Beating the Worst Case

Practical Course – 6th Meeting

Jean-Pierre, Marcus

Sheet 3

Generate Graphs

Did you evaluate the algorithms/metrics on the new graphs?

What models are you using to generate graphs?

Did you find out, how we generated the graphs?

Real World Graphs

Do the previous results apply to the new graphs?

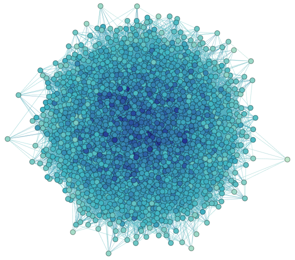
What sources for real world graphs are you using?

Was the GIRG library usable?
Did you understand what the parameters do?

Models for Complex Networks

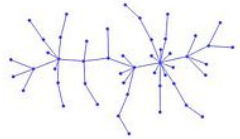
Three characteristics:	ER 1959	Pref. Attach. / Barabási-Albert 1923 / 1999	Chung-Lu 2002	Watts-Strogatz model 1998	GRG	HRG 2010	GIRG 2019
heterogeneous degrees		✓	✓			✓	✓
short distances / „small-world“	✓	✓	✓	✓		✓	✓
high locality / clustering				✓	✓	✓	✓

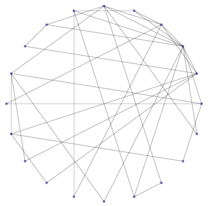
Erdős–Rényi model



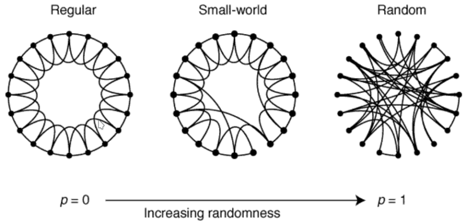
Preferential Attachment

iteratively add vertices, choose edges with probability proportional to current degree





Watts–Strogatz model



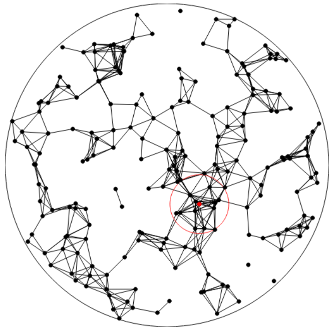
Chung-Lu / Configuration model / IRG

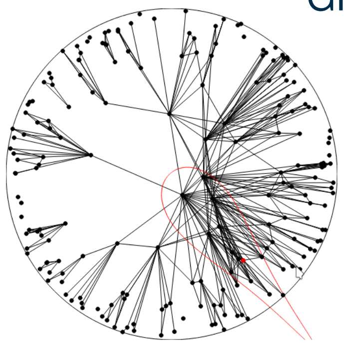
vertices with weights w_i (following power-law distribution);

$\Pr\left[\{e_i, e_j\} \in E\right] \sim \frac{w_i \cdot w_j}{W}$

Geometric Random Graph (Hyperbolic)

sample vertices uniformly in geometry, connect if distance below threshold

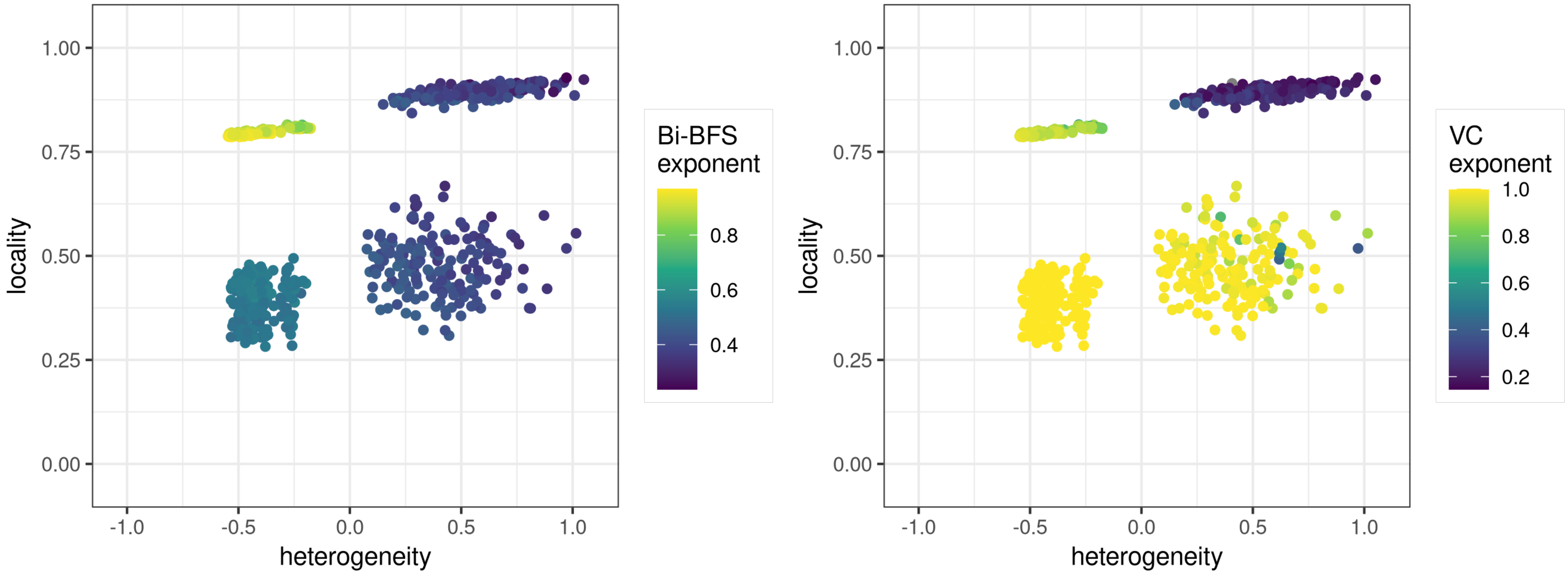




GIRG

GRG + IRG

We generated Some Graphs



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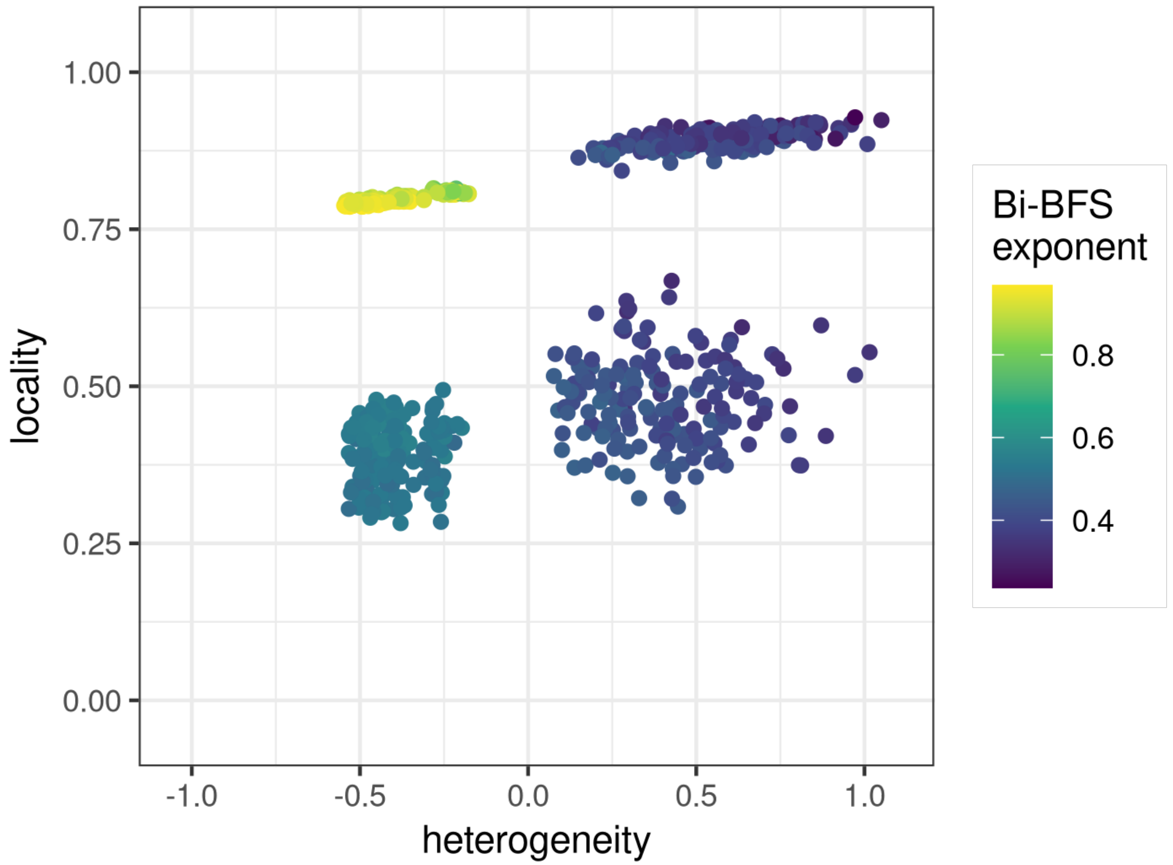
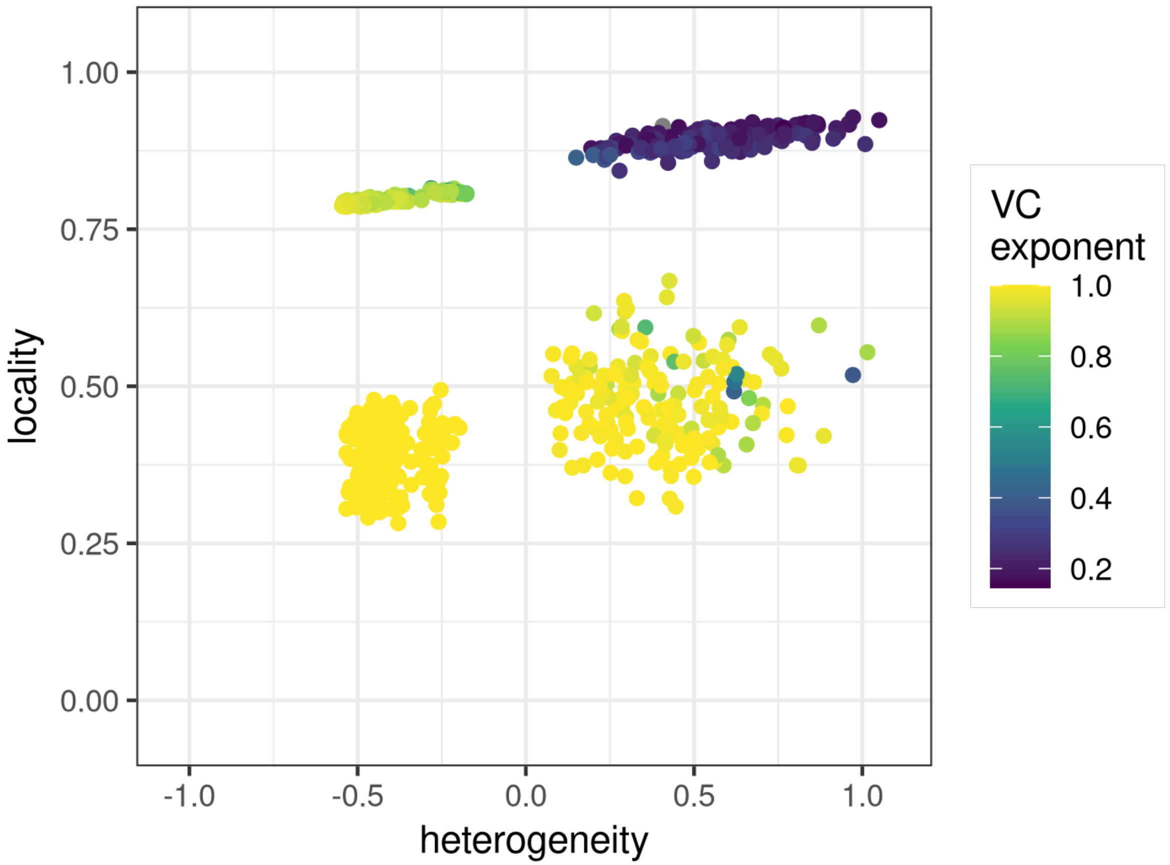
- Barabasi Albert Graphs?





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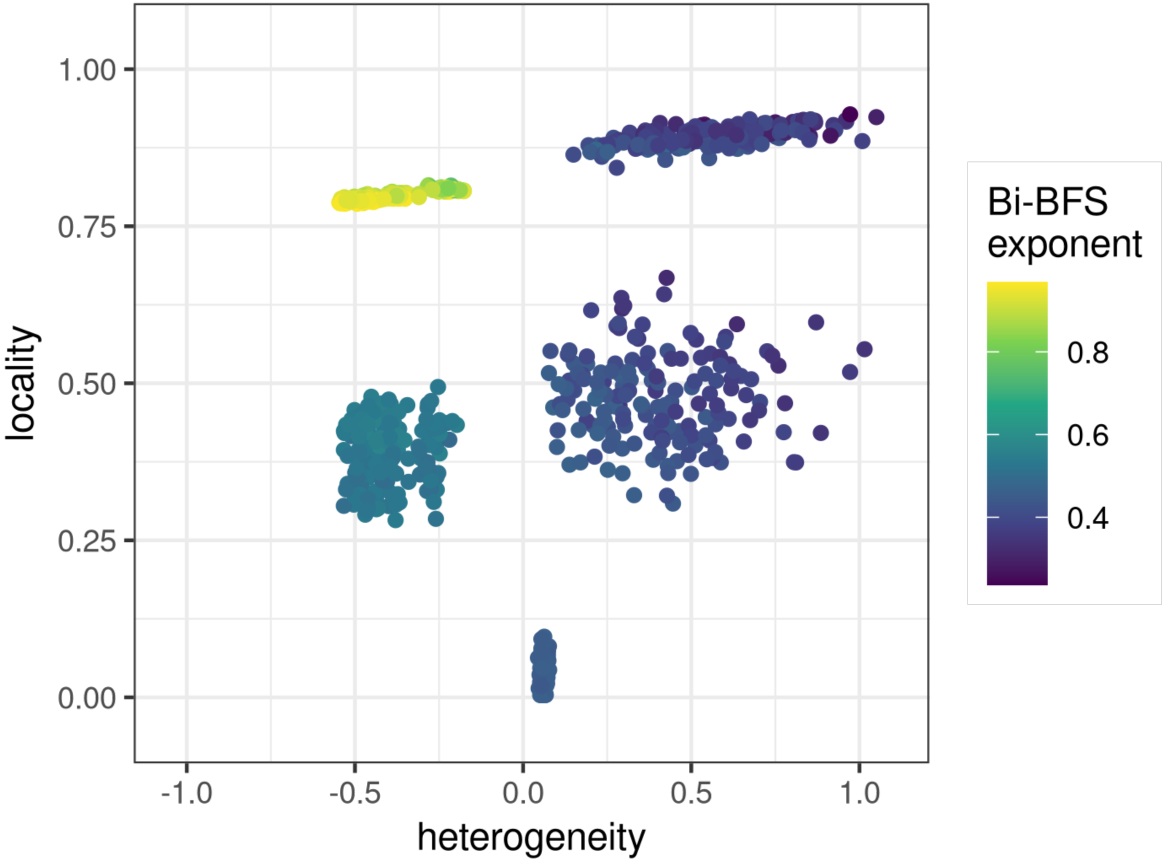

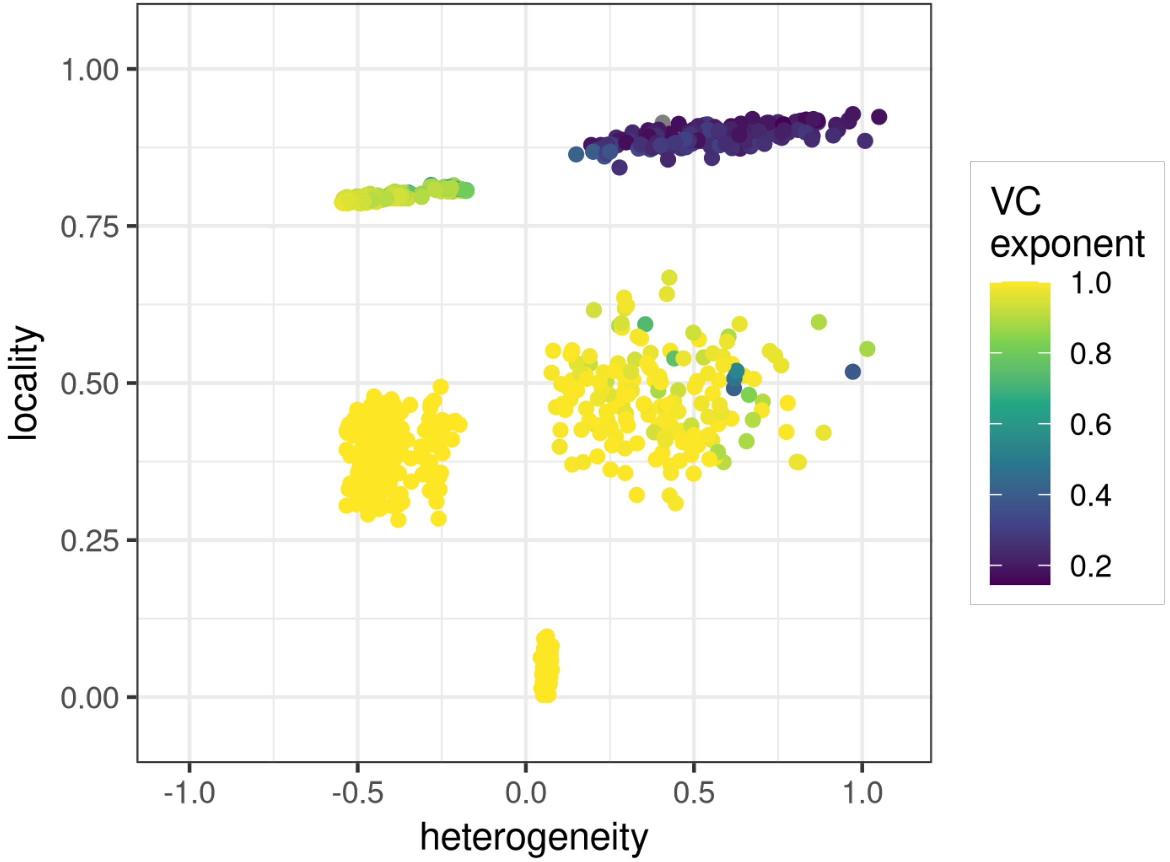

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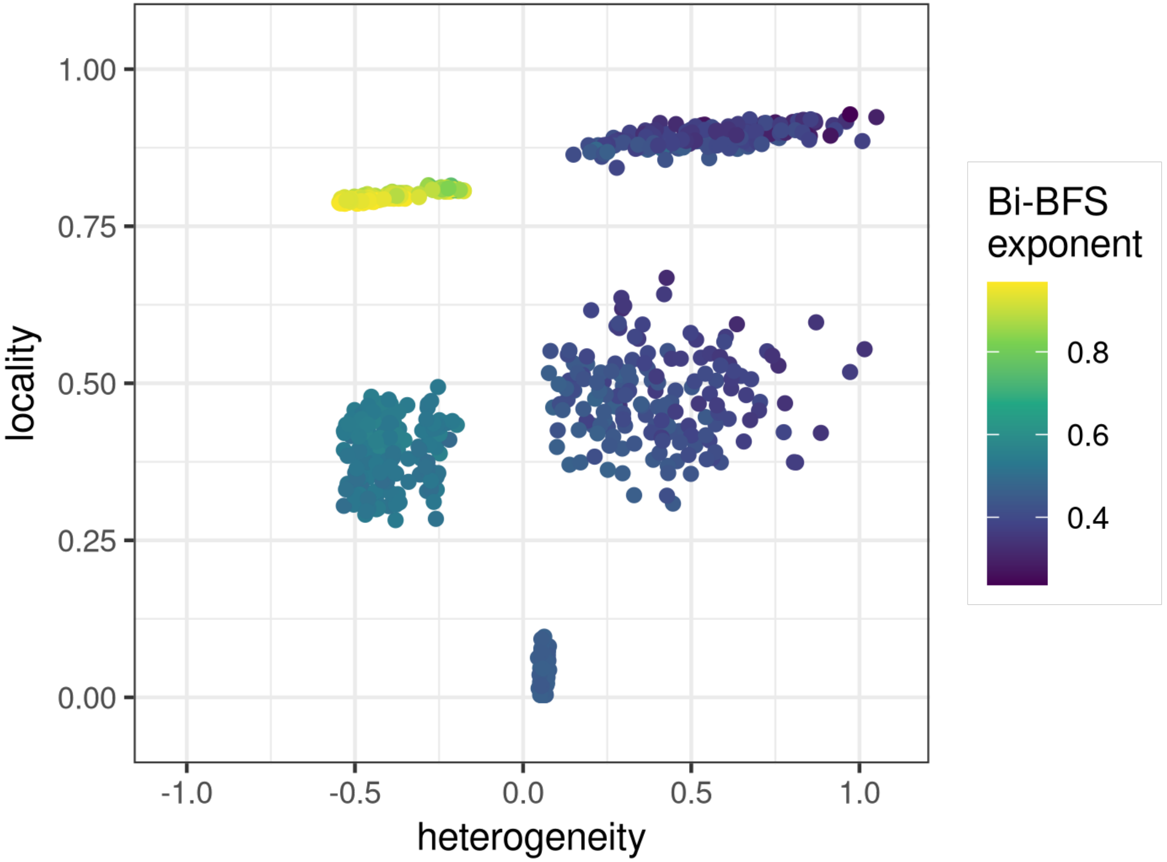

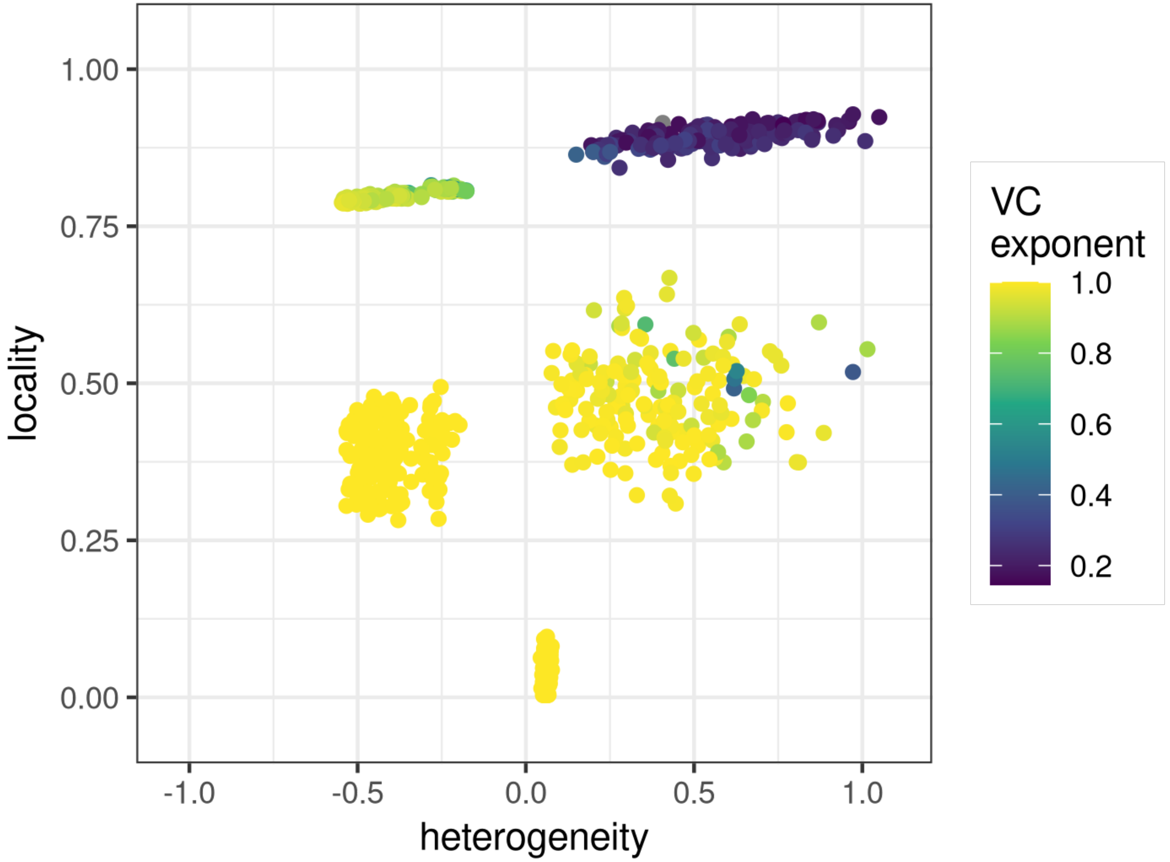

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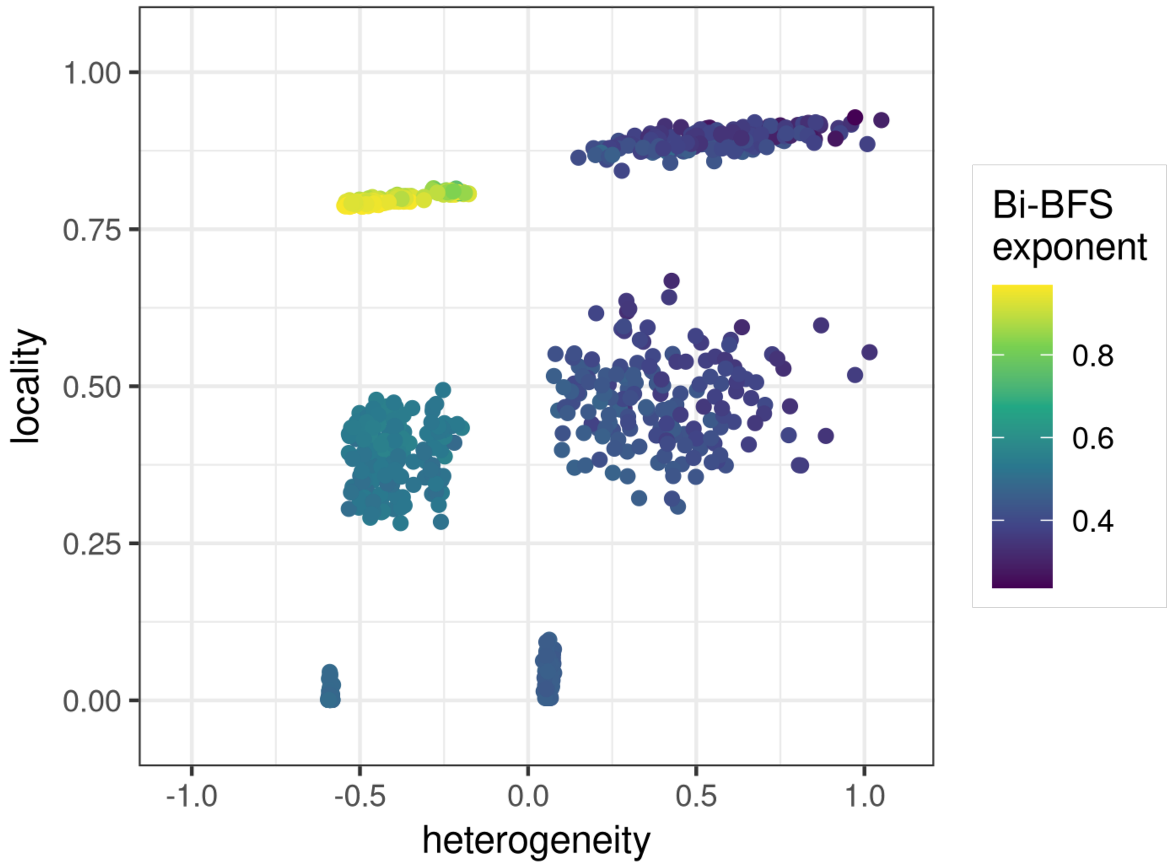

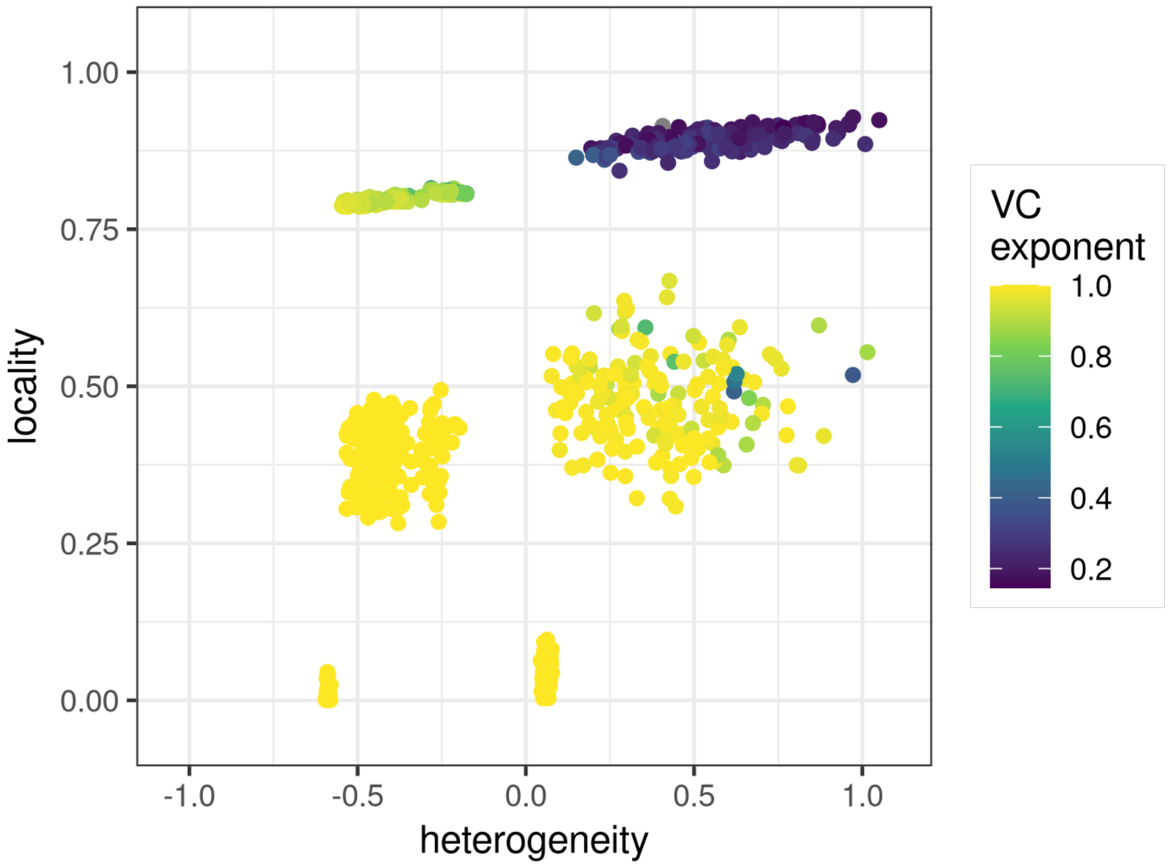

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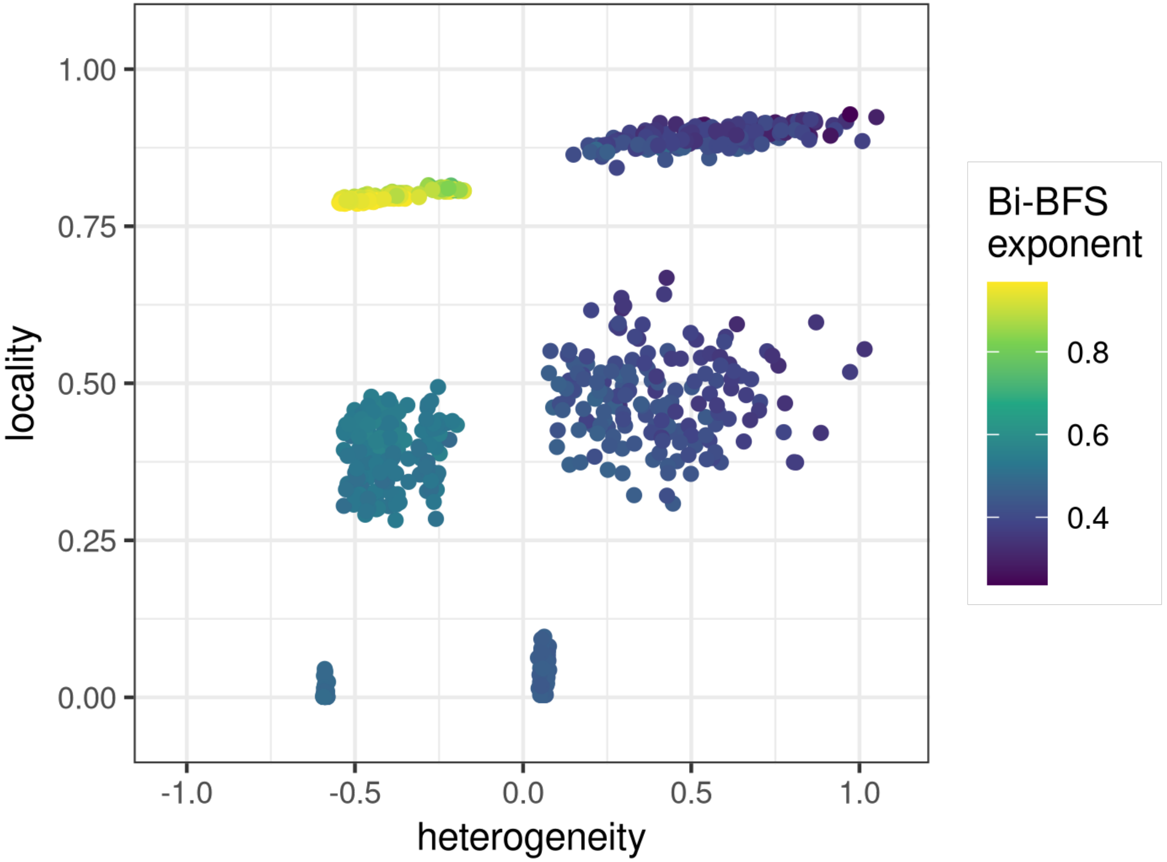

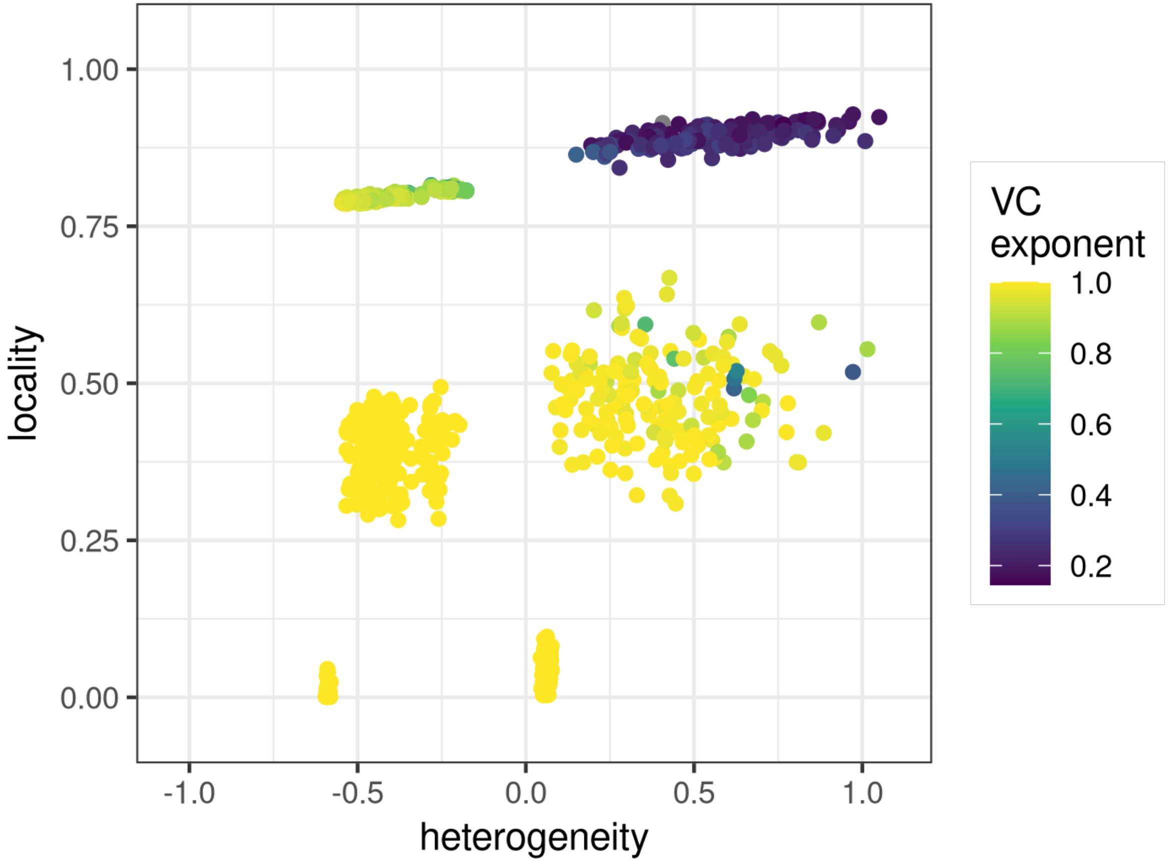

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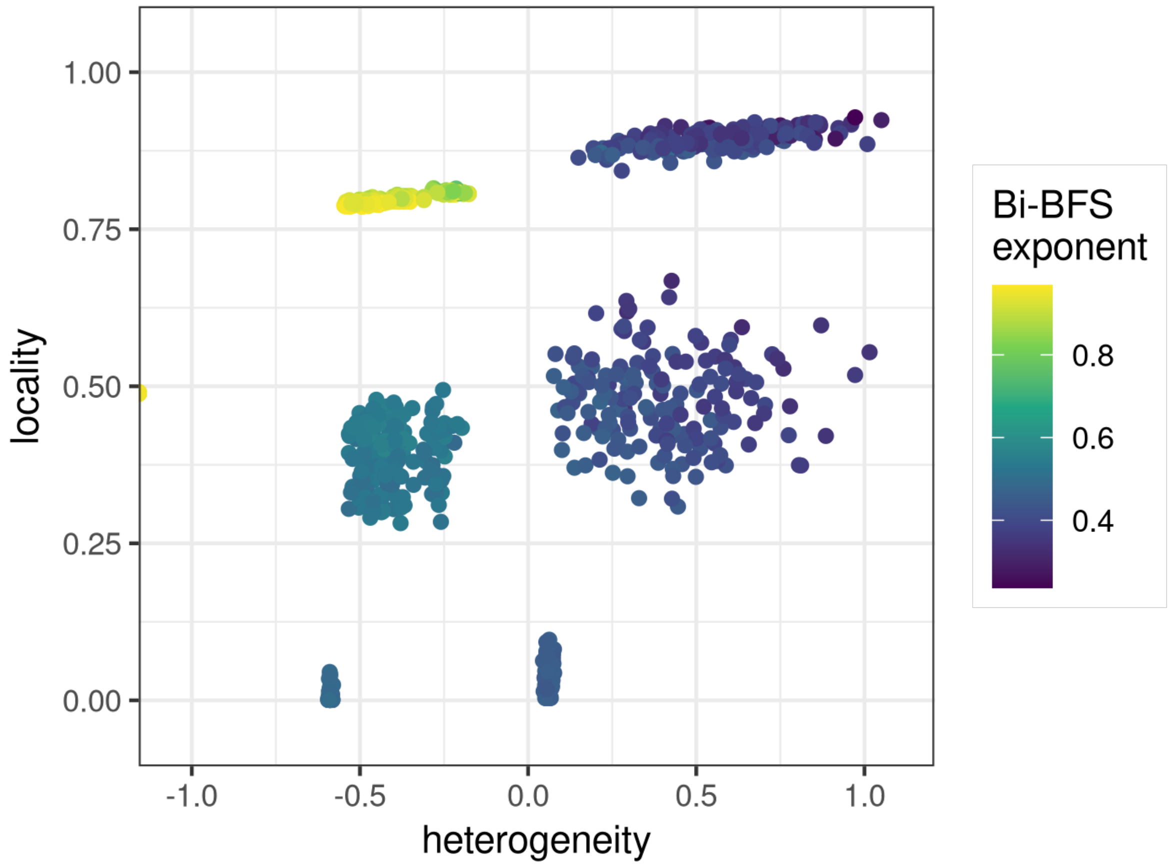

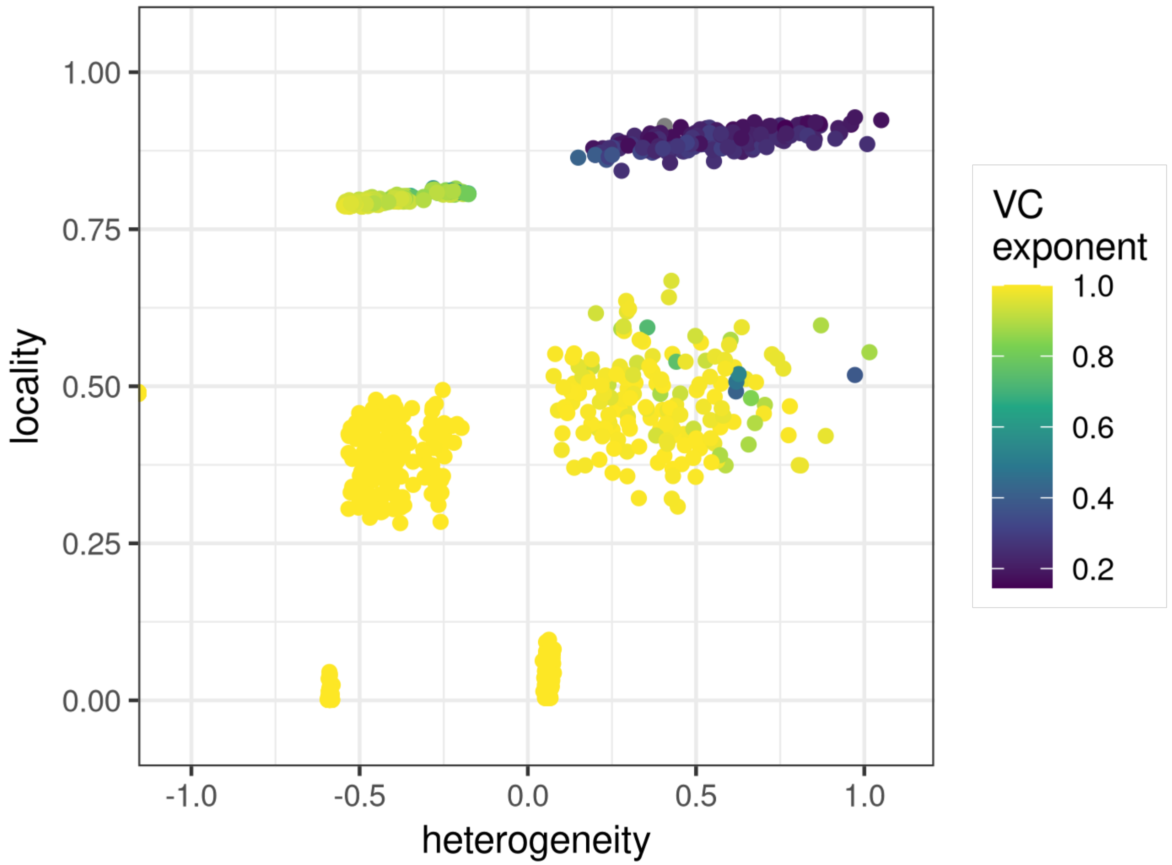

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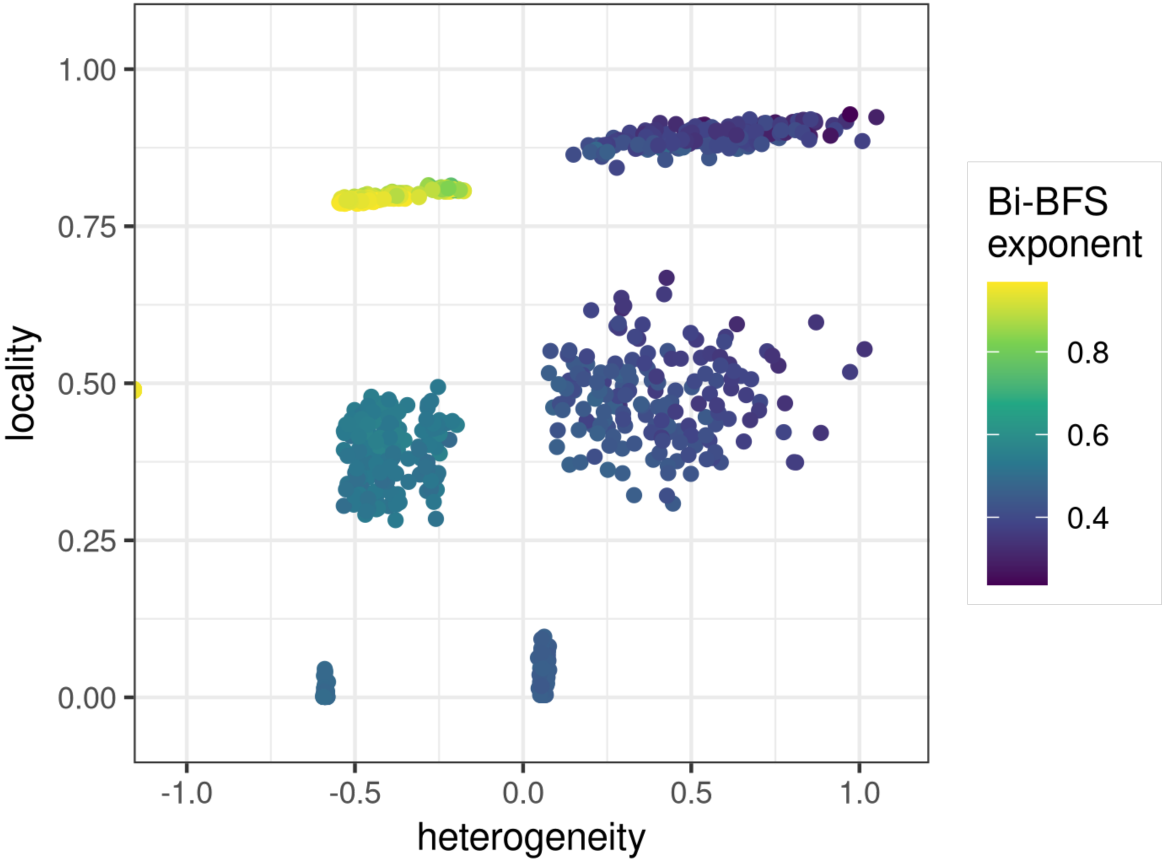

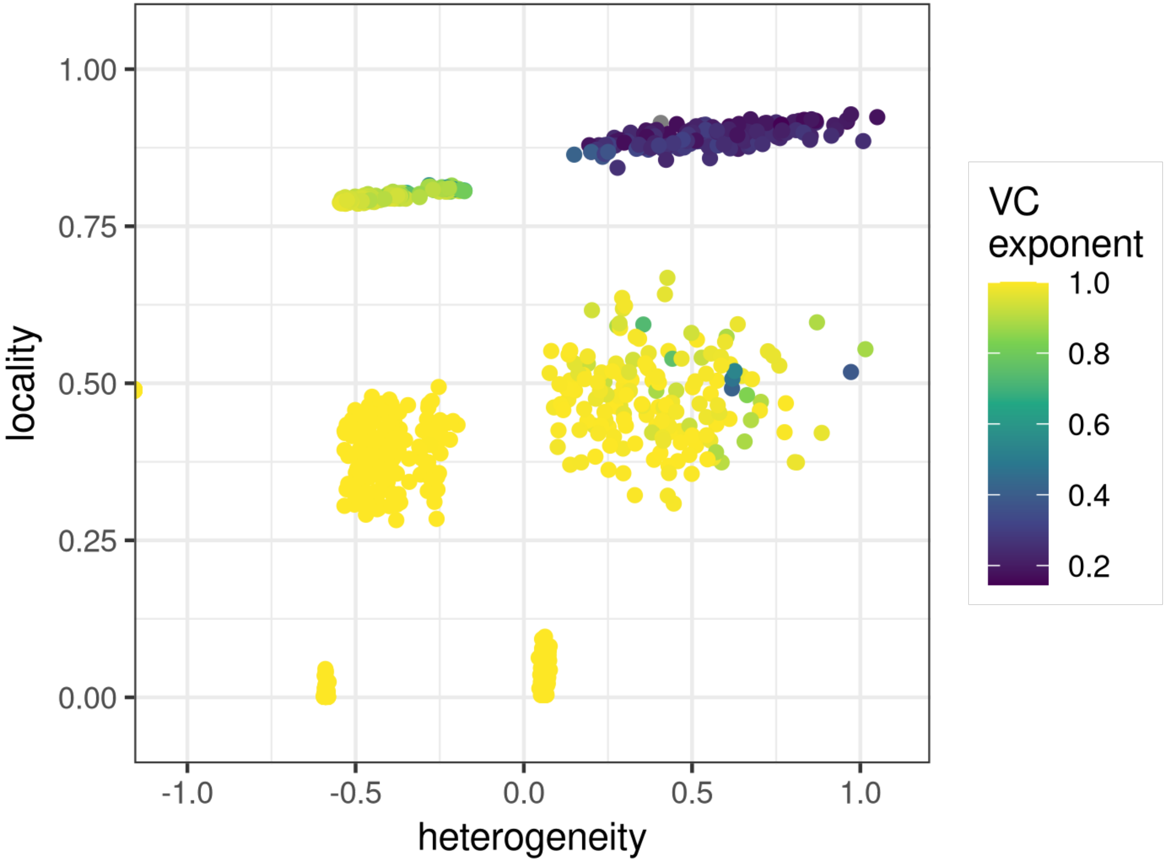

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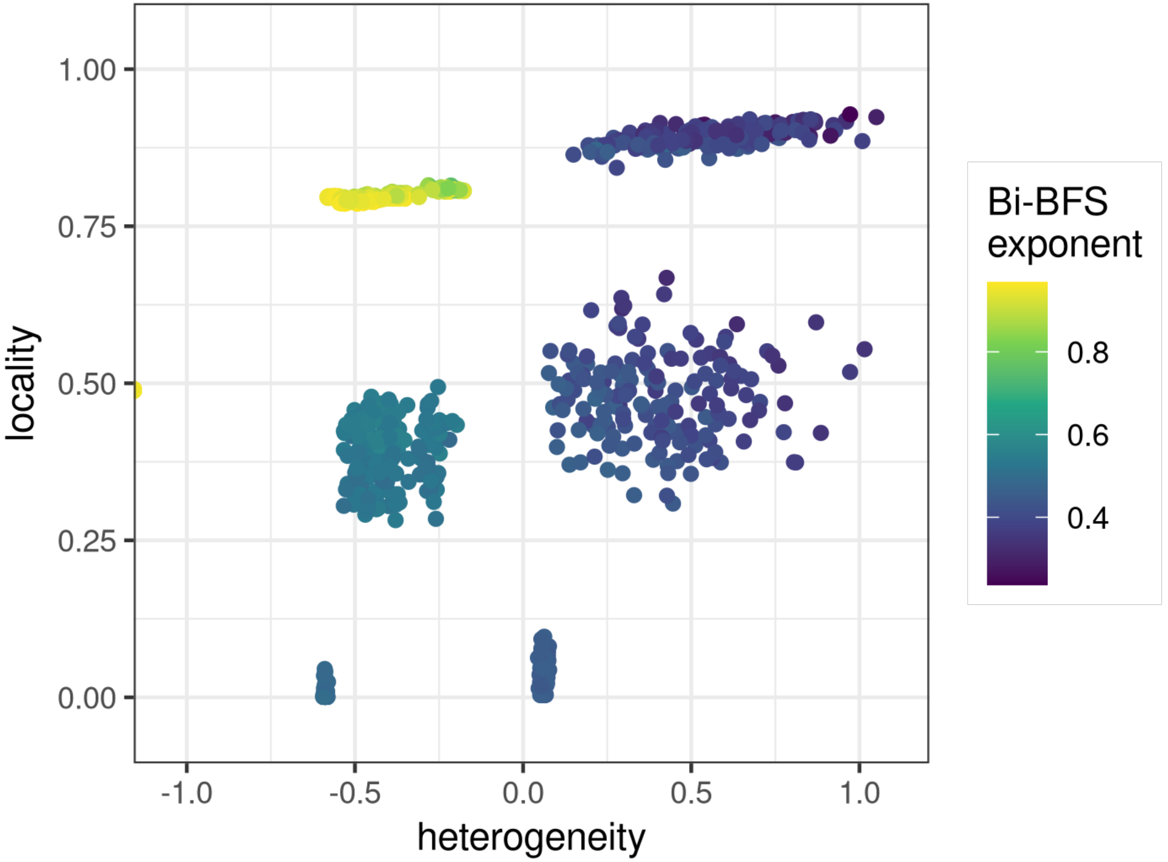

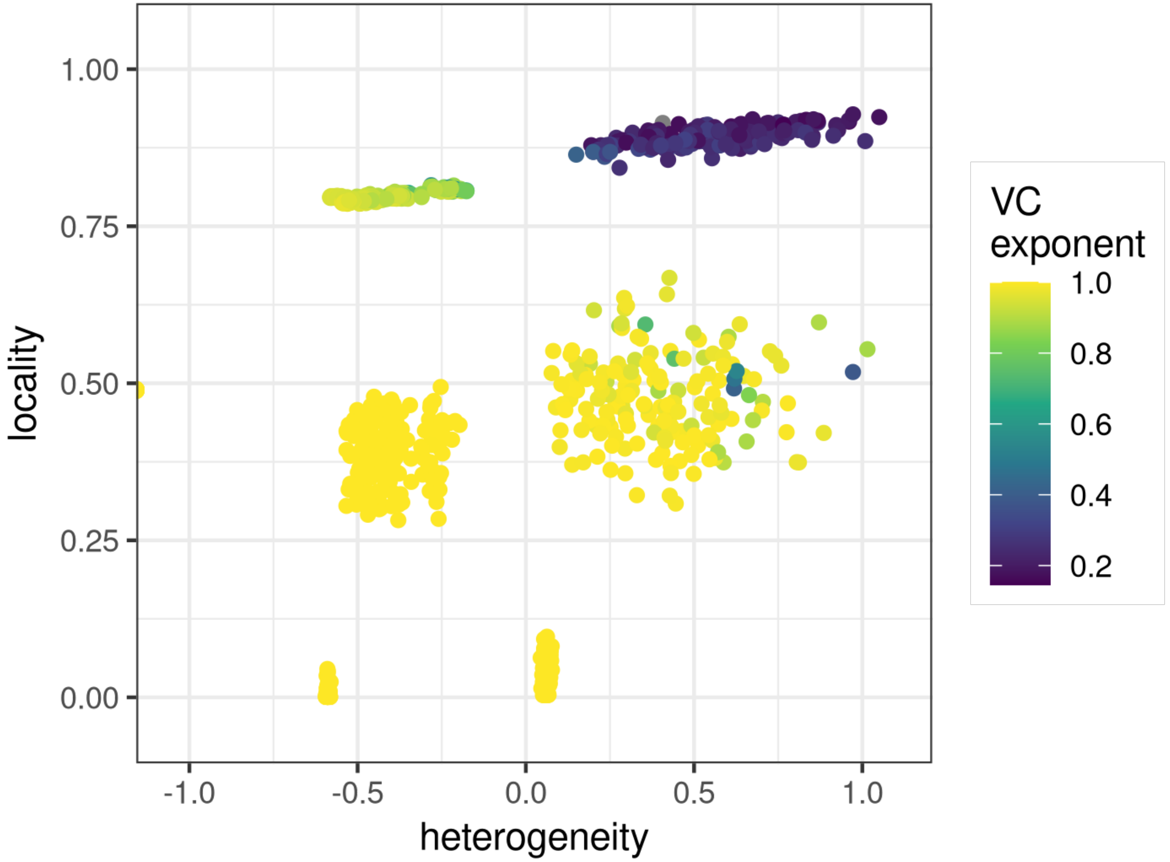

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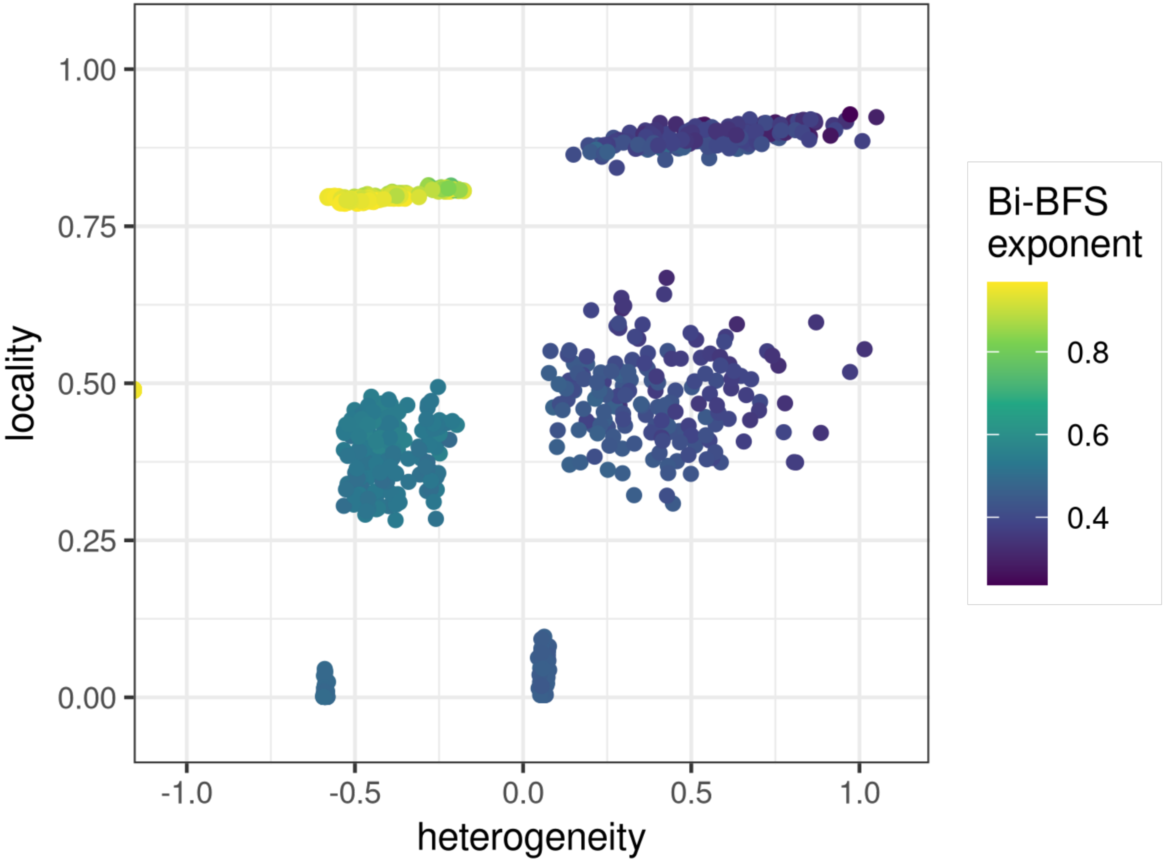

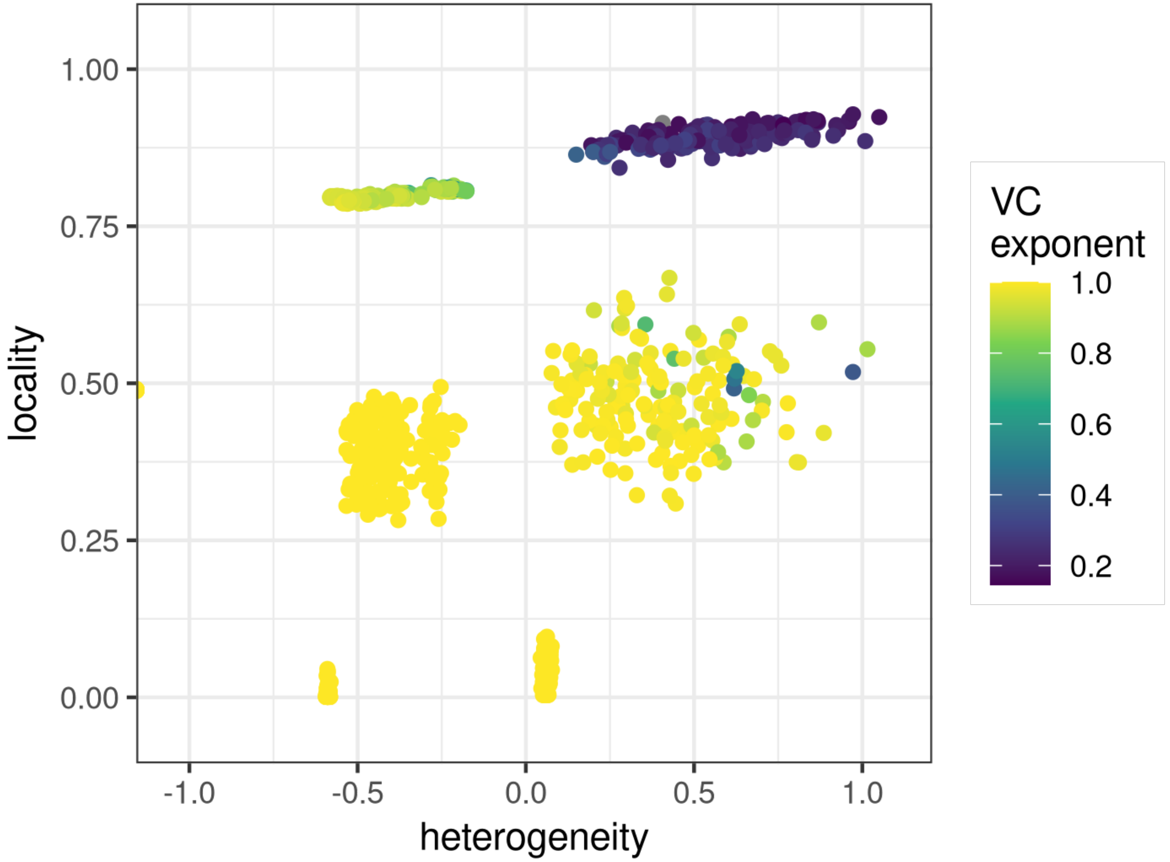

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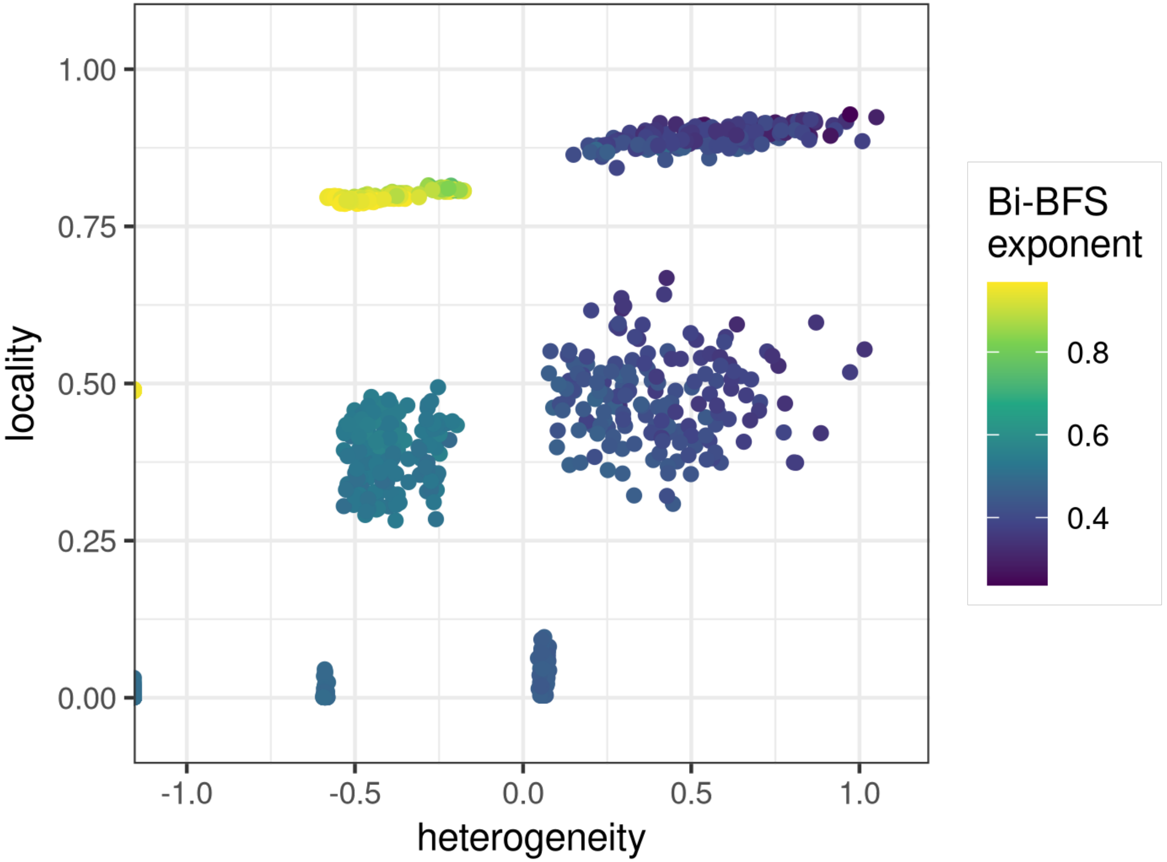

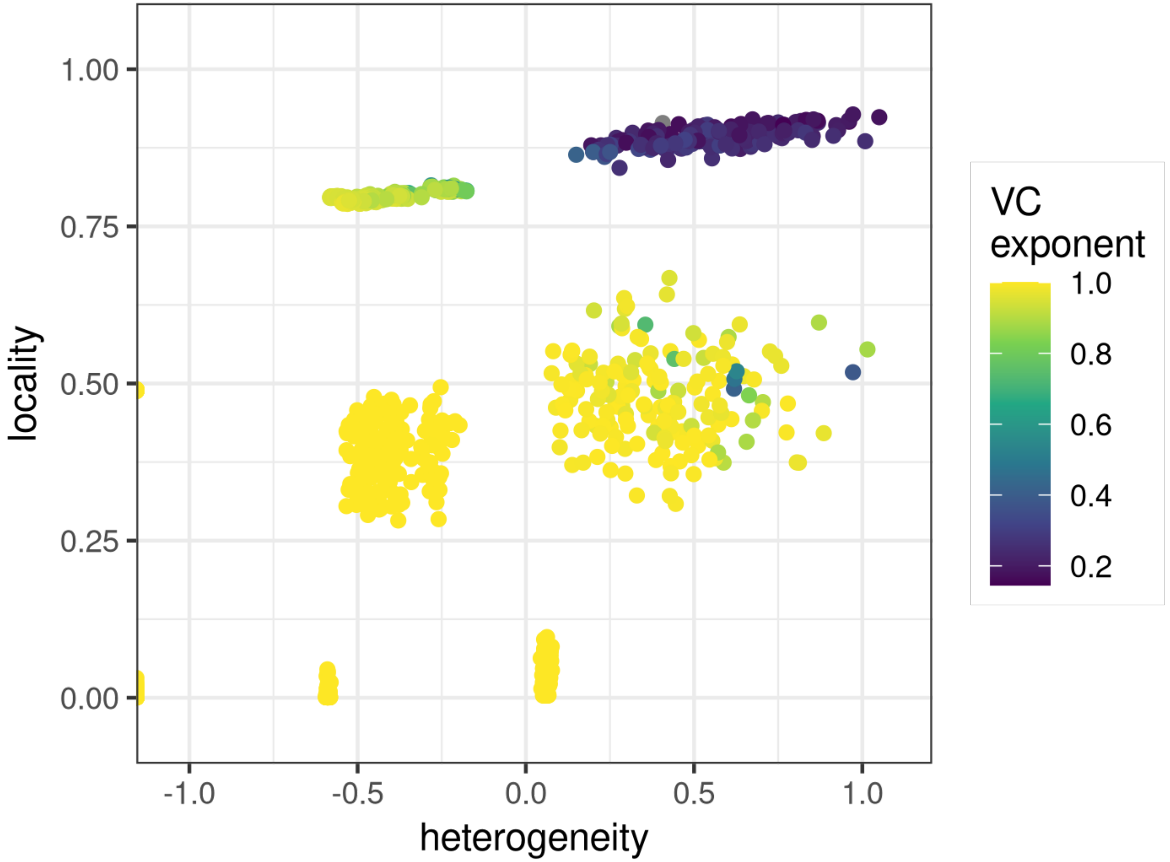

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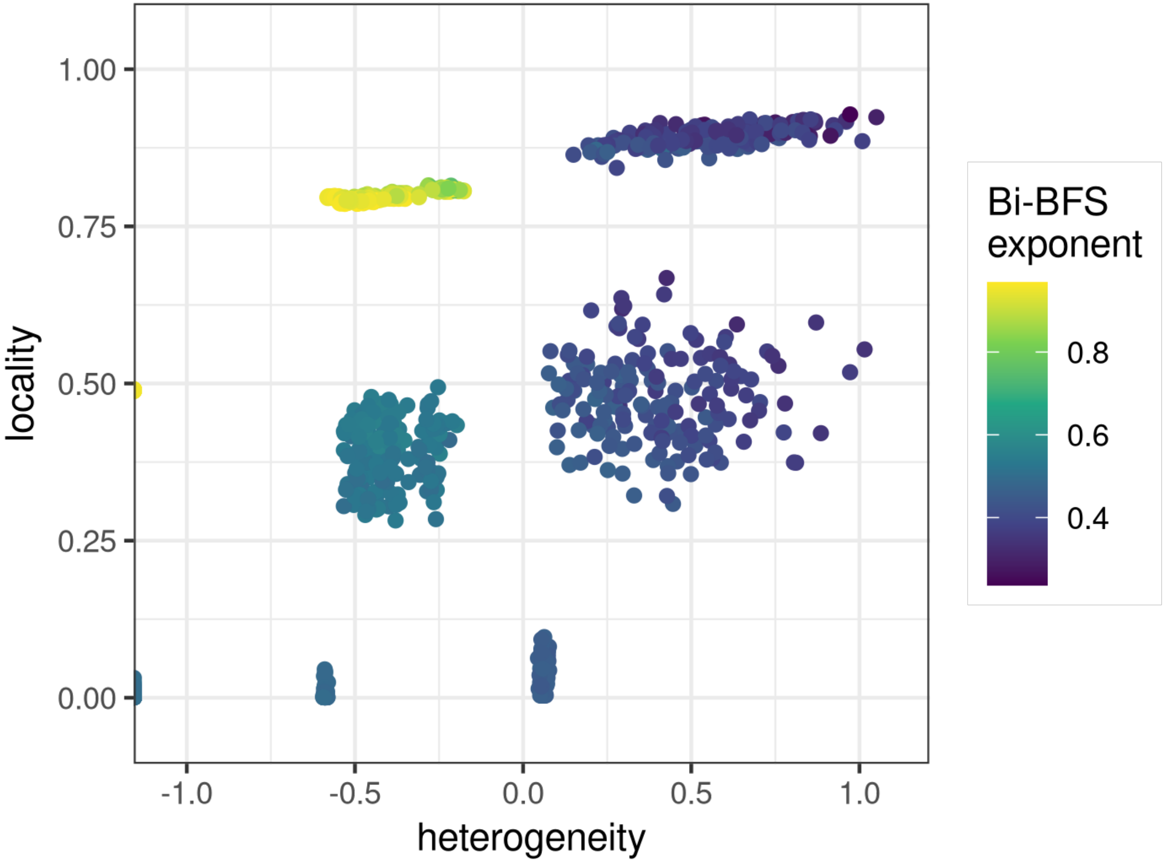
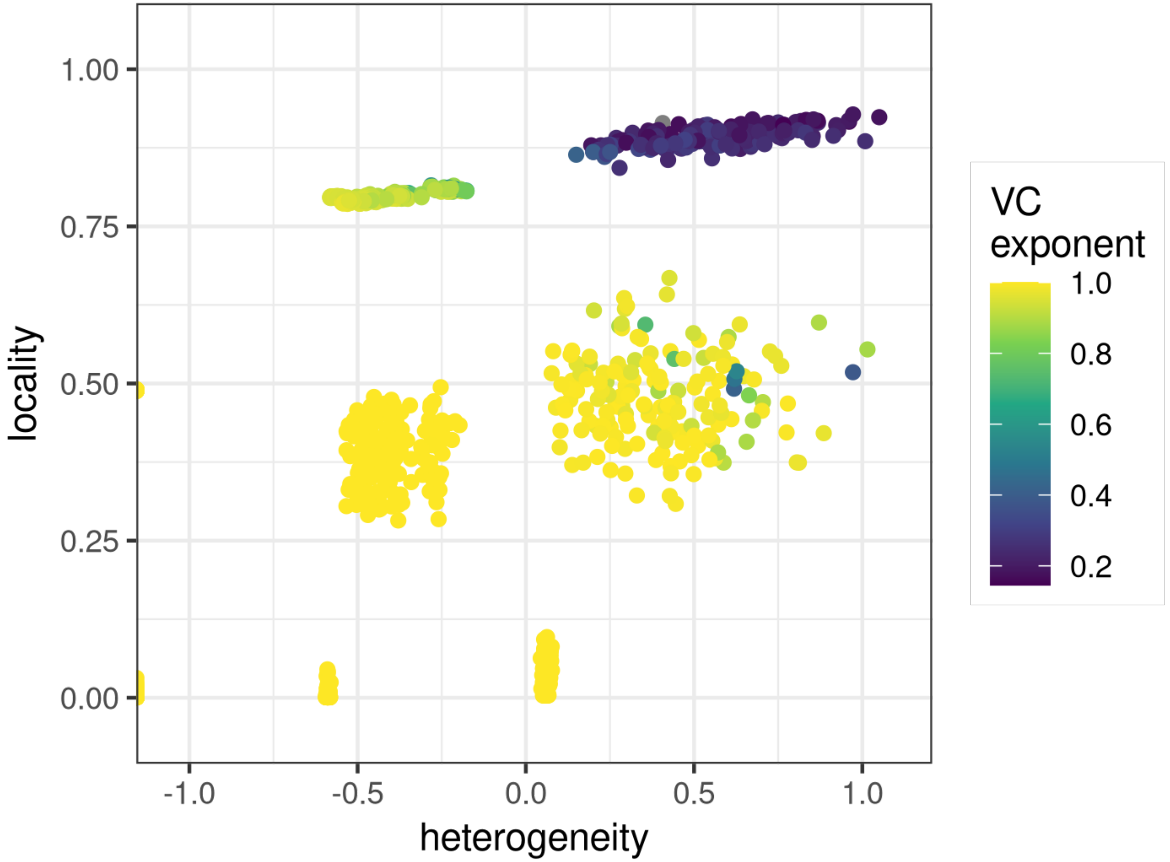
- Stochastic Block Model Graphs?





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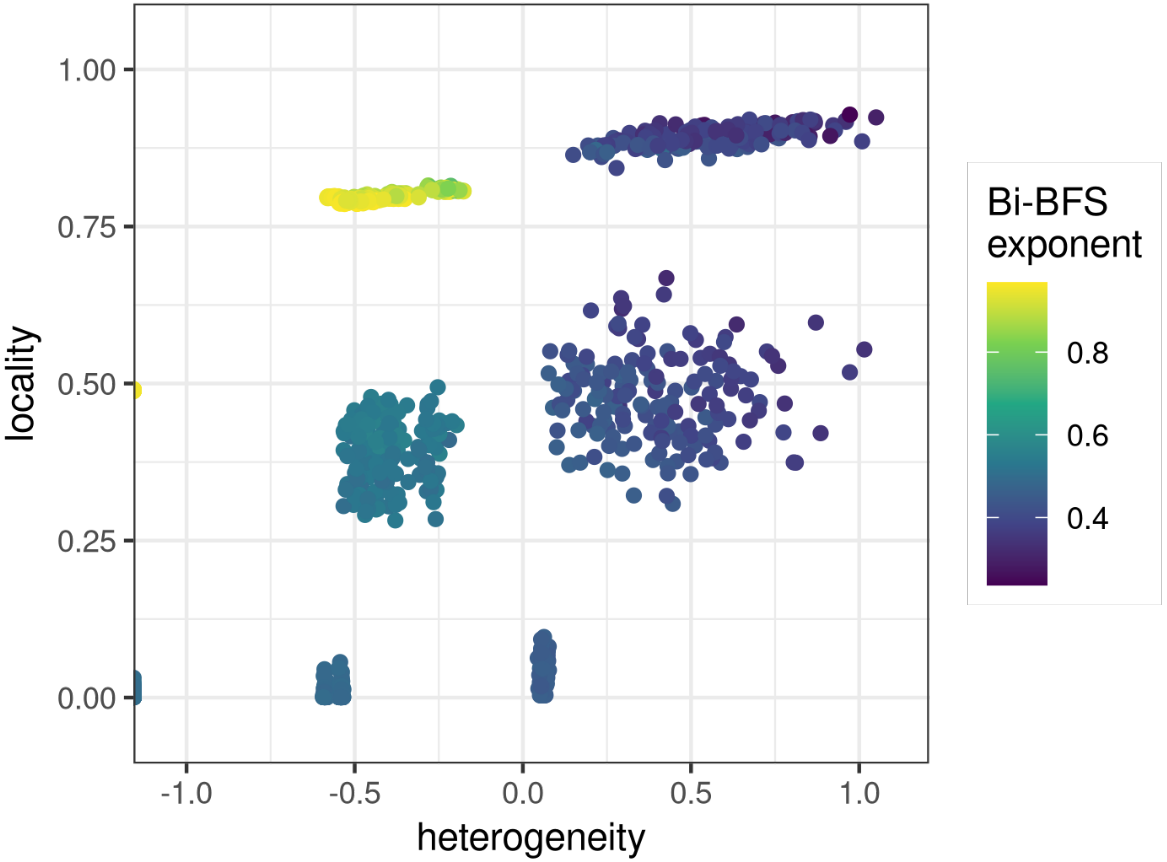

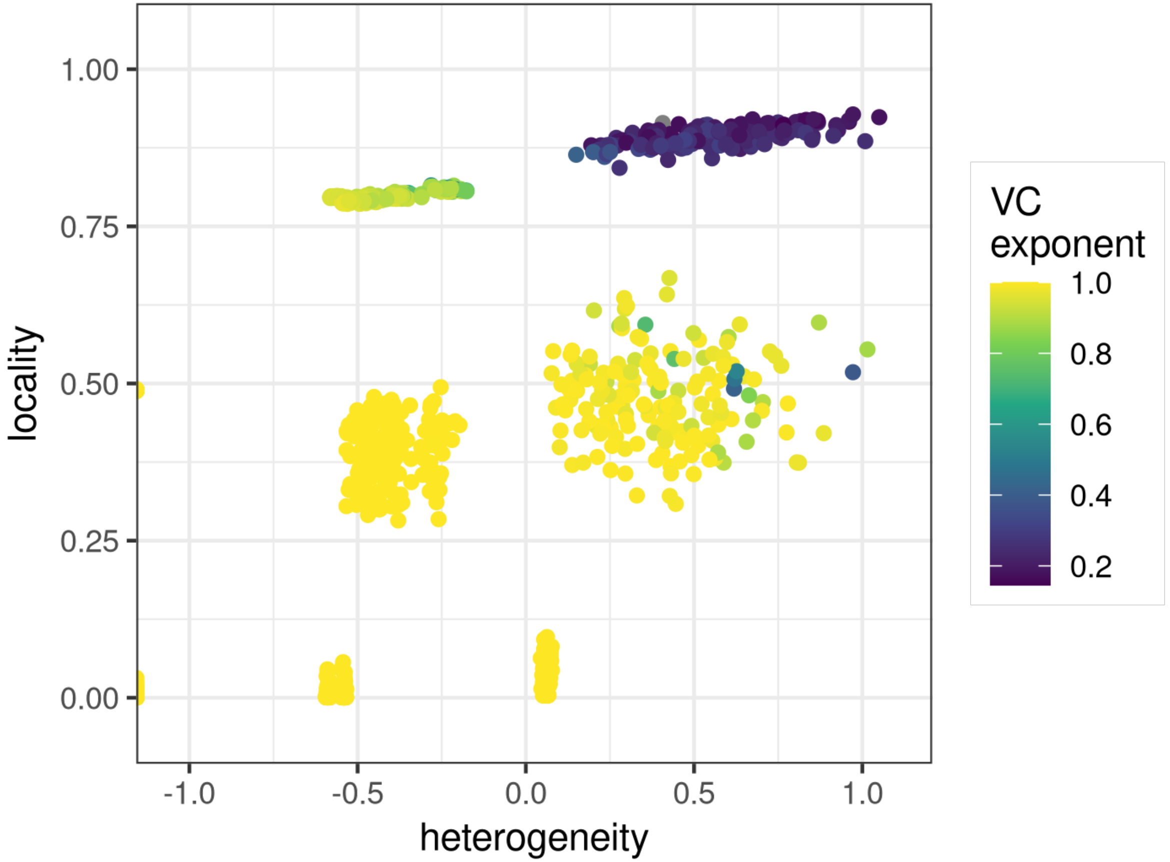

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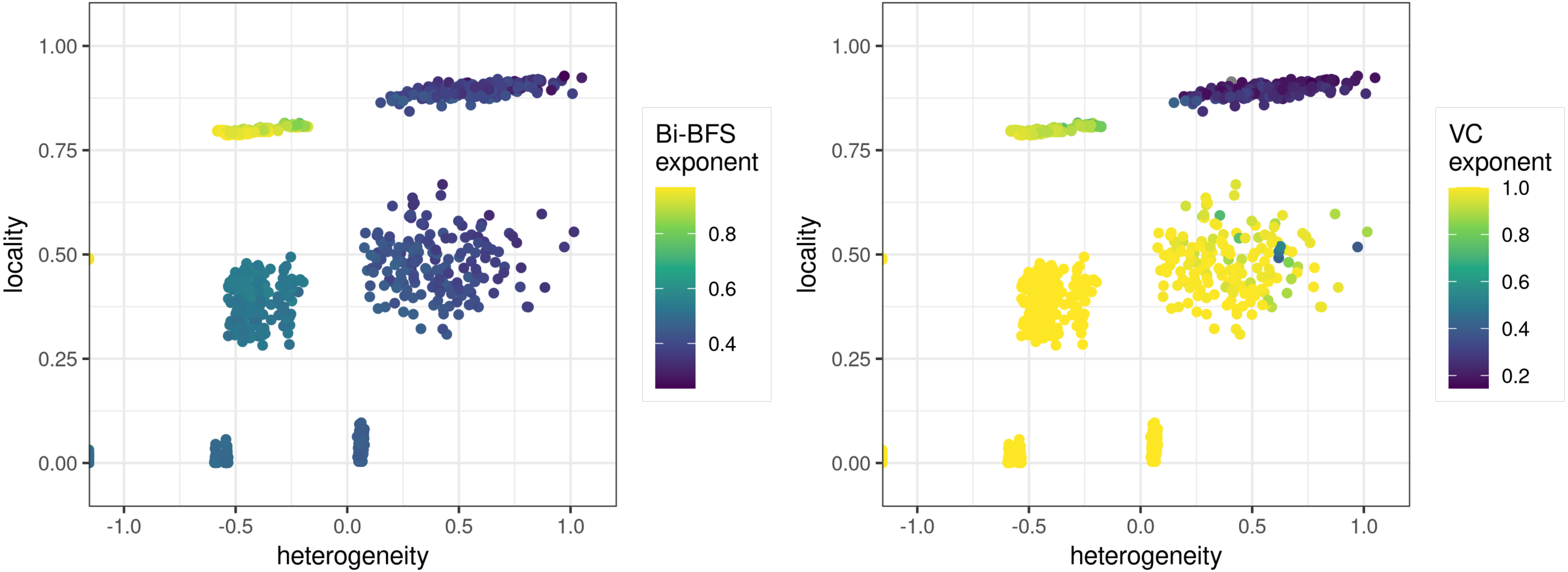

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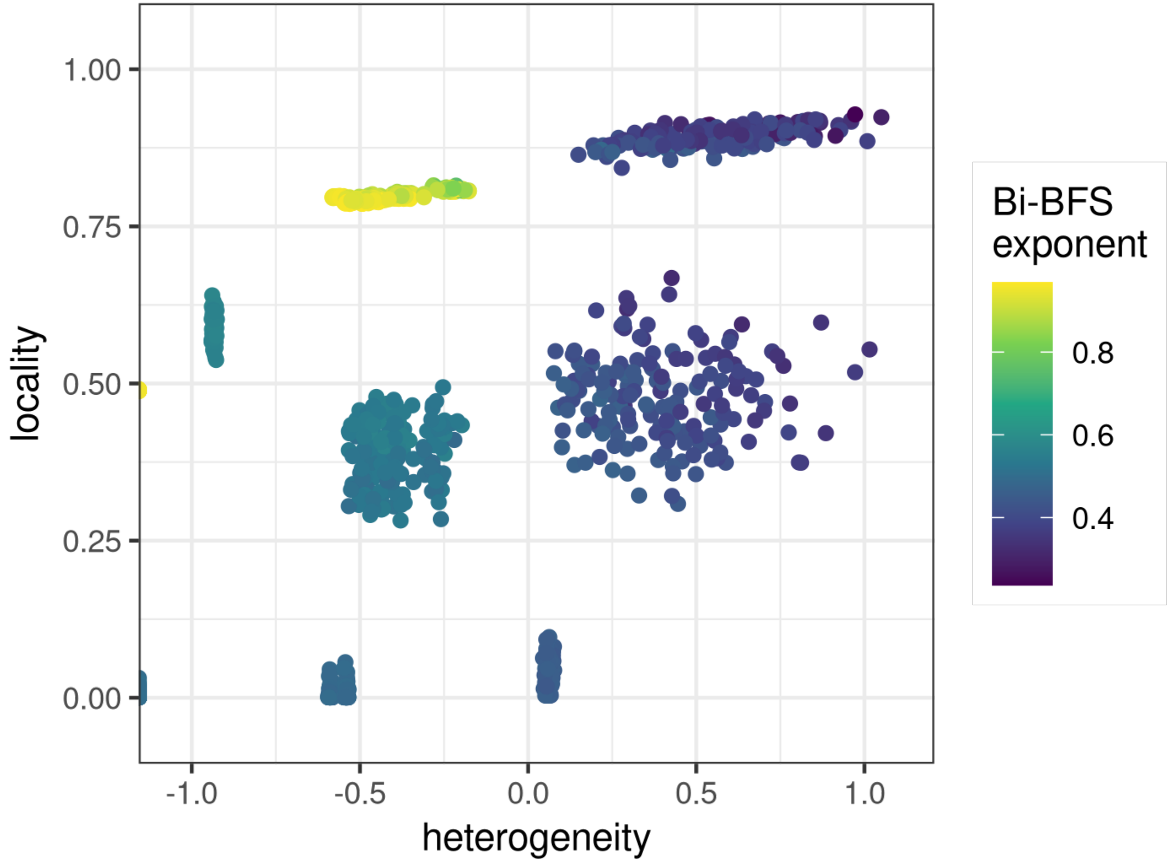

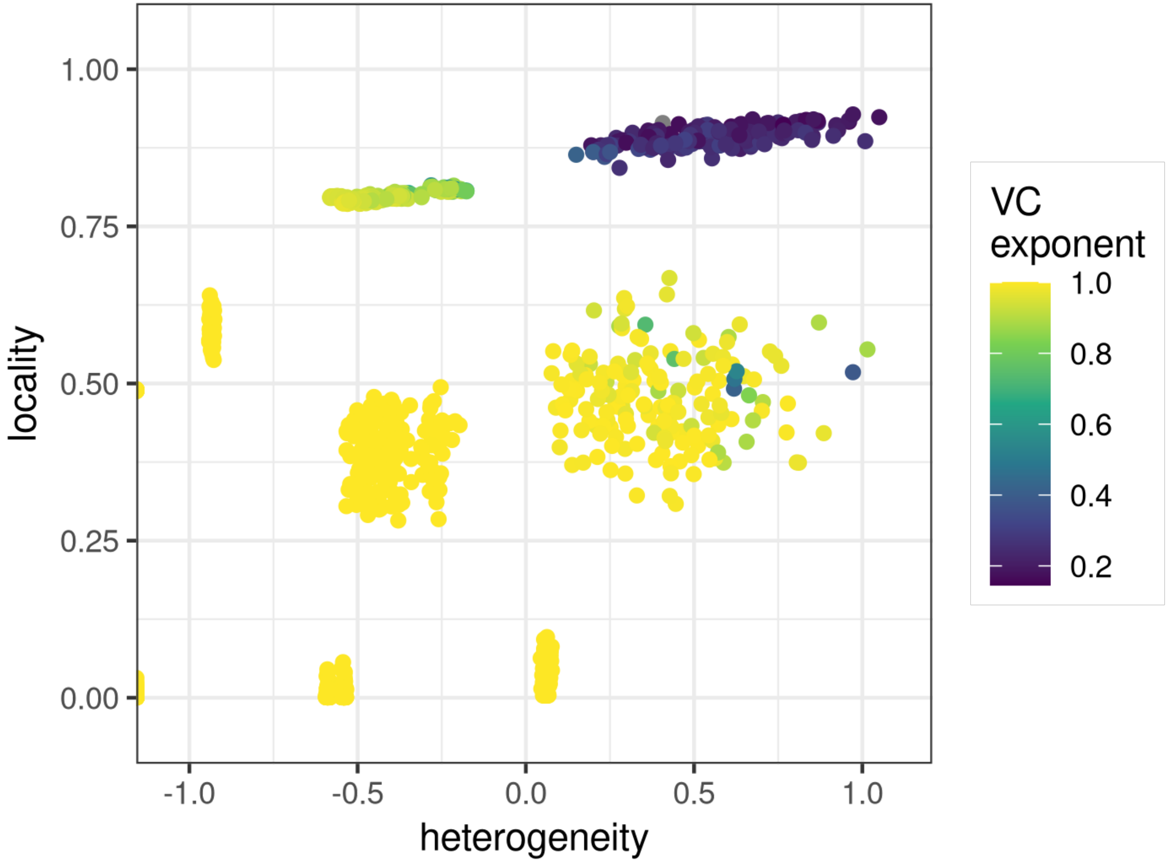

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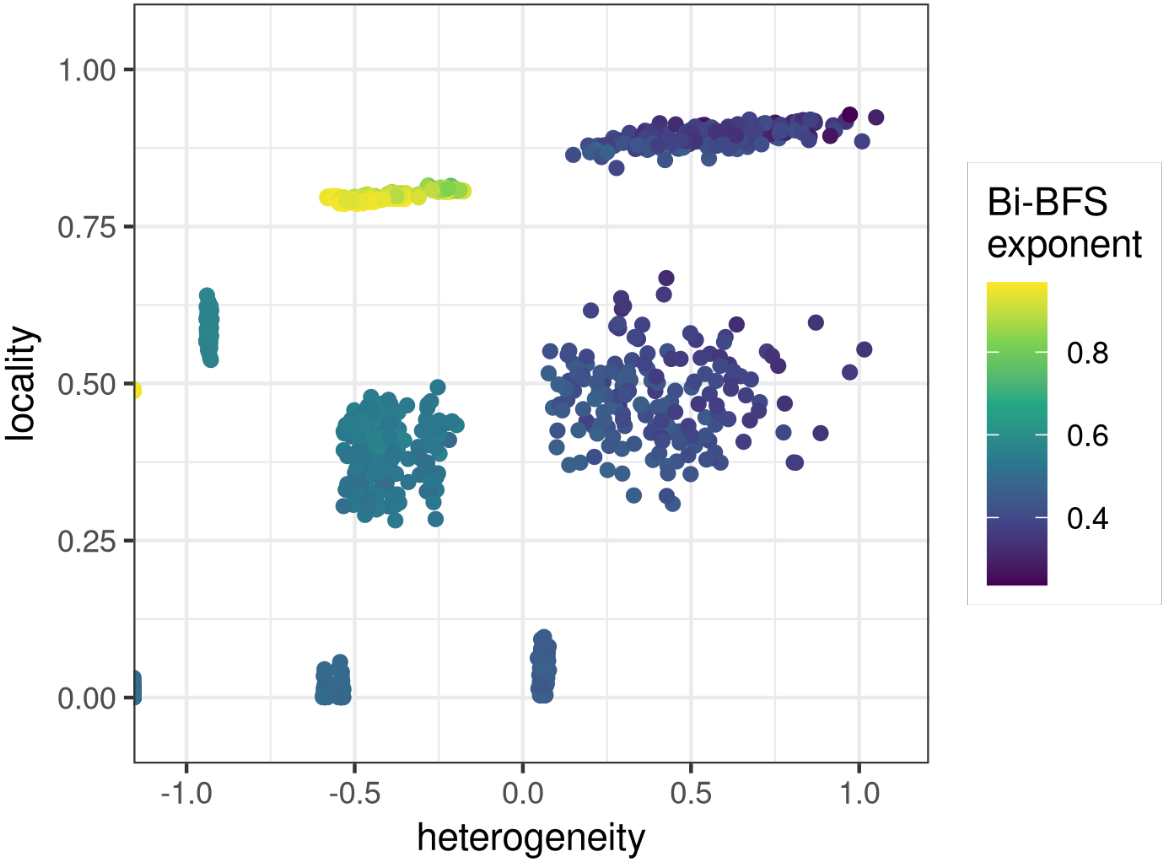
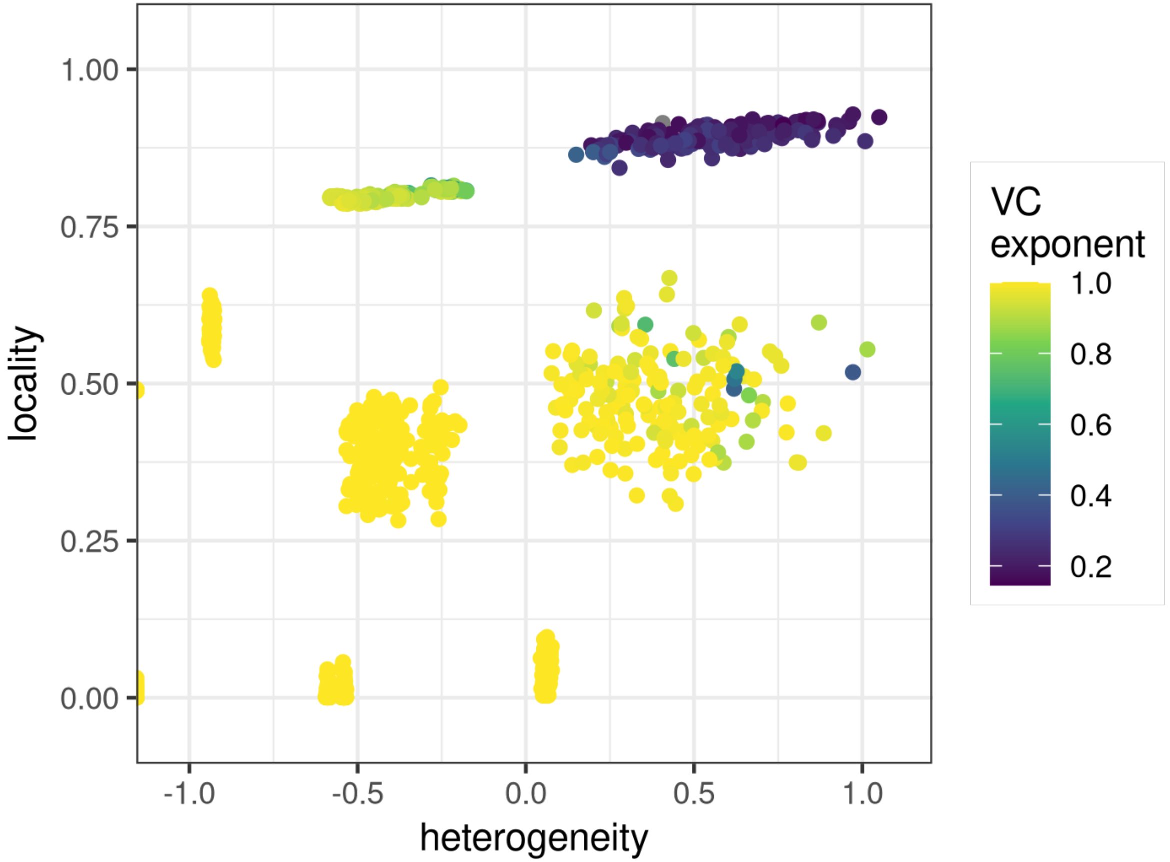
Where are:

- Geometric Inhomogeneous Random Graphs?





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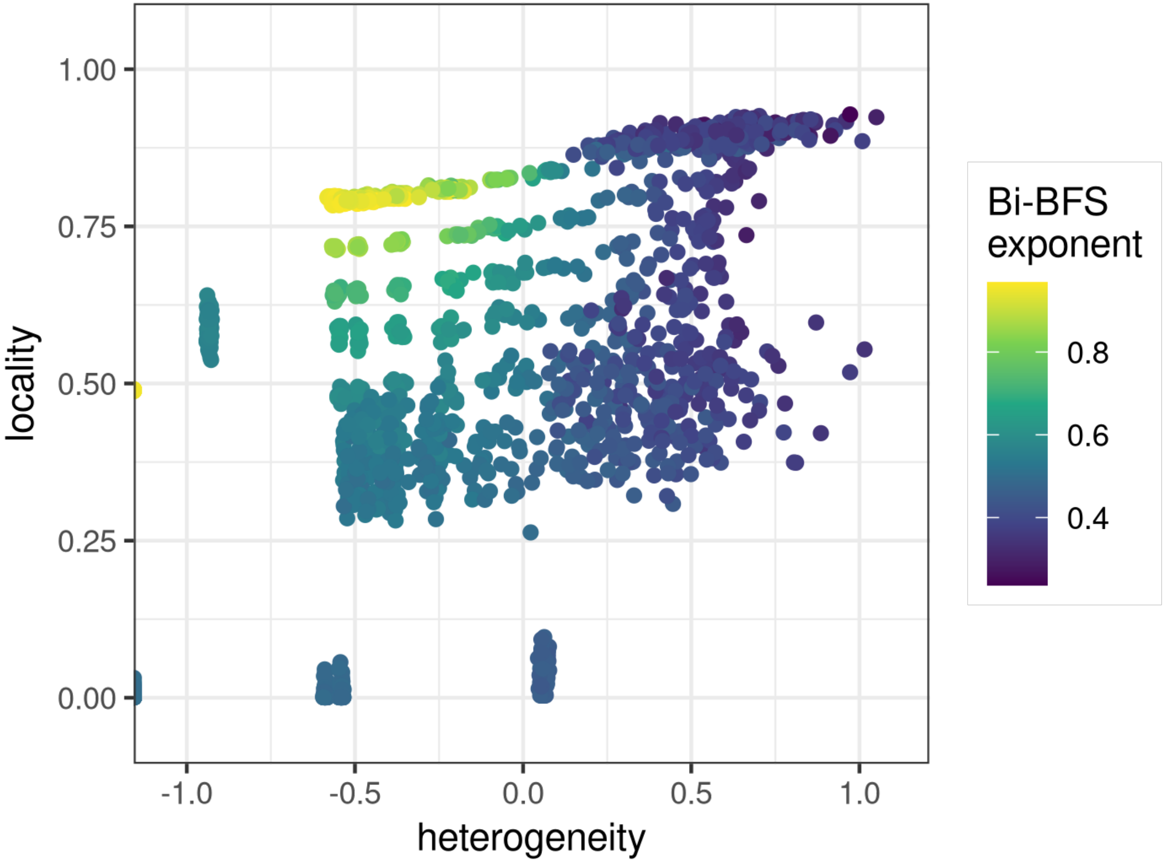

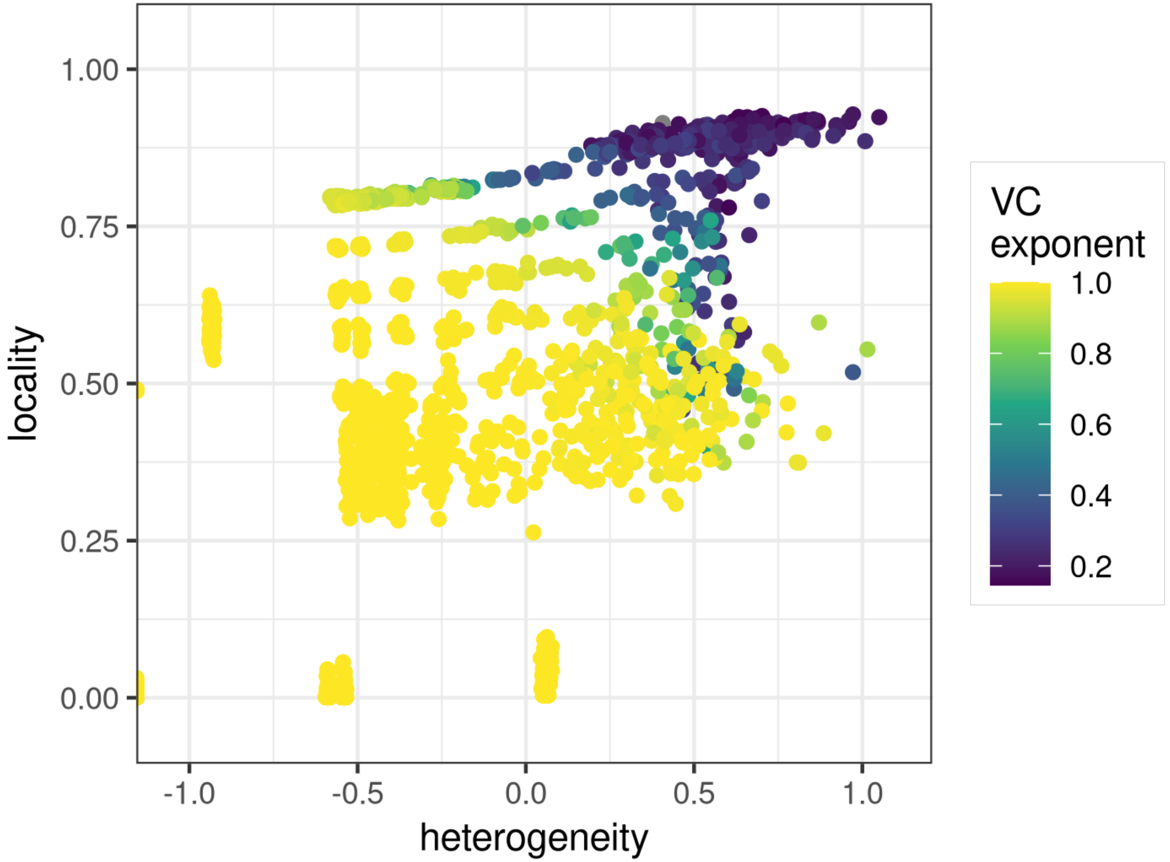

Overview: GIRGs and HRGs

Geometric inhomogeneous random graphs
[Bringmann, Keusch, Lengler, 2019]

Geometric inhomogeneous random graph (GIRG)

- weights $w_1, \ldots, w_n$ (typically power-law distributed)
- random positions for the vertices (d-dimensional ground space)

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connect if sufficiently close

connection probability proportional to $w_u w_v$

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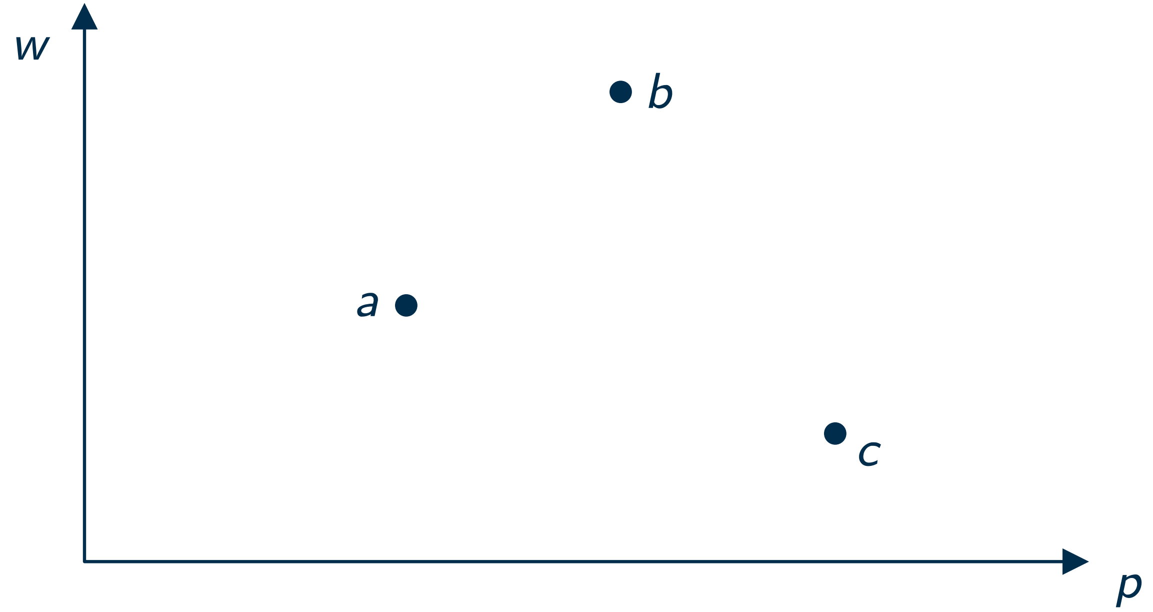

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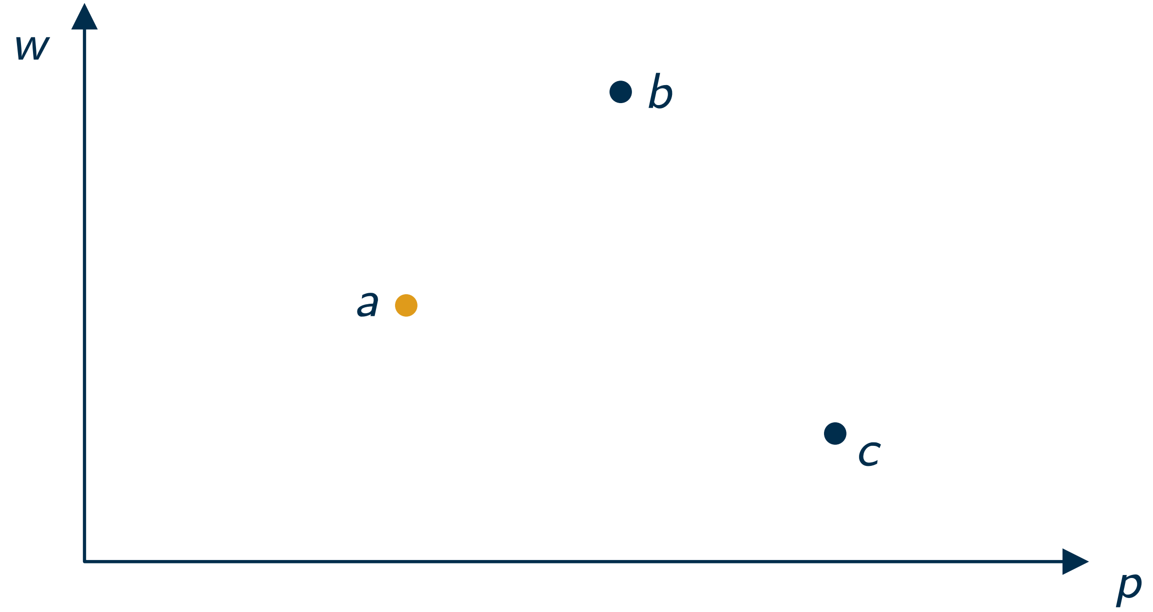

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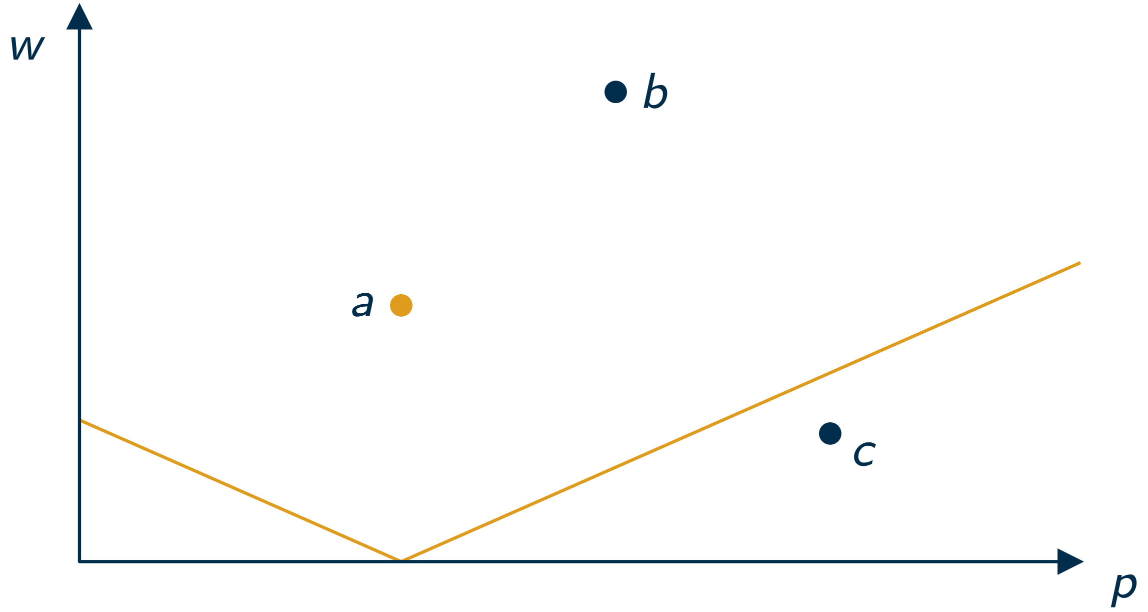

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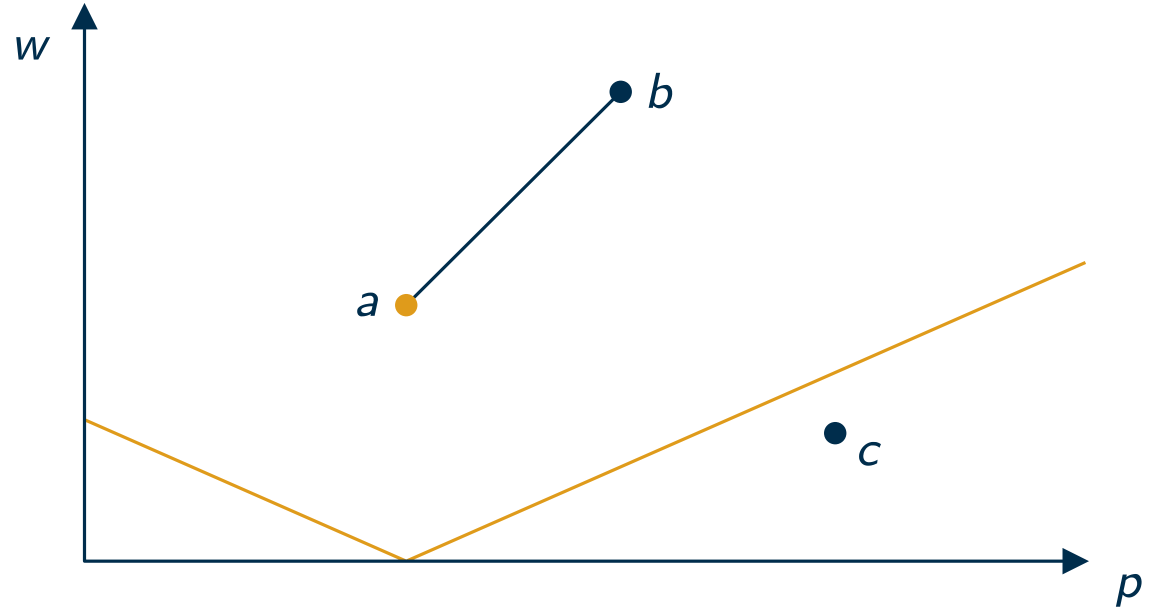

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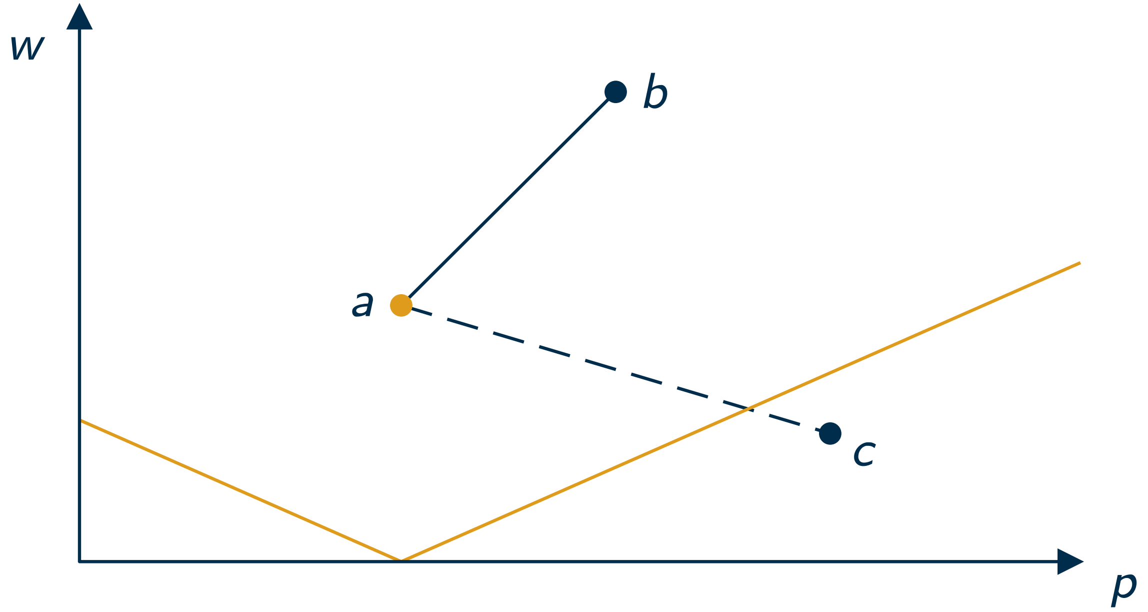

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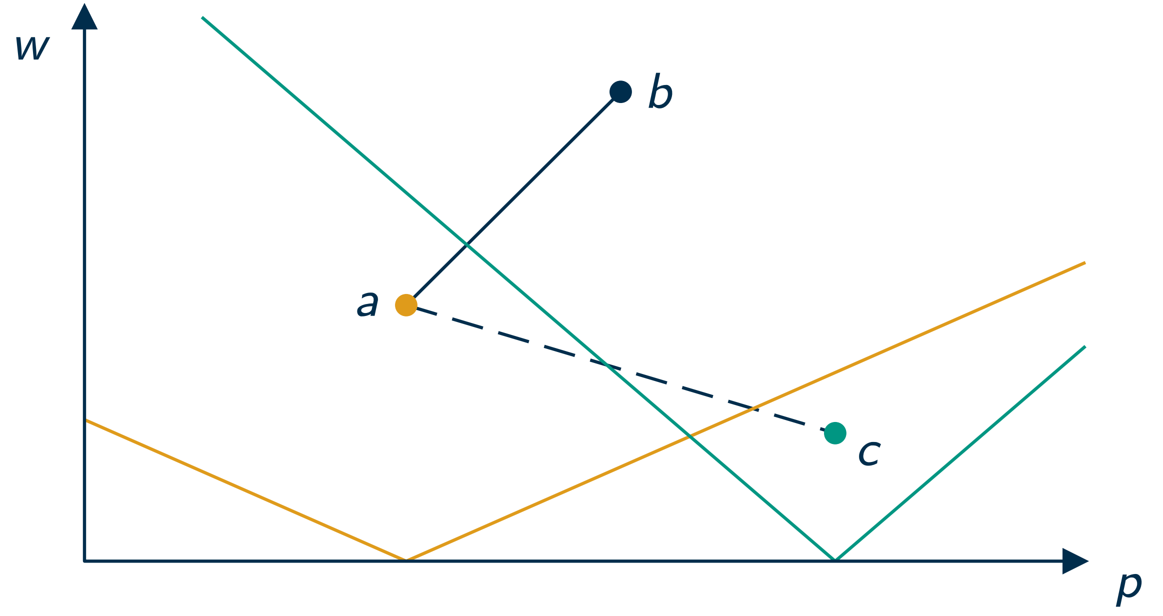

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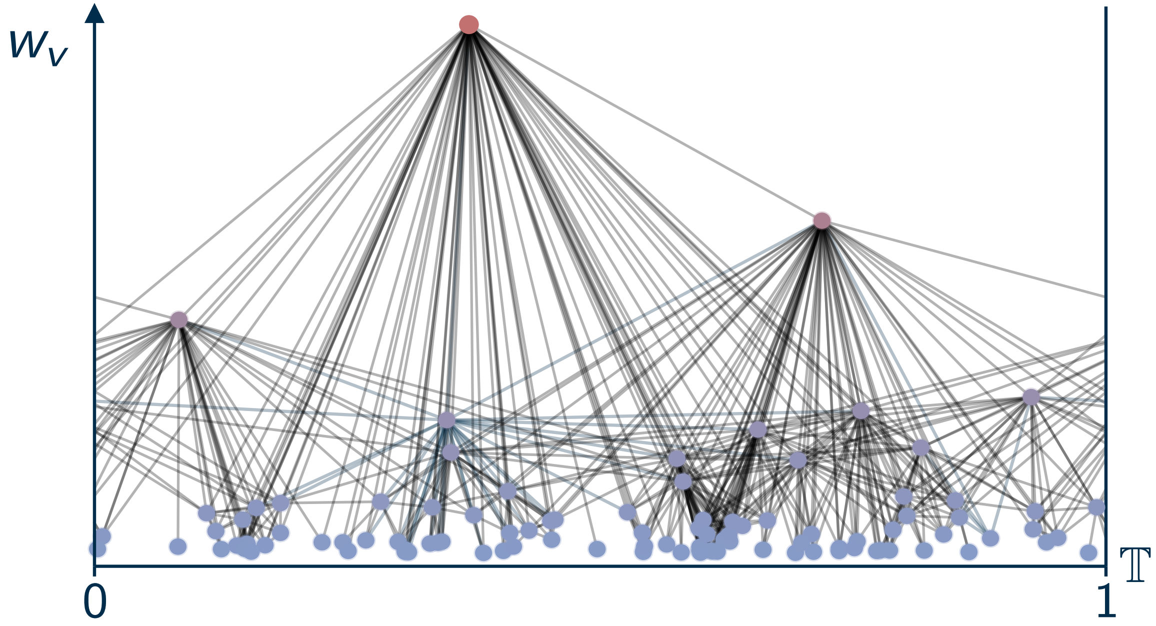

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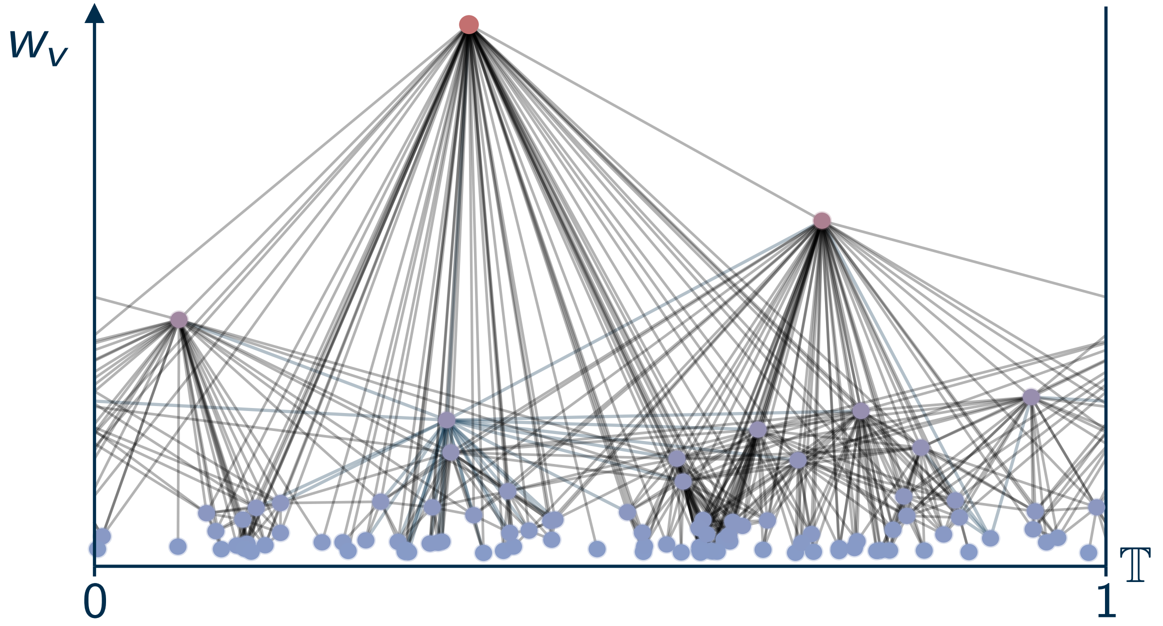
Popularity – Similarity

Popularity versus similarity in growing networks
[**Papadopoulos, Kitsak, Serrano, Boguñá, Krioukov** 2012]



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Hyperbolic random graphs (HRG)

Hyperbolic geometry of complex networks
[**Krioukov, Papadopoulos, Kitsak, Vahdat, Boguñá** 2010]

- random positions in hyperbolic space
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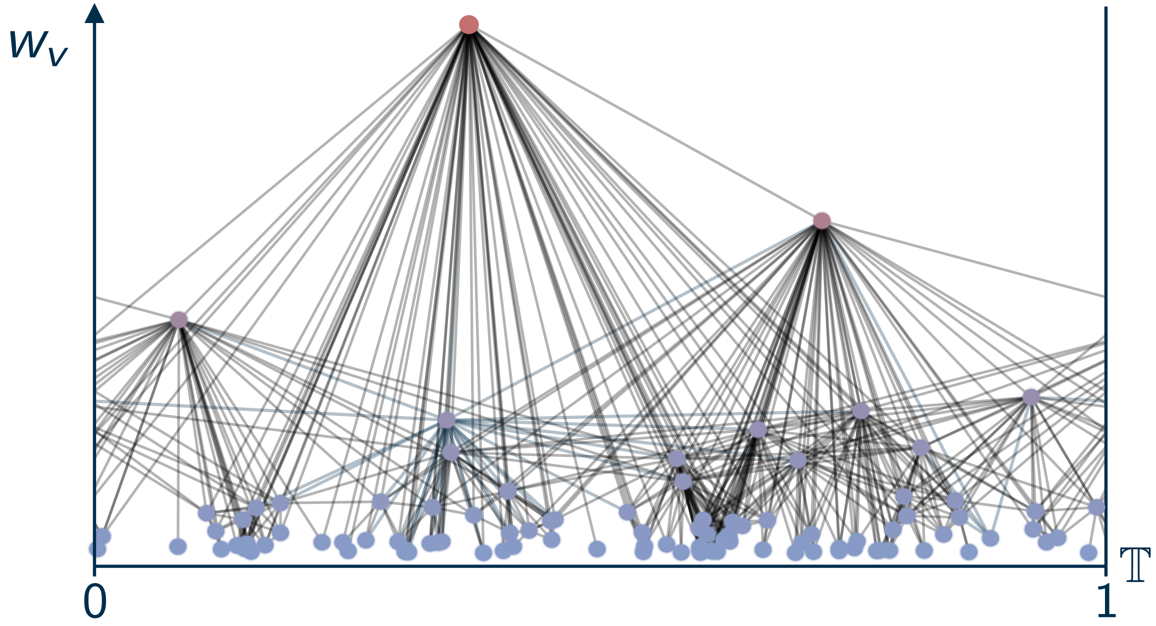
- more or less a special case of GIRG



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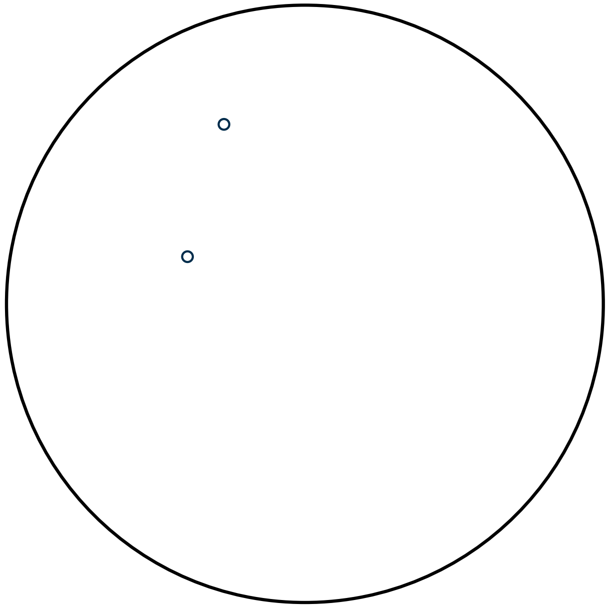

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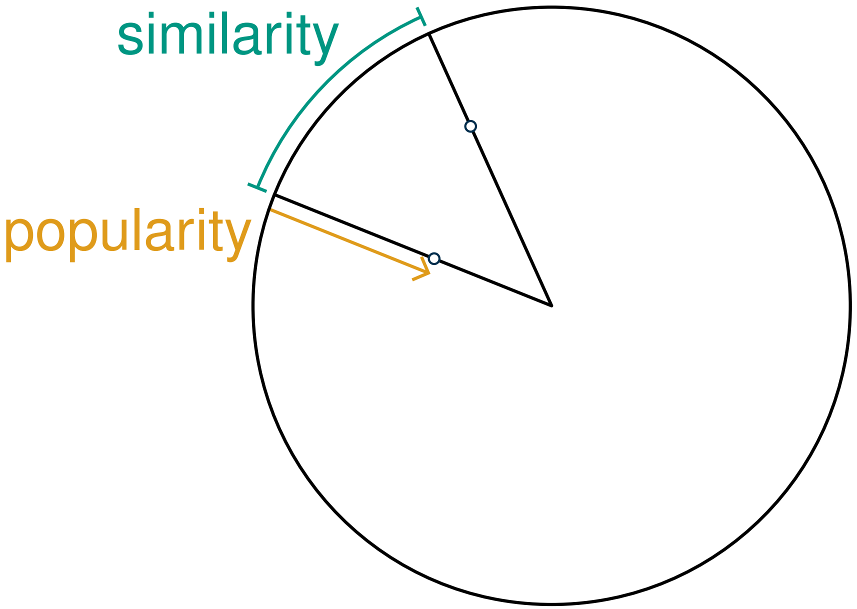

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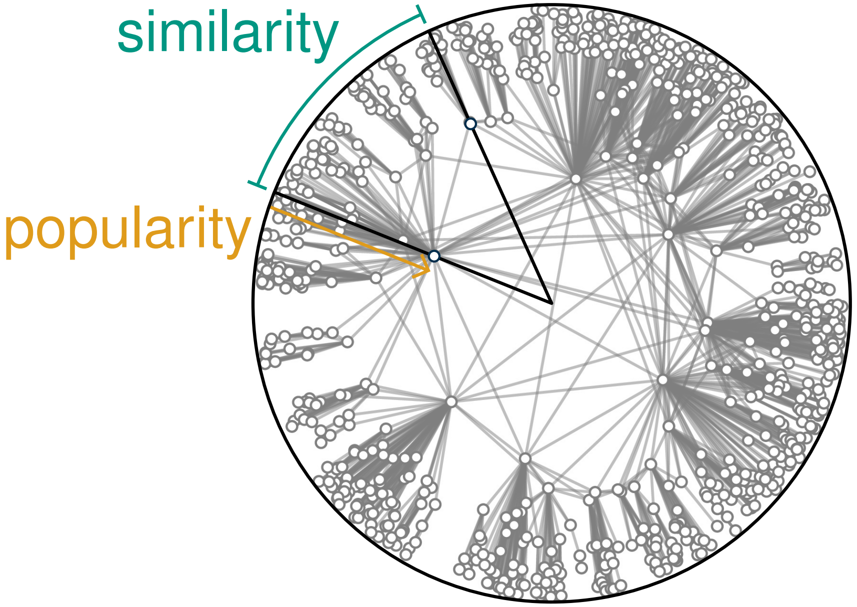

Overview: GIRGs and HRGs

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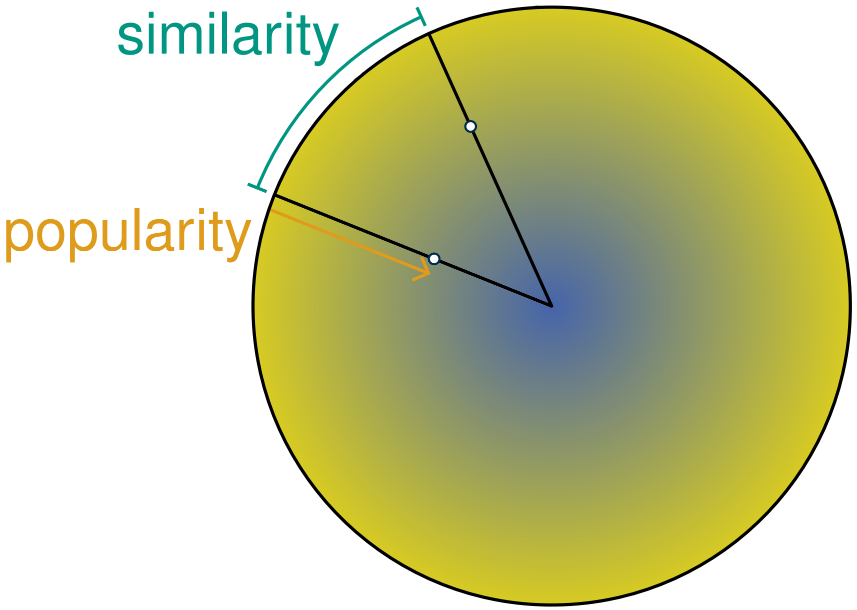

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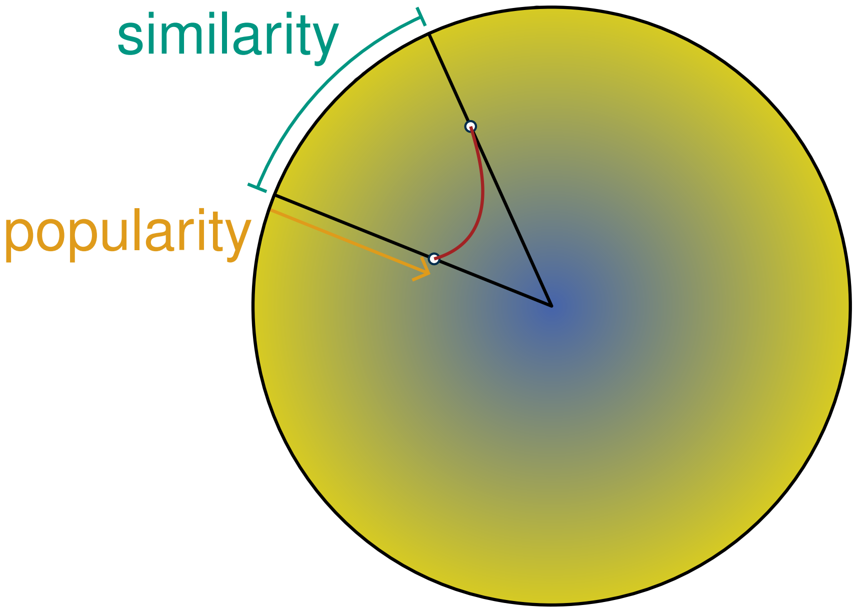

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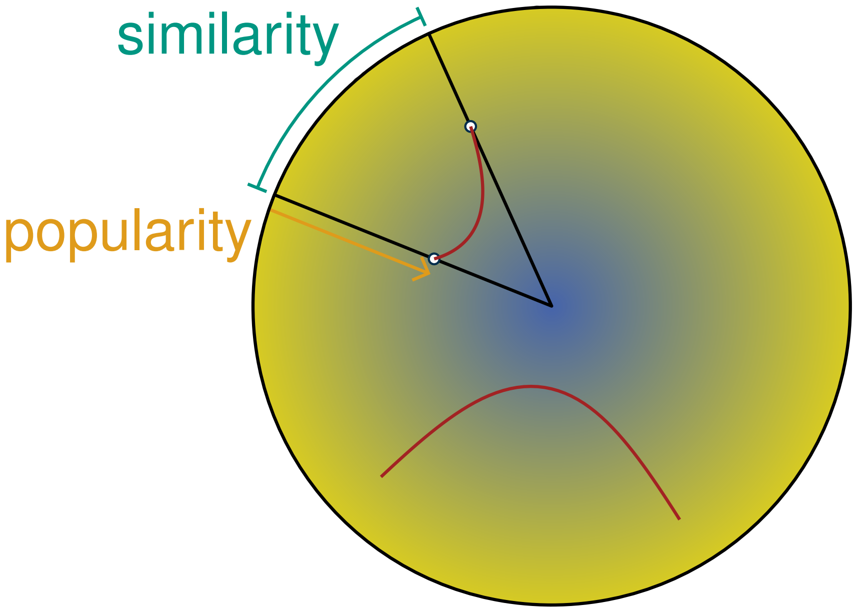

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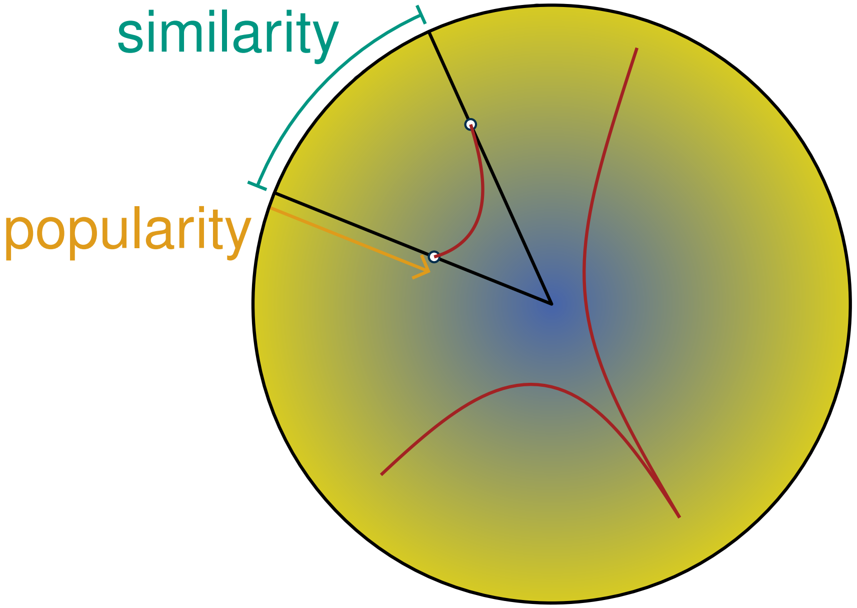

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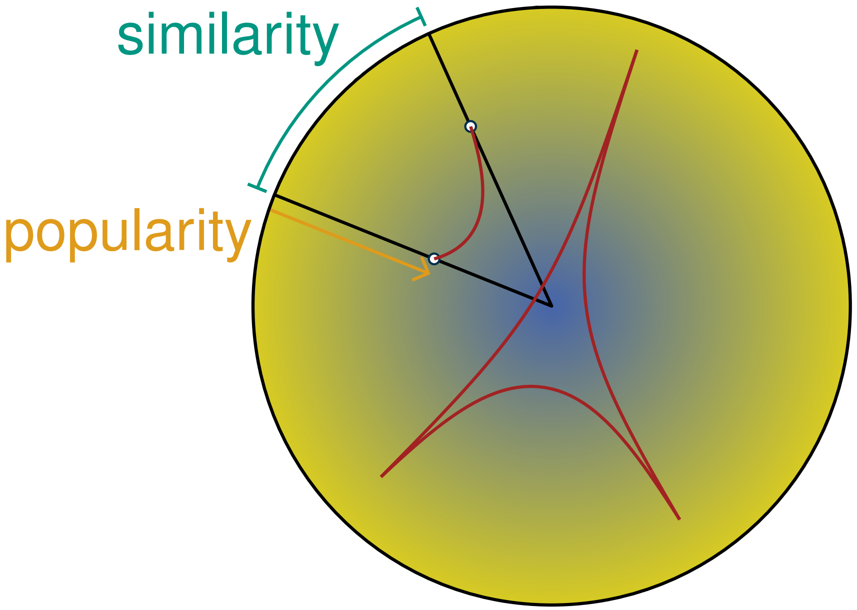

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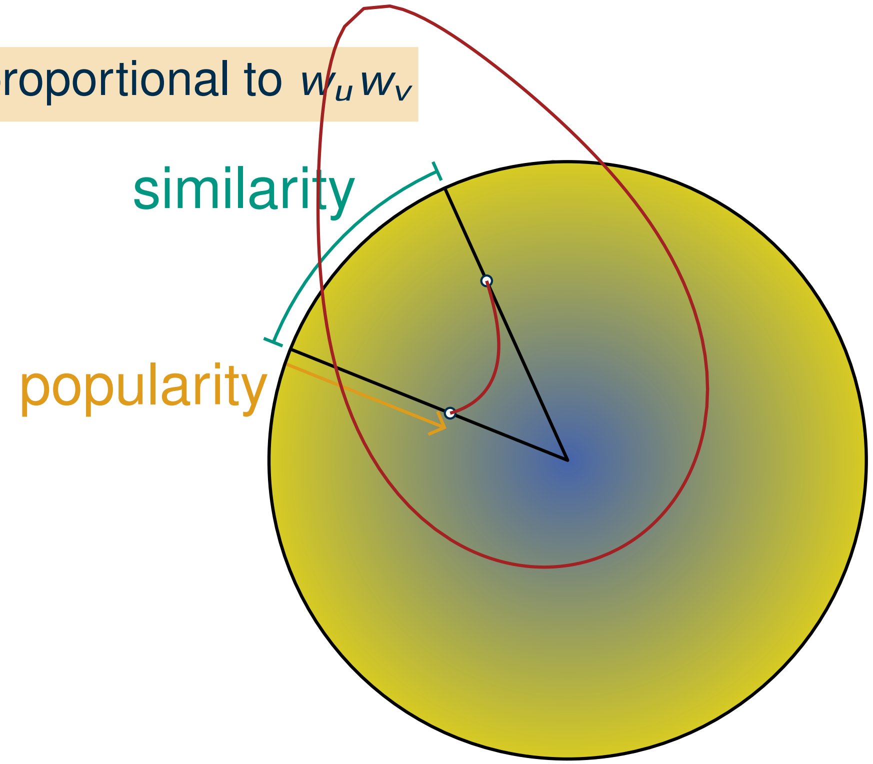

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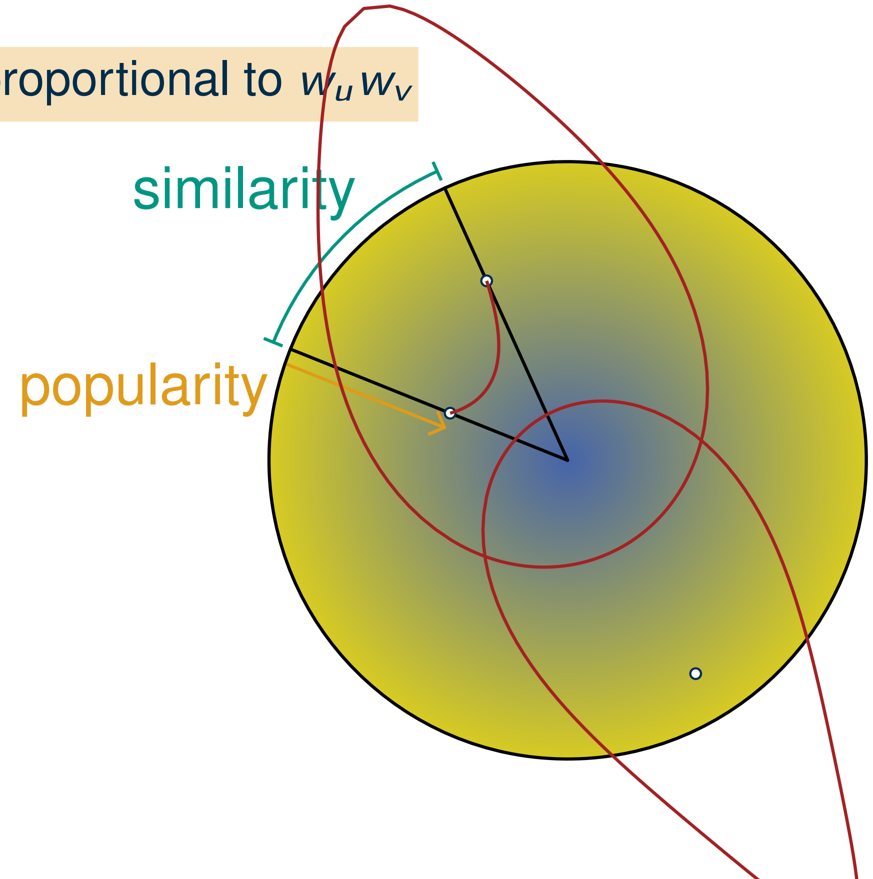

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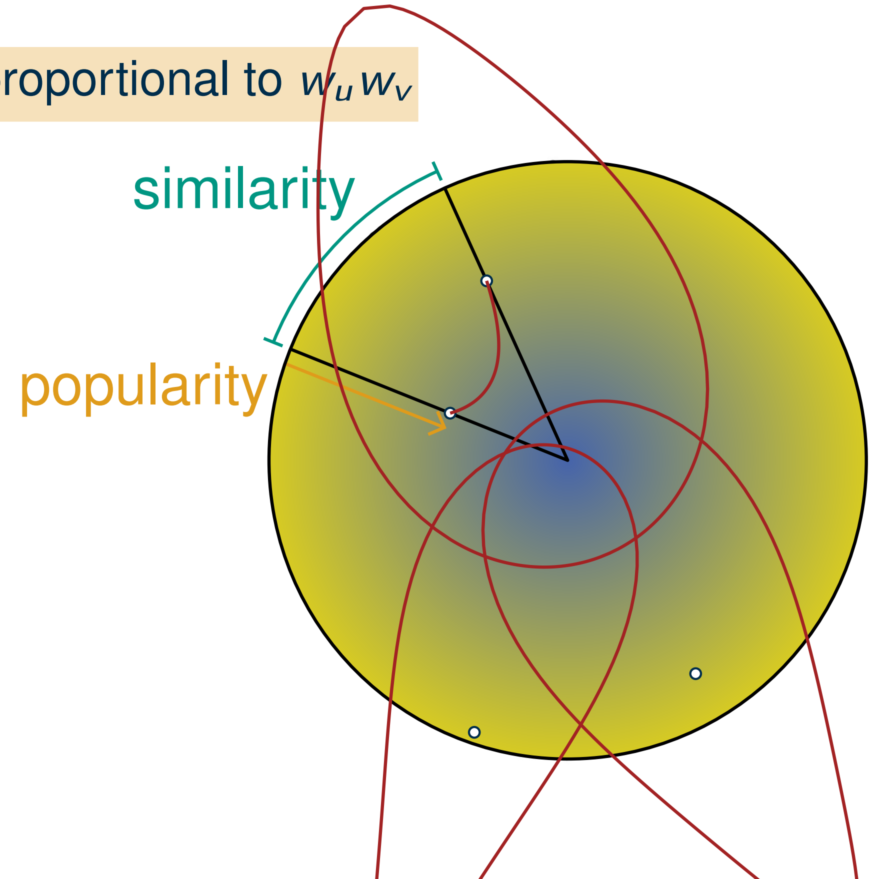

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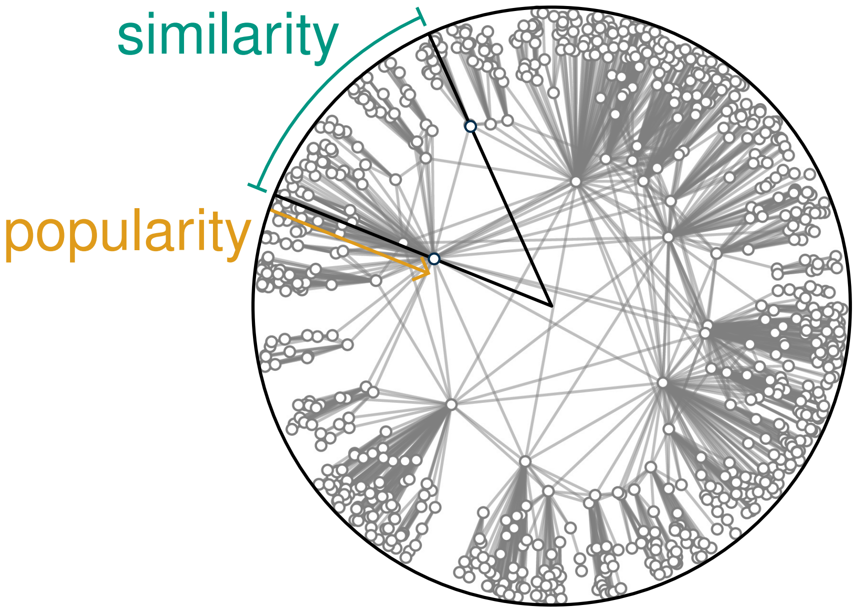

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Weights and positions

- random weights in $[1, \infty)$ with PDF $f(x) = cx^{-\tau}$ or deterministic power-law weights
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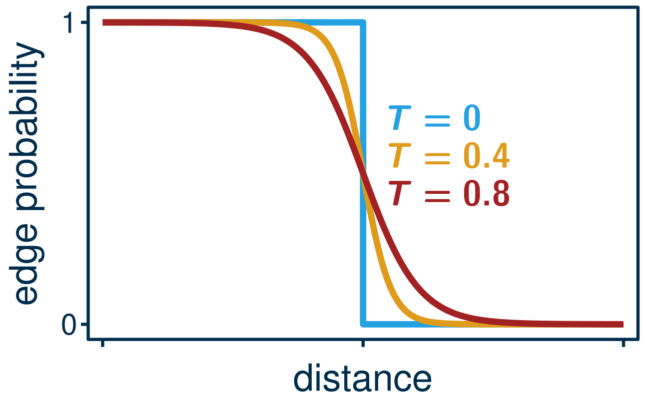
Temperature > 0

- additional parameter $T \in (0, 1)$
- connection probability $p_{uv} = \min\left\{1, \left(\frac{1}{\mathrm{dist}(u,v)^d} \cdot a\frac{w_u w_v}{n}\right)^{1/T}\right\}$
- interpolate between high locality ($T = 0$) and low locality ($T \to 1$)



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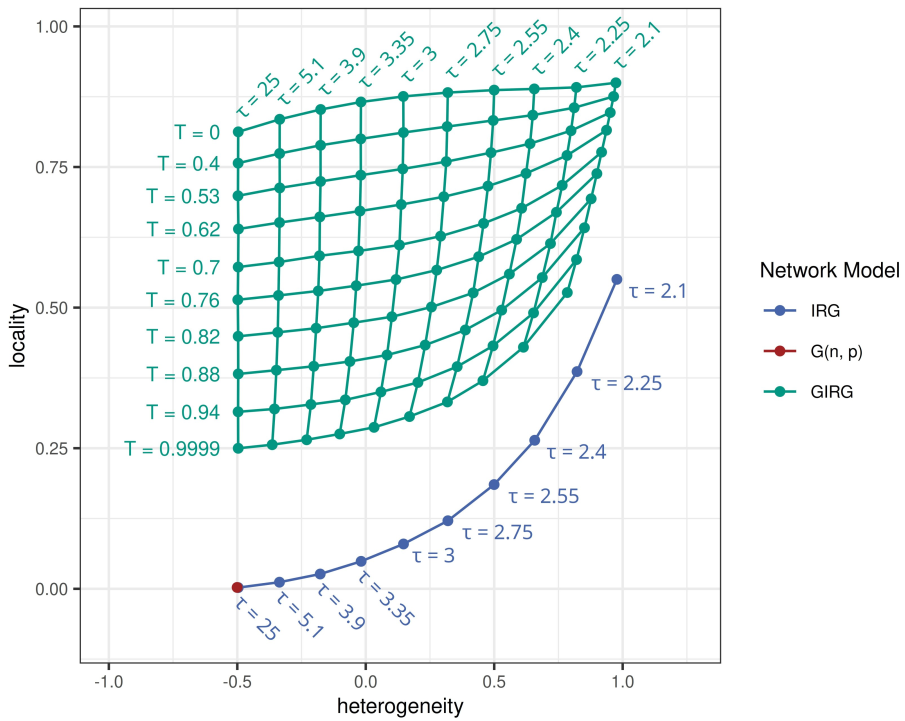

On the external validity of average-case analyses of graph algorithms [**B., Fischbeck** 2022]

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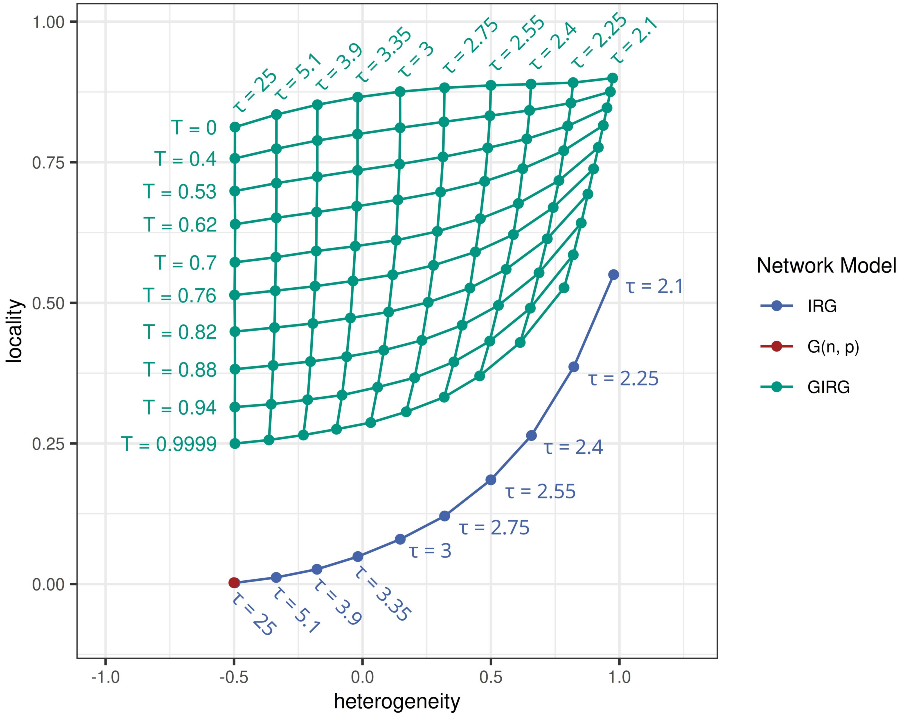

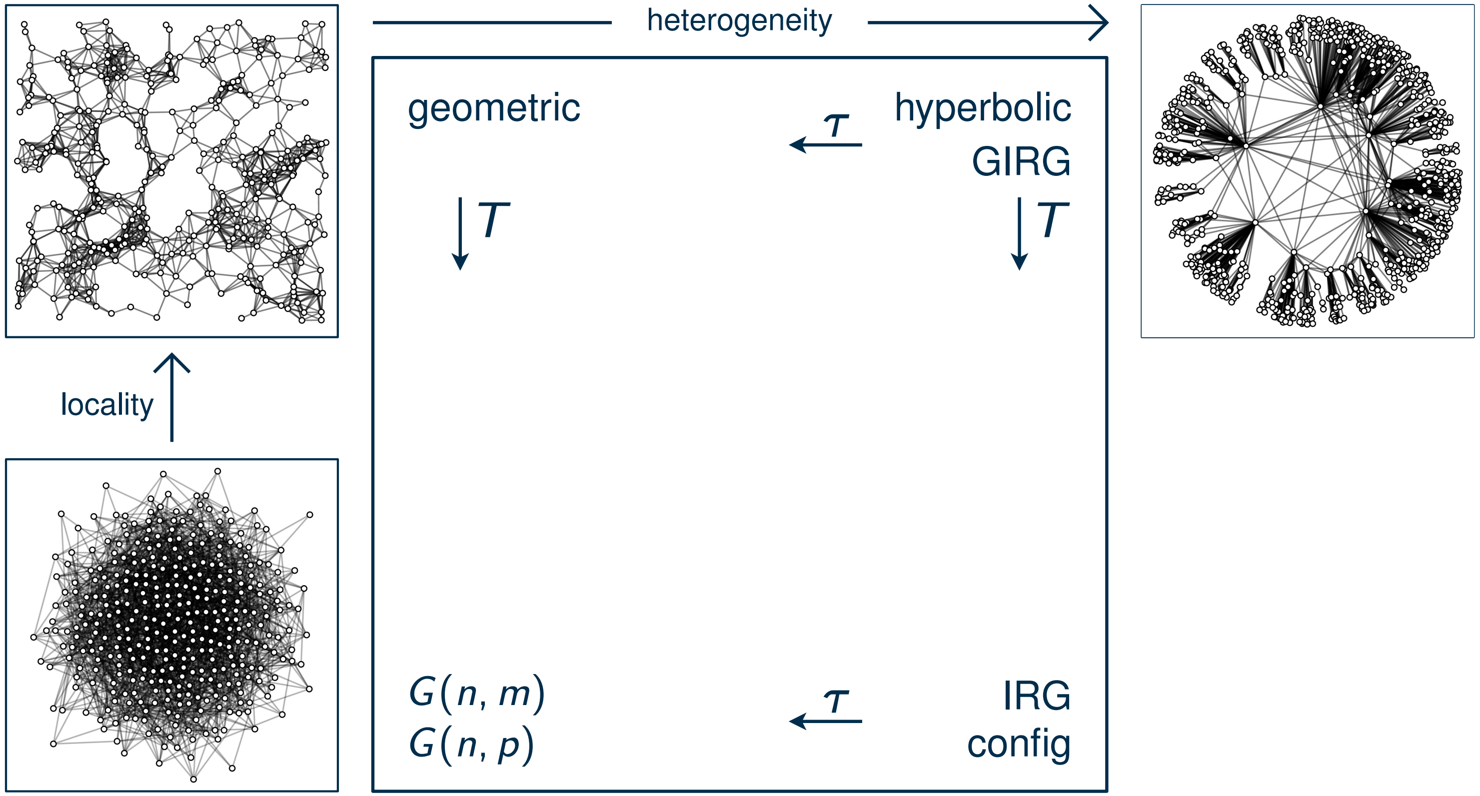

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What’s next?

Next Week

- present you work on sheet 3
- 5 minutes with slides

Sheet 4

- examine an additional algorithm
- optimize your code and workflow

Afterwards: Project

- Goal: investigate and answer research question
- presentation and written report