

Exercise Sheet 2: Network Properties

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In the previous exercise sheet, we observed that the bi-BFS and vertex cover algorithms behave quite differently on some of the networks. Now, we want to examine more closely how the networks differ. Can we quantify these differences? Can we find meaningful properties that allow us to predict or explain running time and core size?

Exercise 1: Degree Distribution

The *degree* of a vertex refers to the number of its neighbors. One way to analyze the networks, is to study the degrees of vertices in more detail. For this purpose, vertex degrees can be regarded as a random variable whose distribution can be examined.

Visualize the degree distribution of individual networks. How do they differ? Try to categorize them into different groups. How can the degree distributions be plotted in a helpful way? If necessary, search online to find good ways to visualize degree distributions.

How can the different groups of degree distributions be characterized? Is it possible to find metrics or parameters that can distinguish between the groups? Can the performance of the algorithms be explained based on these metrics?

Exercise 2: Locality

Another important property describes how much non-random structure the networks possess. For example, road networks have an inherent geometry. This geometry leads, among other things, to the fact that the endpoints of a road are often connected by many short paths, whereas two random vertices tend to have a larger graph-theoretical distance. Similarly, in social networks, the phenomenon occurs that two persons u, v are much more likely to be friends, i.e., connected if they share a common friend. Neither of these properties holds in uniformly random networks¹. In the literature, this property is often referred to as *underlying geometry* or *locality*. Closely related is the concept of *clustering*.

How can locality be measured? Can you come up with your own parameters? Can you find suitable measures online or in the literature? Compare different measures of locality and determine whether the performance of the algorithms can be predicted or explained using locality.

¹See the Erdős-Rényi model, see Wikipedia.

Exercise 3: More Parameters

Select at least two additional graph parameters and examine how well the behavior of Bi-BFS and vertex cover can be predicted based on these parameters. Is there a strong correlation between your graph parameters and the parameters of the previous exercises, or are they independent of each other? Which parameters are most suitable for distinguishing between the networks and predicting the running time of algorithms?