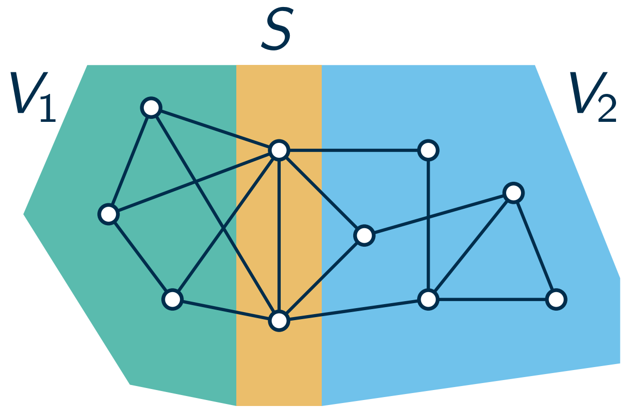
Parameterized Algorithms

Treewidth

Thomas Bläsius

Separators

Definition
Let $G = (V, E)$ be a graph. A separator $S \subseteq V$ that separates V_1 from V_2 is **α-balanced** if $|V_1|, |V_2| \leq \alpha|V|$.



- $|V| = 10$
- $|V_1| = 3$
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$\Rightarrow \frac{1}{2}$-balanced

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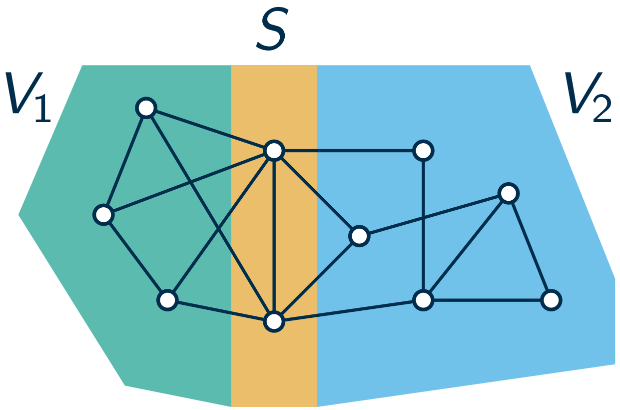

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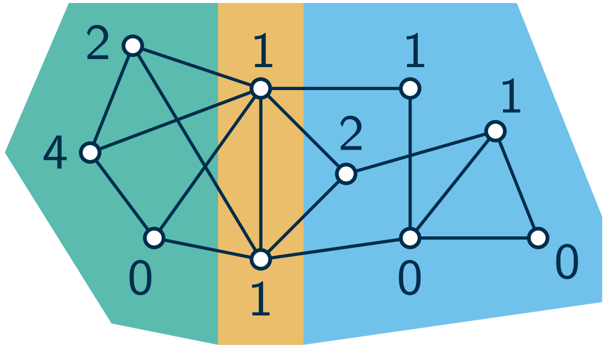
If G has vertex weights $w\colon V \to \mathbb{N}$, then S is **α-balanced** if $w(V_1), w(V_2) \leq \alpha w(V)$.



- $w(V) = 12$
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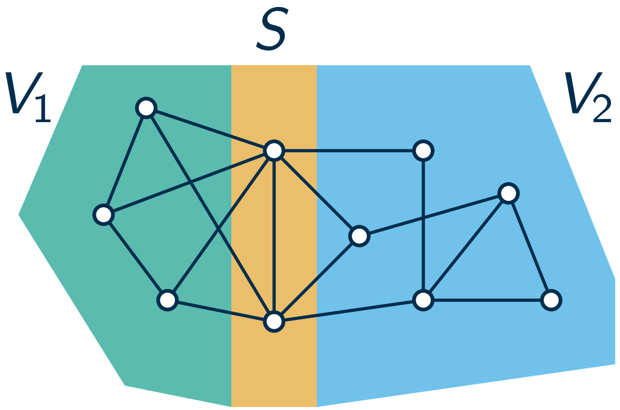
Lemma
Every tree (and every weighted tree) has a $\frac{2}{3}$-balanced separator of size 1.



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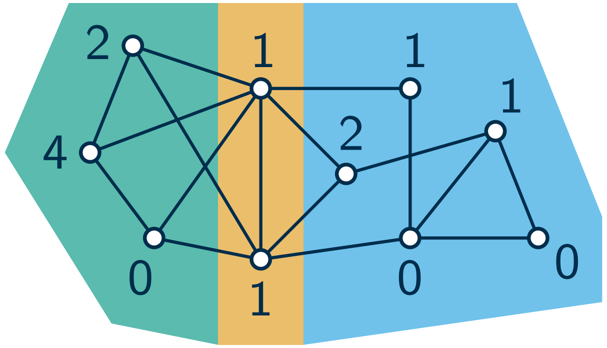

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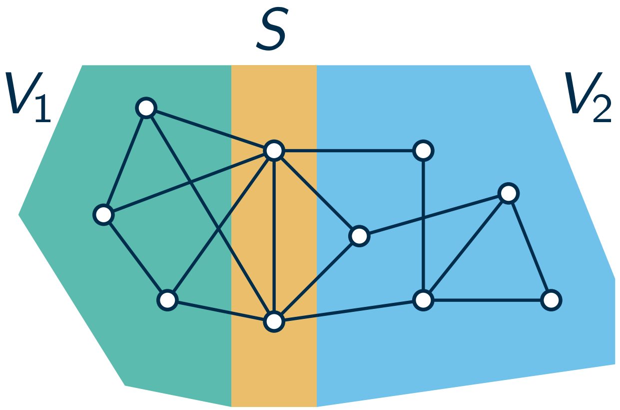

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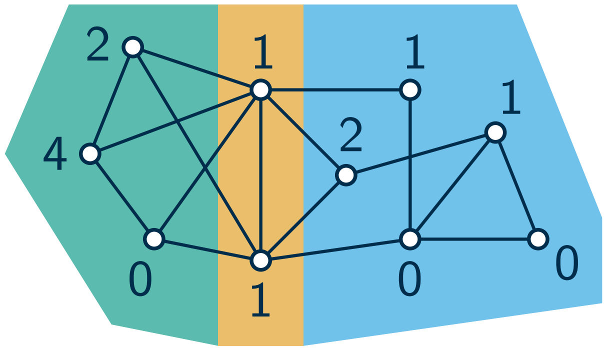

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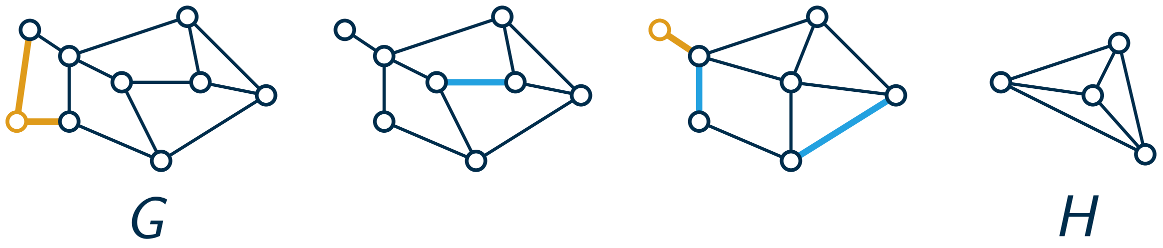
Lemma
Every (weighted) graph with treewidth t has a $\frac{2}{3}$-balanced separator of size at most $t+1$.

VERTEX COVER in planar graphs

Definition
A graph H is a **minor** of G, if H can be obtained from G by taking a subgraph and contracting edges.



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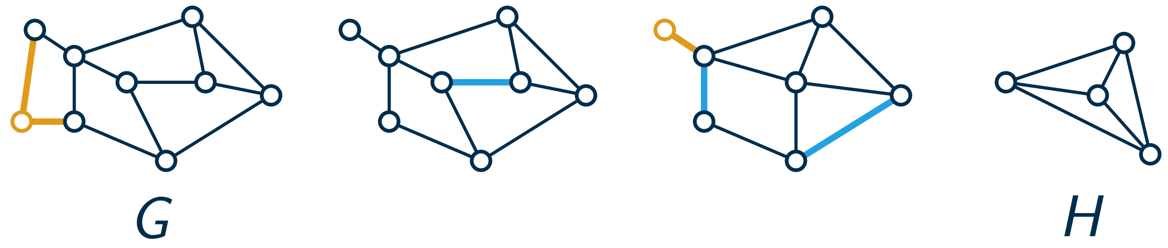

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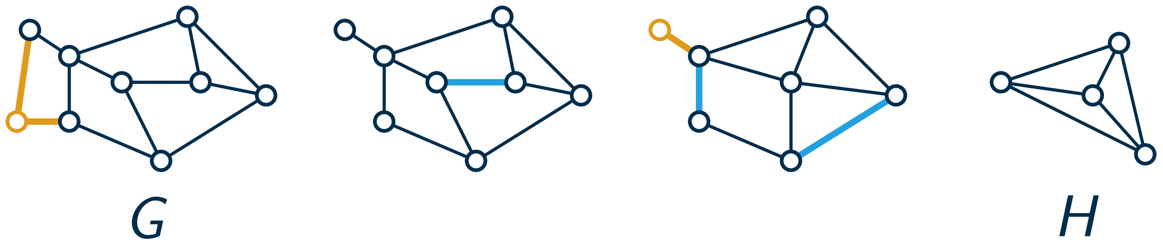

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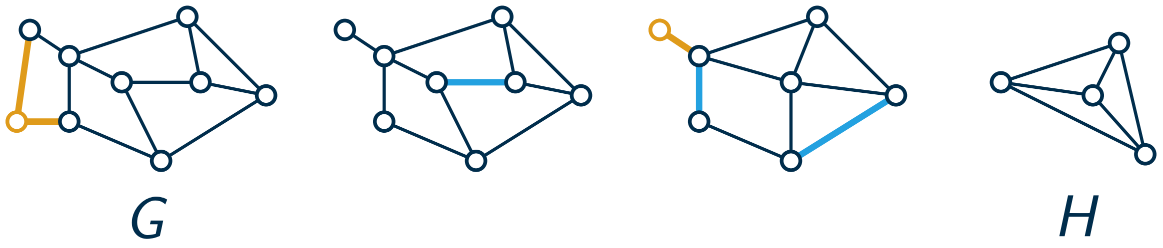

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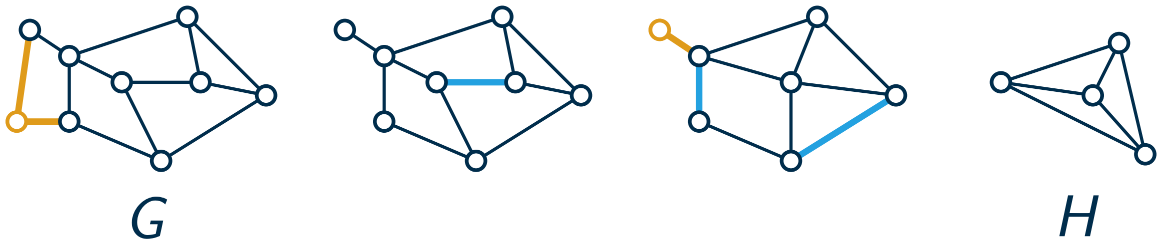

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- for planar graphs: $\text{tw}(G) \geq \frac{9}{2}t \Rightarrow G$ has Γ_t as minor
- this is constructive: n^2 algo constructs a Γ_t minor or a tree decomposition of width roughly $\frac{9}{2}t$

Vertex Cover in planar graphs

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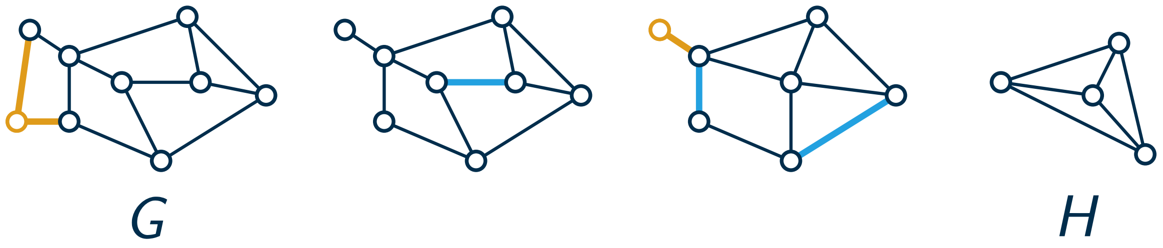

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Theorem: Vertex Cover parameterized by solution size k can be solved in $2^{O(\sqrt{k})} \cdot n^{O(1)}$ on planar graphs.

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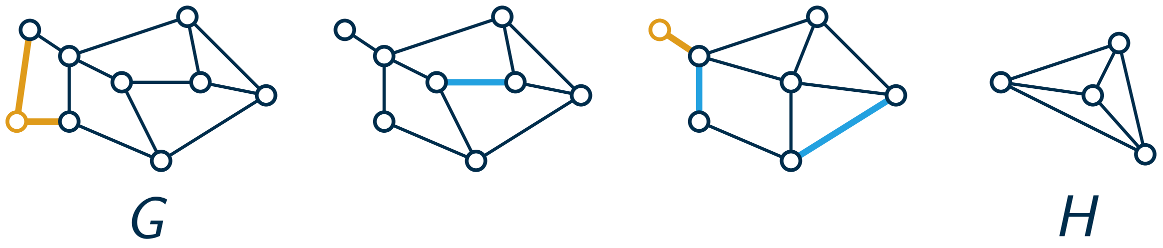

Some facts about treewidth

Can you also show this?

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