Parameterized Algorithms

Exercise 2 – Sheet 1, Sheet 2, Iterative Compression

Elly, Jean-Pierre, Wendy

Sheet 1 – General Stuff

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- anyone looking for a team?

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- always explain why your solution is correct

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Pseudocode

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Pseudocode

- super hard to read

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Sheet 1 - Dominating Set

a) Dominating Set on trees

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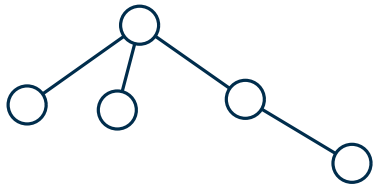

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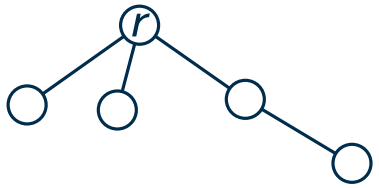

- root tree

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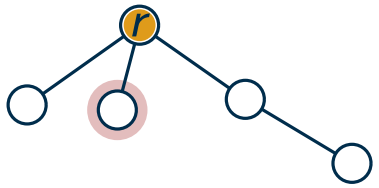

- root tree
- select parent of leaf

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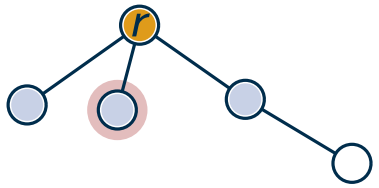

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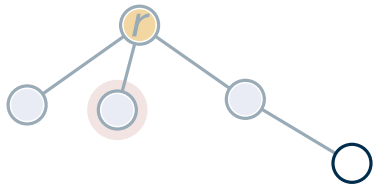

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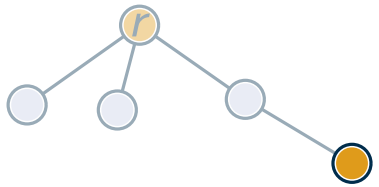

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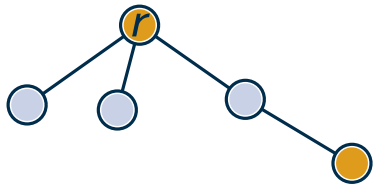

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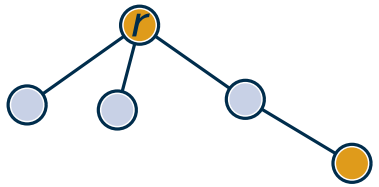

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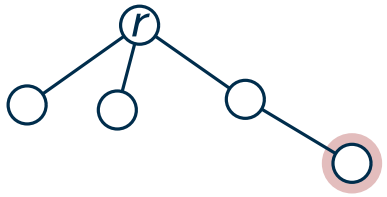

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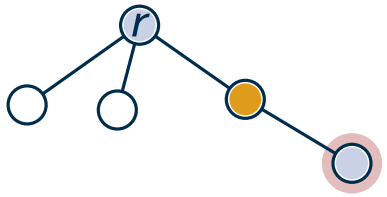

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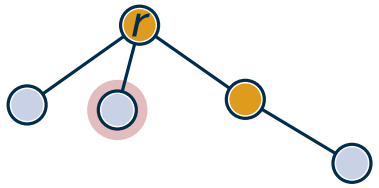

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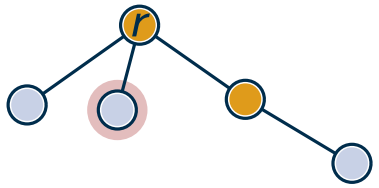

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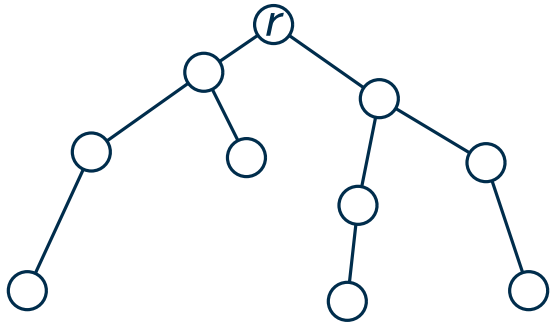

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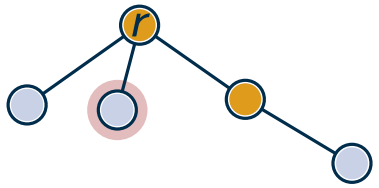

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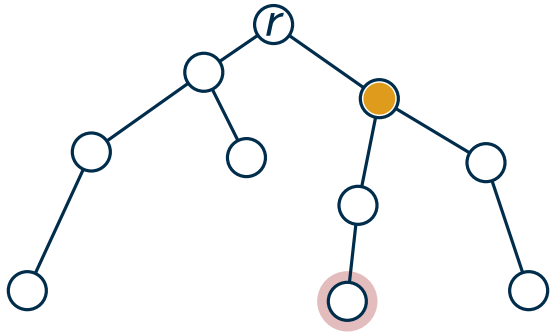

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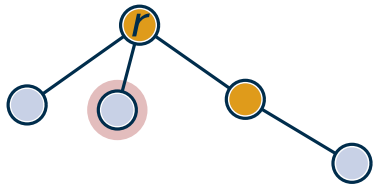

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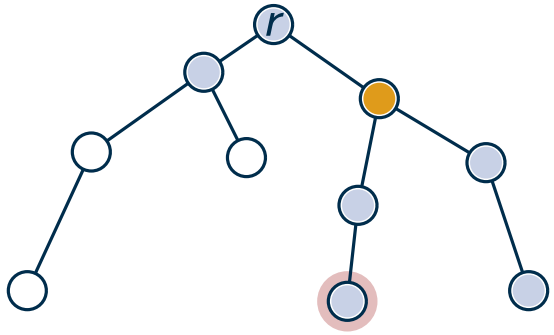

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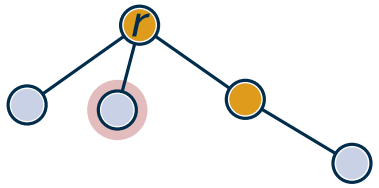

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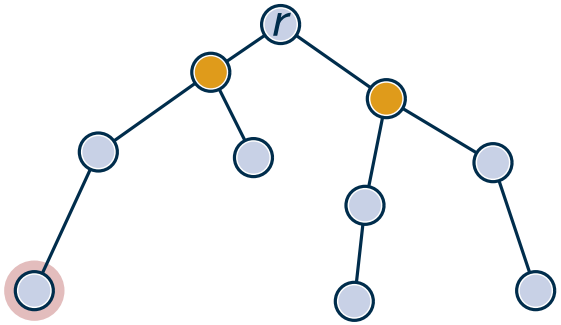

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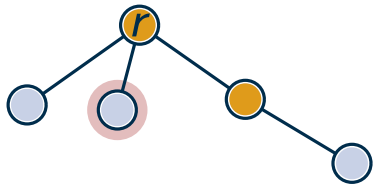

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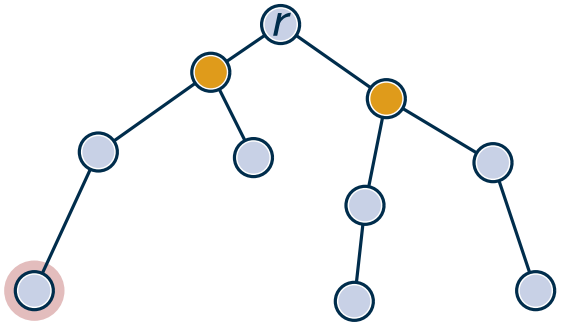

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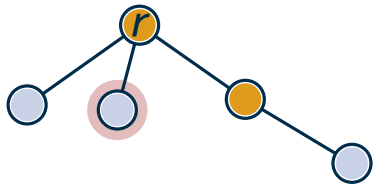

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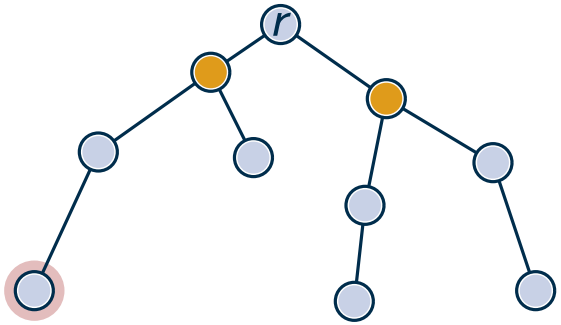

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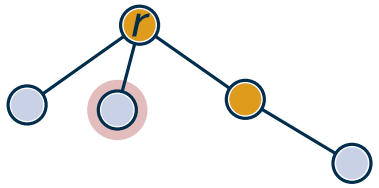

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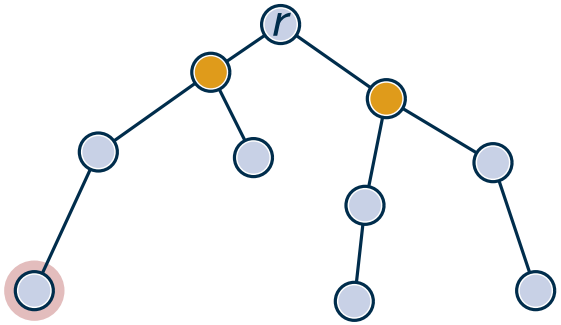

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c) Dominating Set on graphs with max deg $\leq d$

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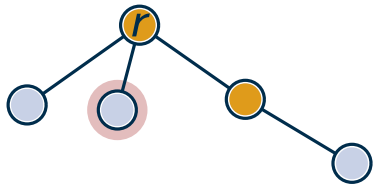

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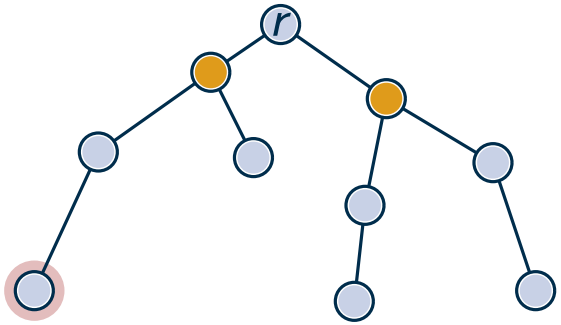

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- search tree: vertex or neighbor $\Rightarrow O((d+1)^k m)$

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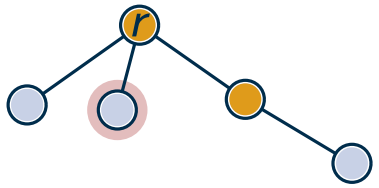

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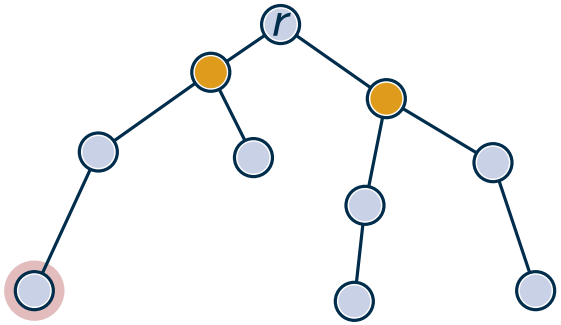

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Cover n points with k lines

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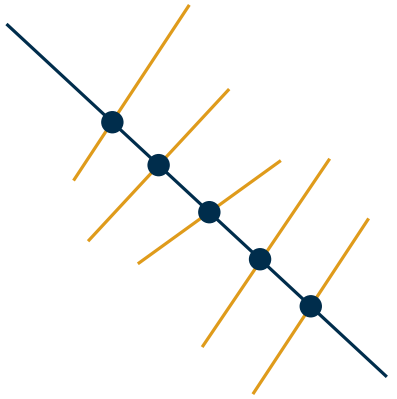
Kernelization

(R1) select line that covers $k + 1$ points



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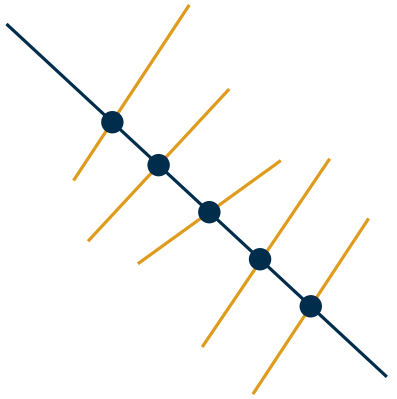

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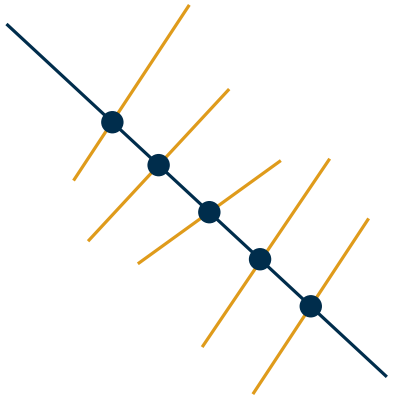

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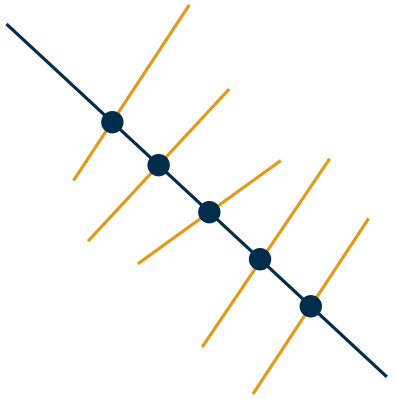

Search Tree

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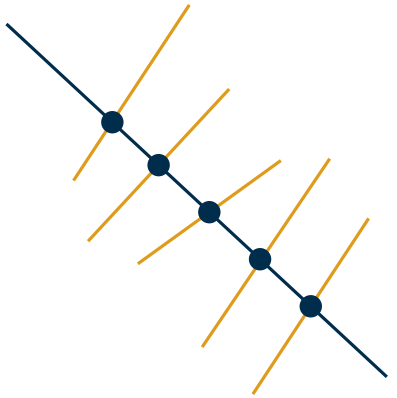

Search Tree

- for $k+1$ points: there is a line that goes through two points

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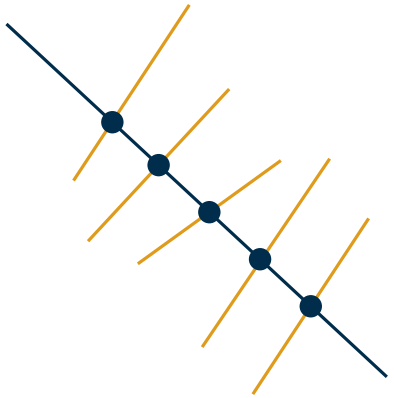

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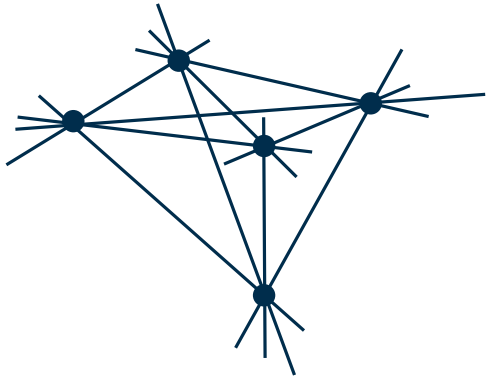

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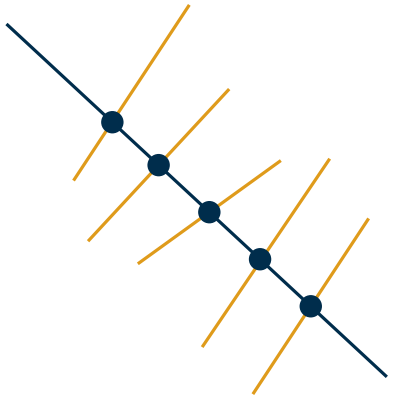

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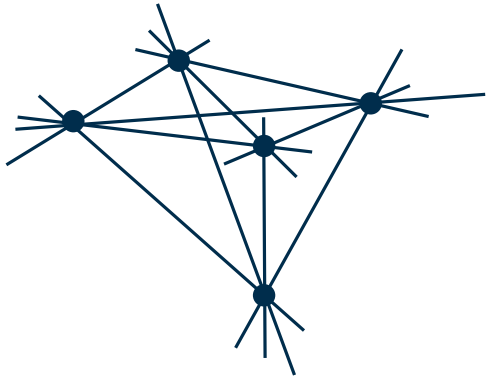
- degree $(k+1)^2$, depth k



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Weighted Vertex Cover

Tim Domnick, 2024/25

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- find WVC with weight $\leq k$
- goal: same running time as VC with size $\leq k$ in $O(1.342^k(n+m))$ time

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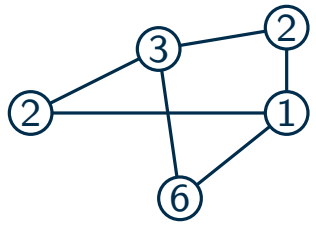

Tim Domnick, 2024/25

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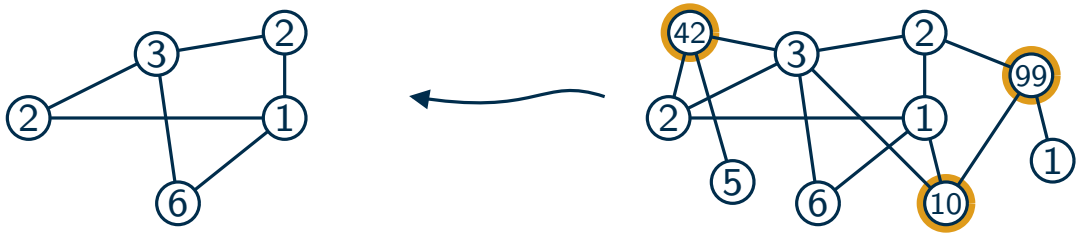
a) constant weights b) get rid of high weights



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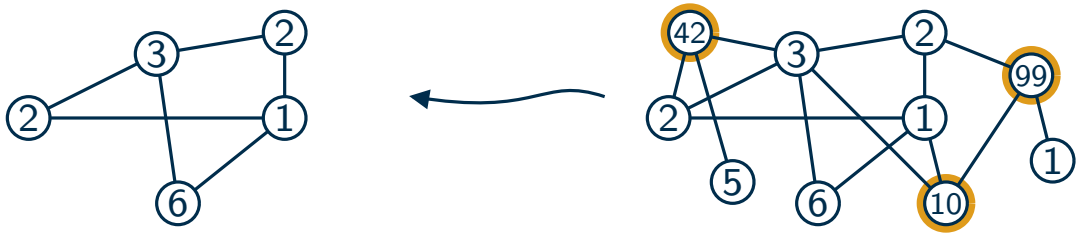

How to choose the constant?

Sheet 2

Tim Domnick, 2024/25

Weighted Vertex Cover

- find WVC with weight $\leq k$
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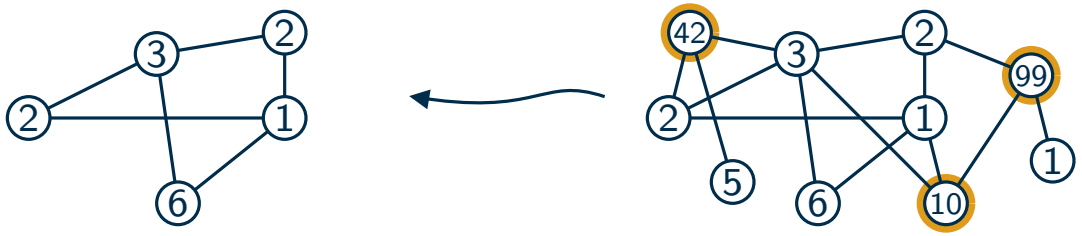

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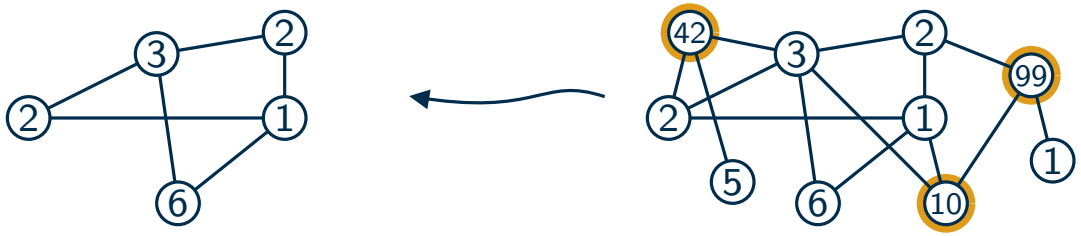

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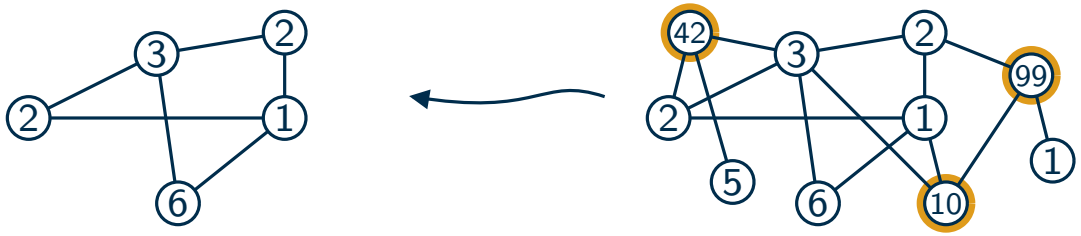

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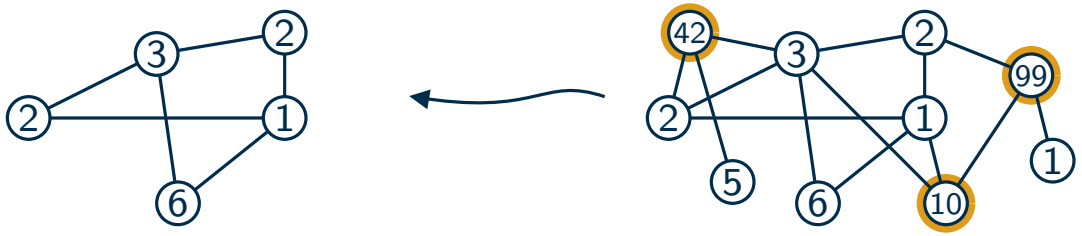

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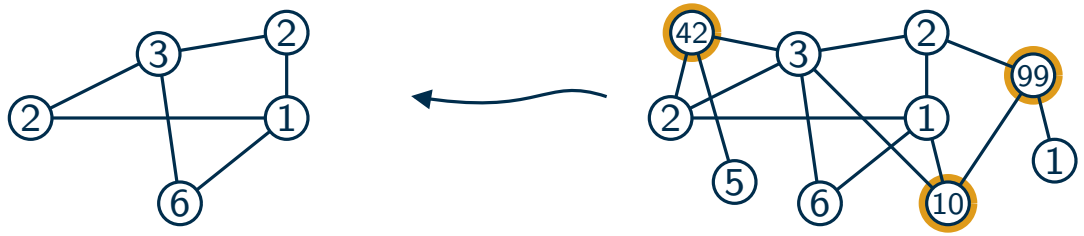

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3-Hitting Set

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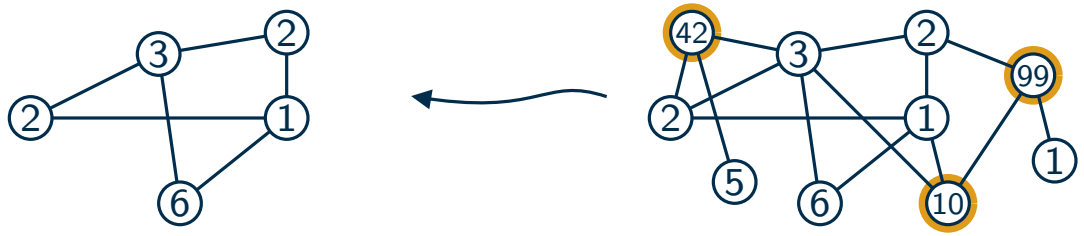

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- better search tree (naive: $3^k n^{O(1)}$)

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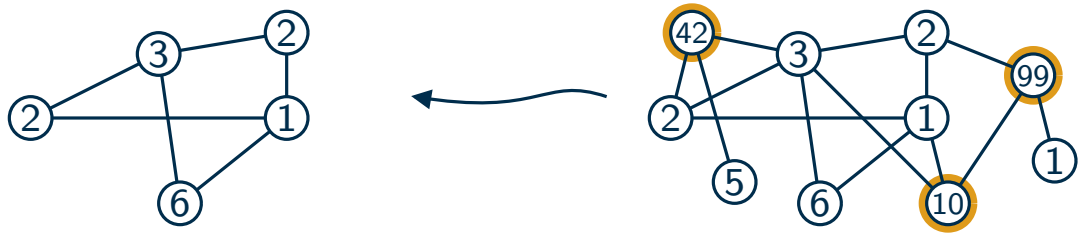

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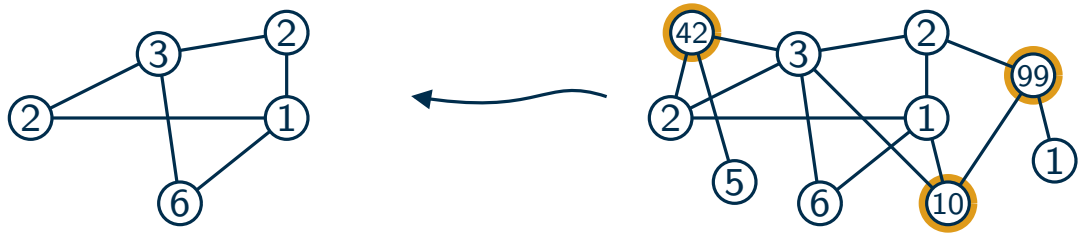

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- solve the Disjoint Problem
- how do we get there? is each step ok?

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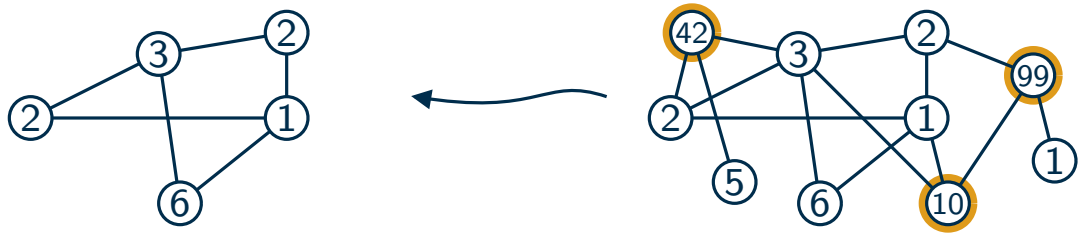

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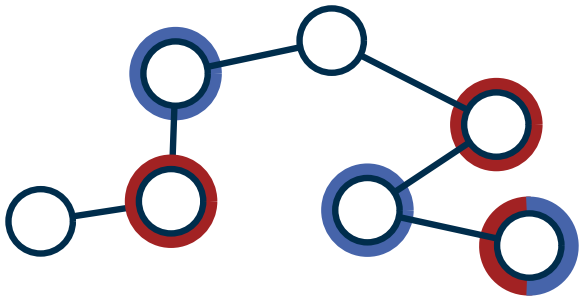

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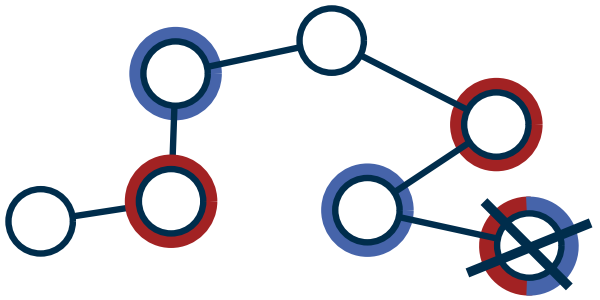

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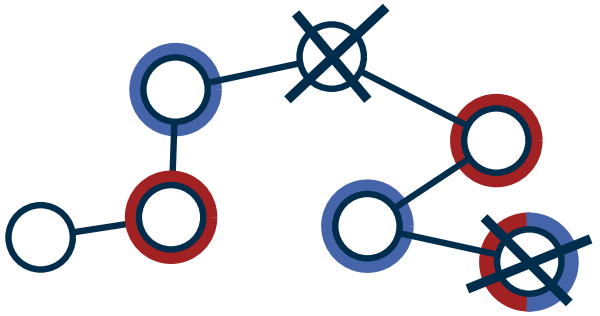

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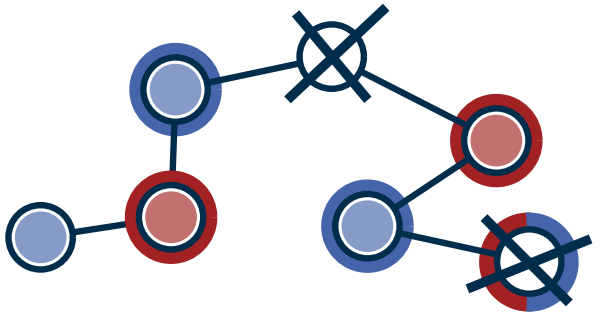

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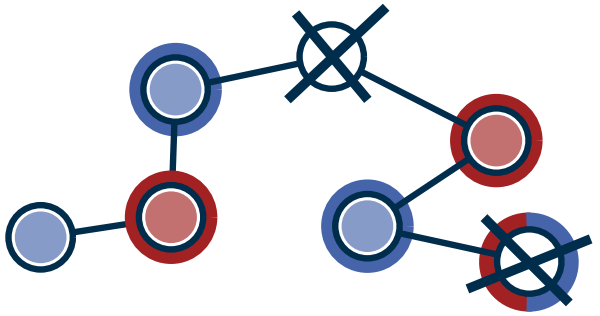

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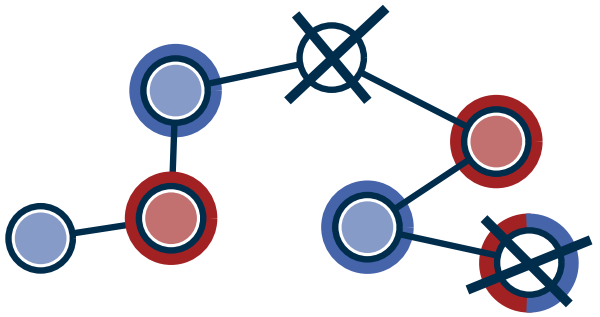

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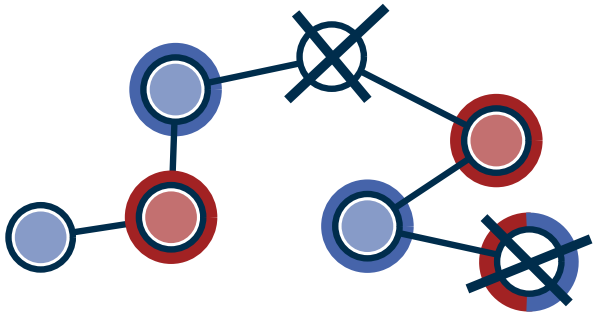

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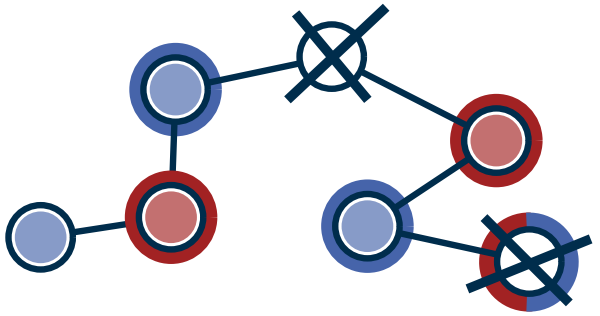

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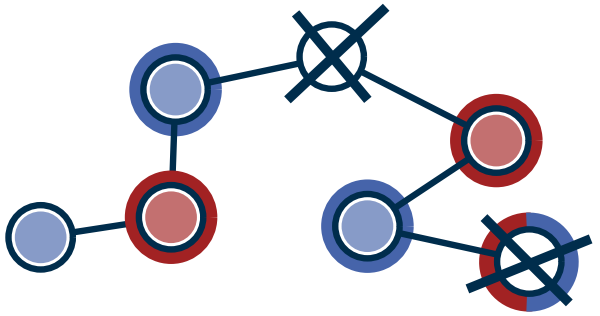

1. Fix a 2-coloring Φ of G (kind of as a reference), how do we get $\Phi \to f$?

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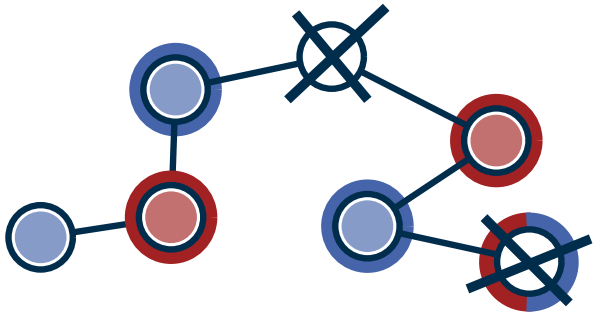

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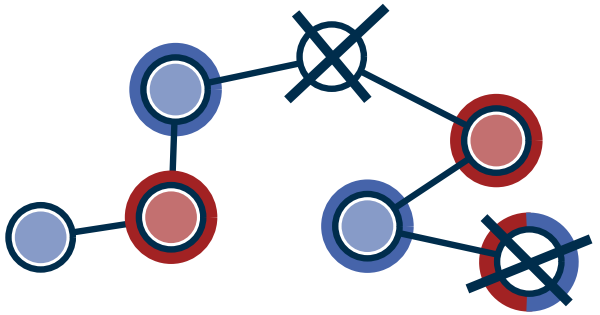

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3. Assume that a vertex separator between two vertex sets can be found in $O(k \cdot (n+m))$

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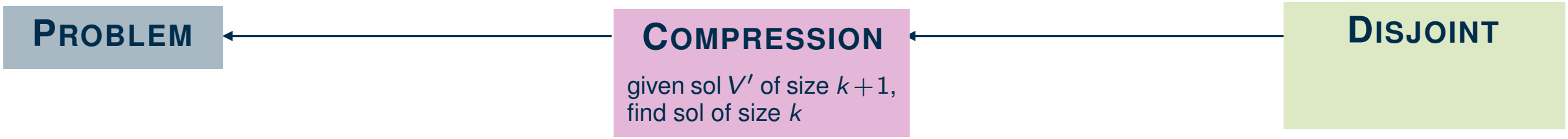

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Compression: given sol V' of size $k+1$, find sol of size k

- start with graph with k vertices $\Rightarrow$ solution

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- start with graph with k vertices $\Rightarrow$ solution
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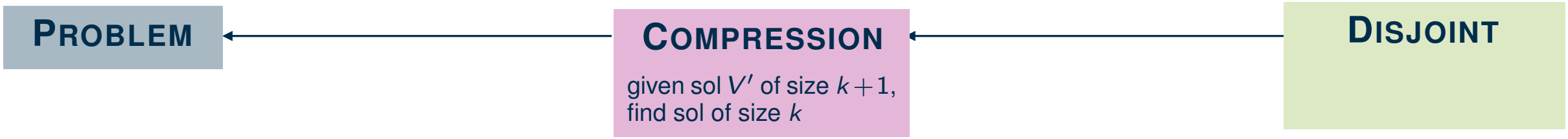

- start with graph with k vertices $\Rightarrow$ solution
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- solve Compression $\Rightarrow$ solution of size k

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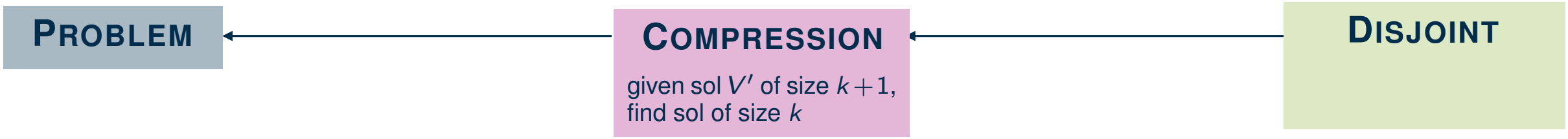

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- repeat

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Problem ← **Compression** ← **Disjoint**

Compression
given sol V' of size $k+1$, find sol of size k

Disjoint
given sol Y of size $k+1$, find disjoint sol of size k

- start with graph with k vertices $\Rightarrow$ solution
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Bipartite Vertex Deletion

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given: graph G, parameter k
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- solve Disjoint for $G - X$ and solution Y
- then, $Y' \cup X$ is solution for G of size k

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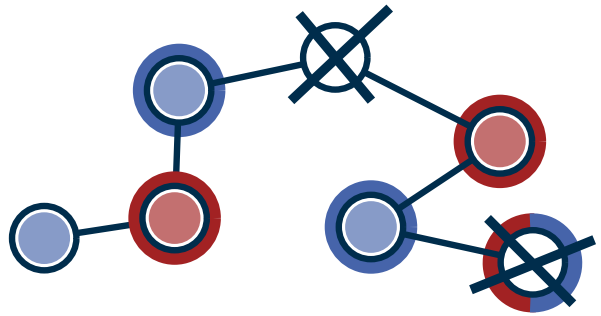

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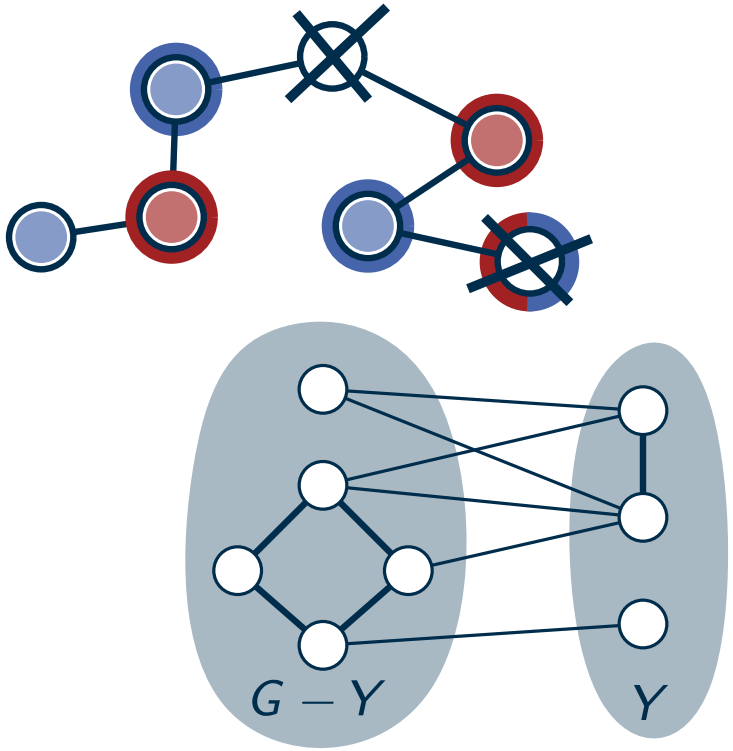

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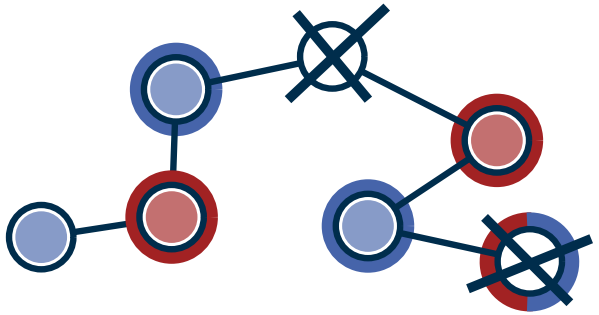

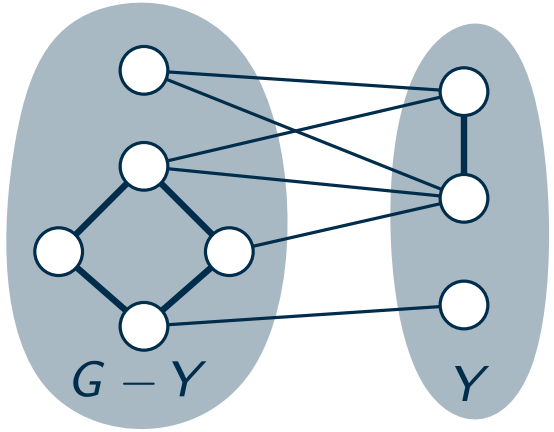

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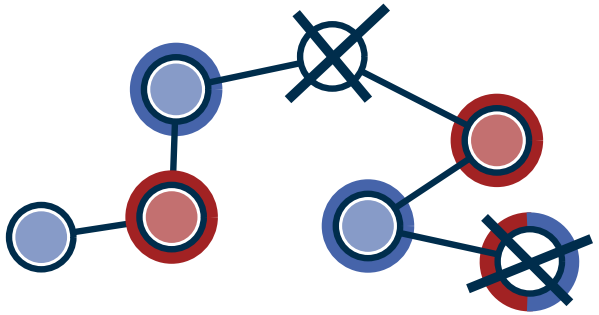

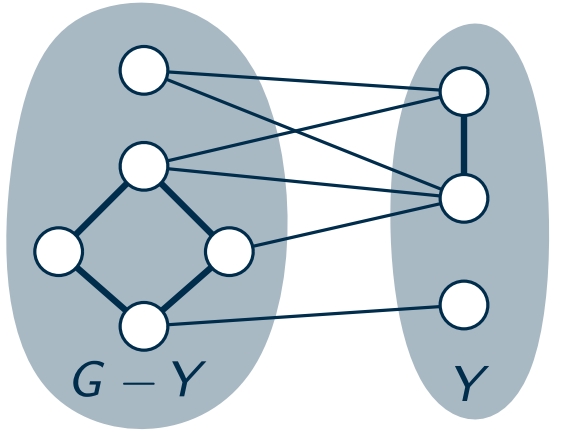

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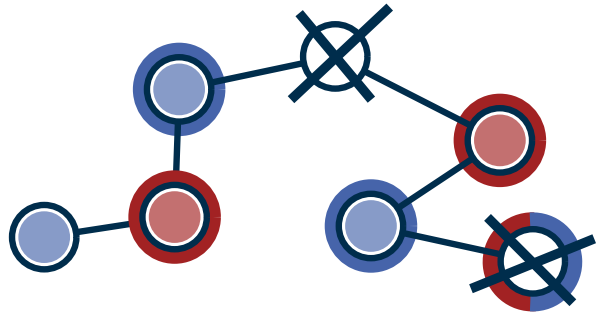

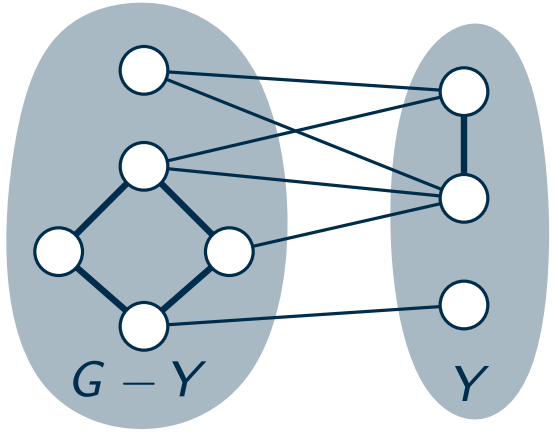

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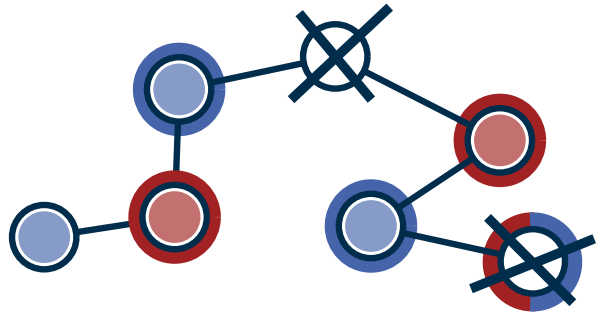

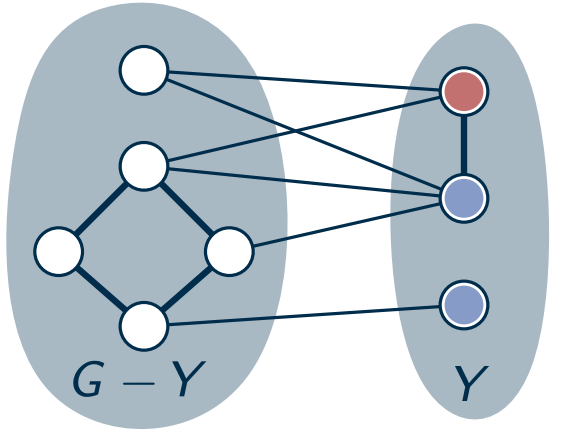

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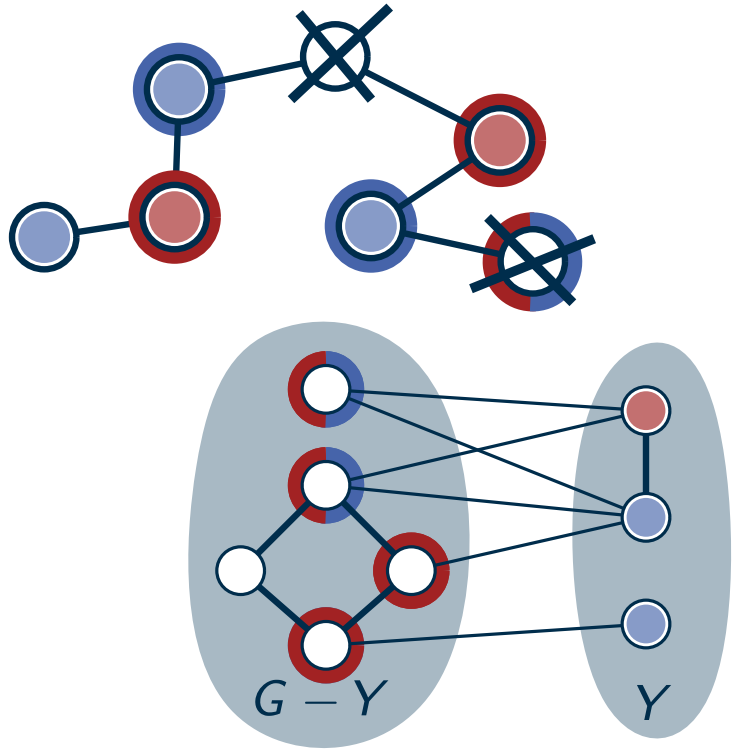

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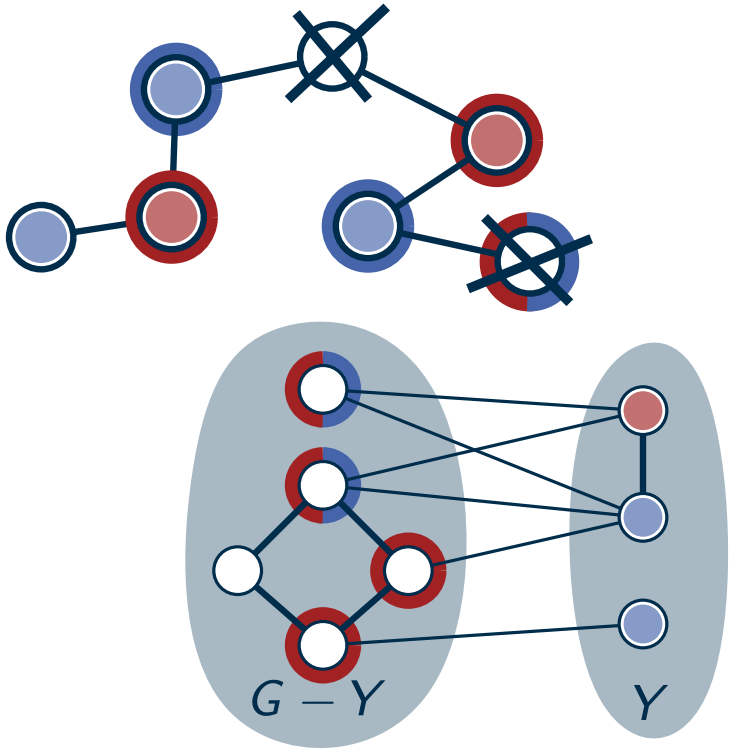

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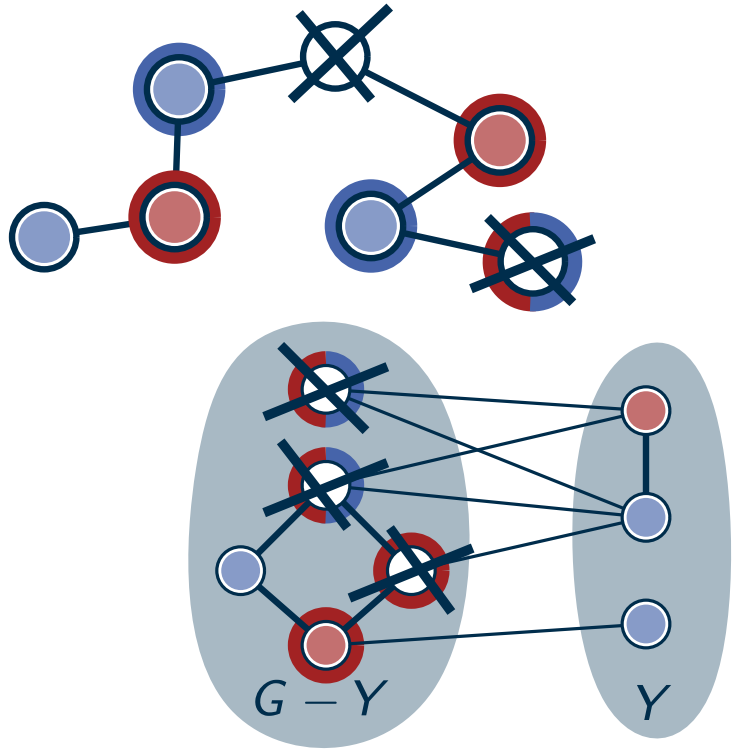

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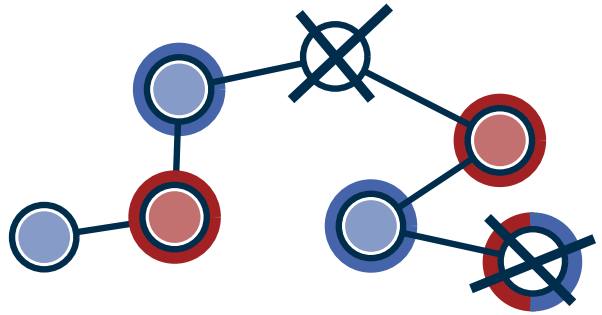

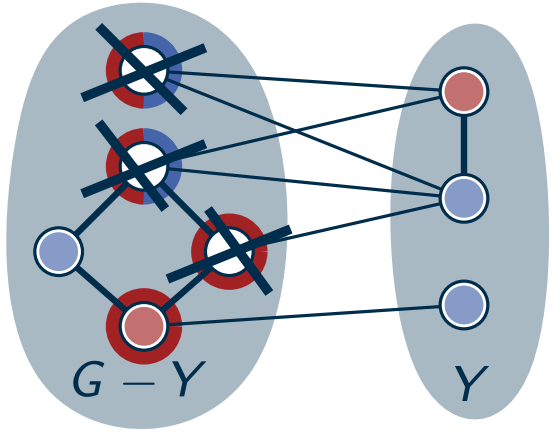

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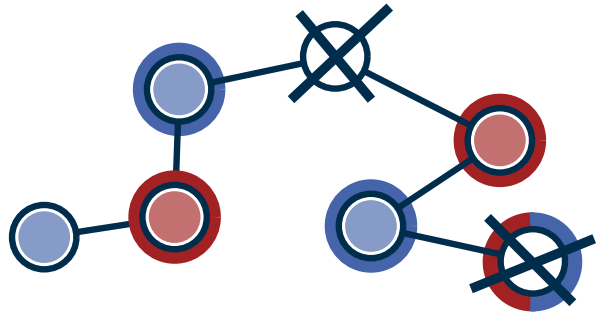

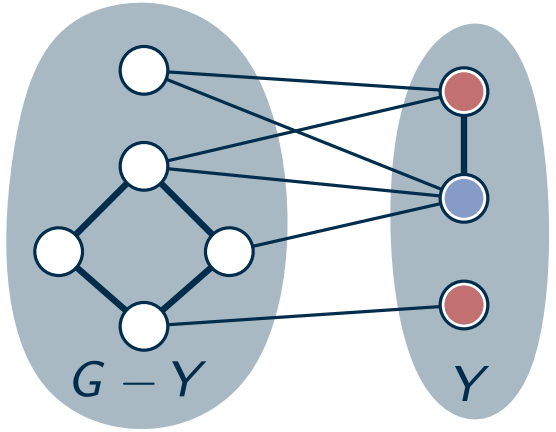

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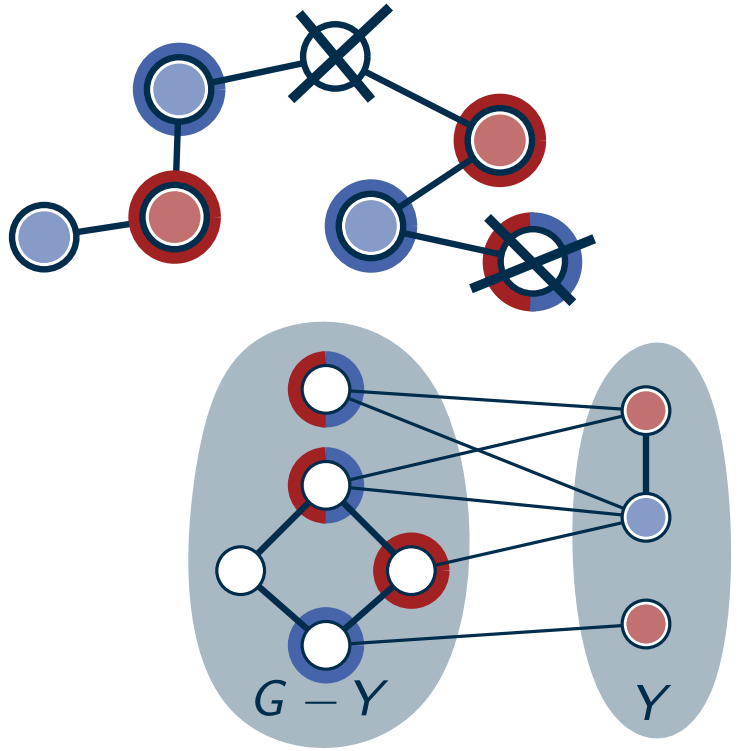

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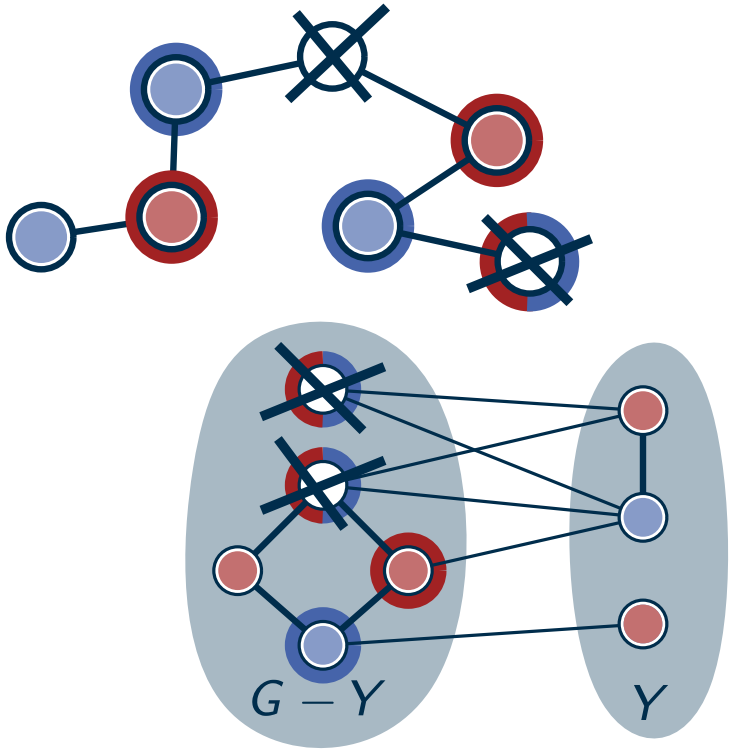

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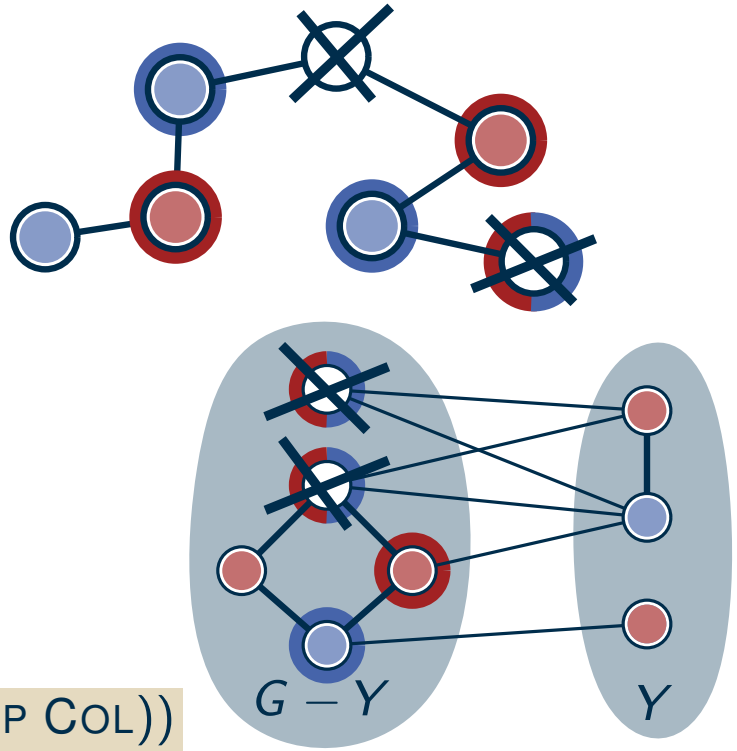

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Running time: $O(2^k \cdot \text{Time}(\text{Ann Bip Col}))$

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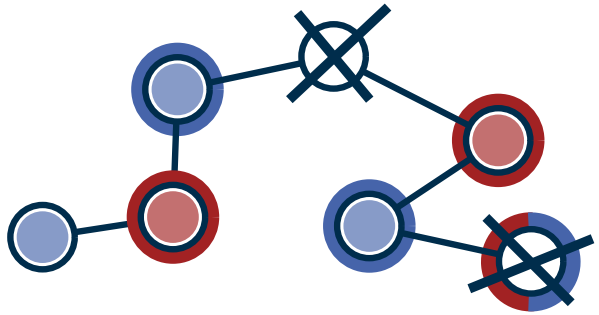

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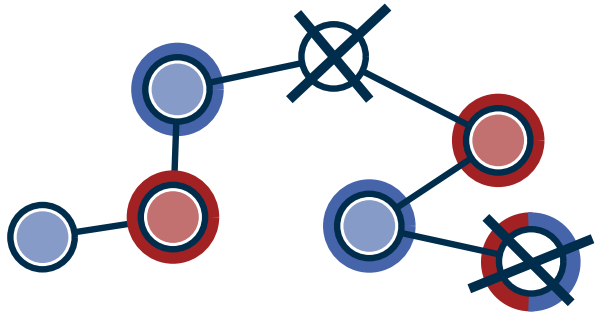

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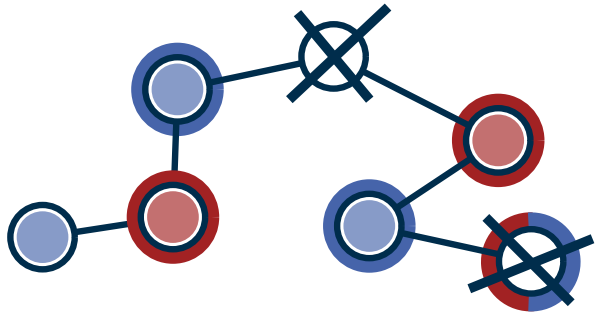

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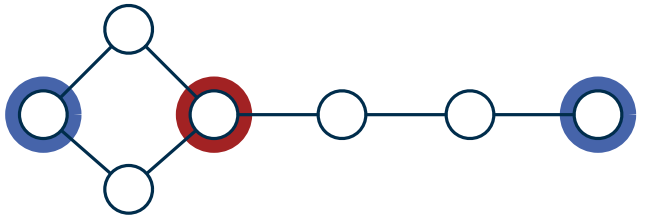

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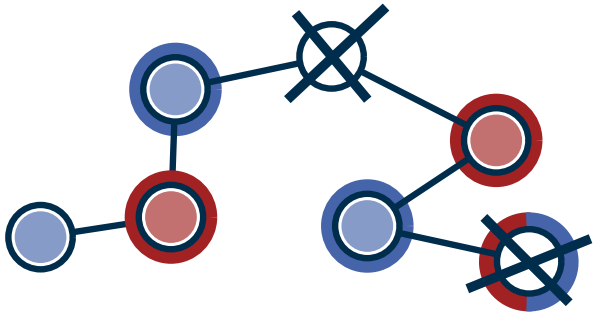

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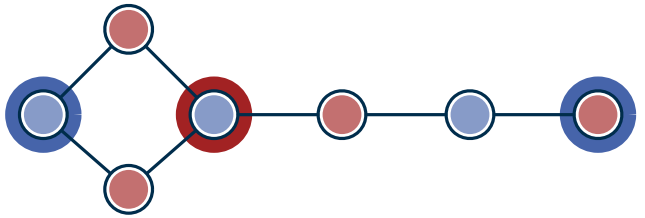

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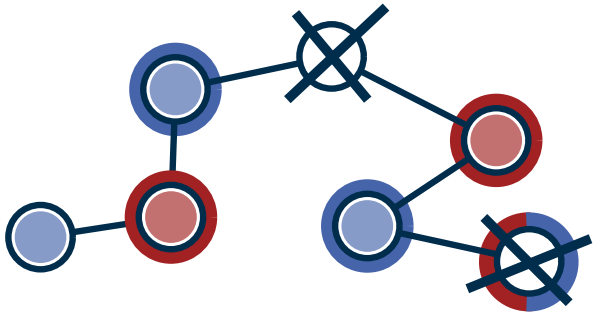

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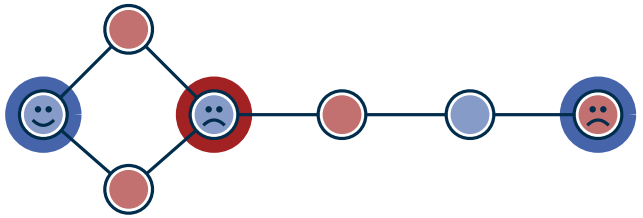

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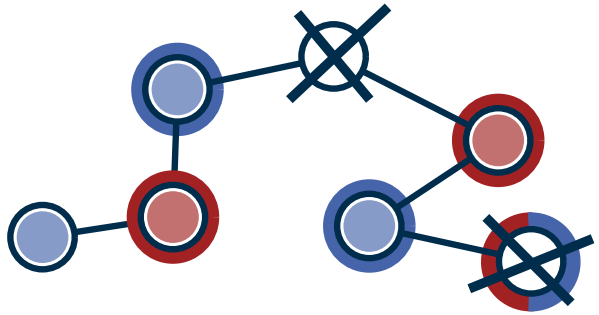

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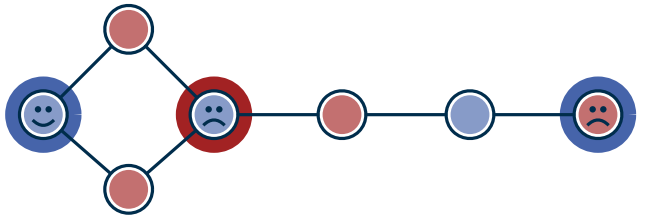

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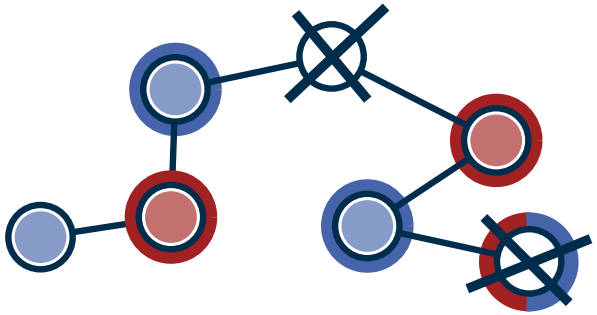

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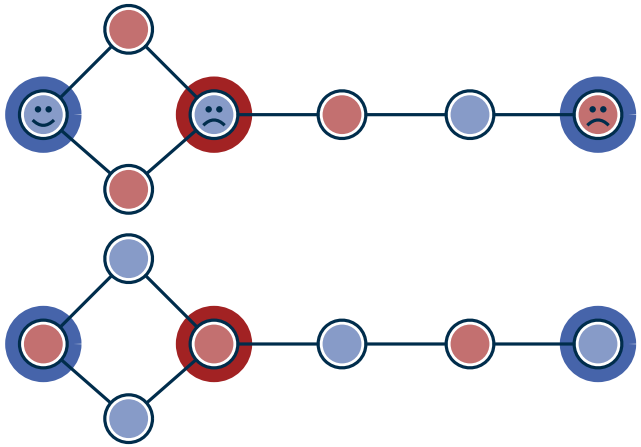

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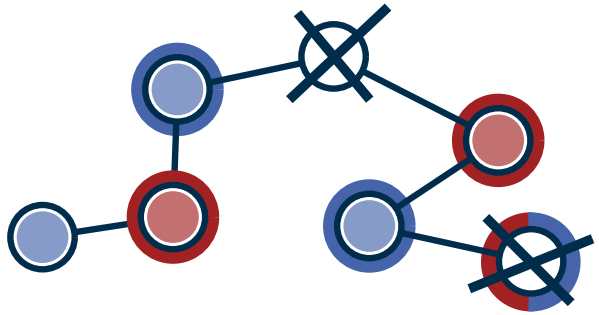

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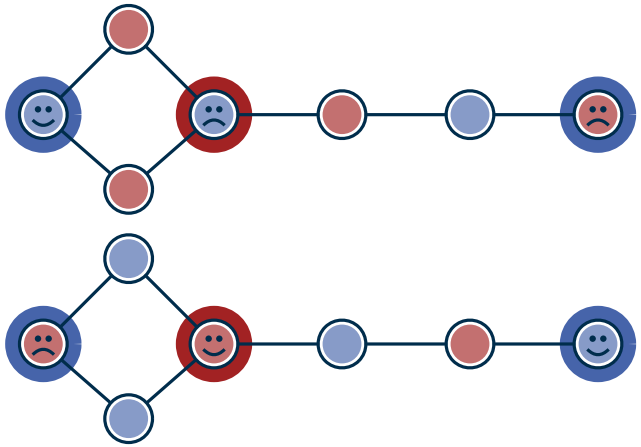

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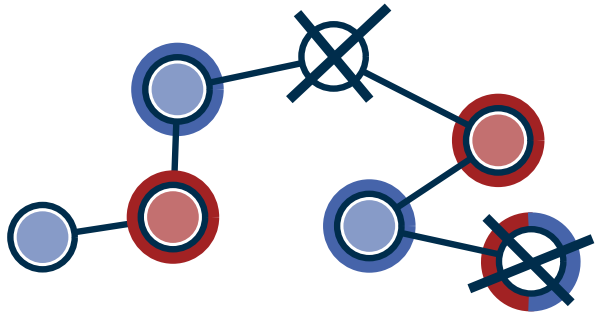

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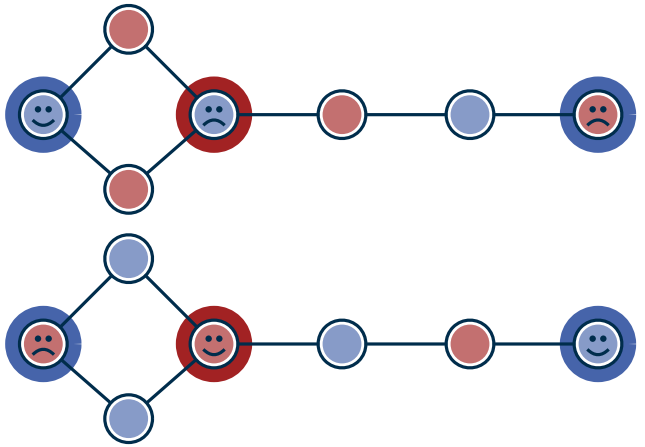

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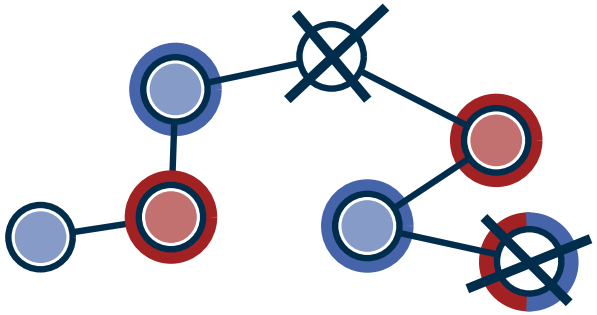

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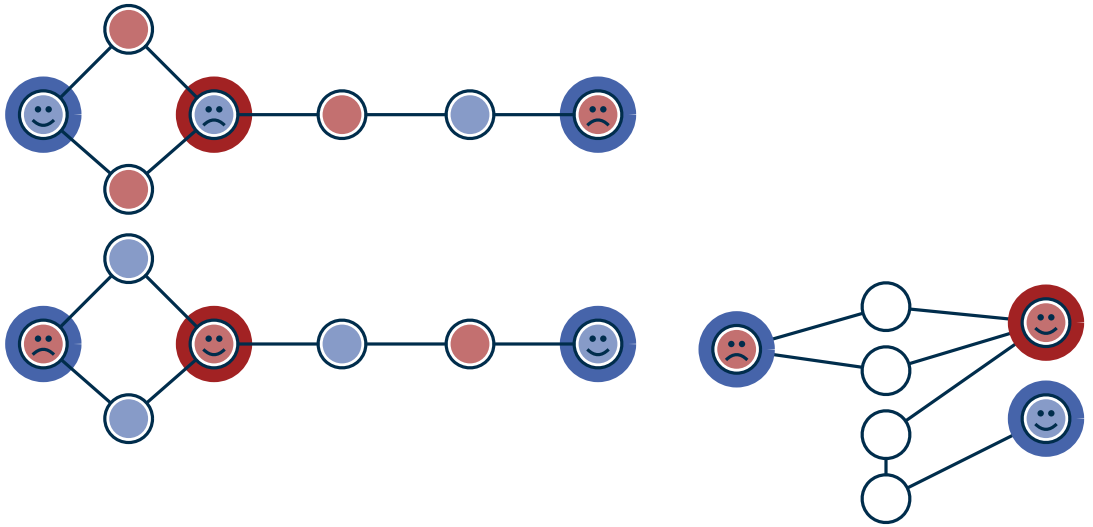

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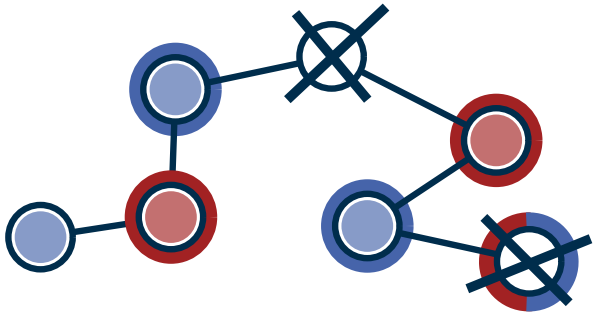

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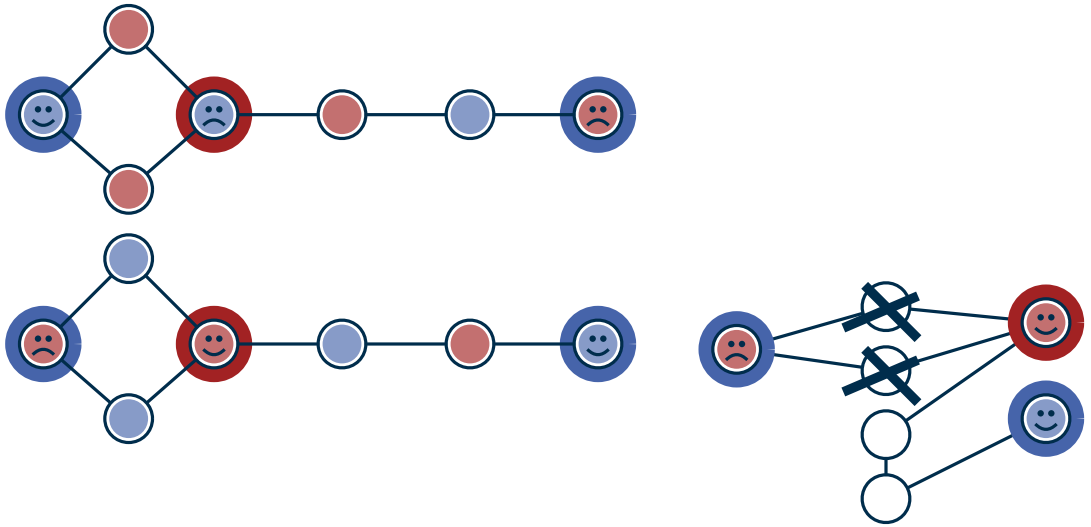

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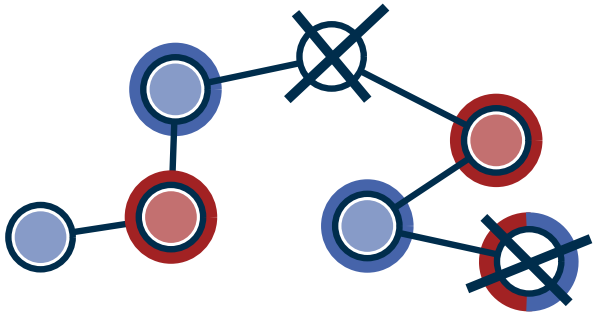

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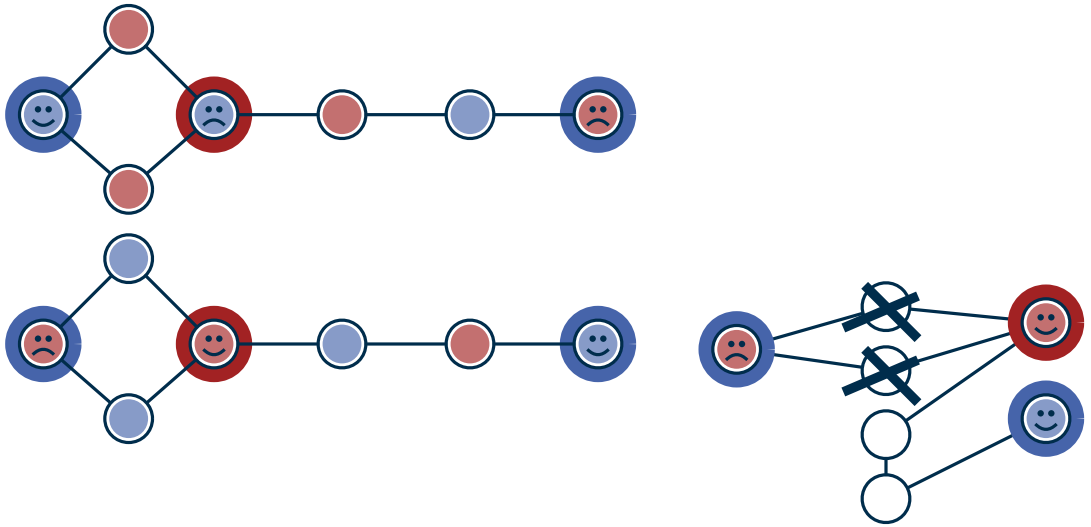

Running time: $O(k \cdot (n+m))$

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given: graph G, parameter k
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- How does it help to solve the problems?
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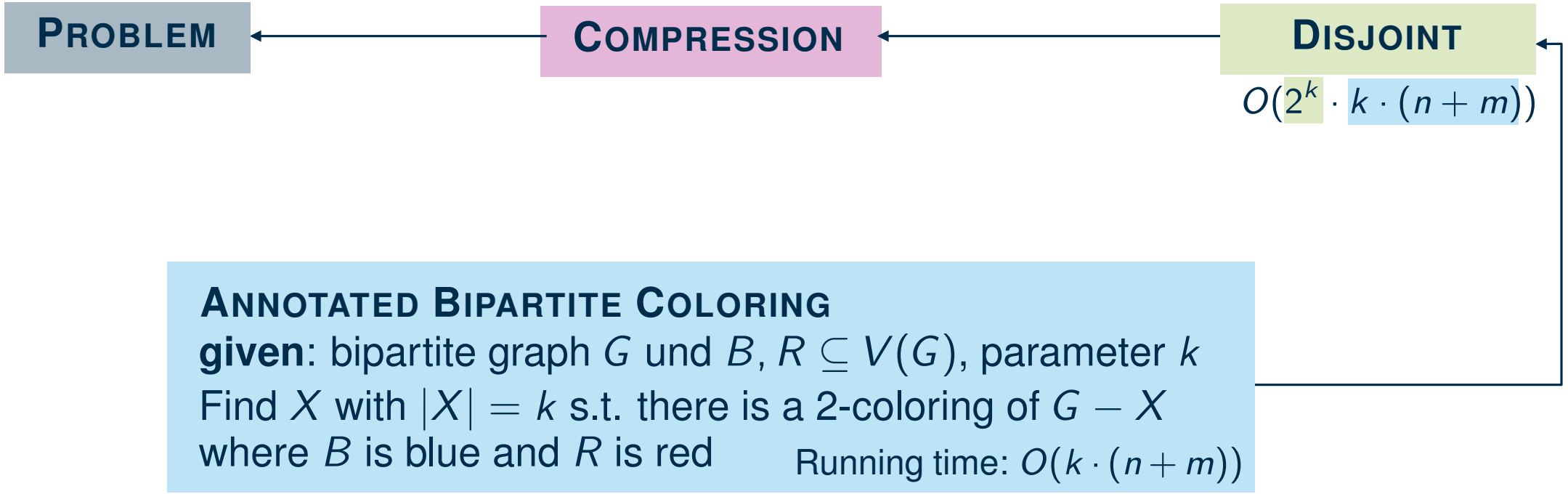

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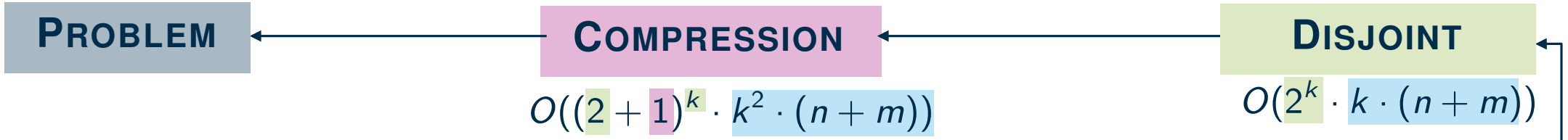

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Problem

$O((2+1)^k \cdot k^2 \cdot (n+m) \cdot n)$

Compression

$O((2+1)^k \cdot k^2 \cdot (n+m))$

Disjoint

$O(2^k \cdot k \cdot (n+m))$

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