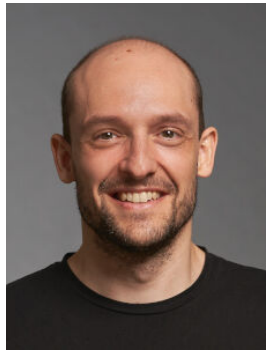
Parameterized Algorithms

Exercise 1 – Orga, Sheet 1, *H*-free Graphs

Elly, Jean-Pierre, Wendy

Orga Stuff



Thomas

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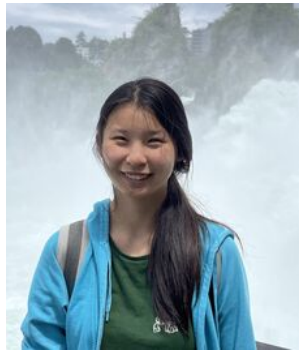
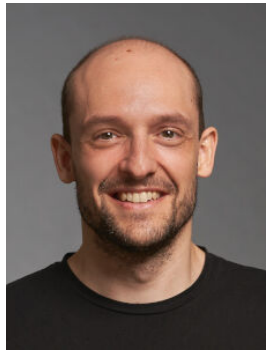


Wendy

Lecture

Exercise

Orga Stuff



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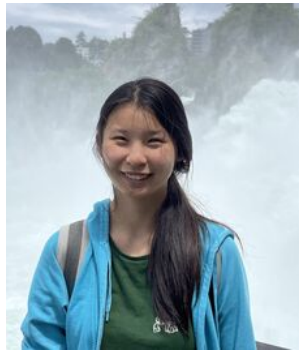

Wendy

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Exercise

?

You

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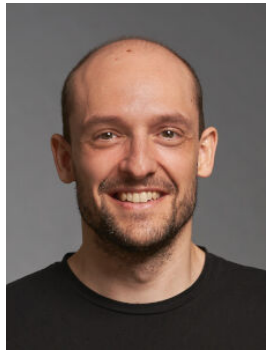

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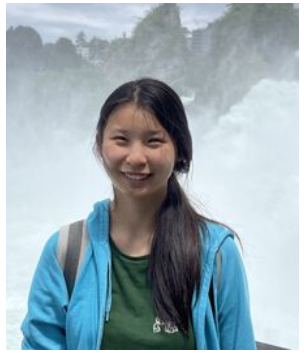

Wendy

You

Lecture

Exercise

Exercise Session

- discuss last exercise sheet
- get hints for next exercise sheet
- work on other problems, get different perspectives

More Orga Stuff

Exercise Sheets

- new sheet every two weeks (Thu – Wed)
- submission in teams via ILIAS (not handwritten, pls) unless your handwriting is really good
- coding submissions (bonus points)

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- at least $\frac{1}{2}$ of total points, $\frac{1}{4}$ of every sheet

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  - choose proper abstraction level

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Points System

0%: did not write anything

25%: had ideas, explained where you are stuck

50%: basic idea is correct (and explained)

75%: small/few details missing

100%: correct and well-written

More Orga Stuff

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Talk to each other, ask for hints on Discord, ask us in person!

send `!help join` to the scale bot

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Sheet 1

Branching Vector

builds on second lecture

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Dominating Set

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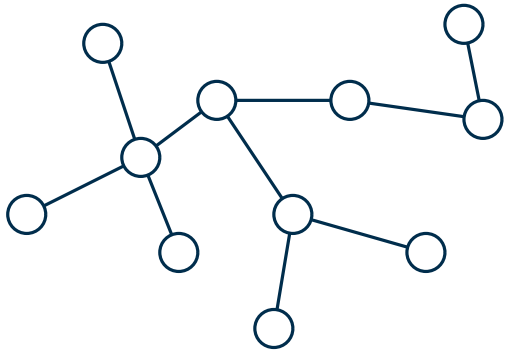
Dominating Set

a) on trees



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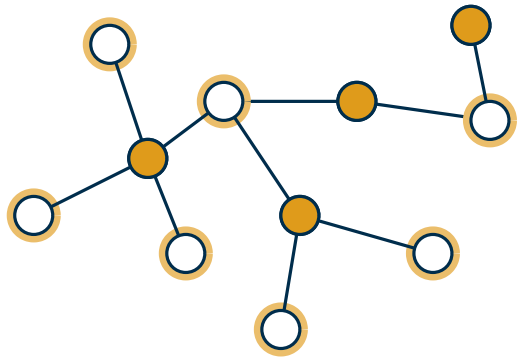

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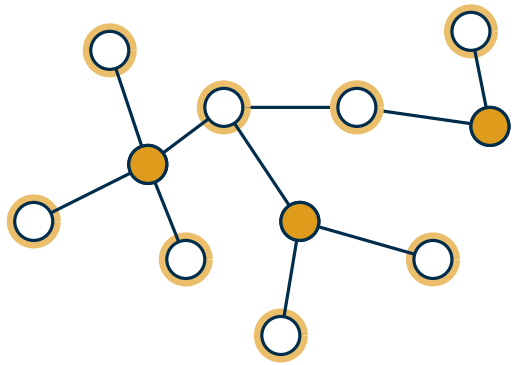

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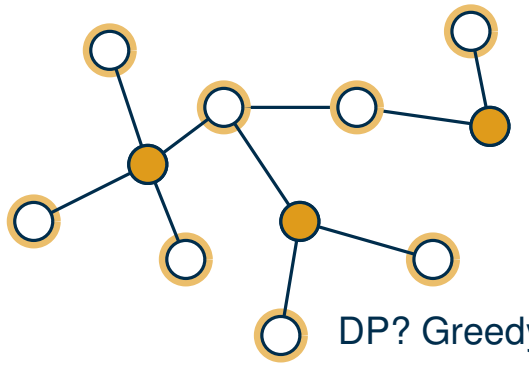

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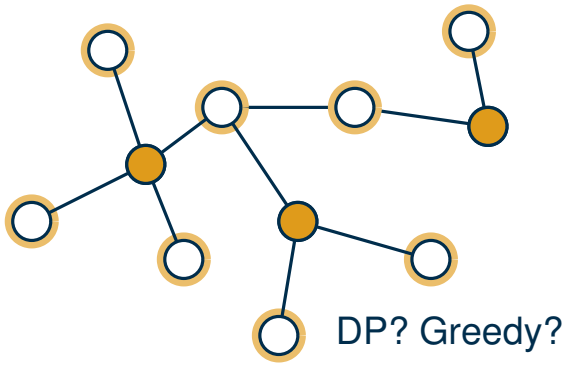
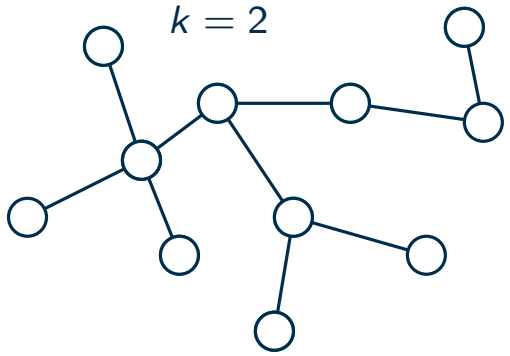


b) k-dominating set on trees



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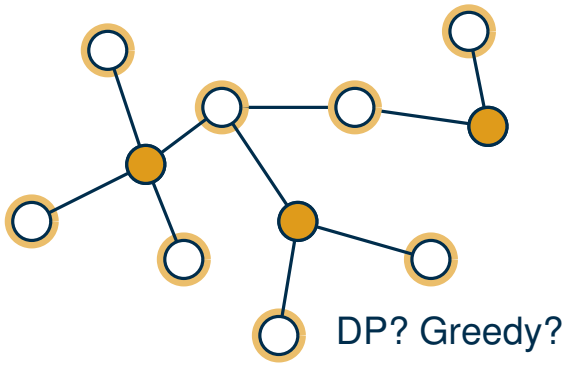

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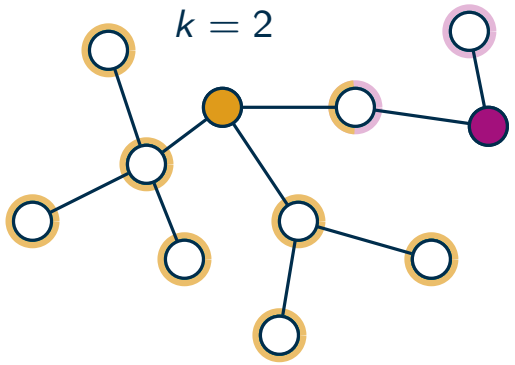

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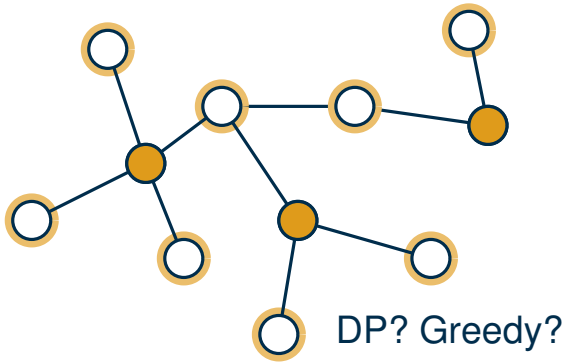

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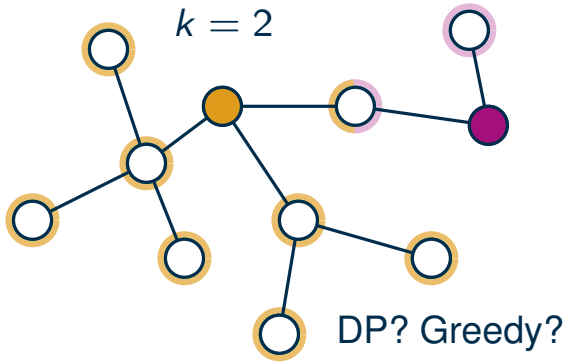

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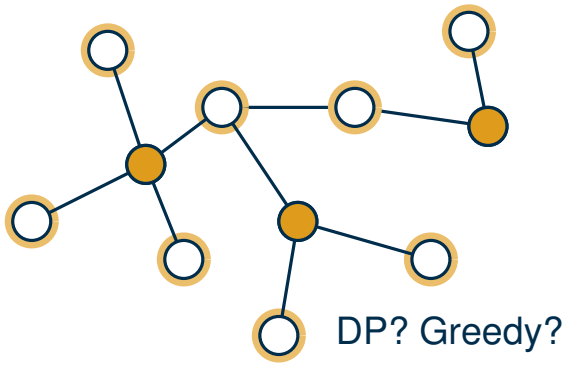

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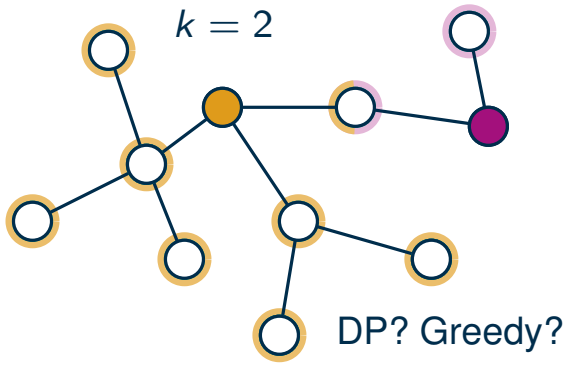

c) FPT in max deg + solution size on general graphs

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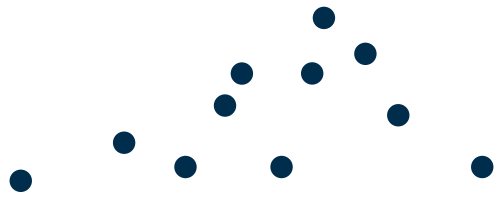

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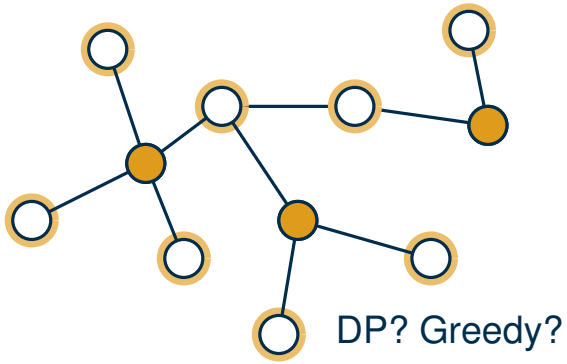

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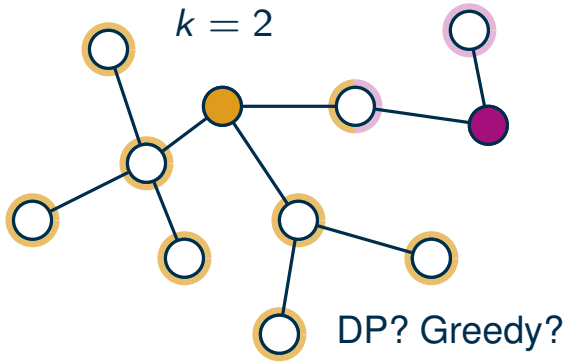

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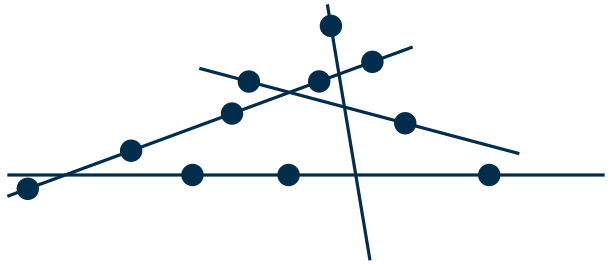

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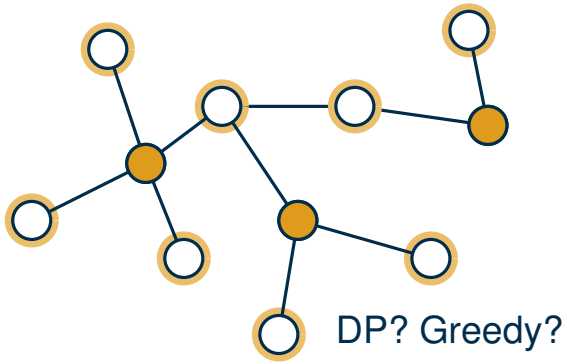

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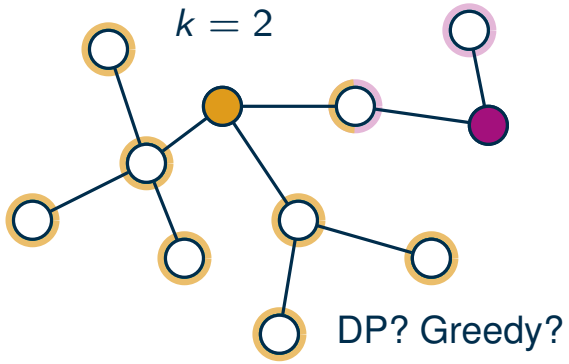

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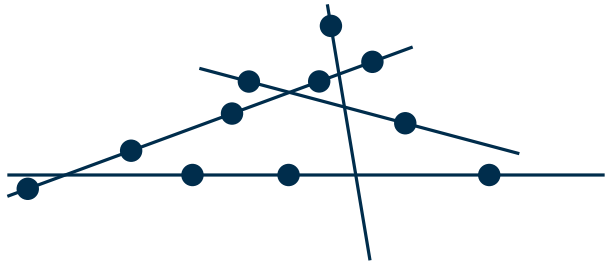

find reduction rules

Dominating Set

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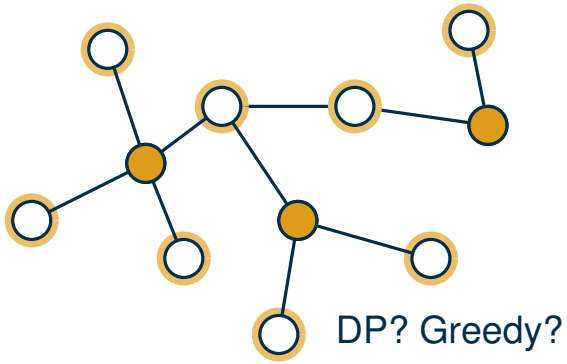
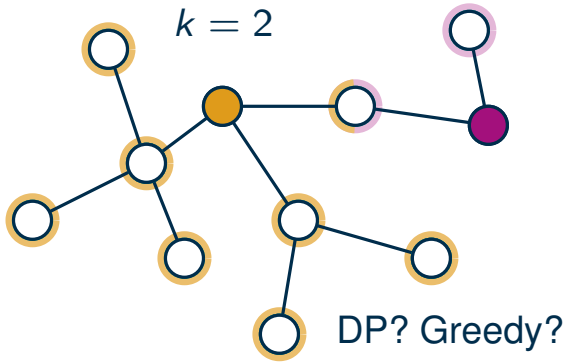


b) k-dominating set on trees



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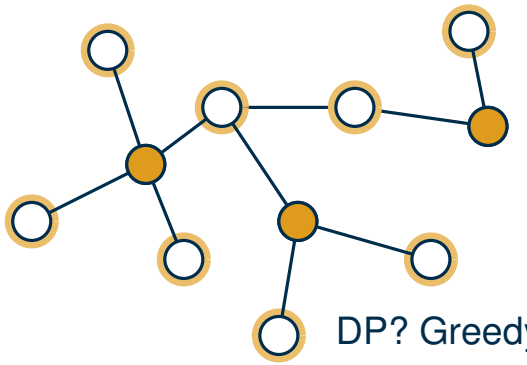

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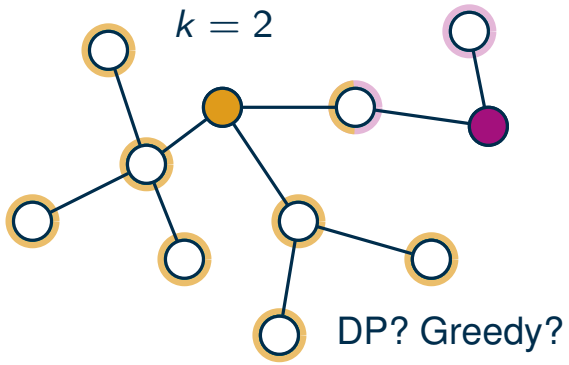

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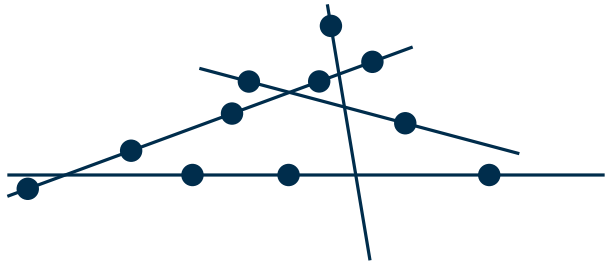

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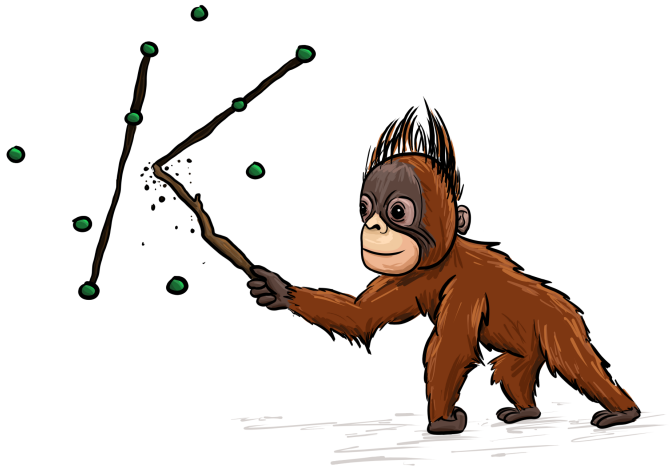
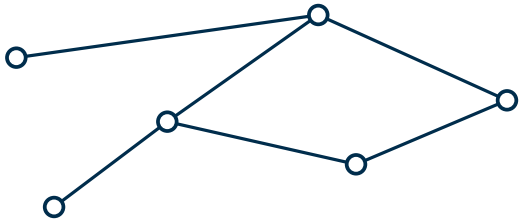


Tim Domnick, 2024/25

Techniques

Bounded Search Tree

Vertex Cover



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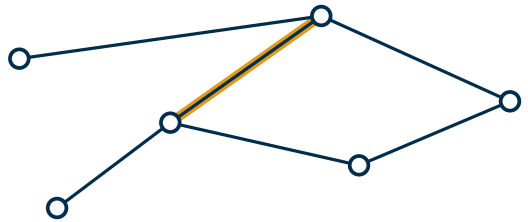

- cover an edge $\rightarrow$ two options to choose

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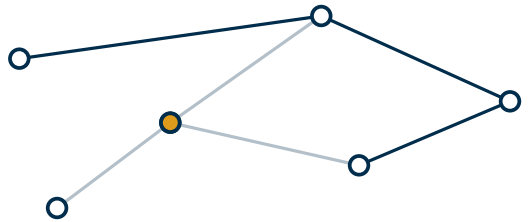

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- choice makes progress (k decreases)

Techniques

Bounded Search Tree

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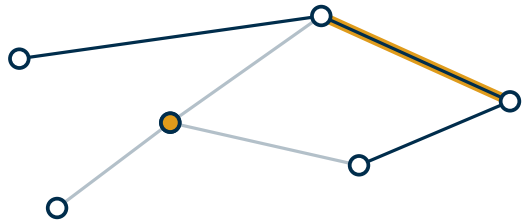

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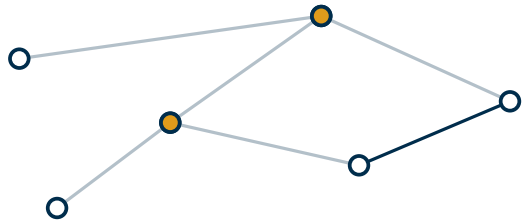

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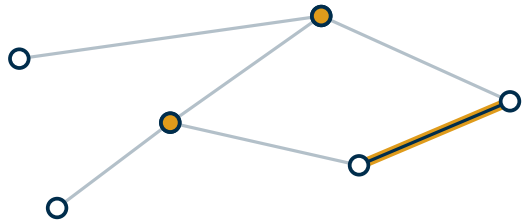

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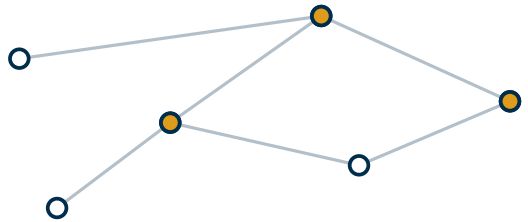

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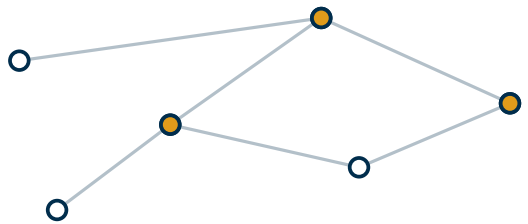
Techniques

Bounded Search Tree

Vertex Cover



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Techniques

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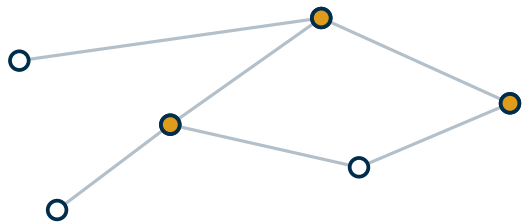

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Techniques

Bounded Search Tree

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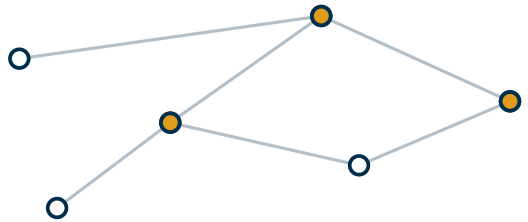

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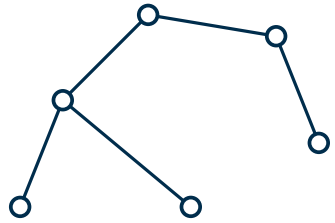
Kernelization

Vertex Cover on trees



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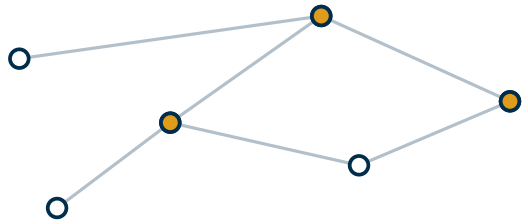

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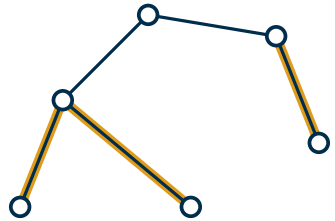
Kernelization

Vertex Cover on trees

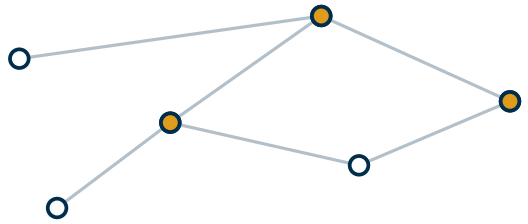


- edges to leaves need to be covered

Techniques

Bounded Search Tree

Vertex Cover



- cover an edge $\rightarrow$ two options to choose
- choice makes progress (k decreases)
- $\text{options}^{\text{depth}} \cdot \text{poly}(n) = 2^k \cdot \text{poly}(n)$ time
- vertex cover in $O(1.6181^k \cdot \text{poly}(n))$ possible

To prove

- in search tree:
  a child yes-instance $\Leftrightarrow$ parent yes-instance

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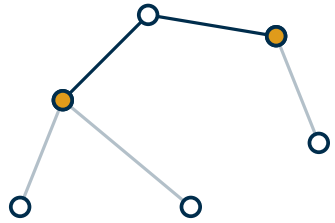

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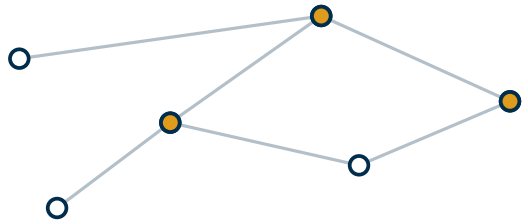

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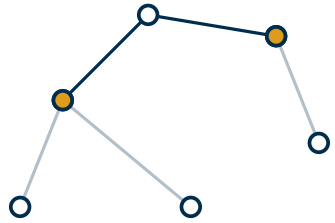

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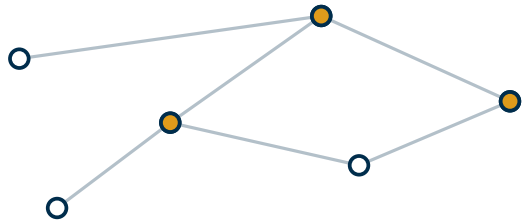

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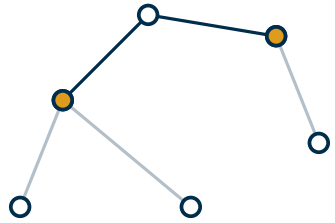

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To prove

- reduction rules correct
- kernel is small

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finite set of graphs → $\mathcal{F}$-FREE VERTEX DELETION

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R1: vertex with super high degree?
R2: delete edge between adjacent vertices with small degrees

How many bad vertices are still ok?
How many vertices with small degree?

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