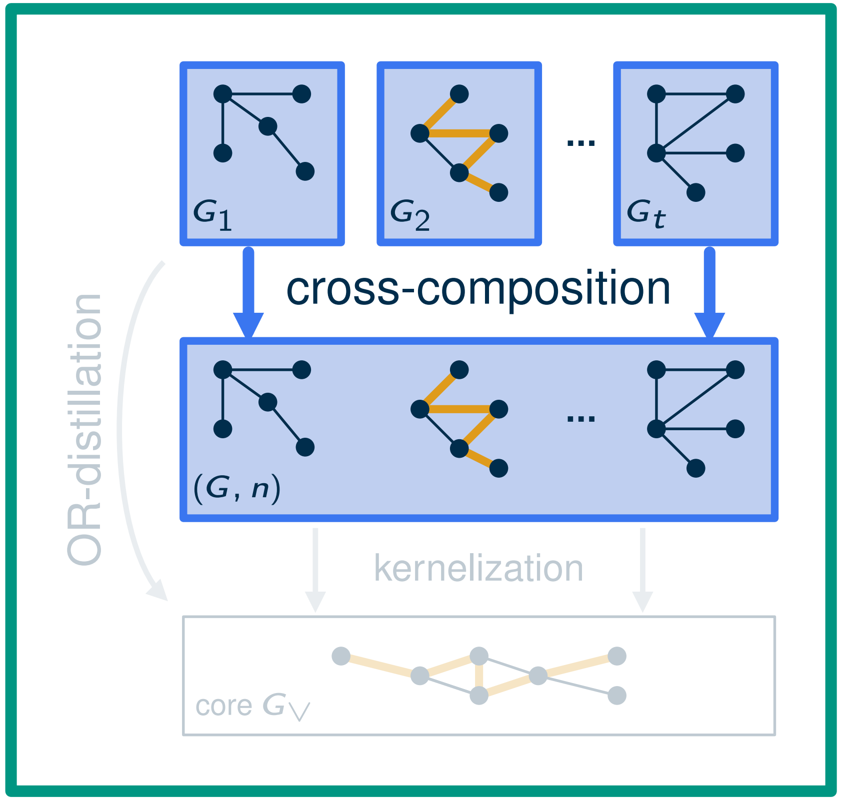

often useful: same "size"

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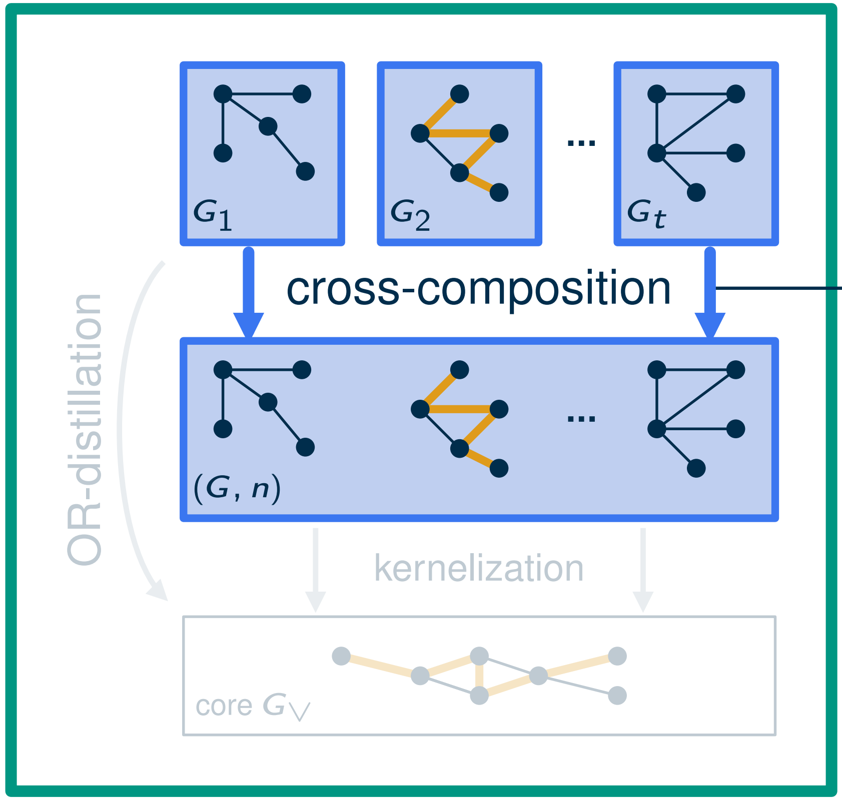

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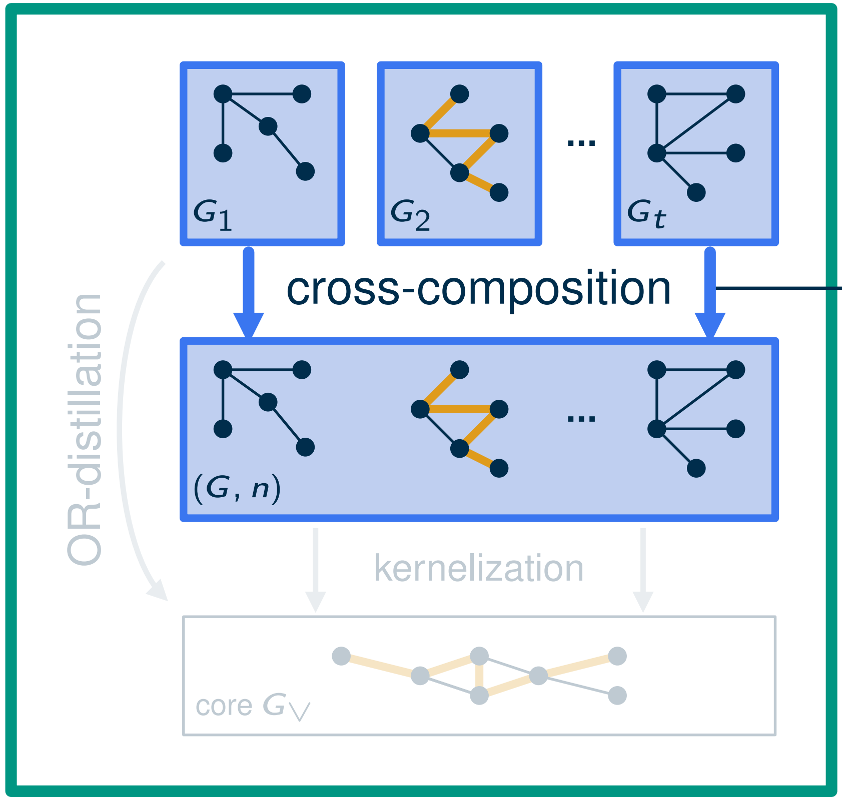

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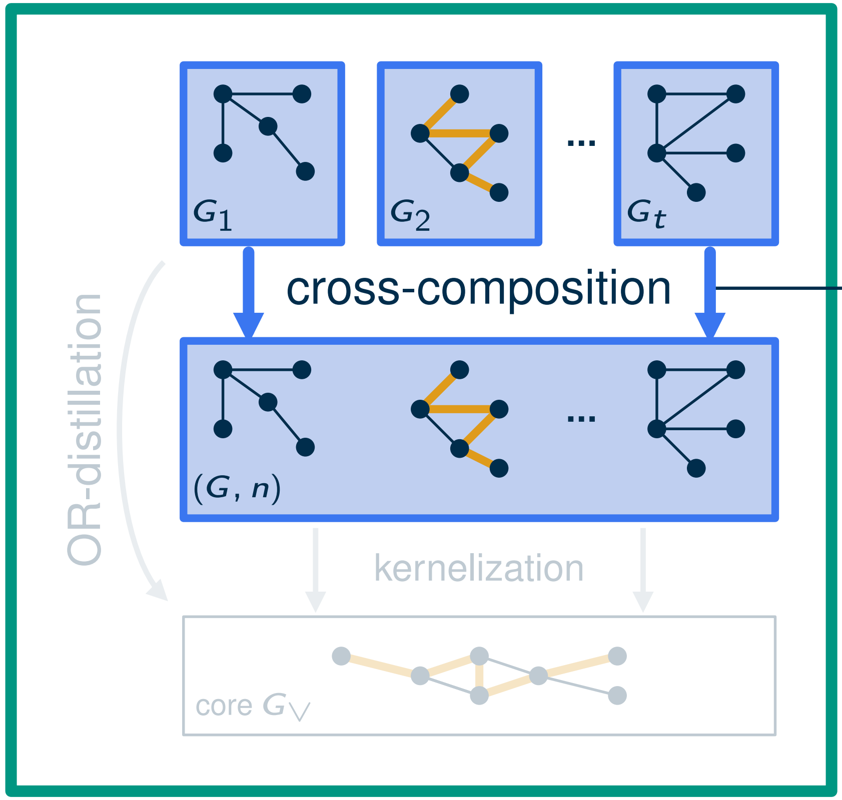

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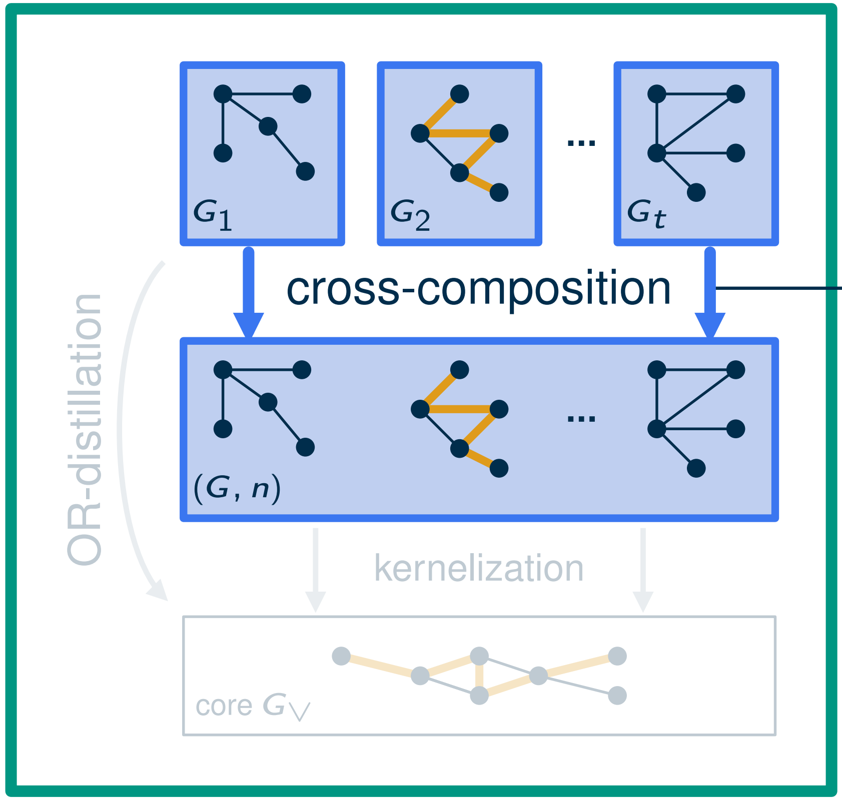

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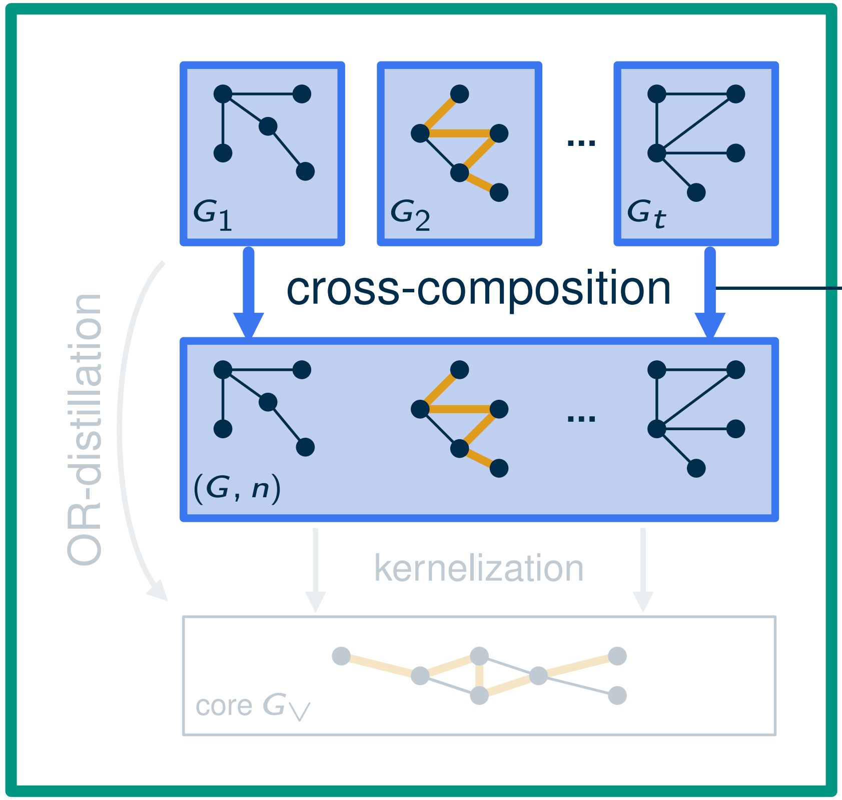

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Do we always have to do this to show a lower bound for a kernel?

Polynomial Parameter Transformations (PPTs)

Idea: use problem from which we already know that it has no polynomial kernel (unless...)

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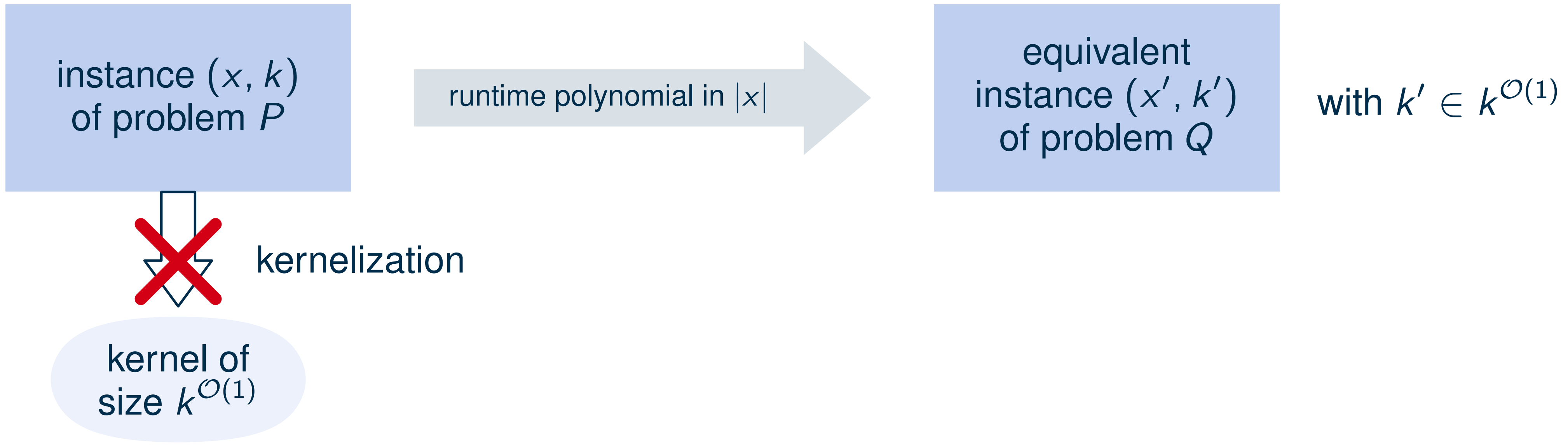

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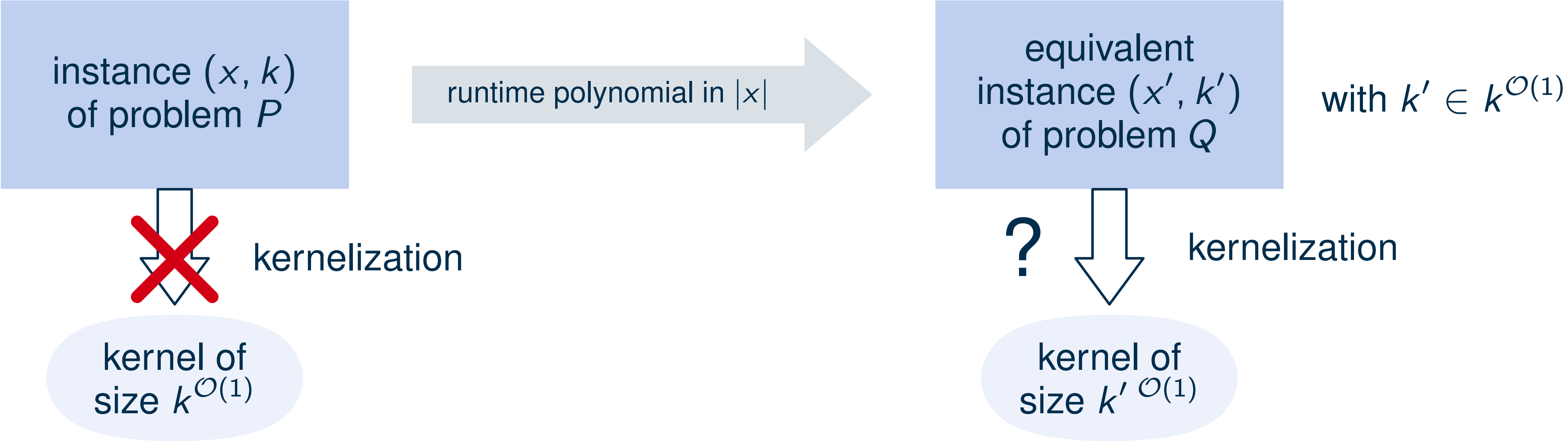

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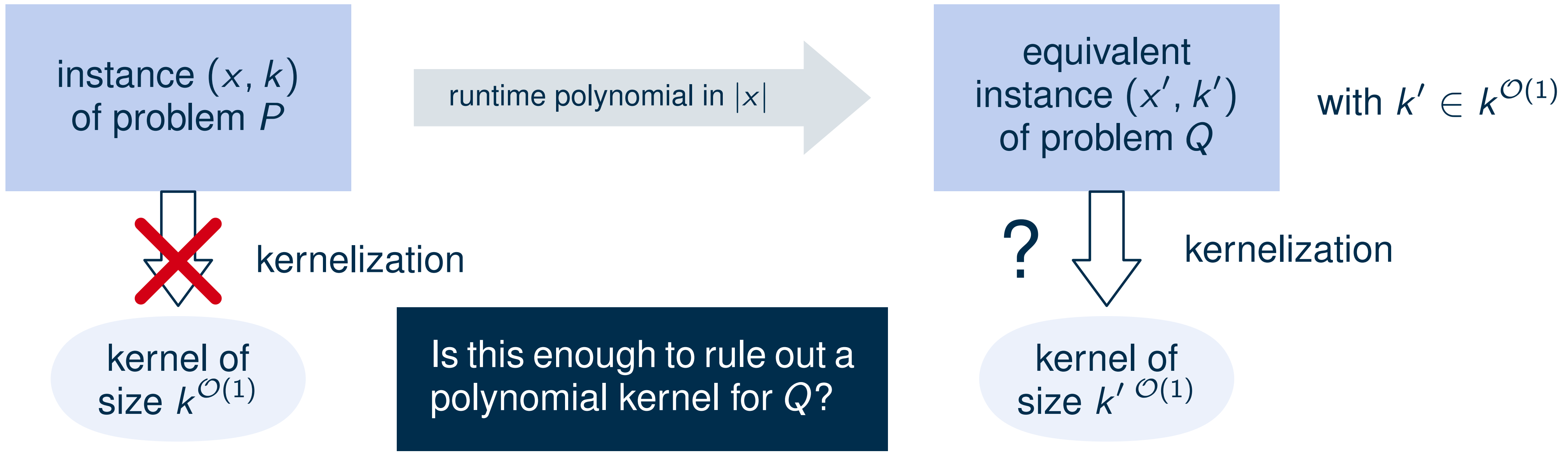

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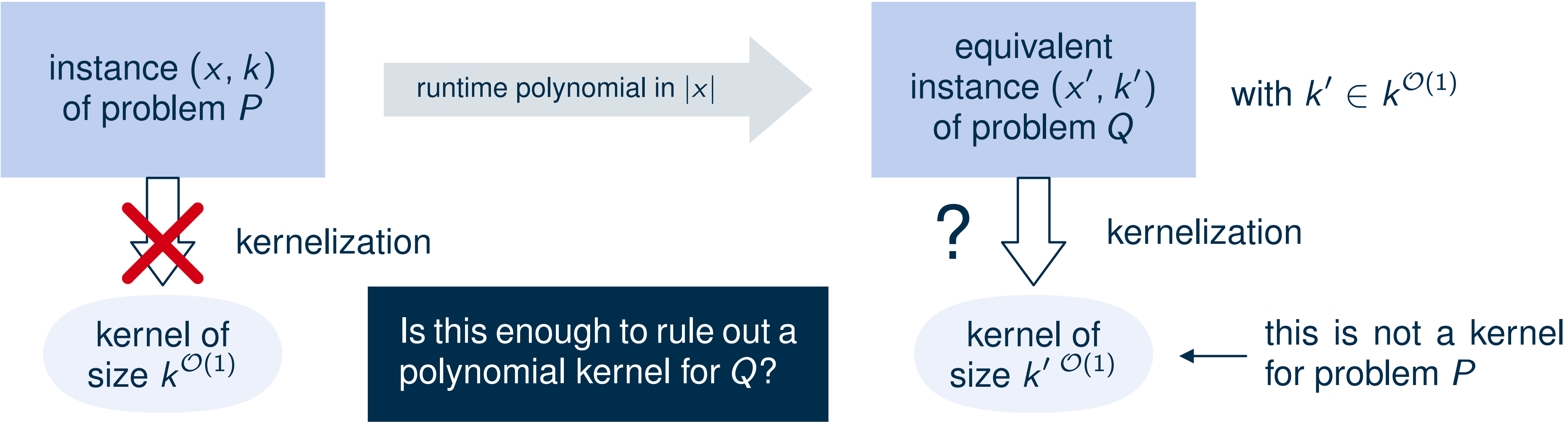

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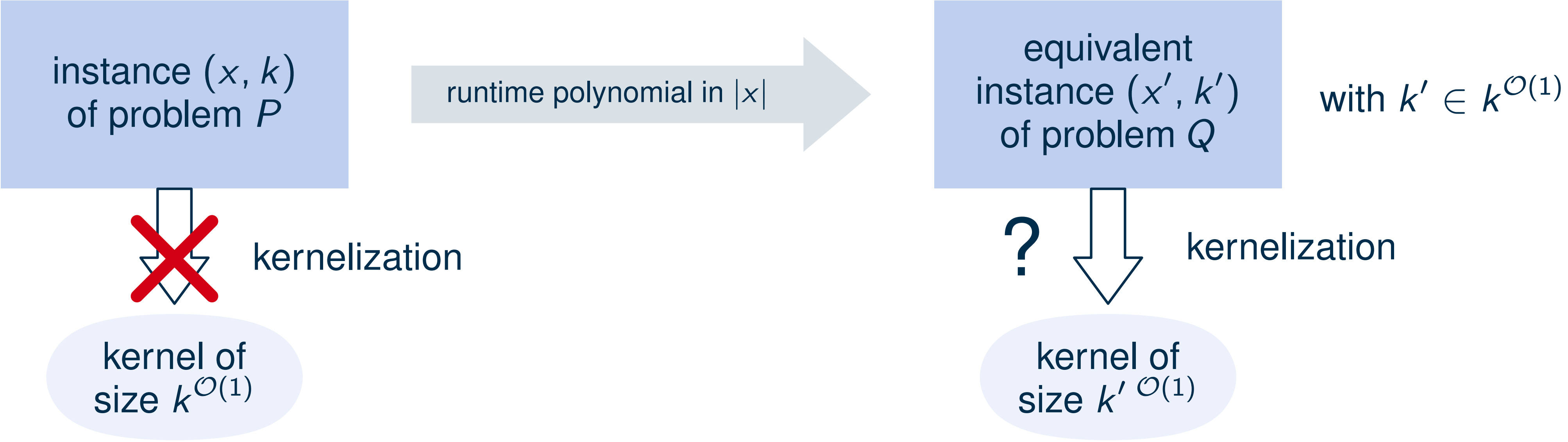

For a parameterized problem L, a **kernelization** is a polynomial-time algorithm that transforms any instance (x, k) into an equivalent instance (x', k') (the **kernel**) such that $|x'| + k' \leq g(k)$.

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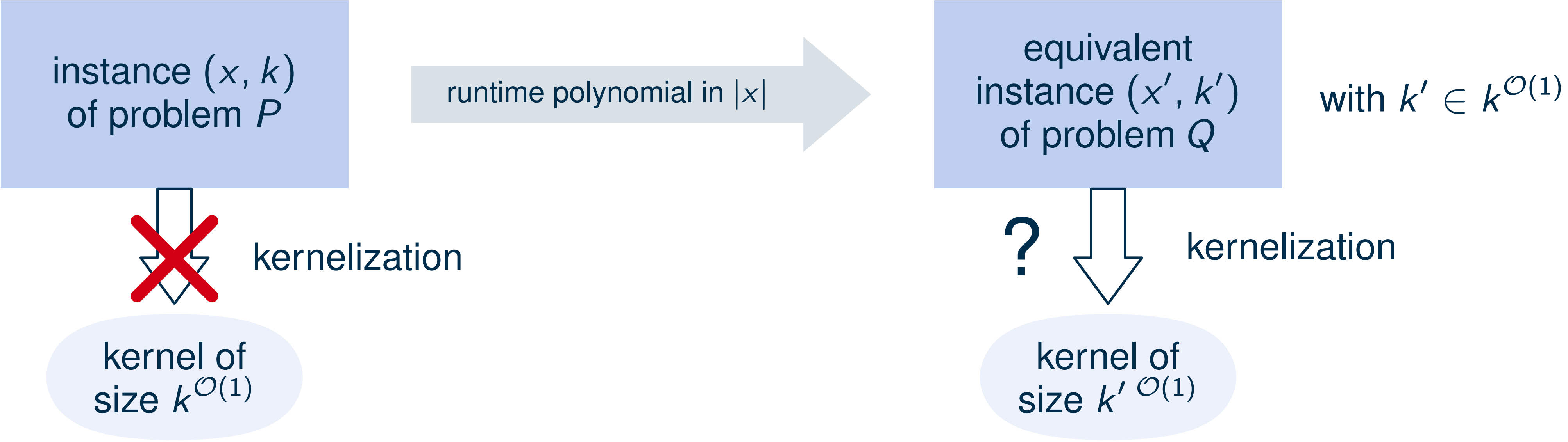

polynomial compression

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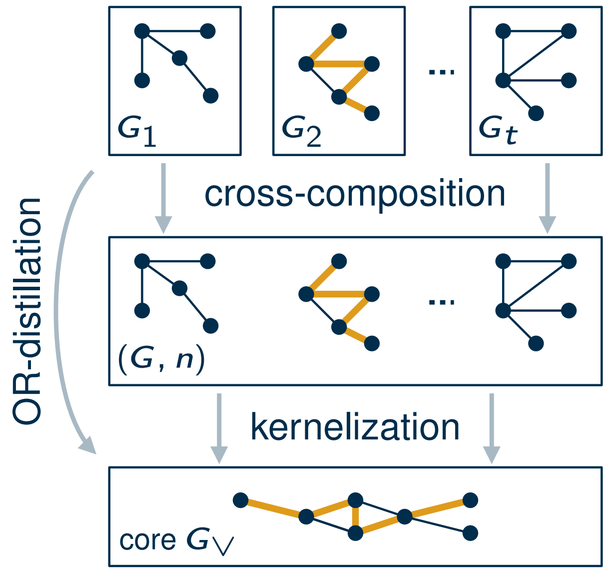
of a parameterized problem L_2

Polynomial Parameter Transformations (PPTs)

Definition
Let L and R be languages. An **OR-distillation** of L into R is an algorithm that takes t similar inputs $x_1, \ldots, x_t$, runs in time $(\sum_{i\in[t]} |x_i|)^{O(1)}$, and produces an output y such that:
$|y| \in \max_{i\in[t]} |x_i|^{O(1)}$ and $y \in R \Leftrightarrow \bigvee_{i\in[t]} x_i \in L$.

Theorem
If there is an OR-distillation of L into R then $L \in \text{coNP}/\text{poly}$.



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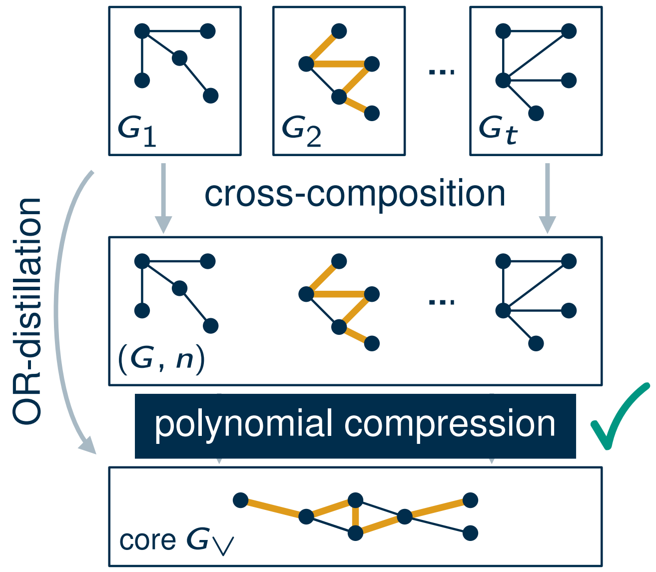

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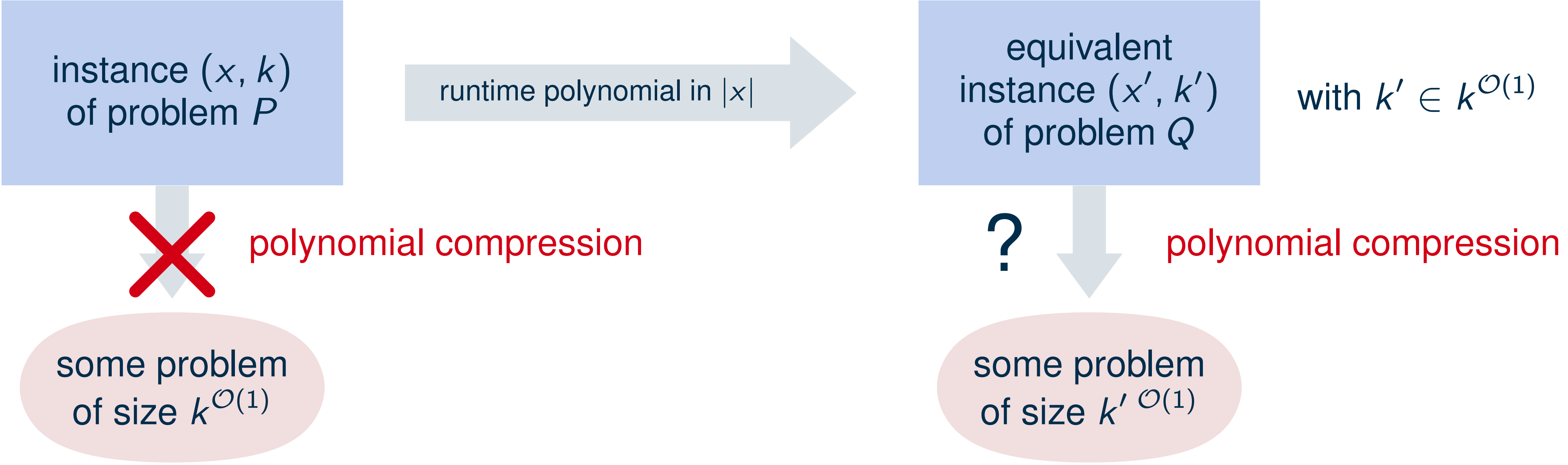

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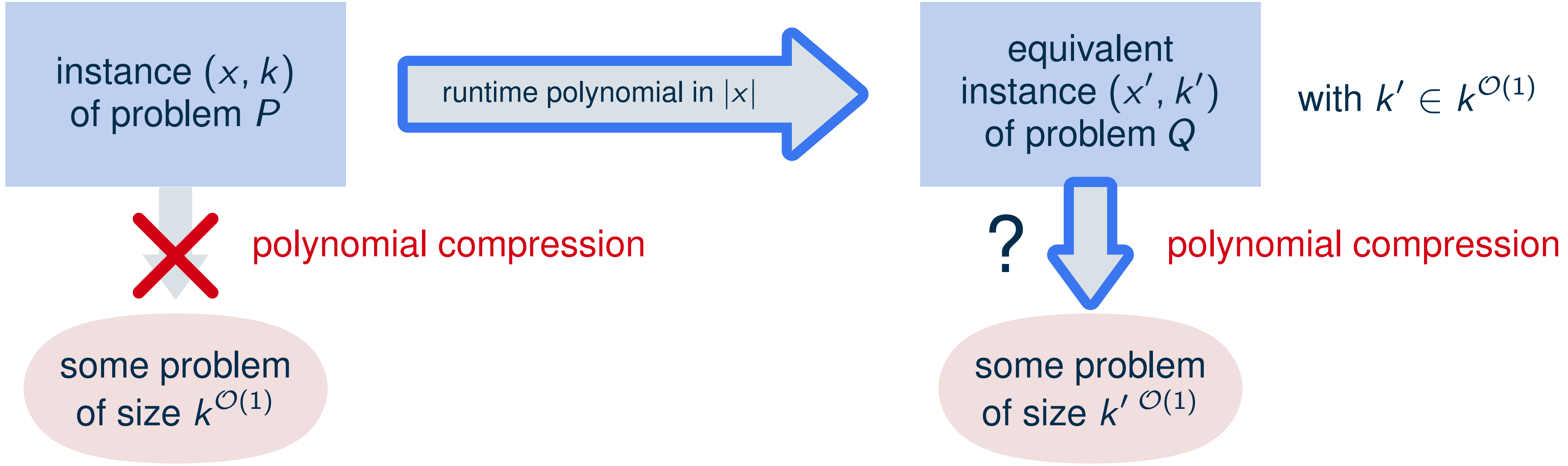

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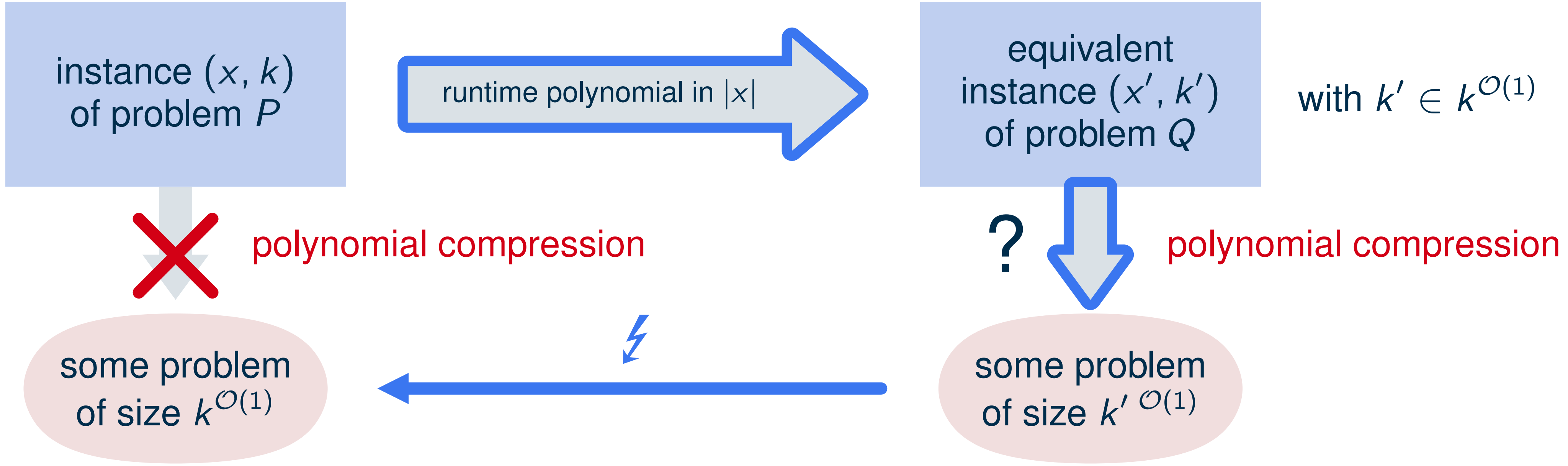

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instance (x, k) of problem P ✗ polynomial compression → some problem of size $k^{\mathcal{O}(1)}$

equivalent instance (x', k') of problem Q ✗ polynomial compression → some problem of size $k'^{\,\mathcal{O}(1)}$

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If problem P does not admit a polynomial compression and there is a PPT from P to Q, then Q does not admit a polynomial compression.

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Polynomial Parameter Transformation (PPT)

If problem P does not admit a polynomial compression and there is a PPT from P to Q, then Q does not admit a polynomial compression.

and if there is no polynomial compression, there is also no polynomial kernel!

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PPT: Colorful Graph Motif $\rightarrow$ Steiner Tree

instance (x, k)
of problem P

PPT
runtime polynomial in $|x|$

equivalent
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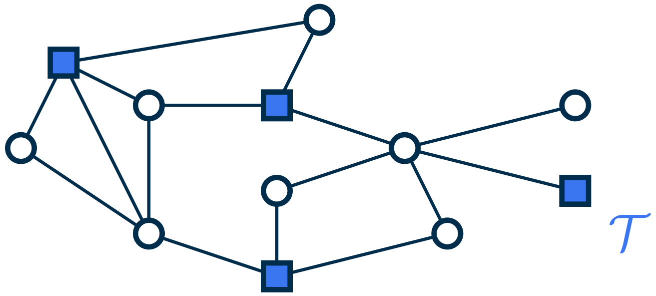

$k = 6$

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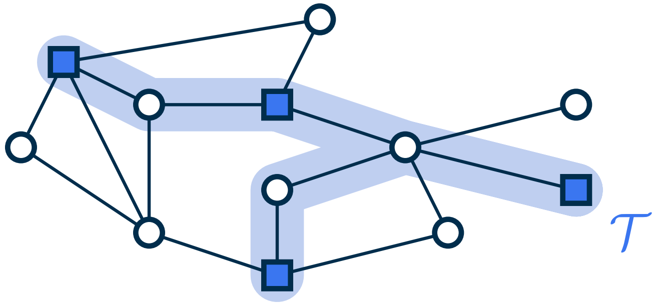

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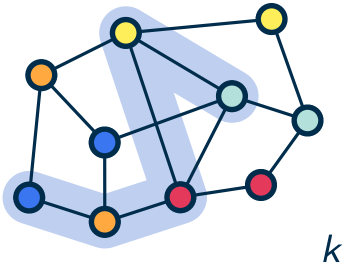

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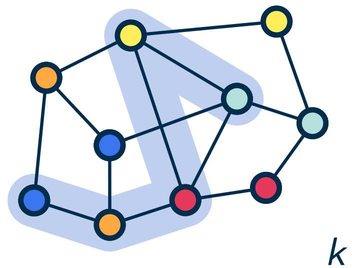

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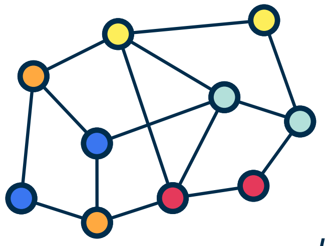
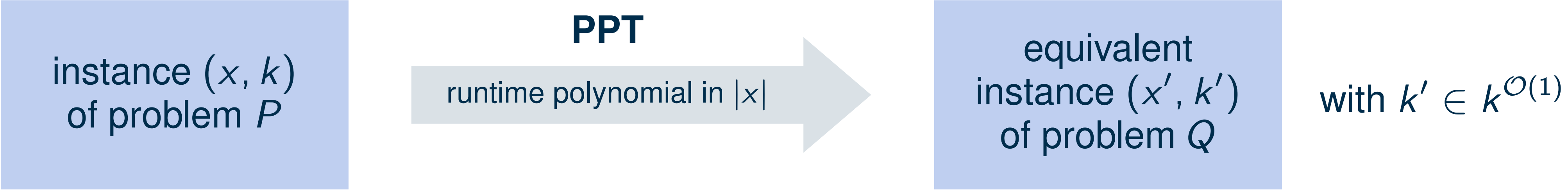


$k' = 2k - 1$

PPT: Colorful Graph Motif → Steiner Tree



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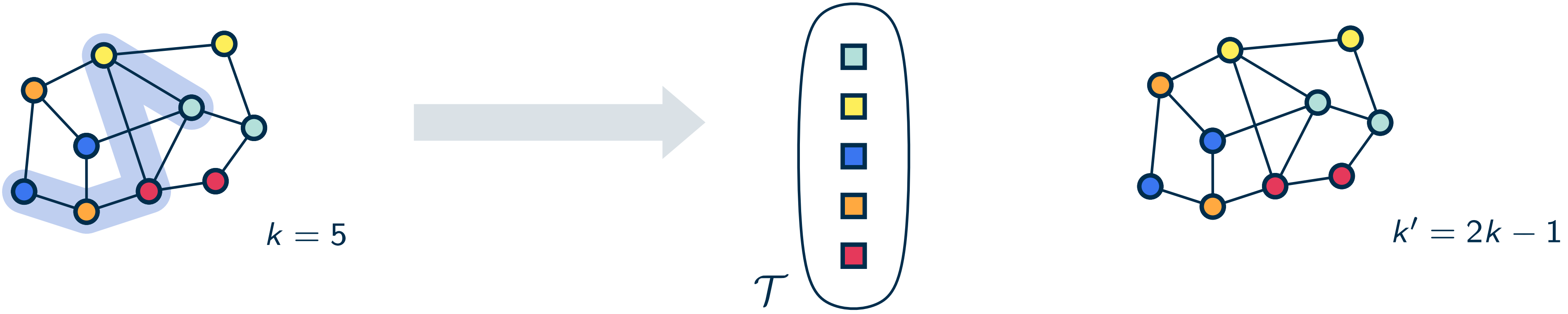

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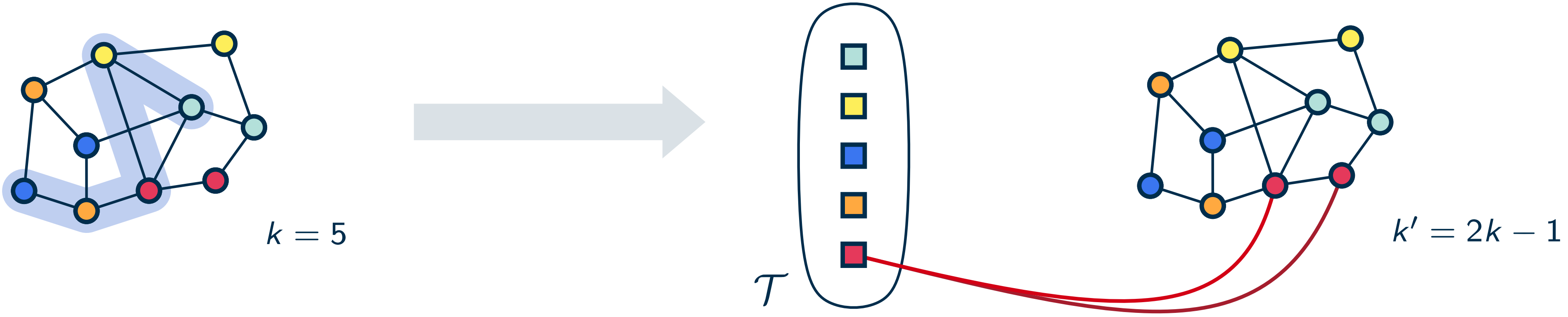

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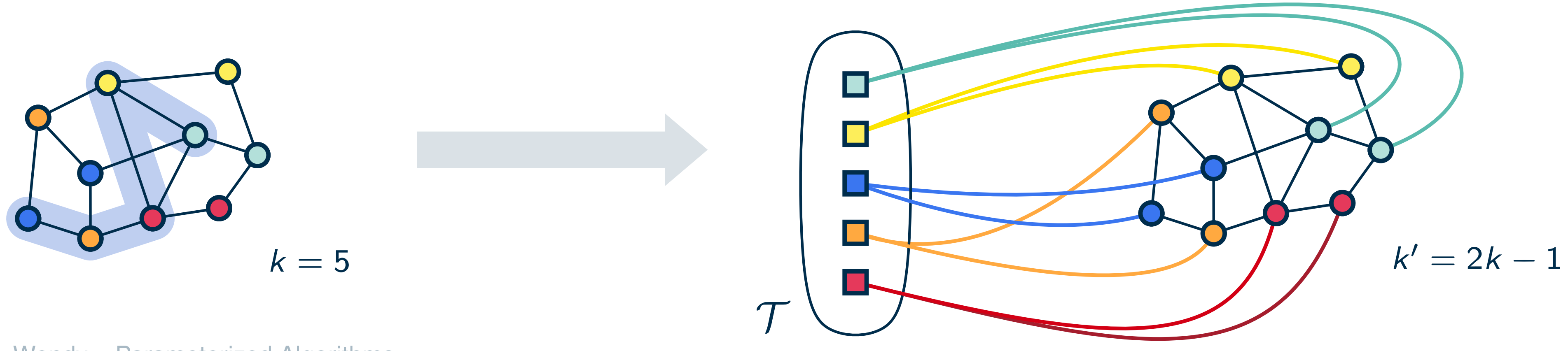

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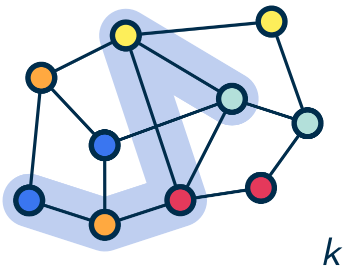

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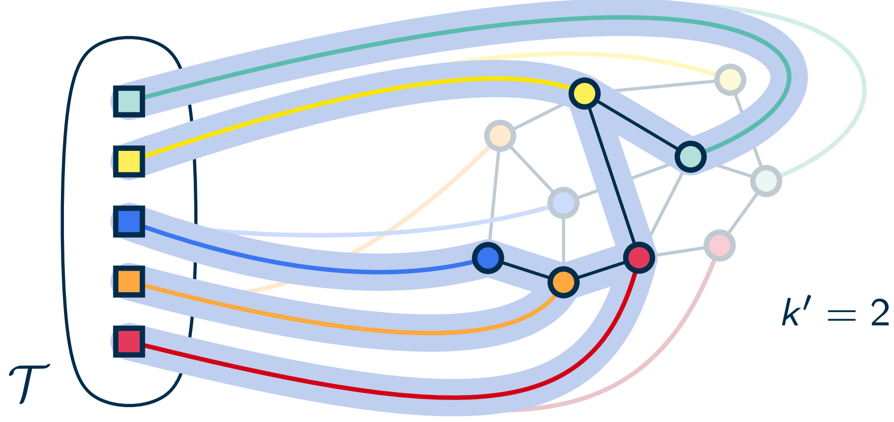

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Cross Compositions and PPTs

Show that there are no polynomial kernels (unless...)

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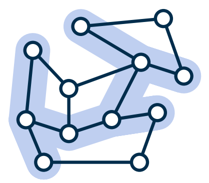

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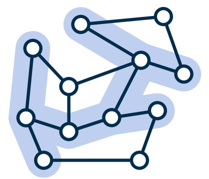

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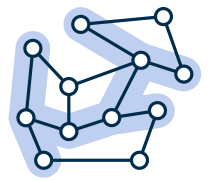

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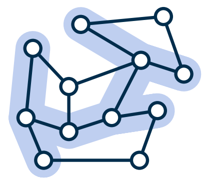

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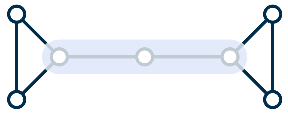

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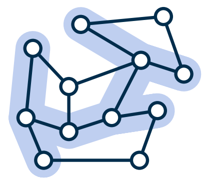

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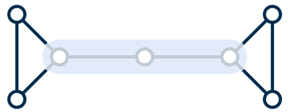

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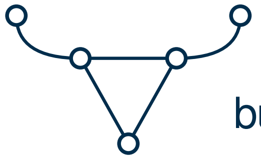

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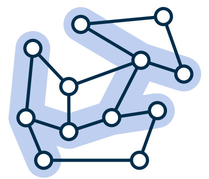

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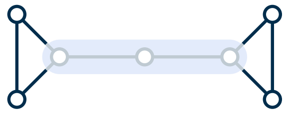

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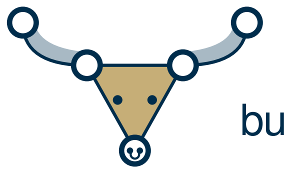

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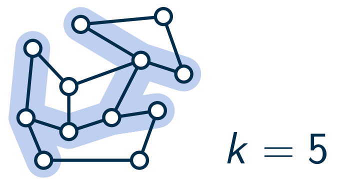

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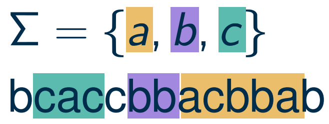

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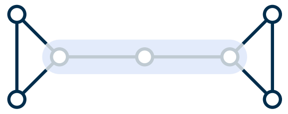

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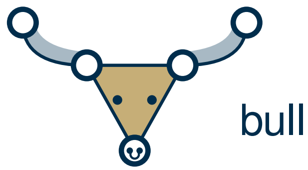

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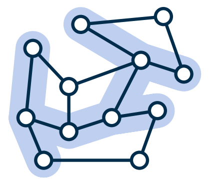

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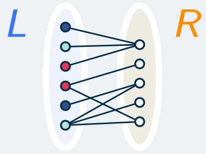

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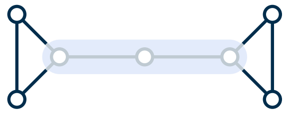

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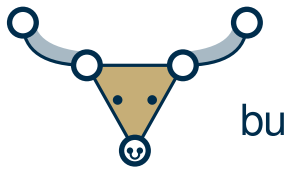

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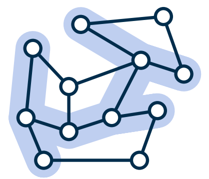

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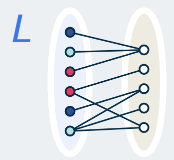

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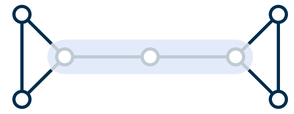

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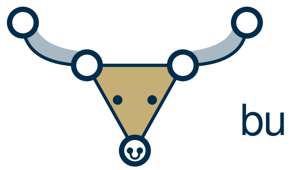

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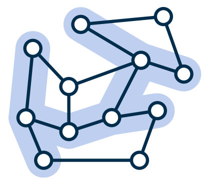

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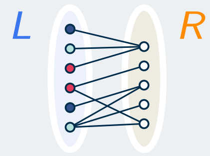

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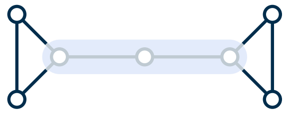

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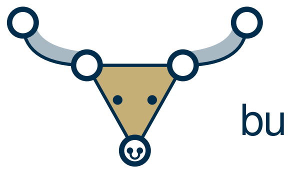

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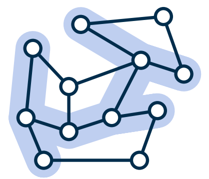

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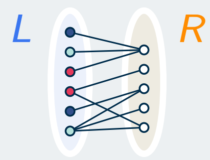

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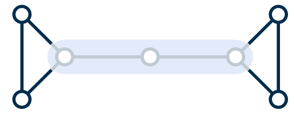

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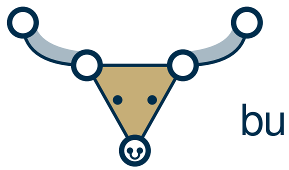

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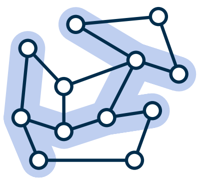

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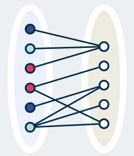

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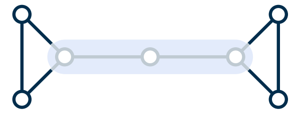

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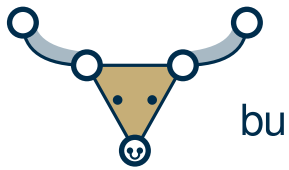

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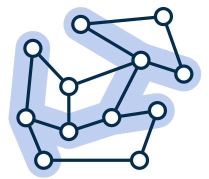

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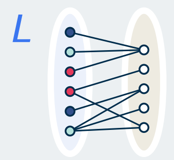

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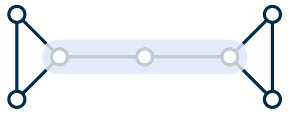
★★



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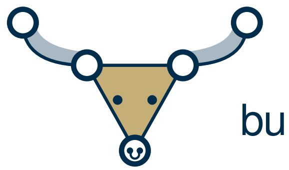

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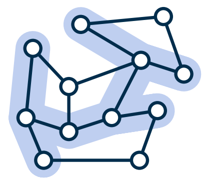

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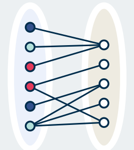
COLORFUL LEFT-RIGHT DOMSET

$k = |R| + \ell$



⋆⋆

CONNECTED VERTEX COVER (k): Given a graph $G = (V, E)$ and an integer k, does G contain a vertex cover of size k that induces a connected subgraph of G?

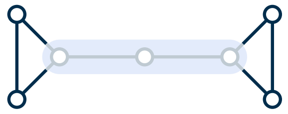
start with same vertex set and add one for each color

⋆⋆

CONNECTED FEEDBACK VERTEX SET (k): Given a graph G and an integer k, can we delete a connected subgraph induced by k vertices such that G becomes acyclic?

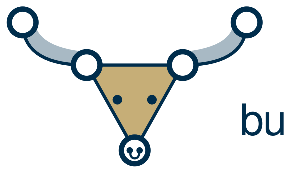
how could edges be extended to cycles?

⋆⋆



CONNECTED BULL HITTING (k): Given a graph G and an integer k, can we delete a connected subgraph induced by k vertices s.t. G does not contain any bull after the deletion?

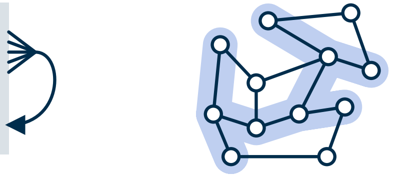
... to bulls?



bull

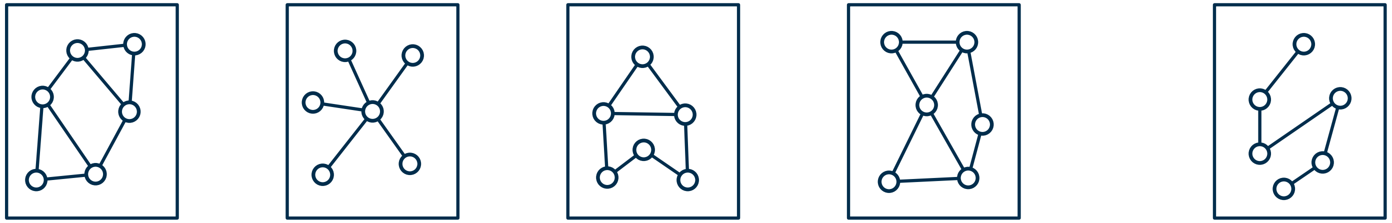
Max Leaf Subtree – Cross Composition

MAX LEAF SUBTREE (k): Given a graph $G = (V, E)$ and an integer k, does G contain a subtree that has at least k leaves?



$k = 5$

G_1 G_2 G_3 G_4 G_5



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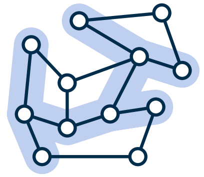

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"similar"?

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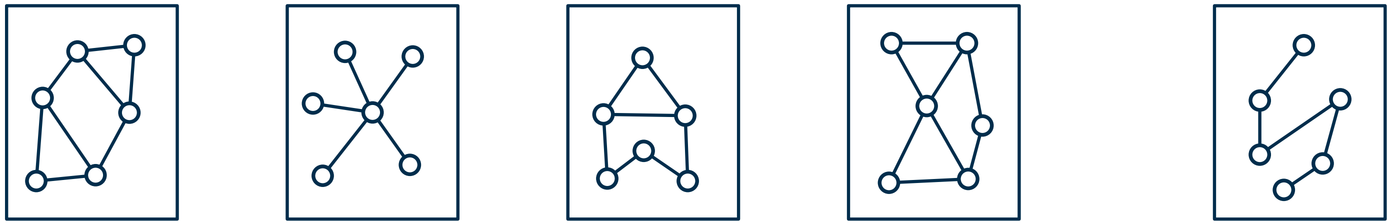

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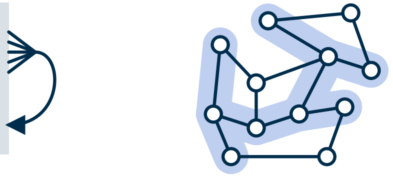

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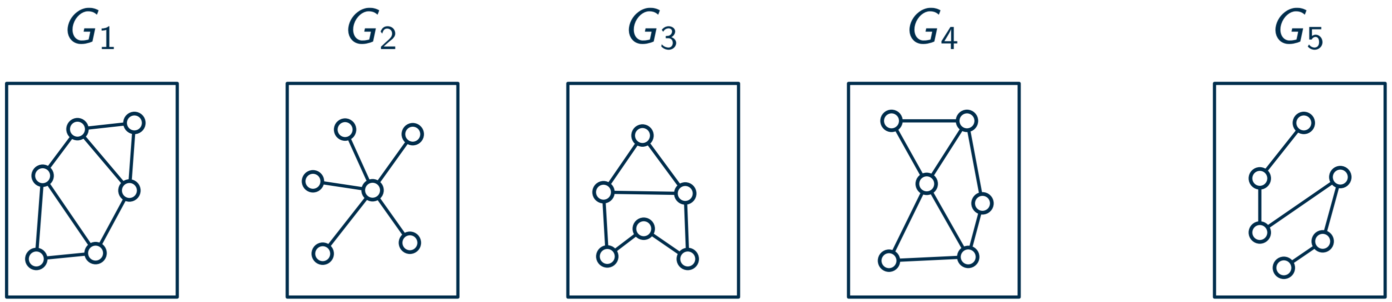

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e.g. same $|V|$

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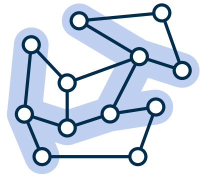

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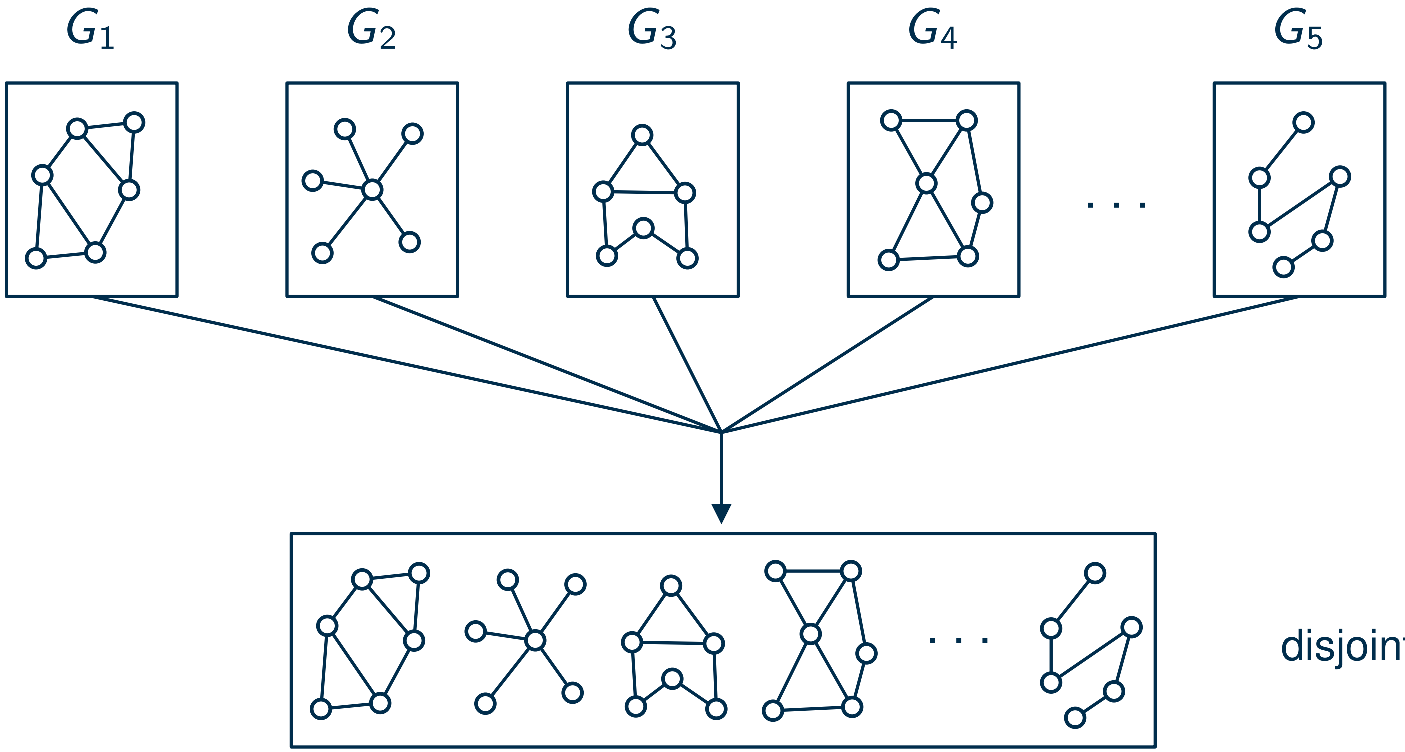

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disjoint union

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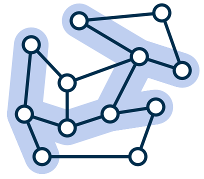

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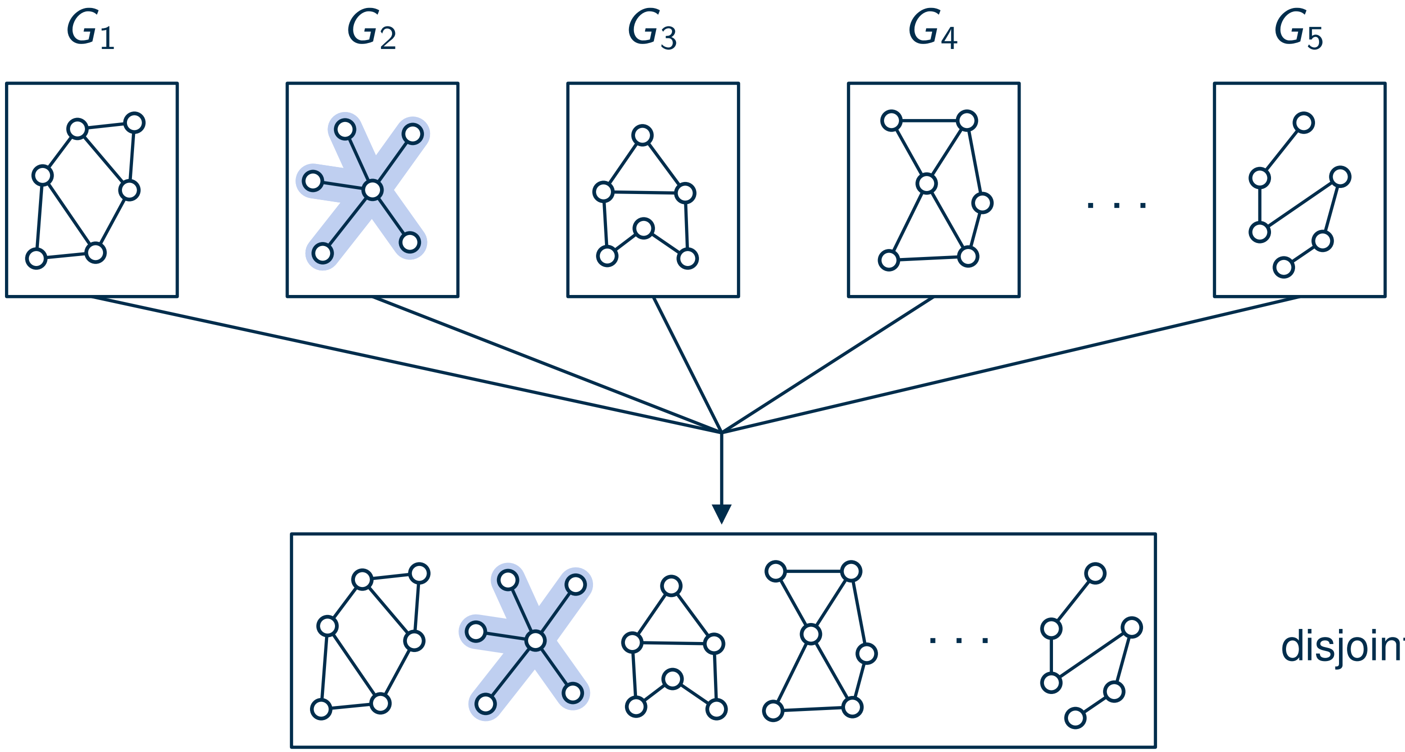

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Disjoint Factors – Cross Composition

DISJOINT FACTORS (|Σ|): Given a word w over an alphabet $\Sigma = \{\gamma_1, \gamma_2, \ldots, \gamma_s\}$, does it contain pairwise disjoint subwords $u_1, u_2, \ldots, u_s$ such that each u_i has length ≥ 2 and begins and ends with γ_i ?

$\Sigma = \{a, b, c\}$

bcacabbacbbab

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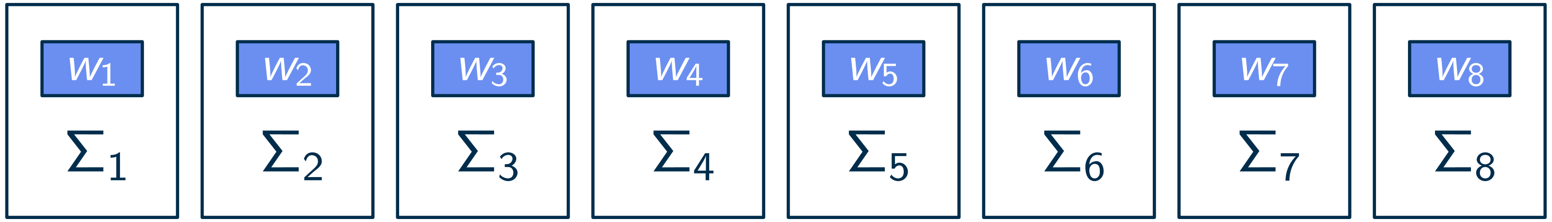

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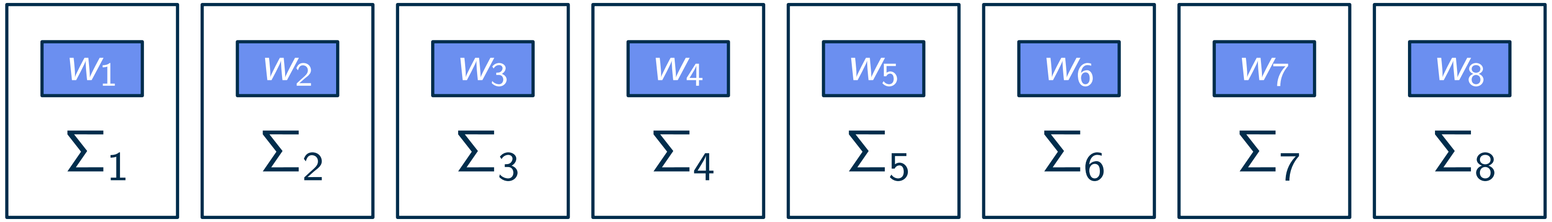

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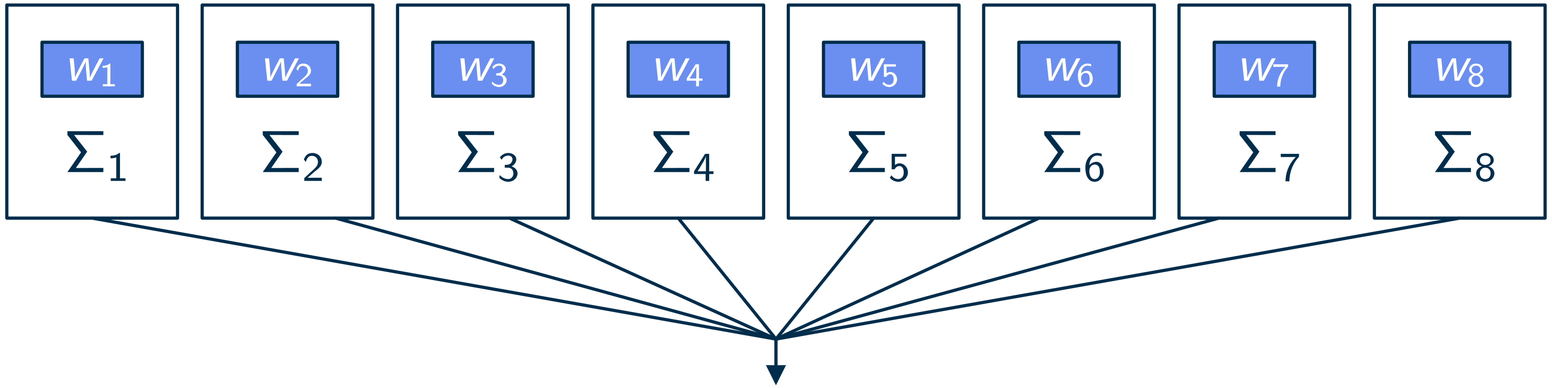

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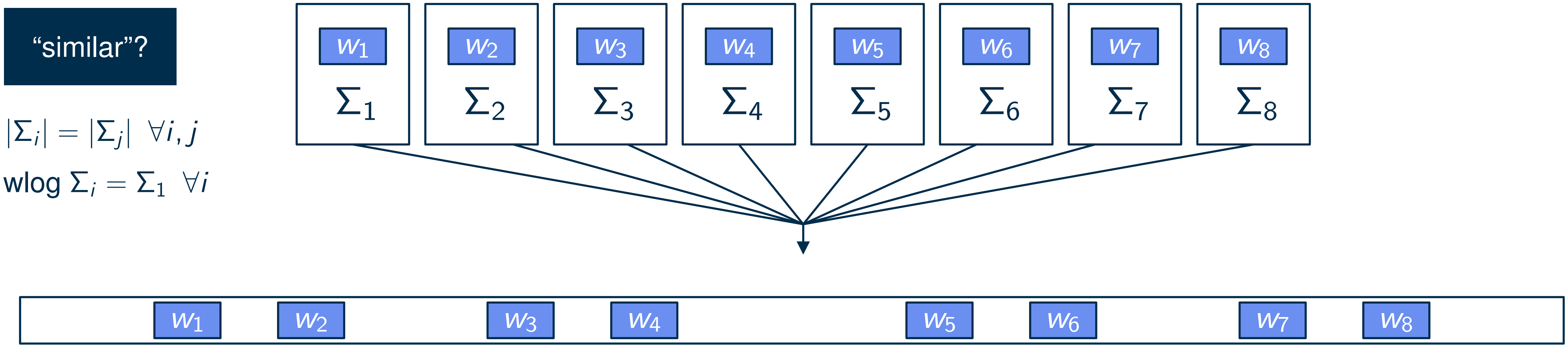

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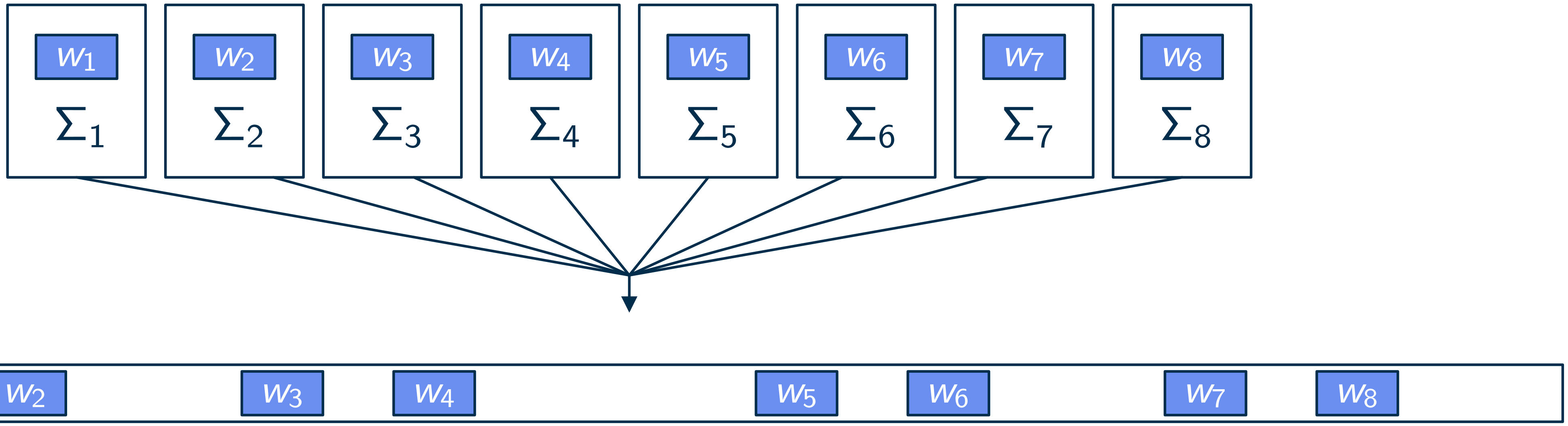

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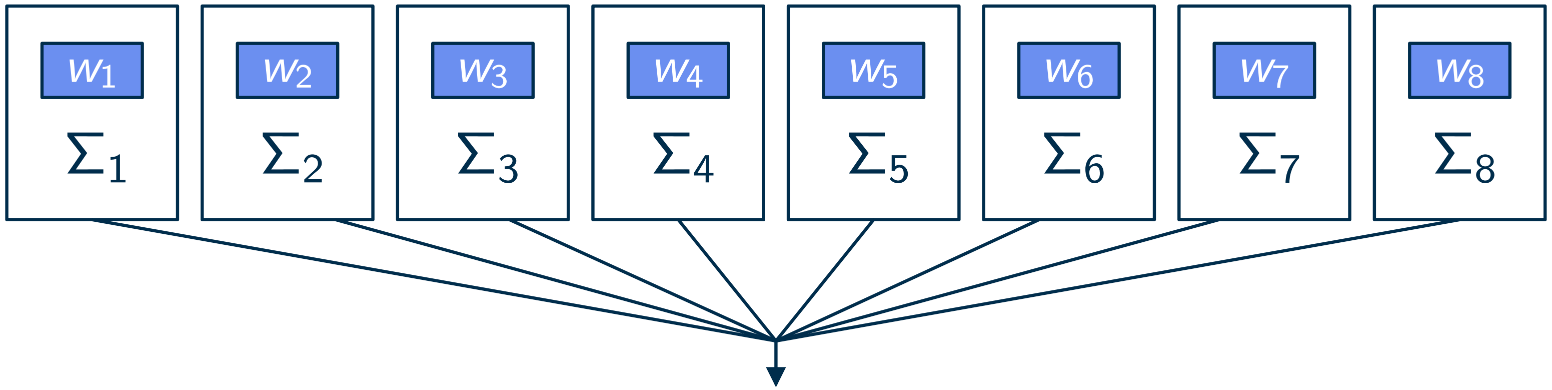

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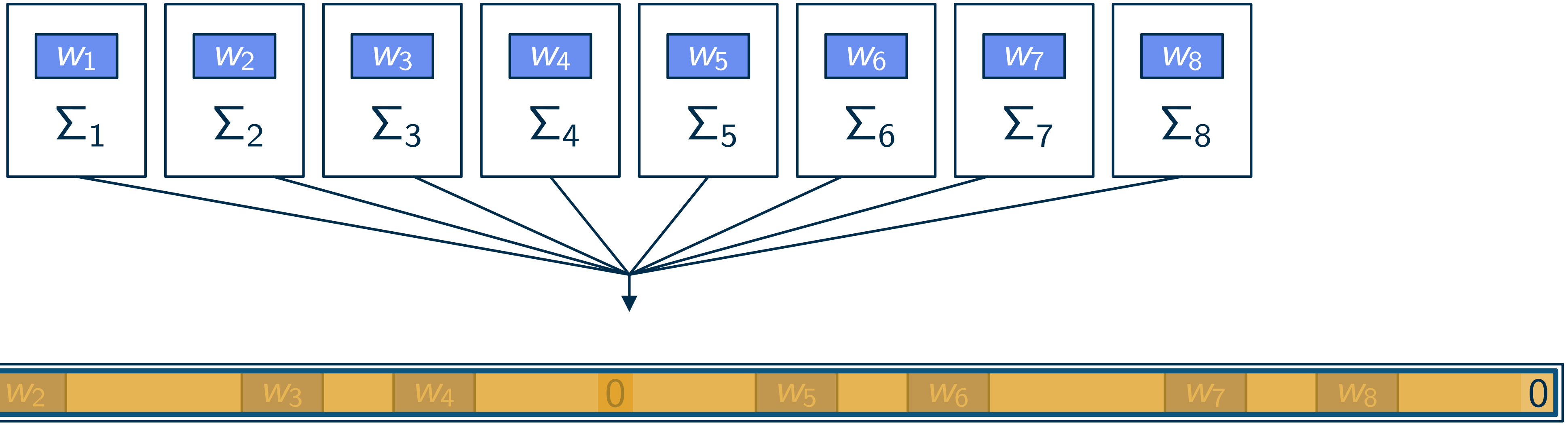

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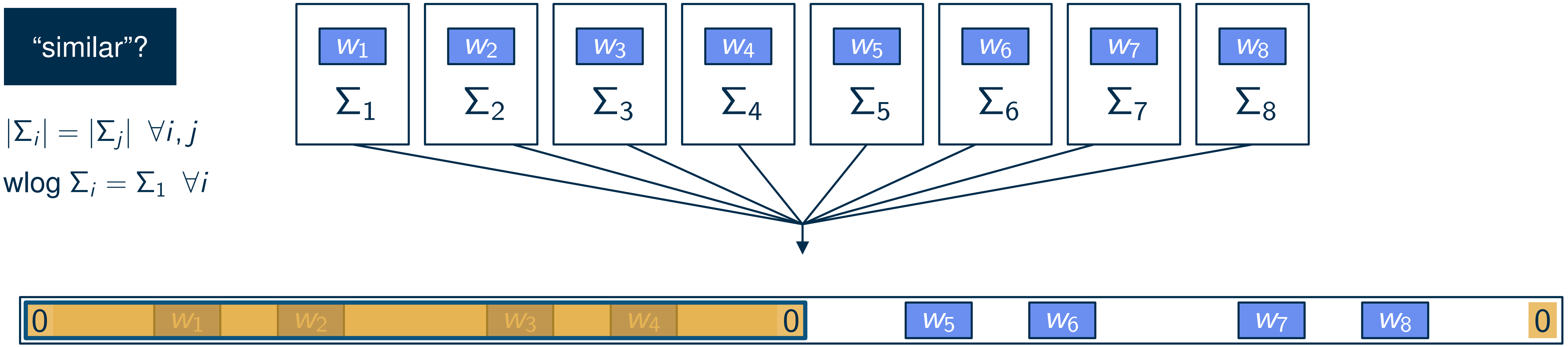

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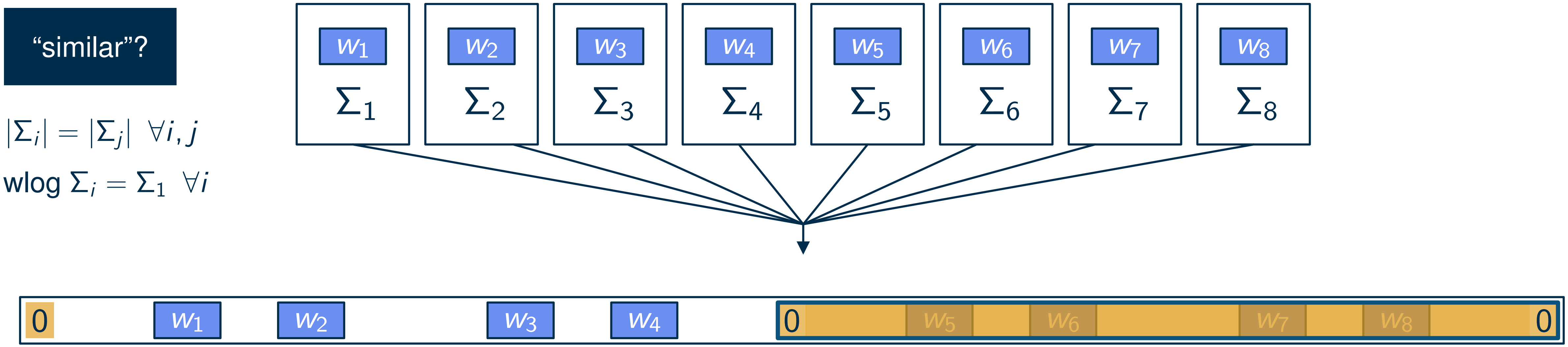

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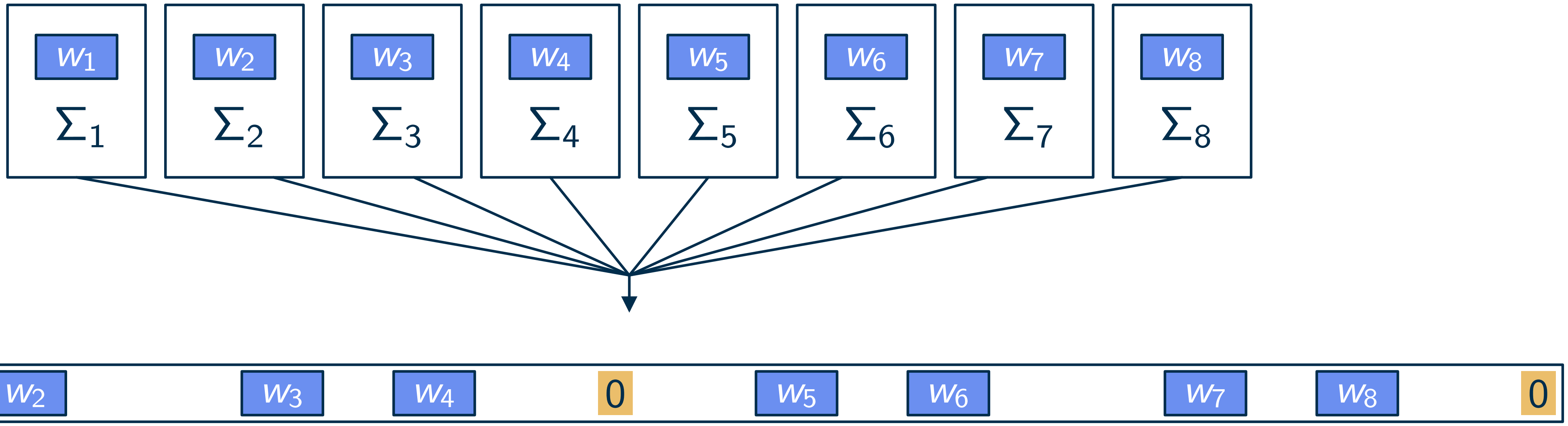

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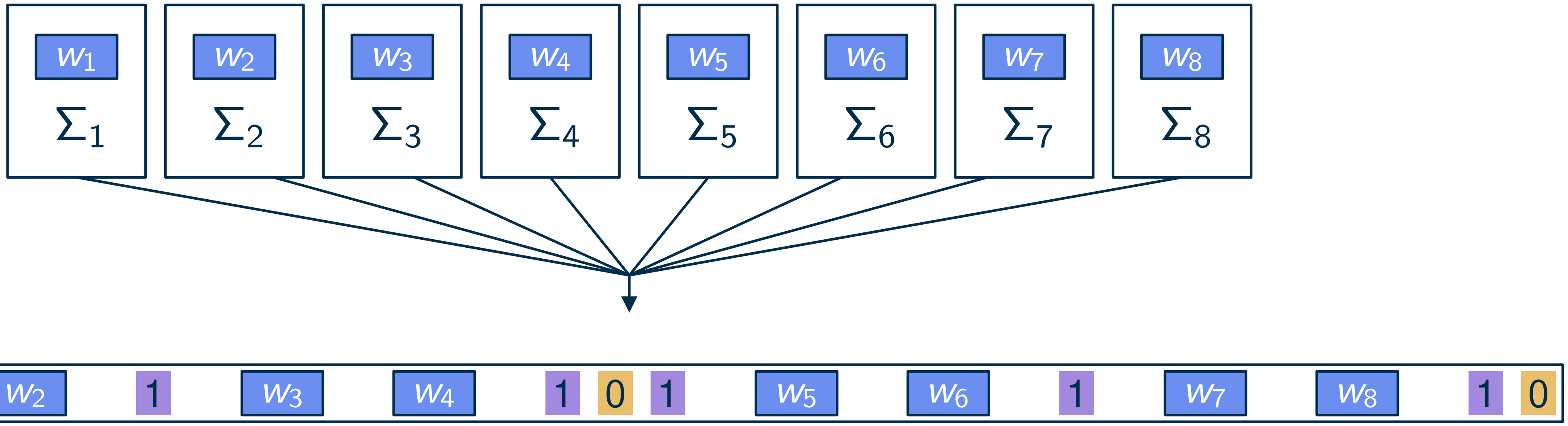

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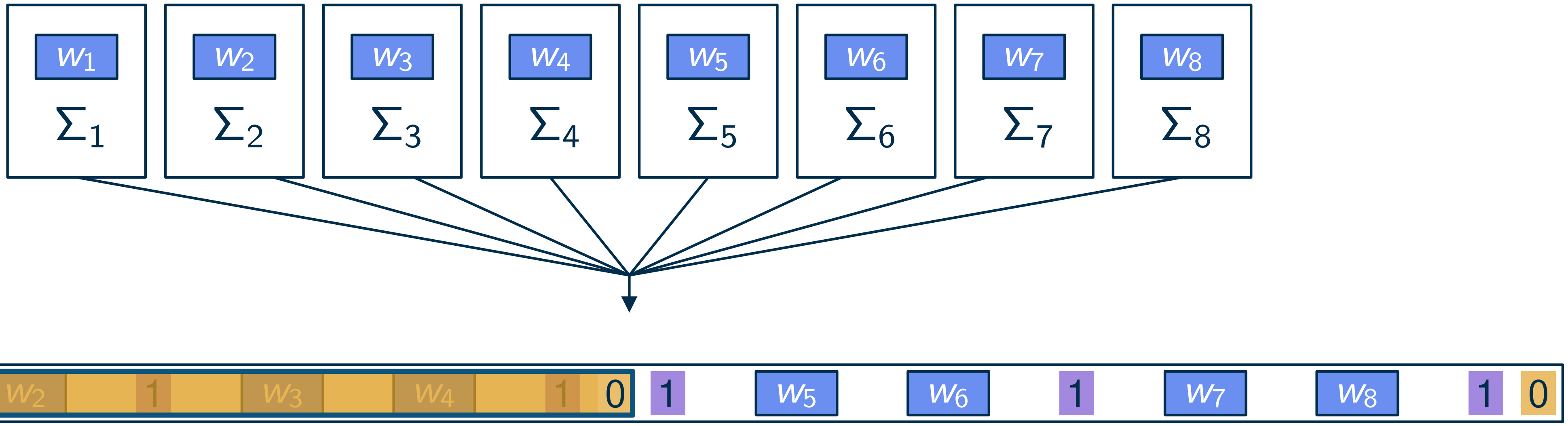

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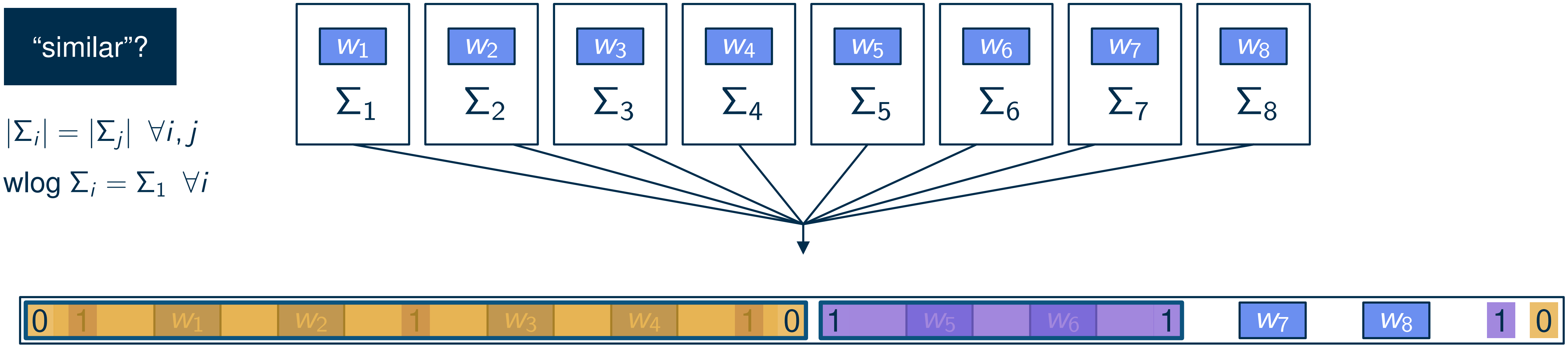

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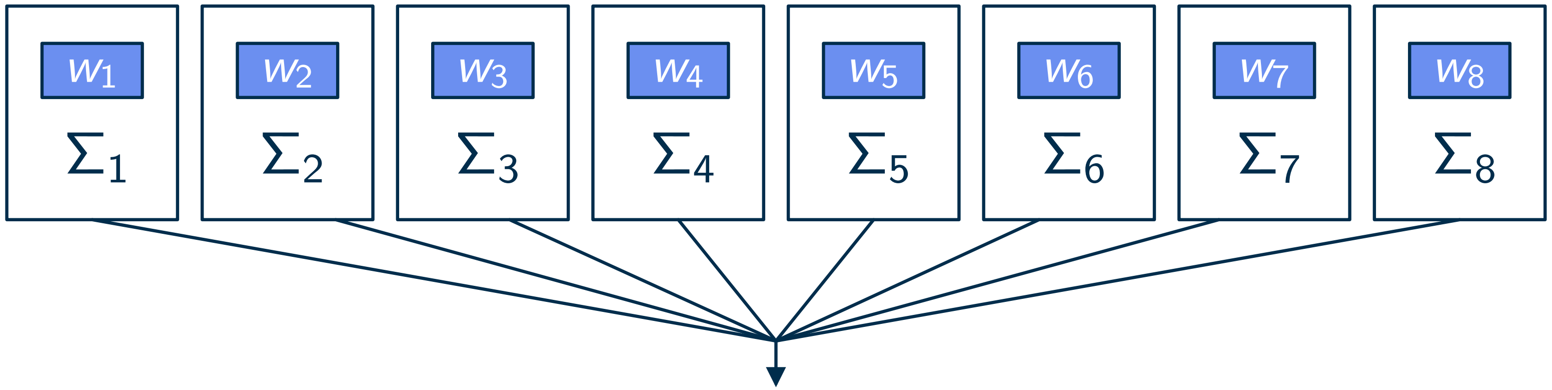

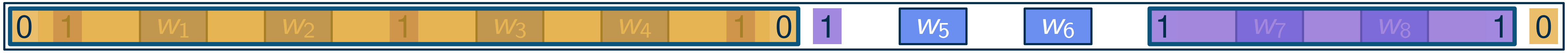

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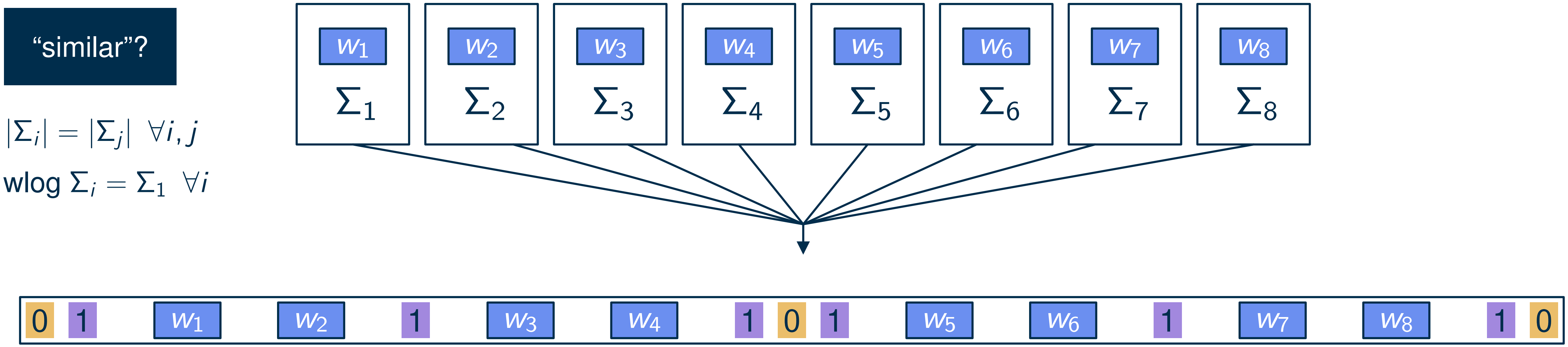

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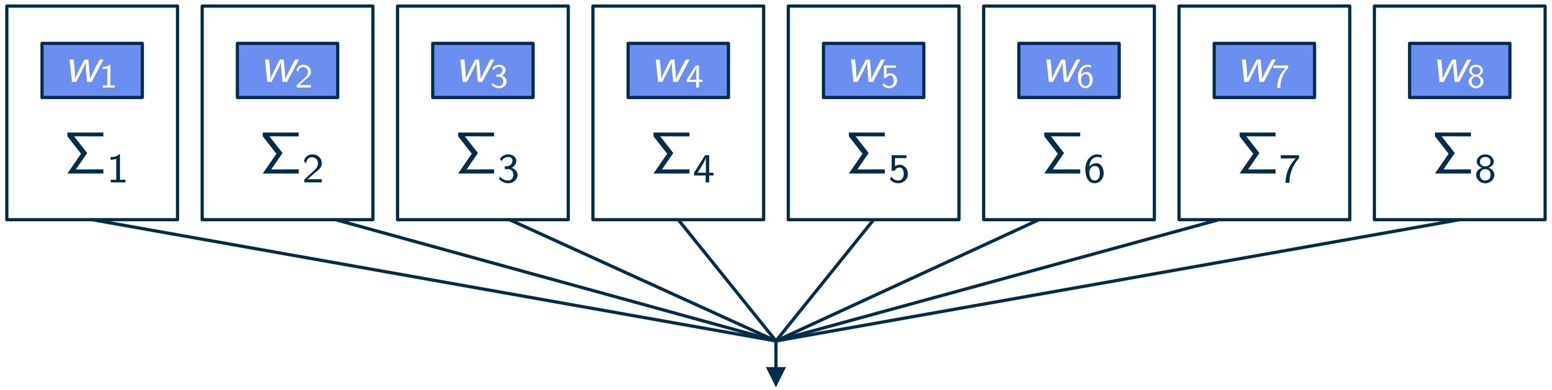

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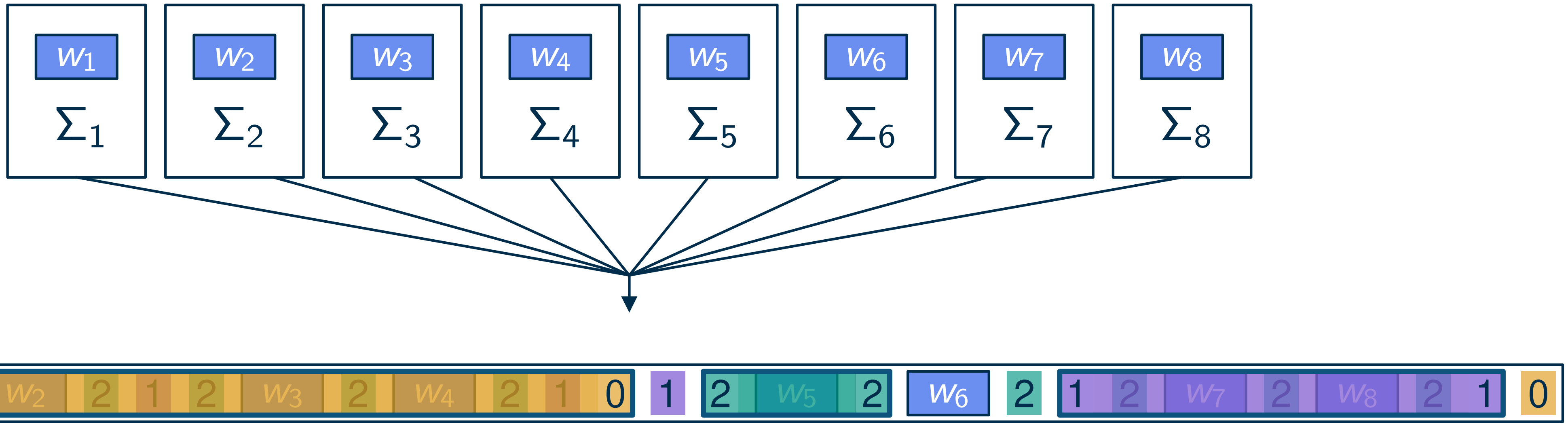

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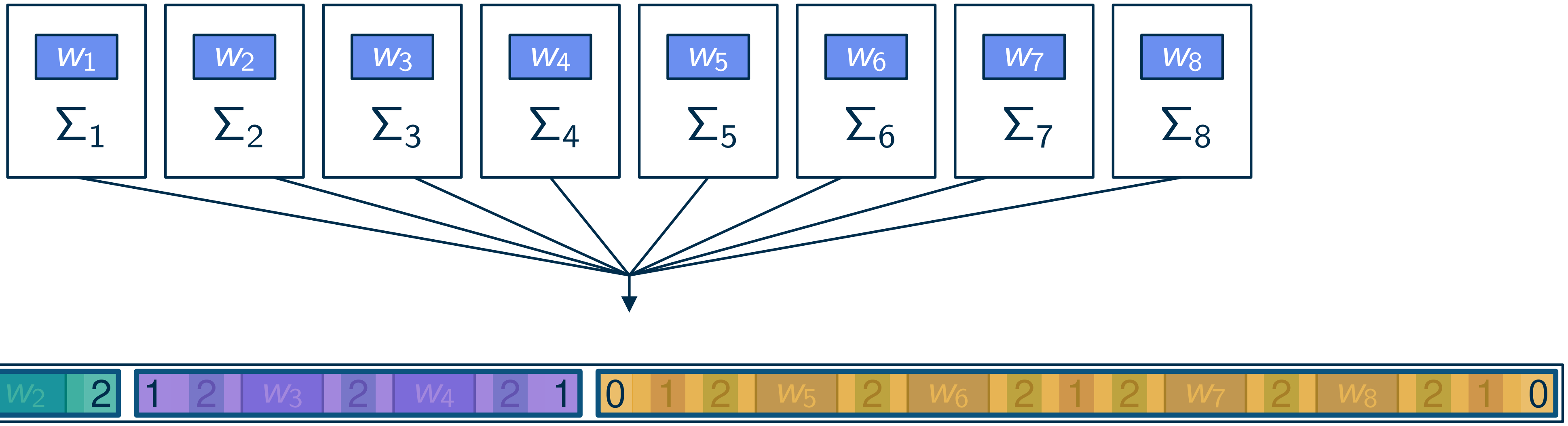

- any choice of subwords u_0, u_1, u_2 leaves exactly one word w_i uncovered

$\Longrightarrow$ solution iff one input instance has solution

$\Sigma' = \Sigma_1 \cup \underbrace{\{0, 1, 2\}}$

log t new letters that don't appear in Σ_1

Polynomial Parameter Transformation

CONNECTED VERTEX COVER (k): Given a graph G and an integer k, does G contain a vertex cover that induces a connected subgraph of G?

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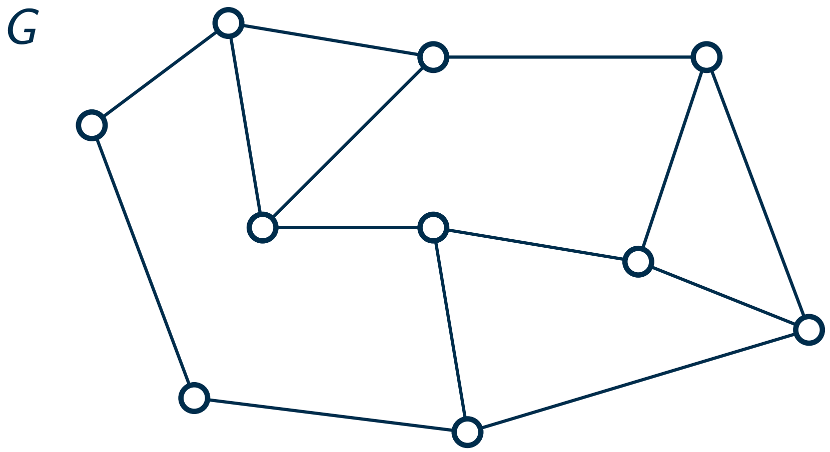

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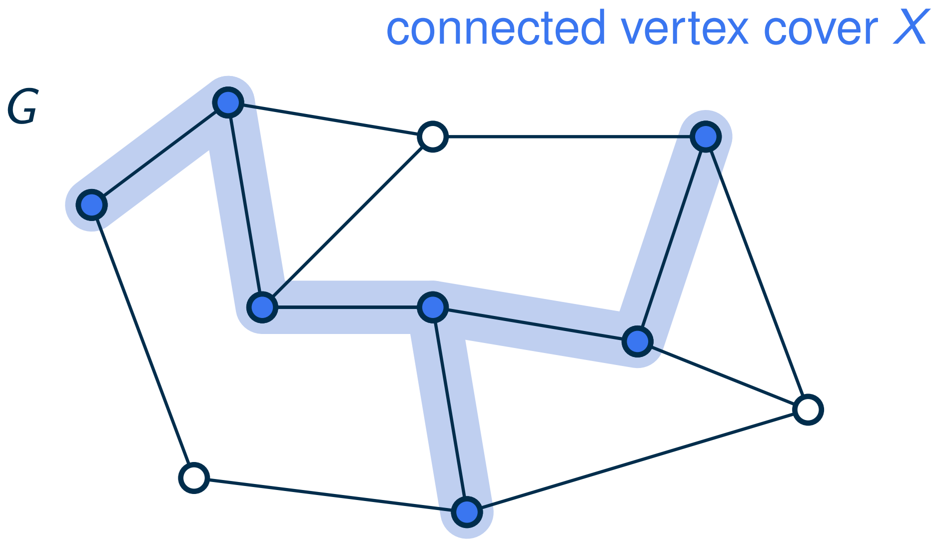

$V \setminus X$ is independent $\implies$ no cycles in $G[V \setminus X]$

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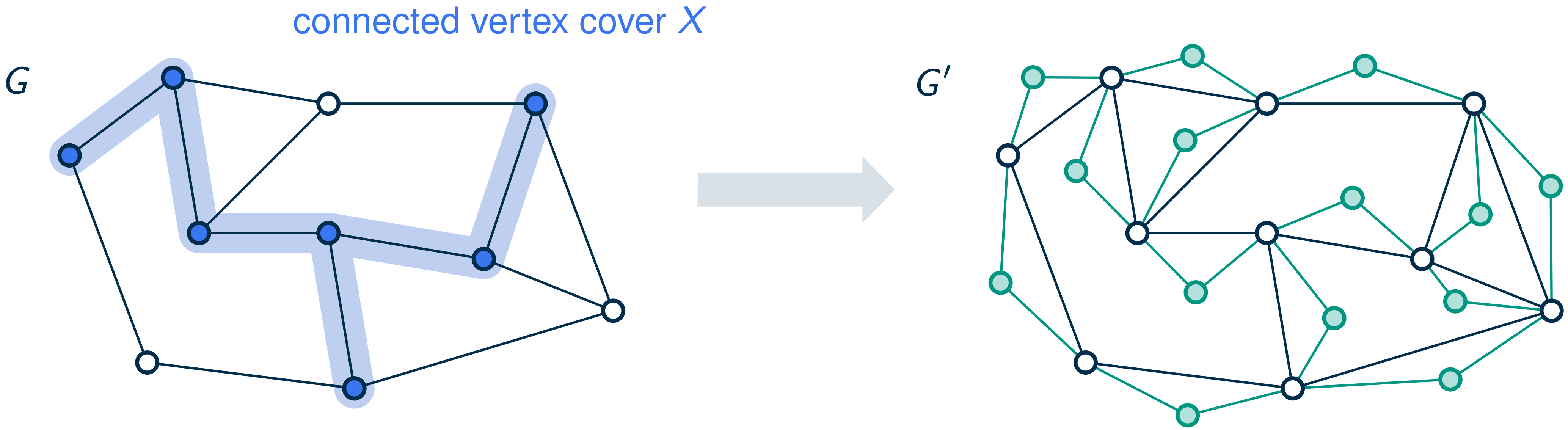

$V \setminus X$ is independent $\implies$ no cycles in $G[V \setminus X]$

extend every edge to a triangle

Polynomial Parameter Transformation

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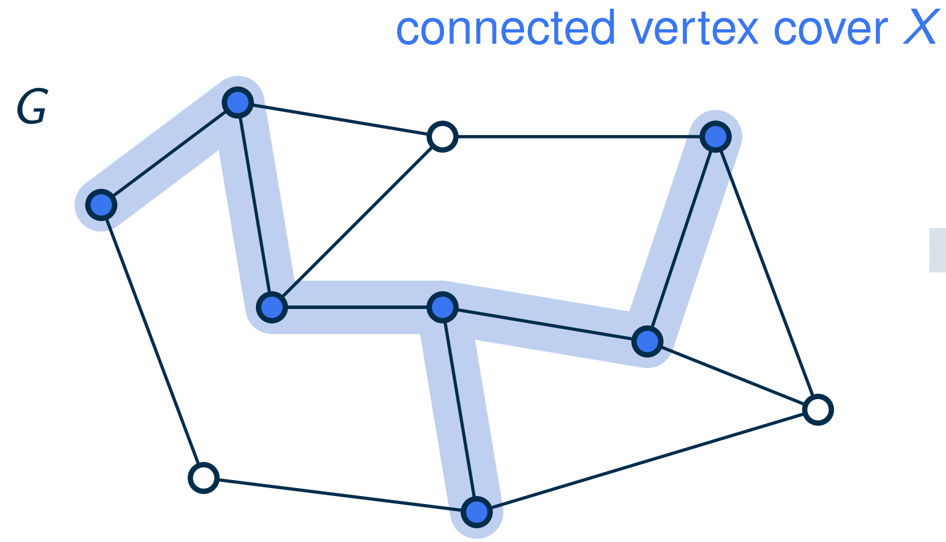

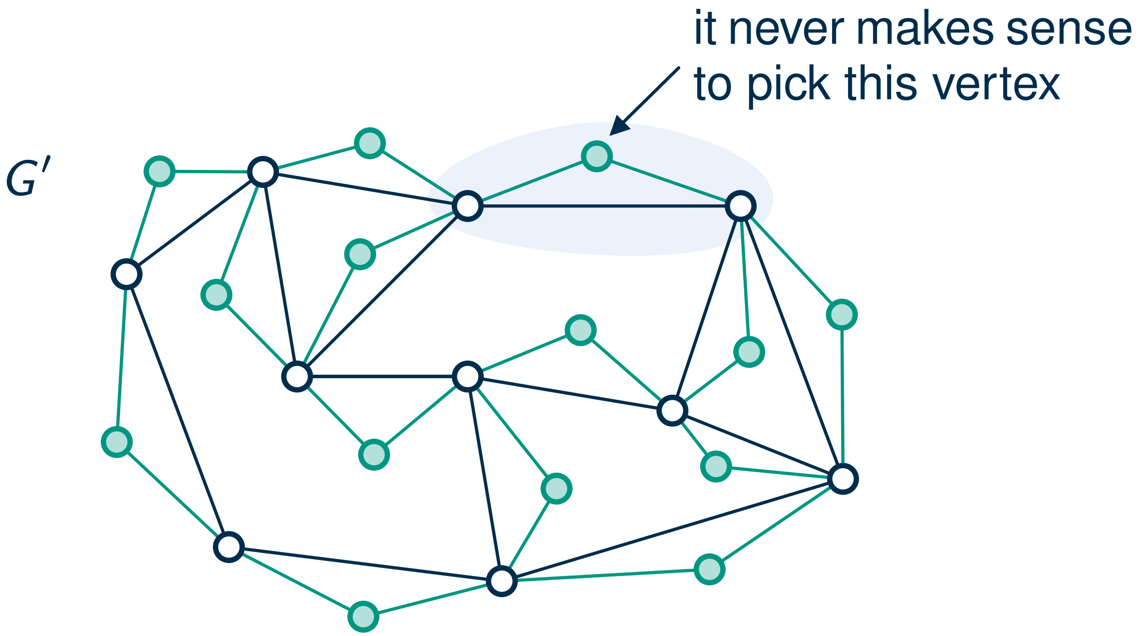

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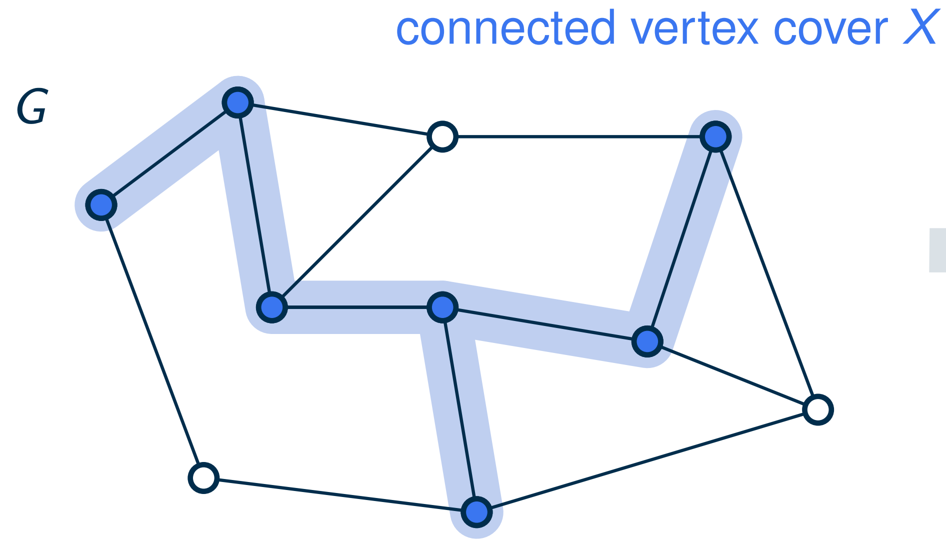

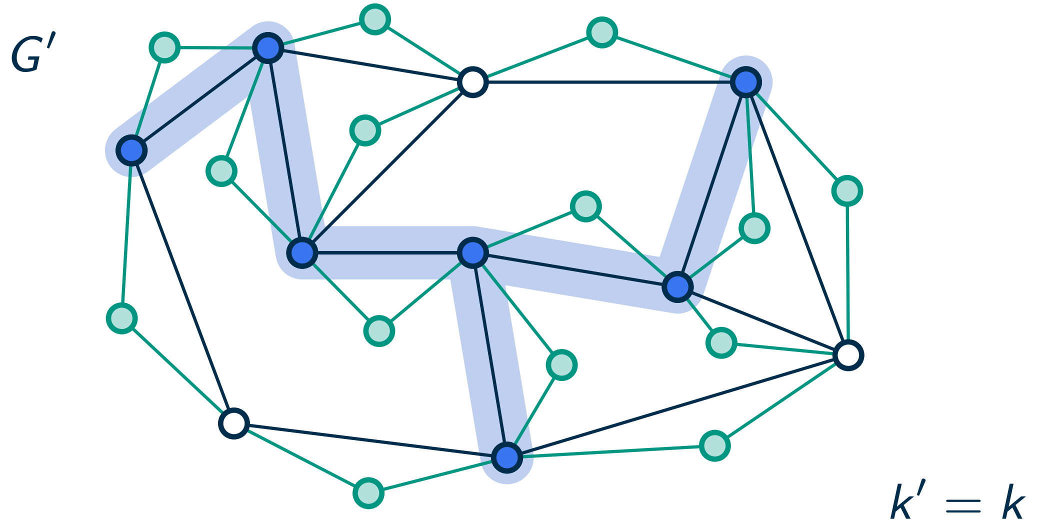

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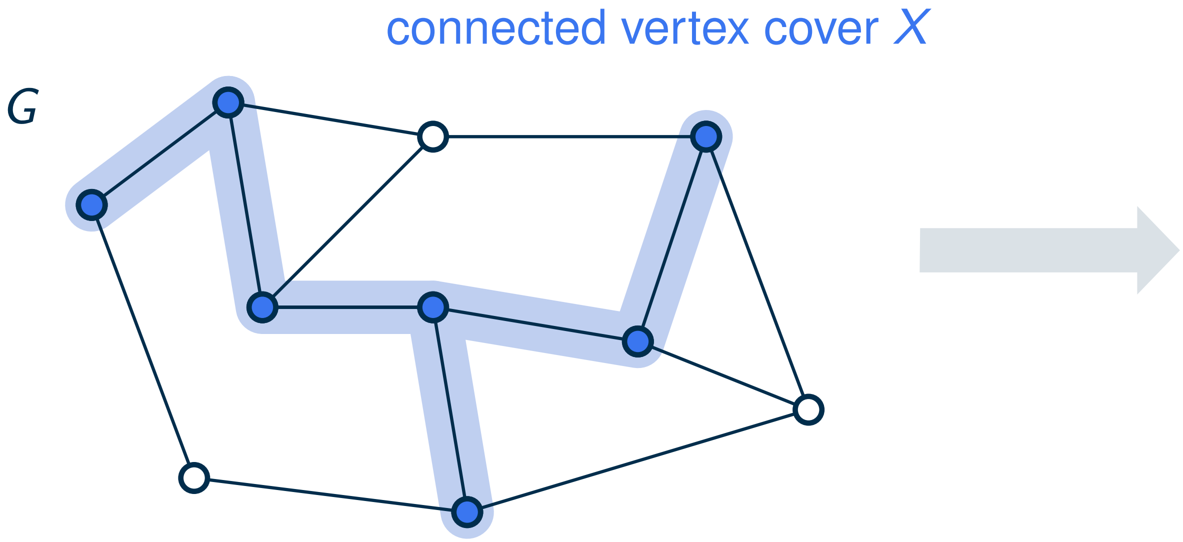

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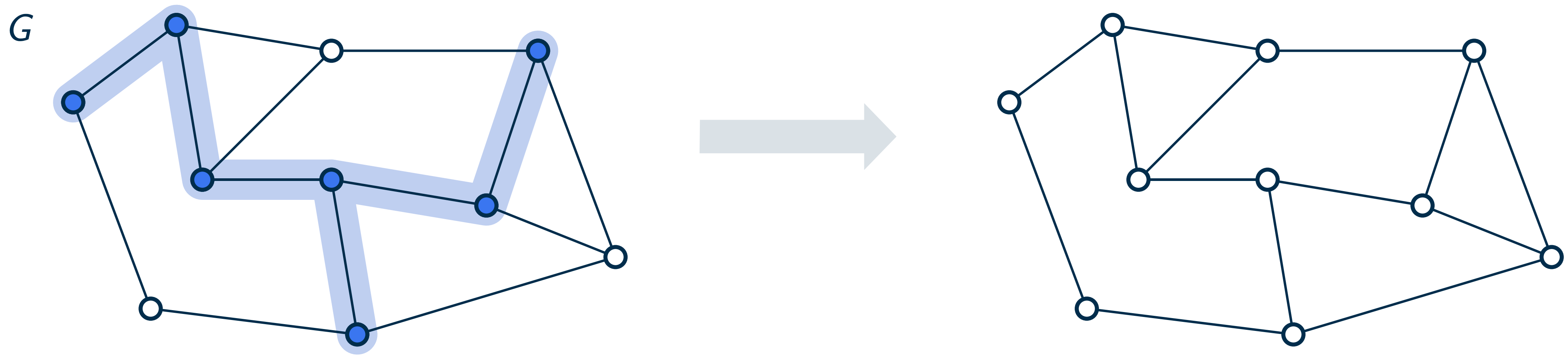
connected vertex cover X



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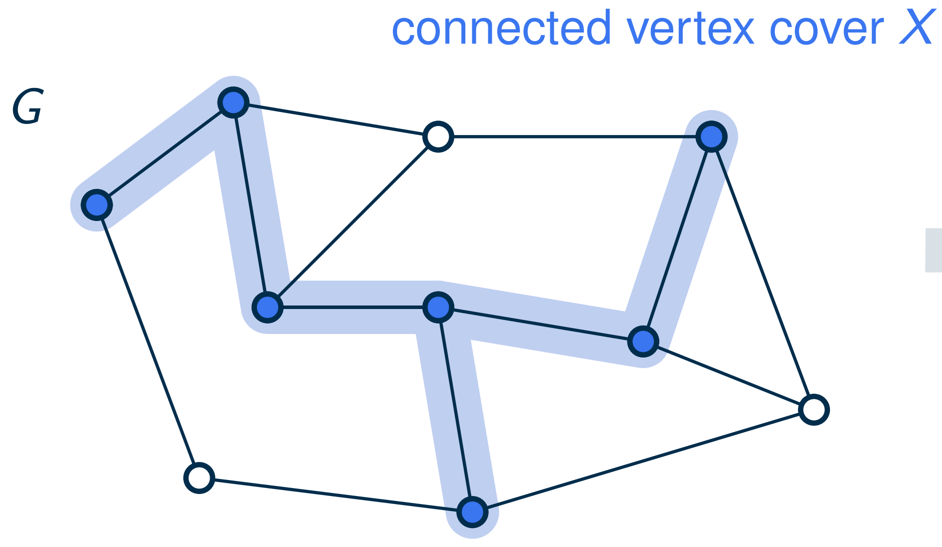

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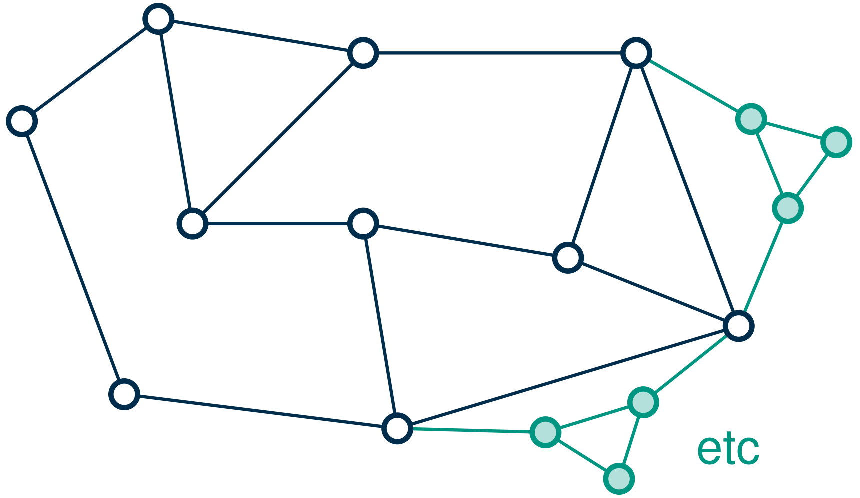

extend every edge to a bull

Polynomial Parameter Transformation

Colorful LR DomSet ($k = |R| + \ell$): Given a bipartite graph G with sides L and R and a coloring of L into ℓ colors. Is there a colorful set $X \subseteq L$ of size exactly ℓ that dominates all vertices in R?

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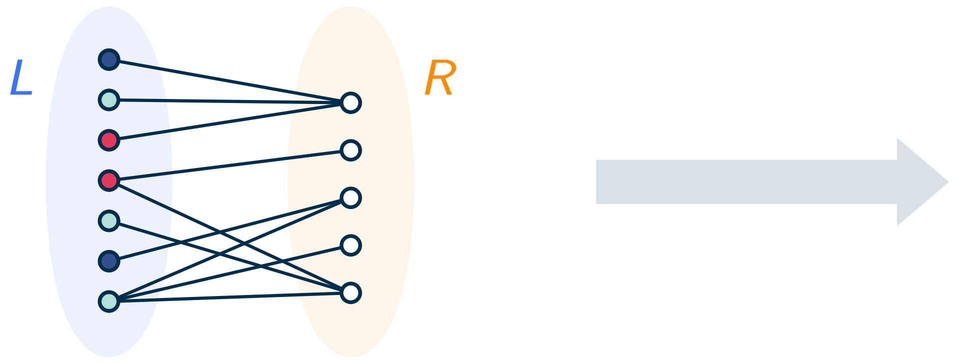

$\ell = 3$

w.l.o.g. there are no isolated vertices in L (otherwise reduce instance)

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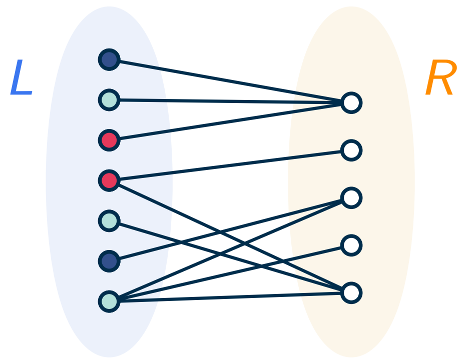

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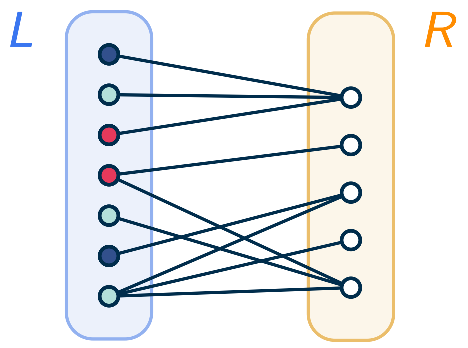

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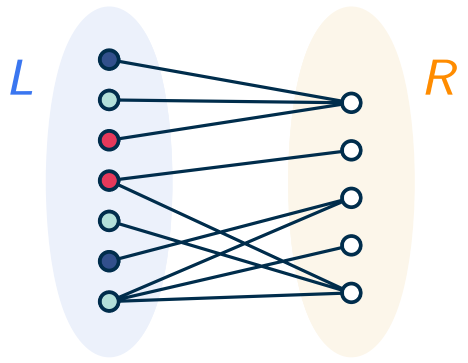

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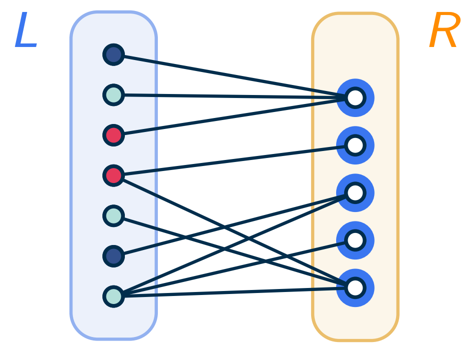

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always has vertex cover of size $|R|$
... but not necessarily connected
... and instance not equivalent

Polynomial Parameter Transformation

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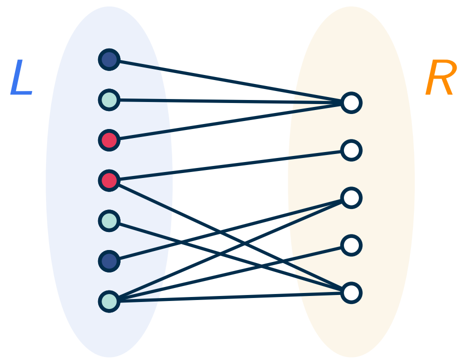

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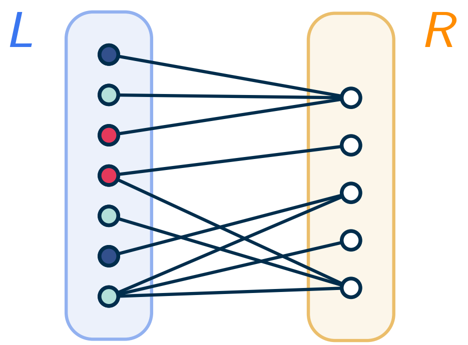

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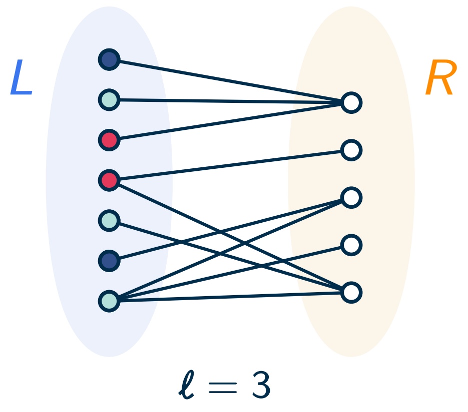

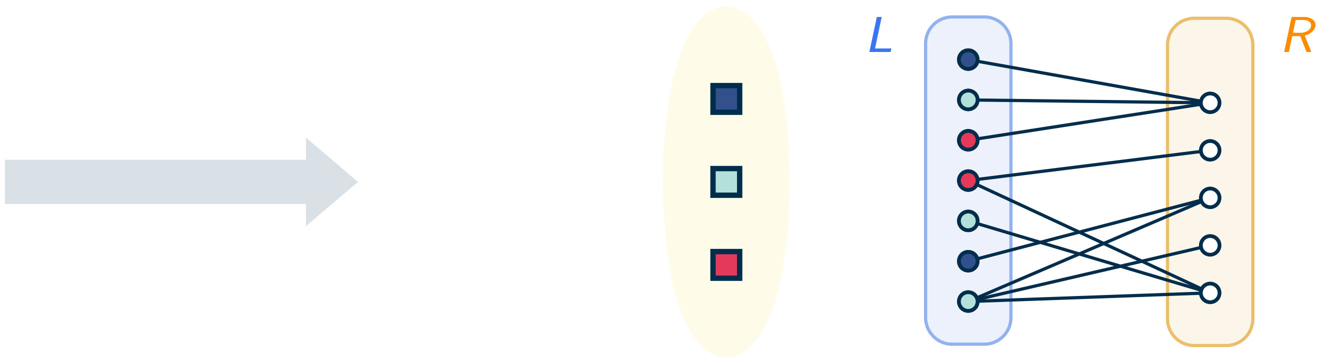

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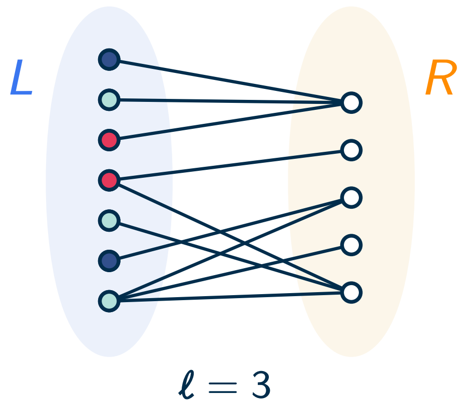

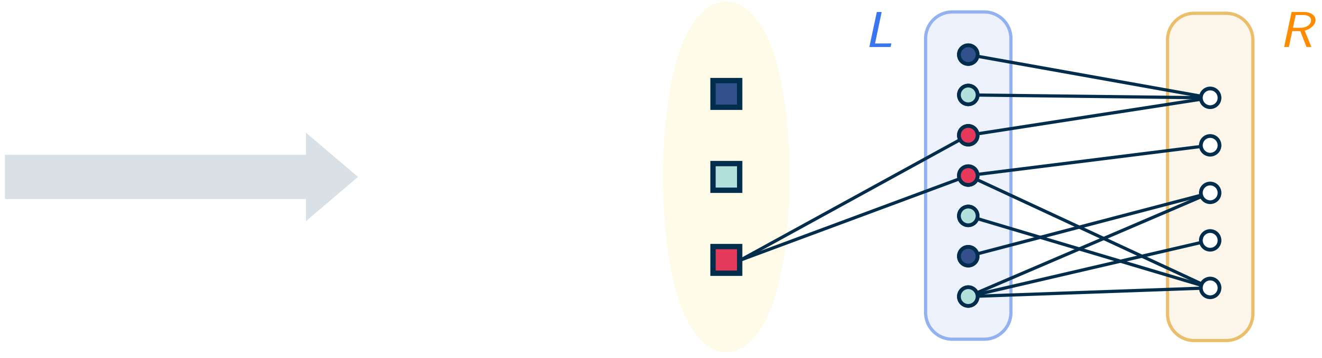

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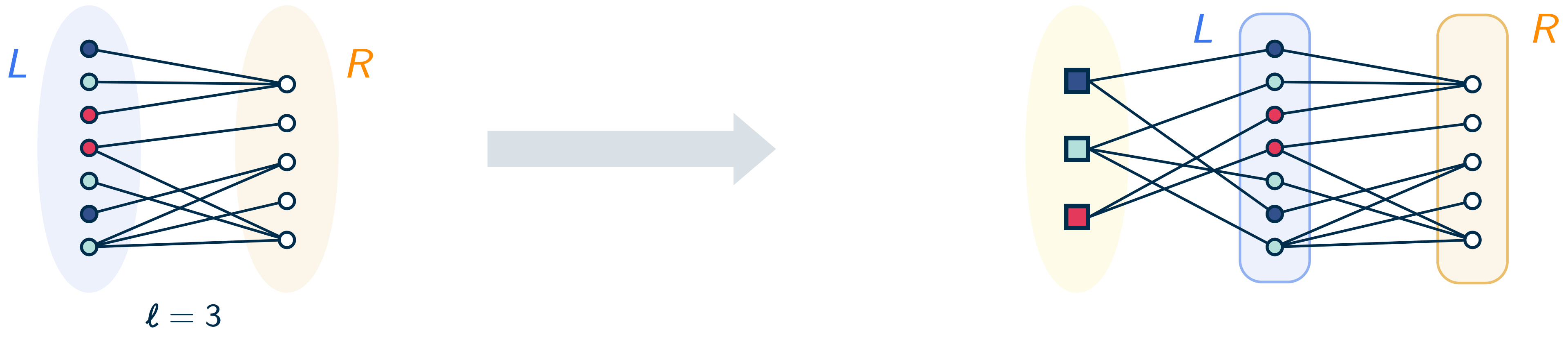

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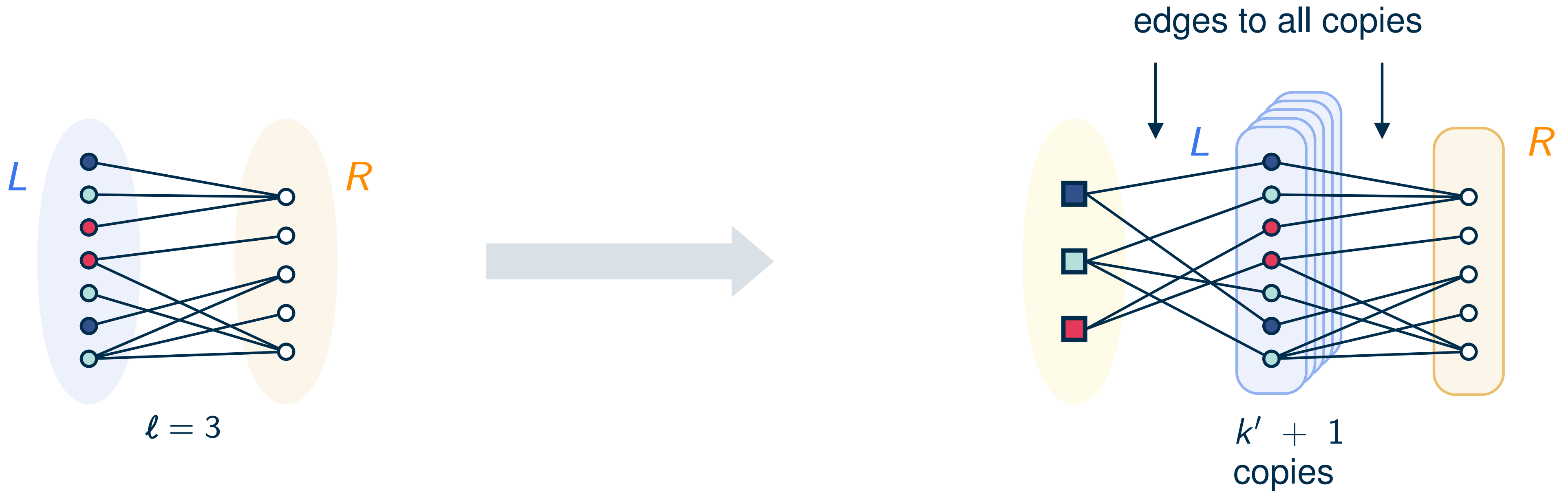

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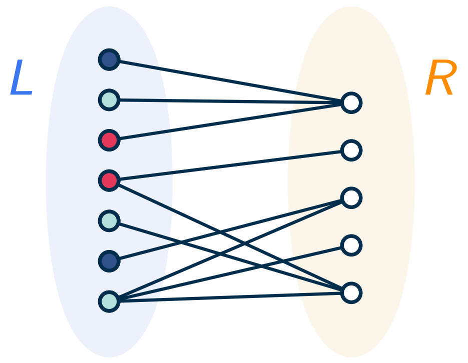

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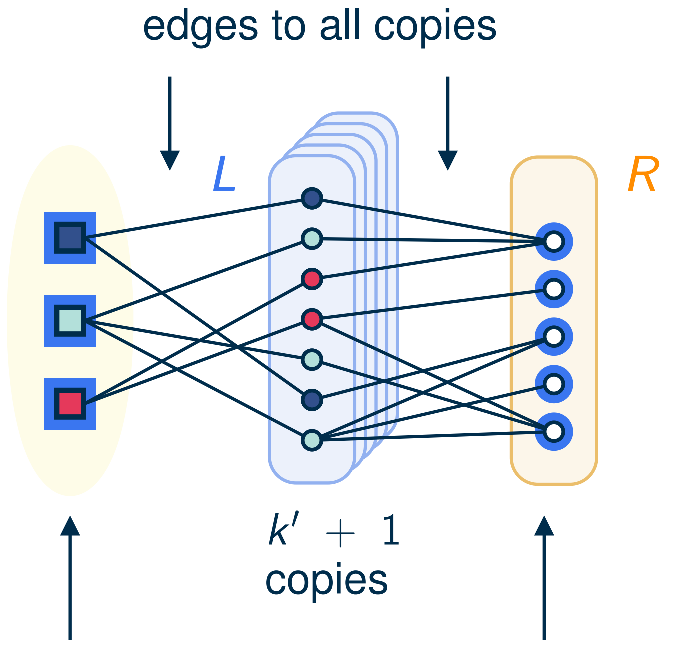

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all these vertices have degree $\geq k' + 1$

$\Longrightarrow$ must be selected, yields vertex cover

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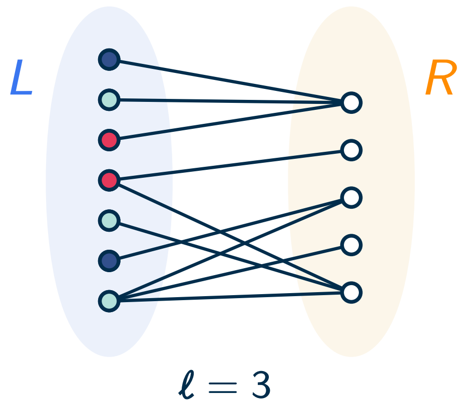

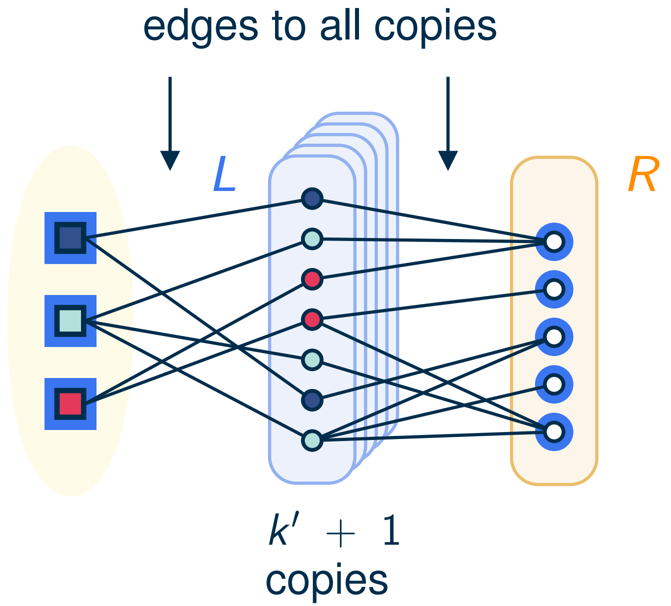

w.l.o.g. there are no isolated vertices in L (otherwise reduce instance)

to achieve connectedness:

- in L: choose one of each color
- in L: choose set that dominates R

Polynomial Parameter Transformation

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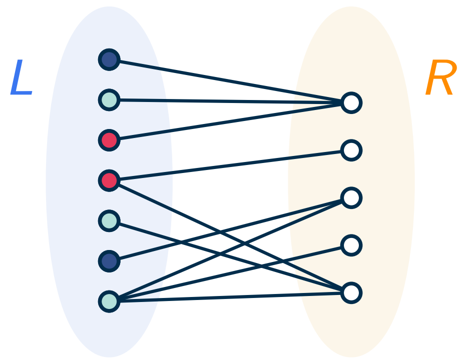

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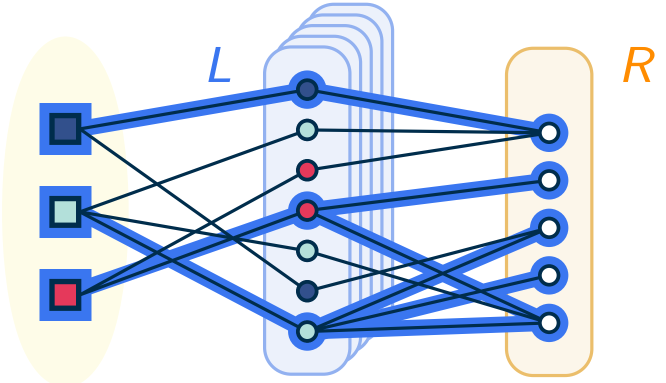

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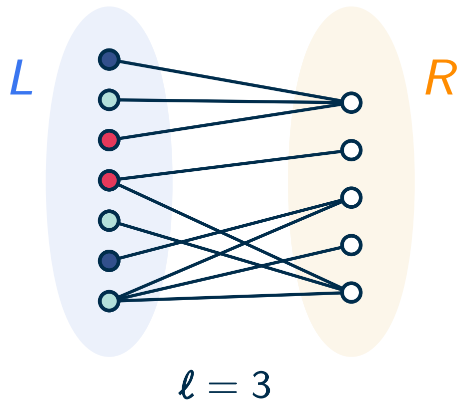

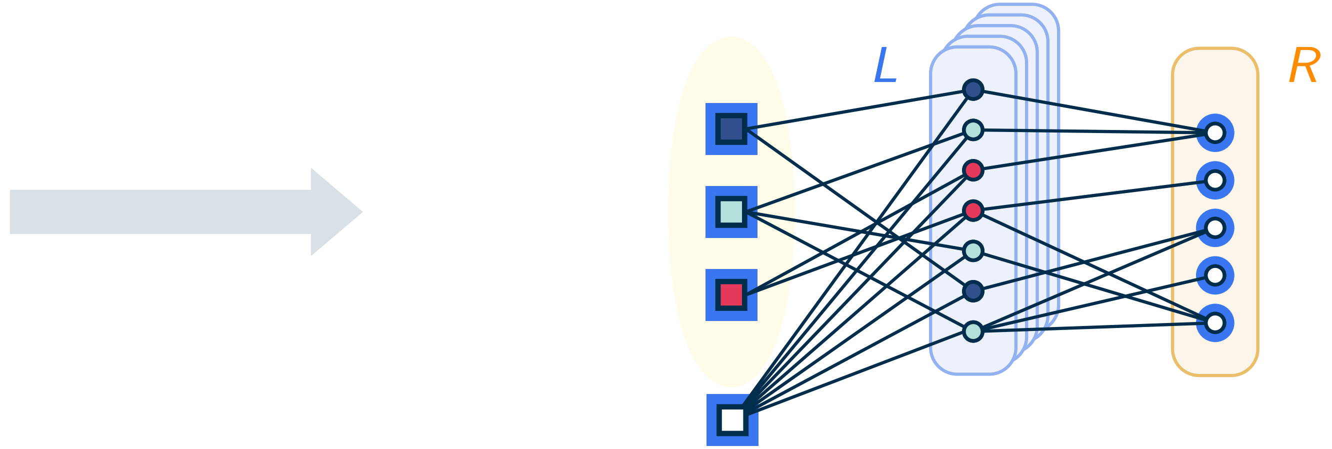

w.l.o.g. there are no isolated vertices in L (otherwise reduce instance)

to achieve connectedness:

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- in L: choose set that dominates R
- add universal vertex for L

Polynomial Parameter Transformation

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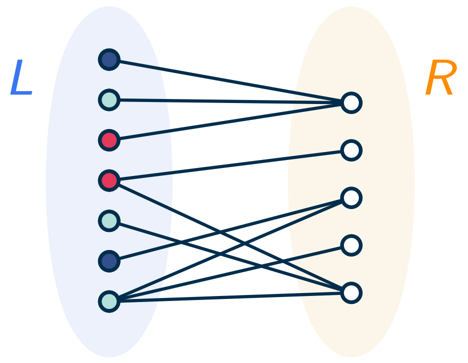

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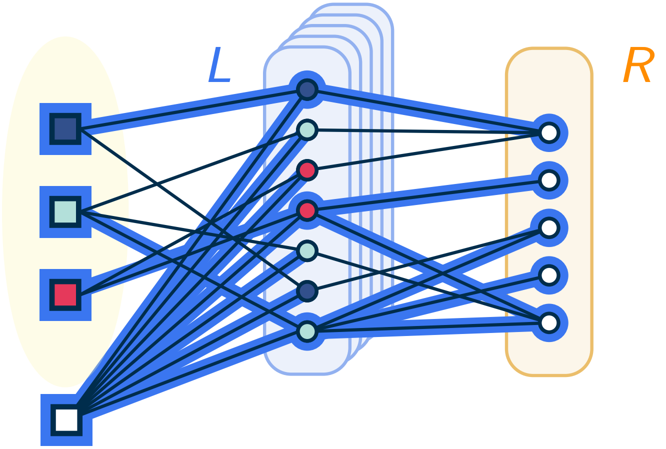

w.l.o.g. there are no isolated vertices in L (otherwise reduce instance)

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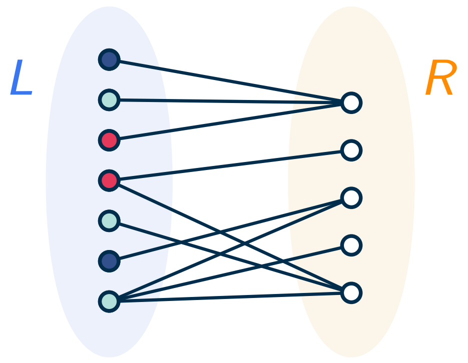
Connected Vertex Cover (k): Given a graph G and an integer k, does G contain a vertex cover that induces a connected subgraph of G?

$k' = |R| + 2\ell + 1 \leq 2k + 1$

$k = |R| + \ell$



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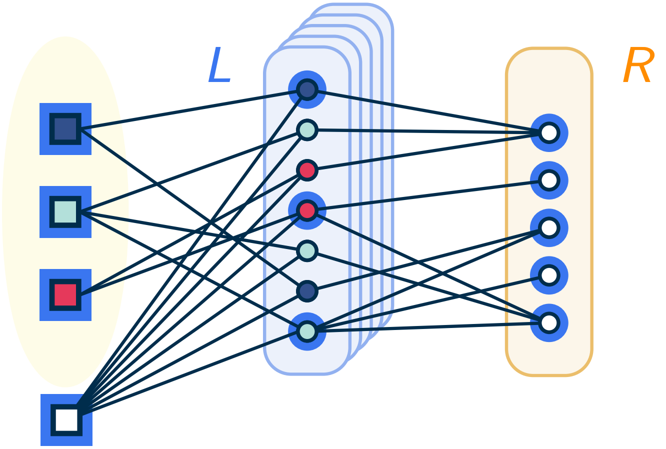

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to achieve connectedness:

- in L: choose one of each color
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