

# Beating the Worst Case

Practical Course – 5<sup>th</sup> Meeting

Jean-Pierre, Marcus

# Sheet 2: Recapitulation

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## Concepts

- explain the performance of bi-BFS and VC with graph parameters
- degree distribution
- locality

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## Task

- Determine suitable measures for heterogeneity and locality
- can they predict algorithm performance
  - analyze 2 other metrics



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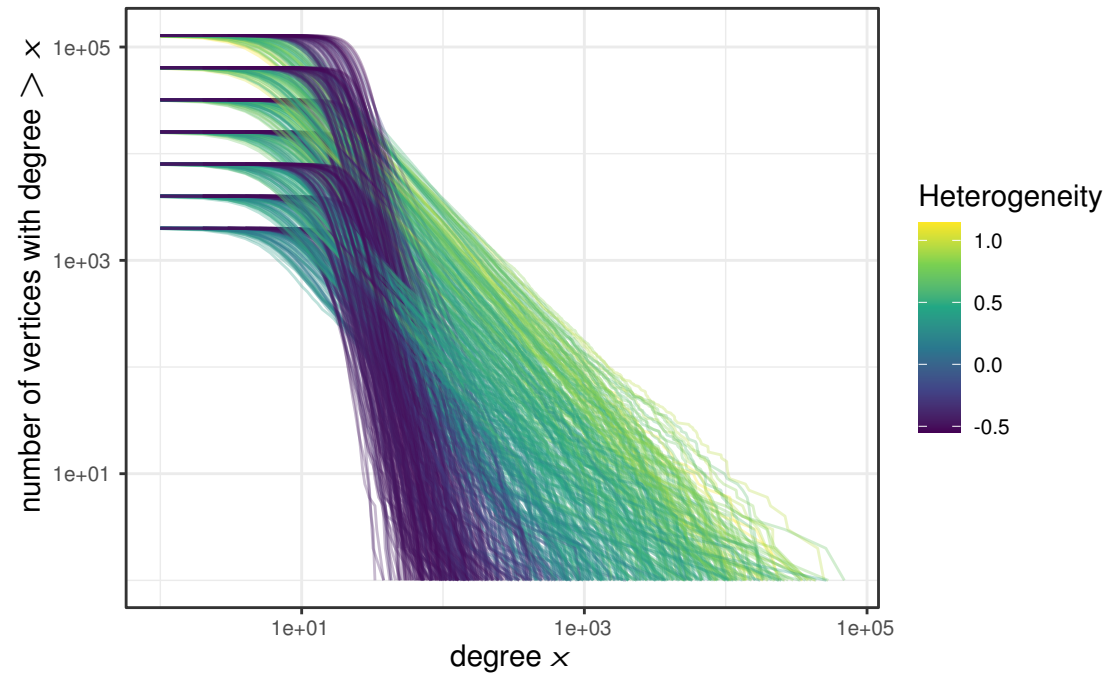
- Determine suitable measures for heterogeneity and locality
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## Methods / Tools

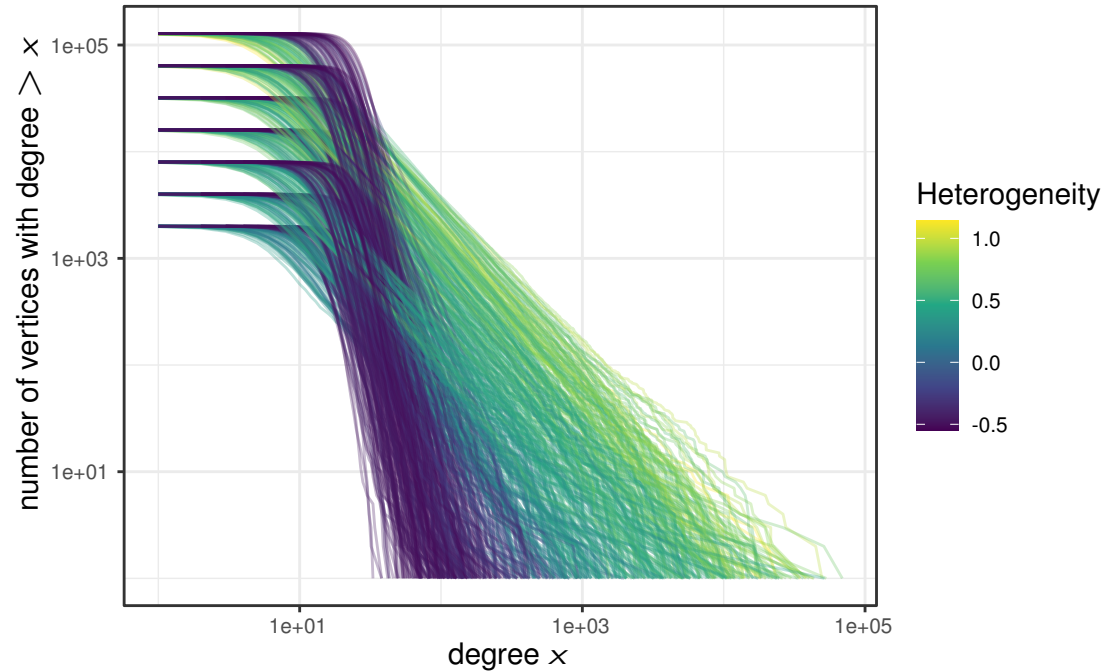
- did you change your experiment setup?
- did you establish a useful pipeline?

# Presentations

# Heterogeneity

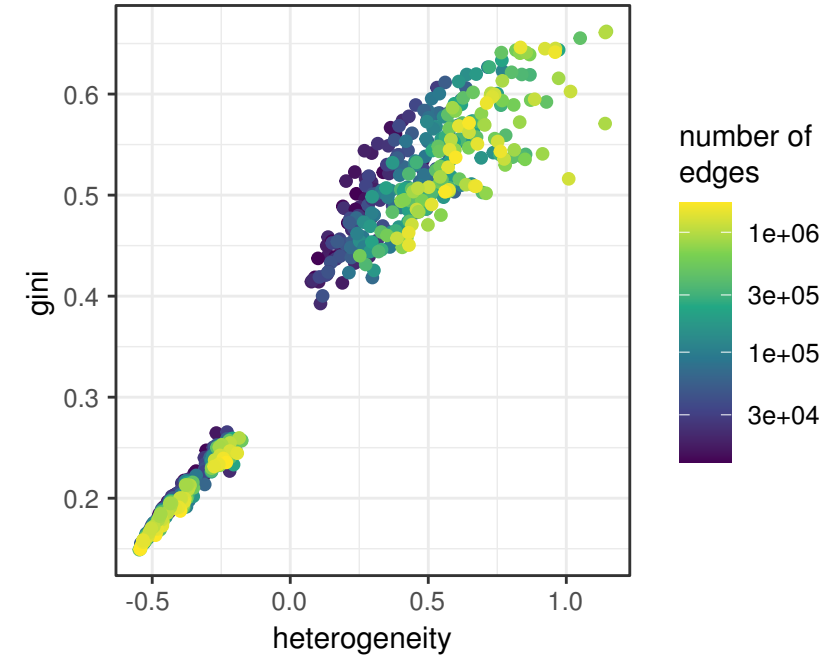
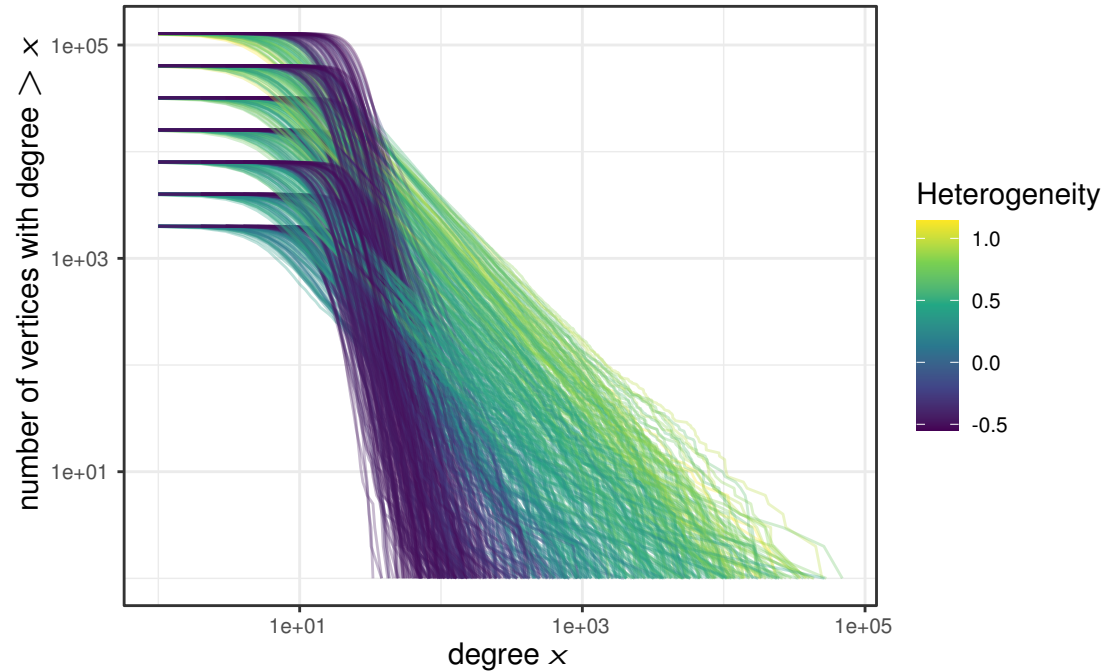


# Heterogeneity



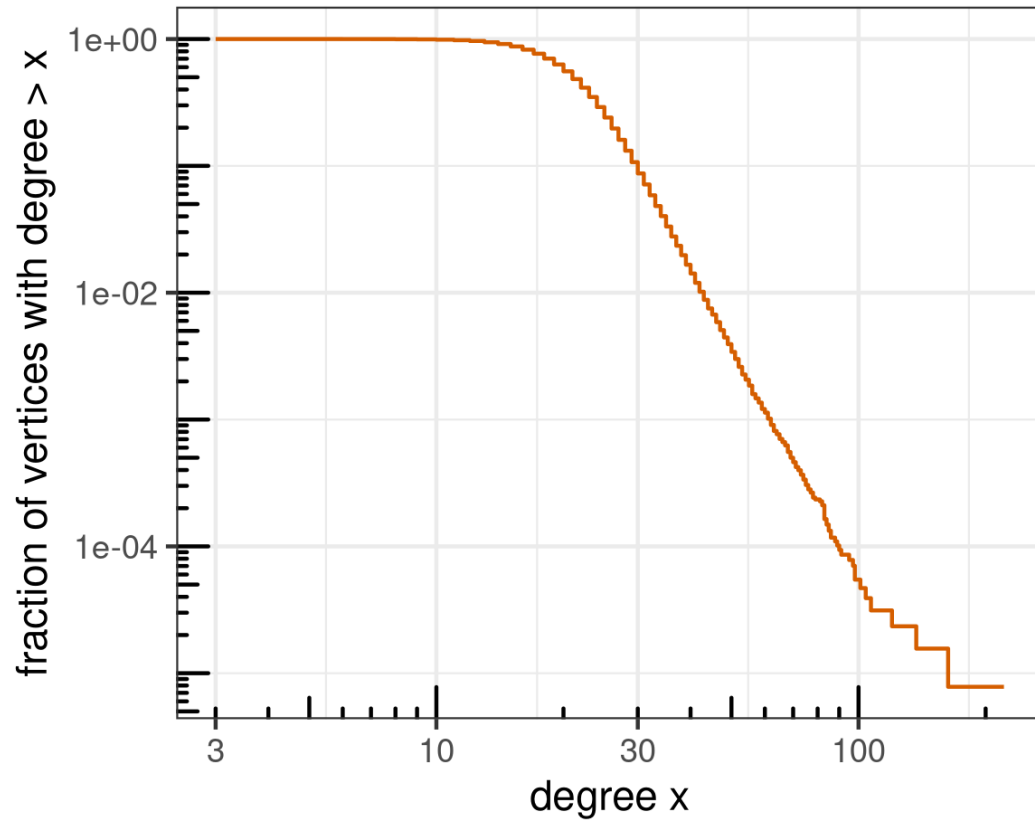
- Compute the standard deviation  $\sigma$  average  $\mu$  of the degree distribution
- Coefficient of Variation:  $\frac{\sigma}{\mu}$
- Heterogeneity:  $\log\left(\frac{\sigma}{\mu}\right)$

# Heterogeneity

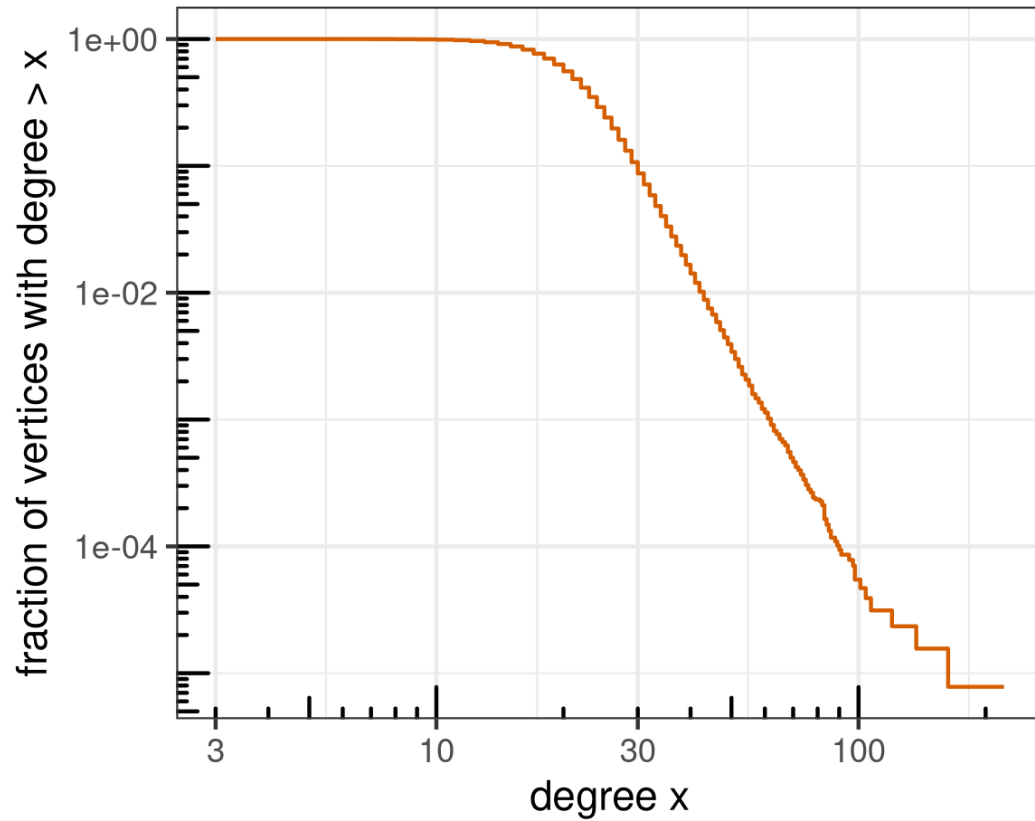


- Compute the standard deviation  $\sigma$  average  $\mu$  of the degree distribution
- Coefficient of Variation:  $\frac{\sigma}{\mu}$
- Heterogeneity:  $\log\left(\frac{\sigma}{\mu}\right)$
- strong correlation with gini coefficient
- slight dependence on graph size

# Why is the CCDF Flat?

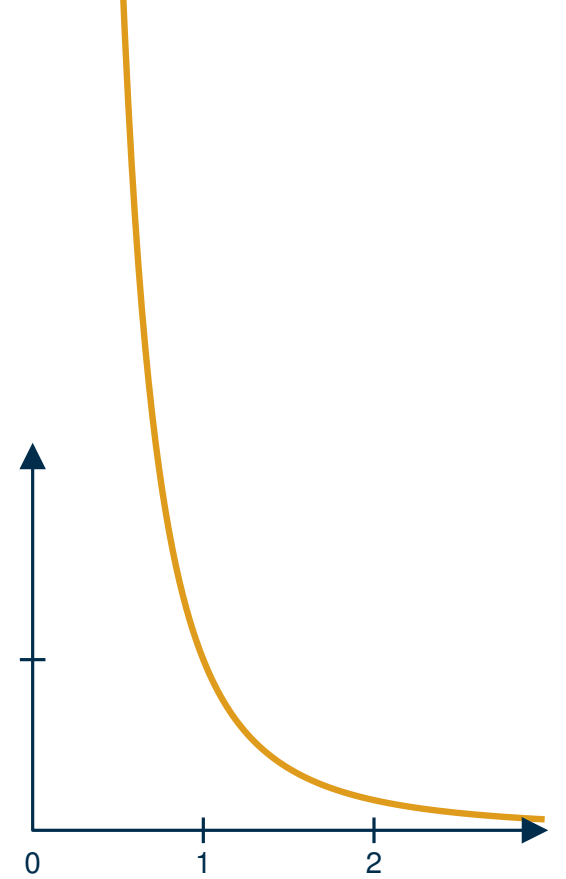


# Why is the CCDF Flat?

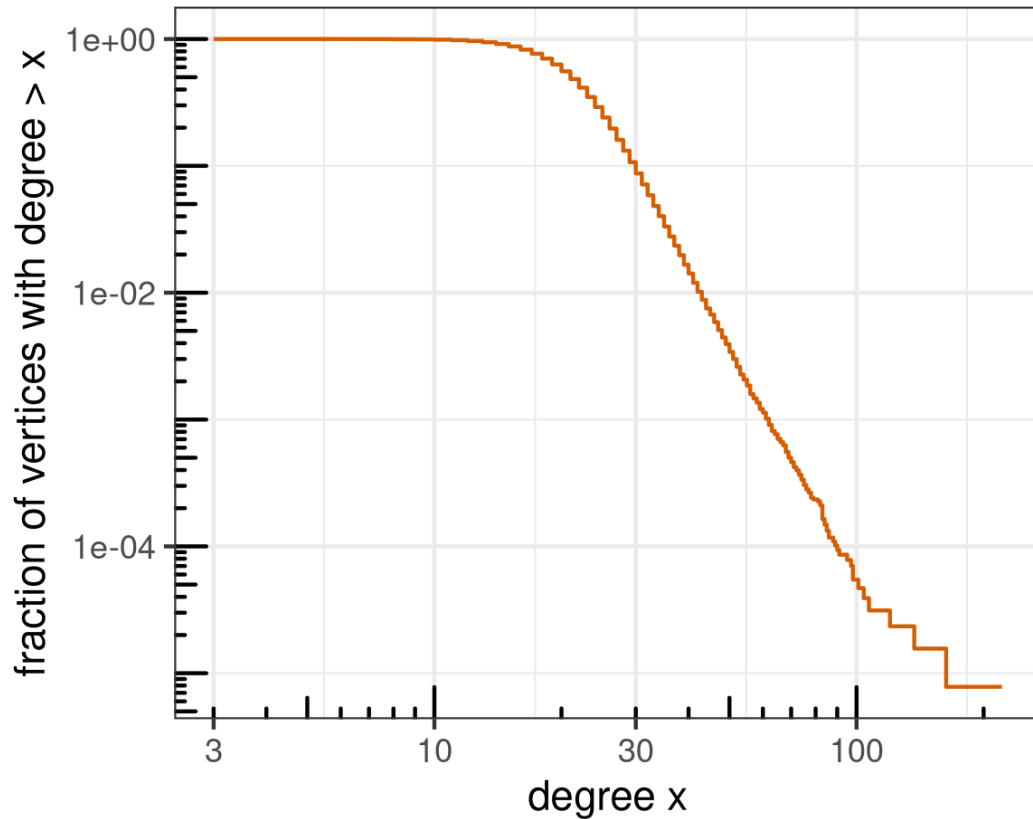


## Power-Law Distribution

■  $f(x) = cx^{-\tau}$

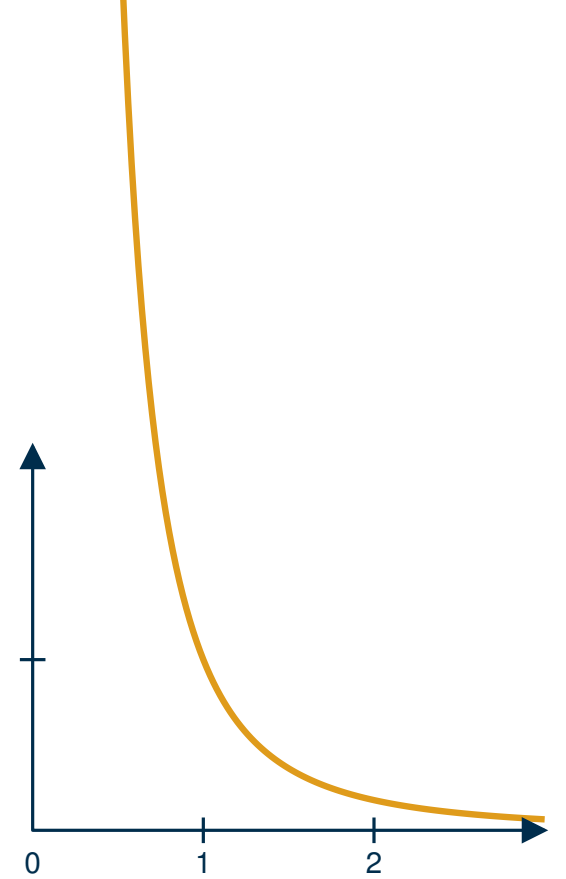


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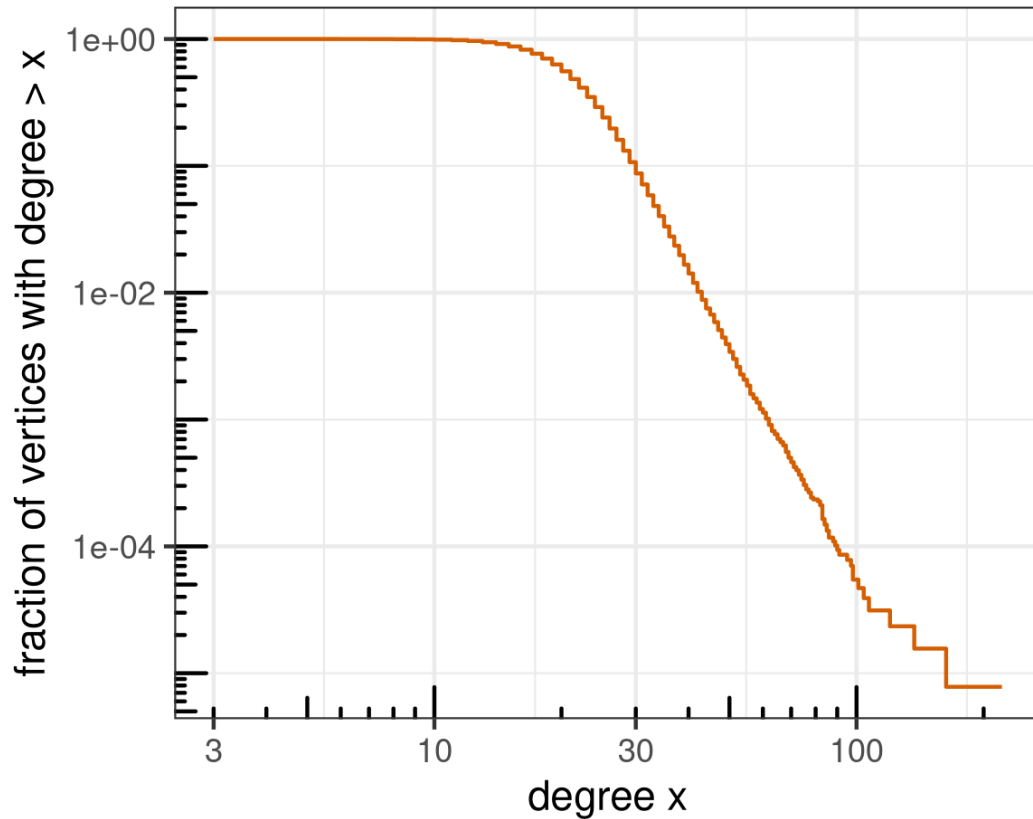
## Power-Law Distribution

- $f(x) = cx^{-\tau}$
- needs  $x_{\min}$  to be well defined, otherwise probability goes towards infinity



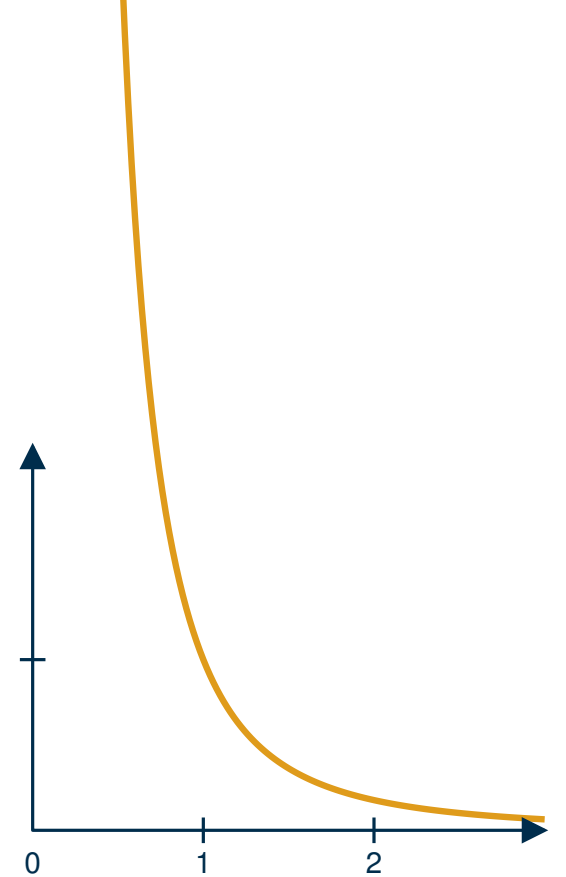


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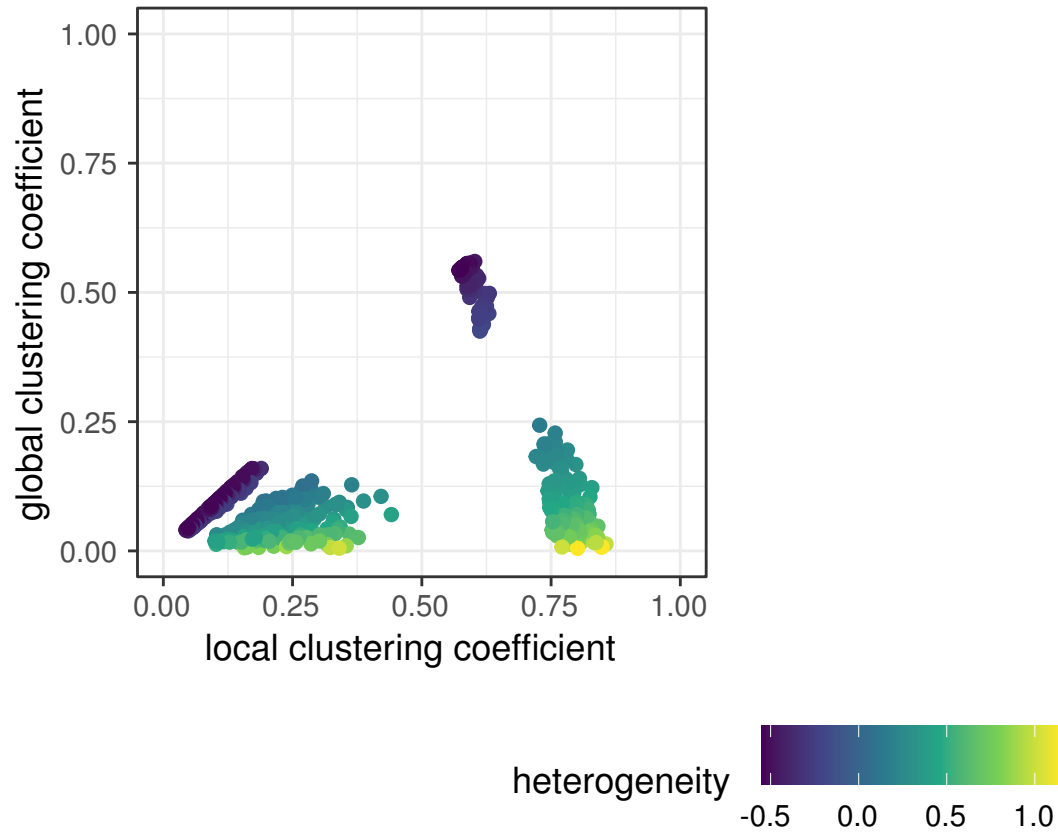
## Power-Law Distribution

- $f(x) = cx^{-\tau}$
- needs  $x_{\min}$  to be well defined, otherwise probability goes towards infinity
- Distribution follows a power law: it follows  $f(x)$  for  $x > x_{\min}$ 
  - $x \leq x_{\min}$  are irrelevant



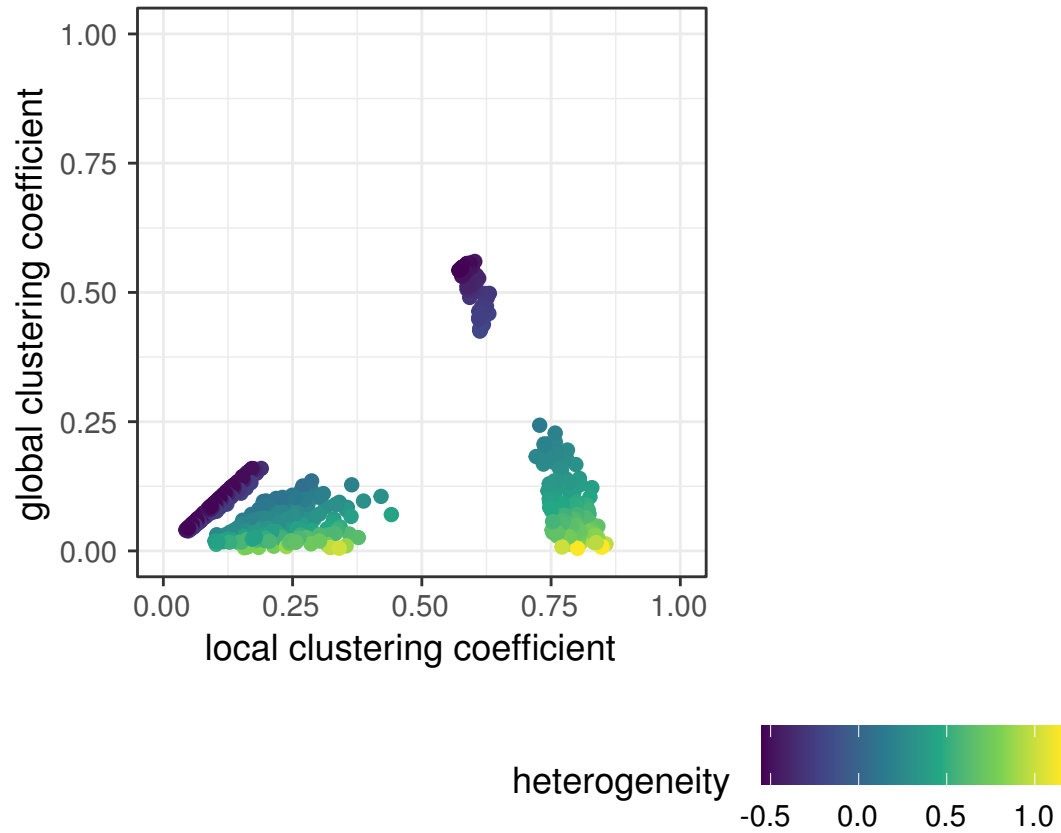
# Locality

Clustering Coefficient Correlations

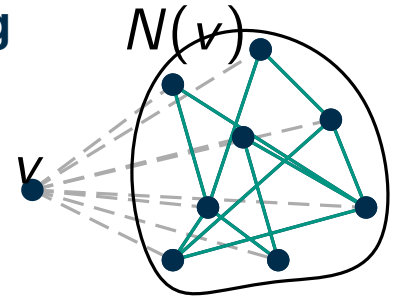


# Locality

Clustering Coefficient Correlations

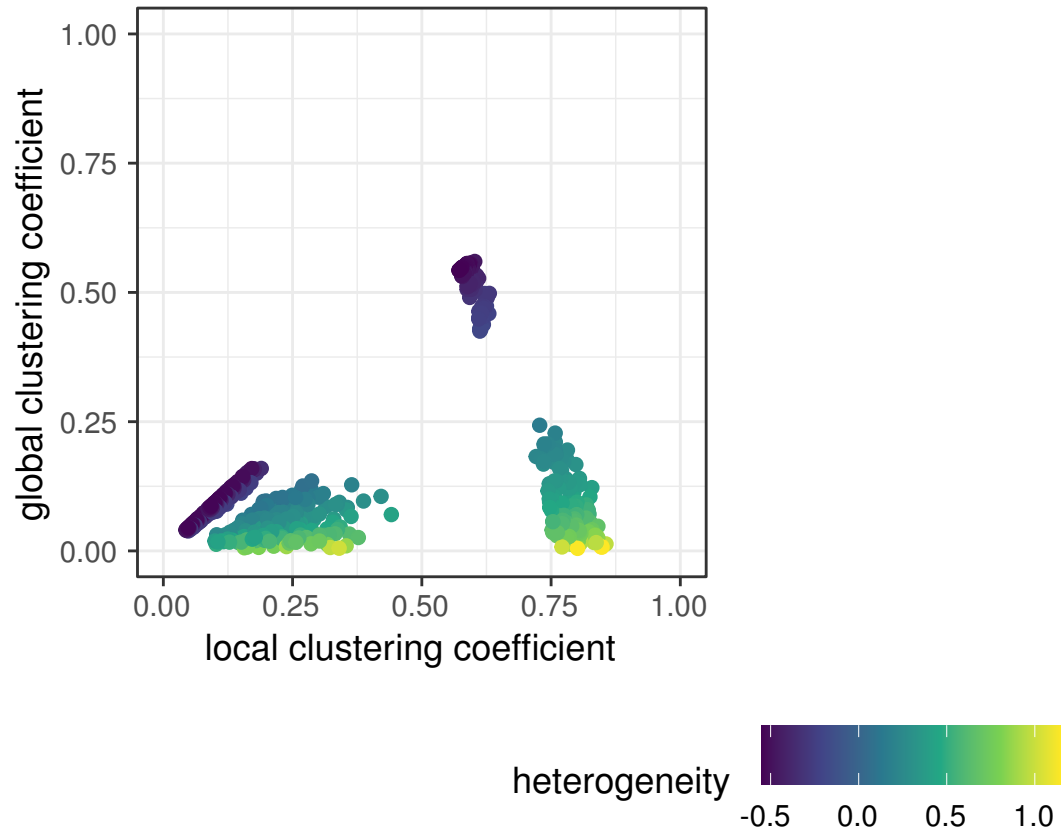


Local Clustering Coefficient

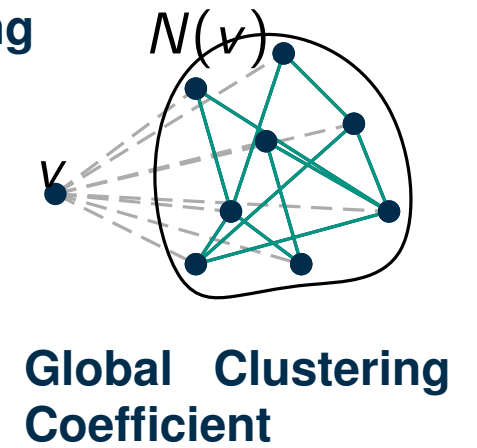
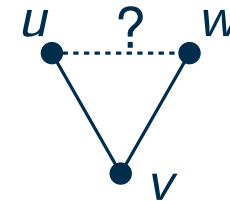


# Locality

Clustering Coefficient Correlations



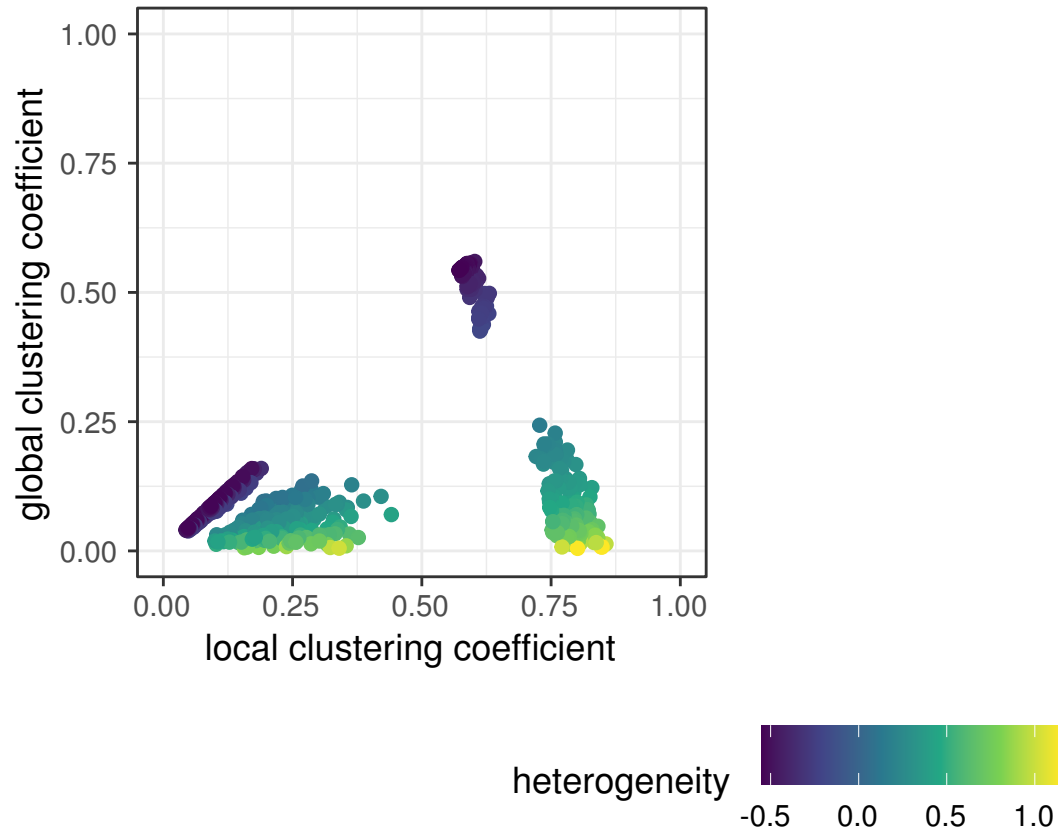
**Local Clustering Coefficient**



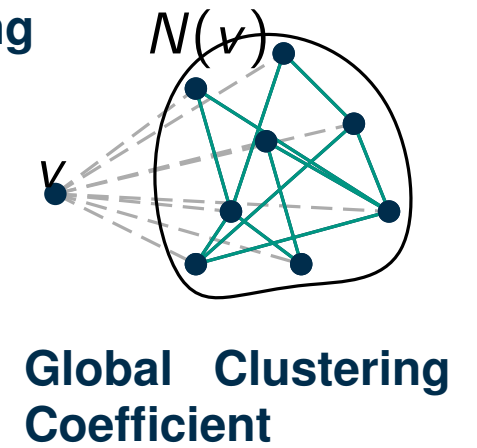
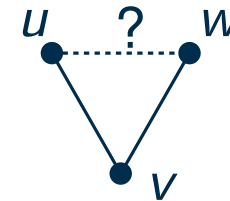
**Global Clustering Coefficient**

# Locality

Clustering Coefficient Correlations



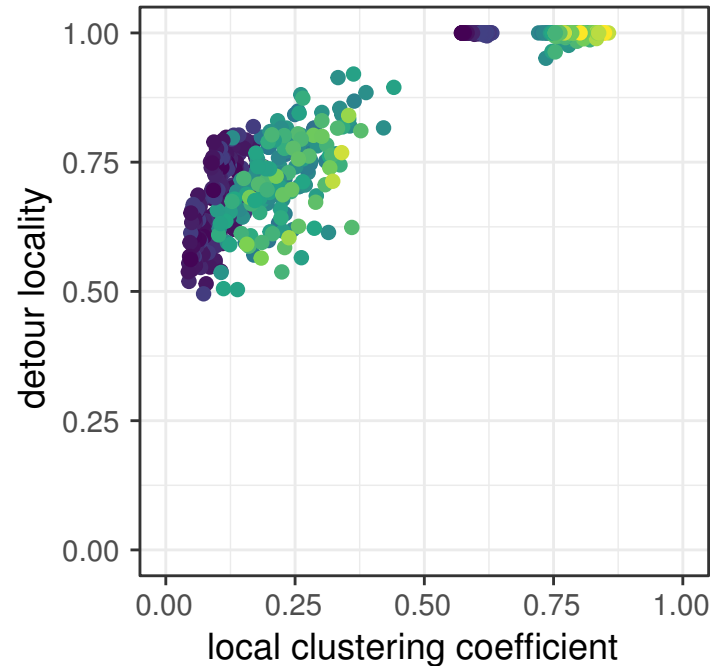
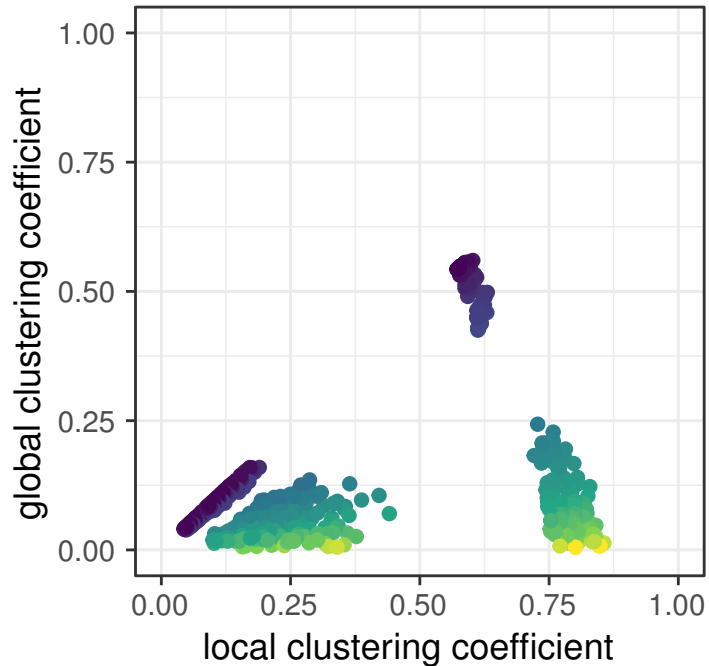
**Local Clustering Coefficient**



- global clustering coefficient does not work on heterogeneous graphs

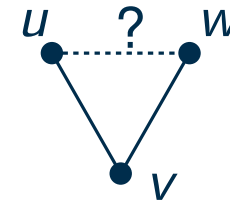
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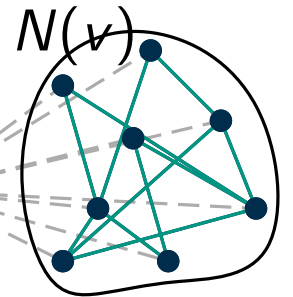


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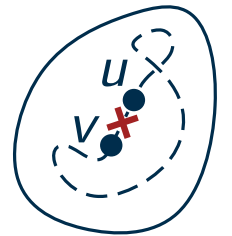
Local Clustering Coefficient



Detour Locality

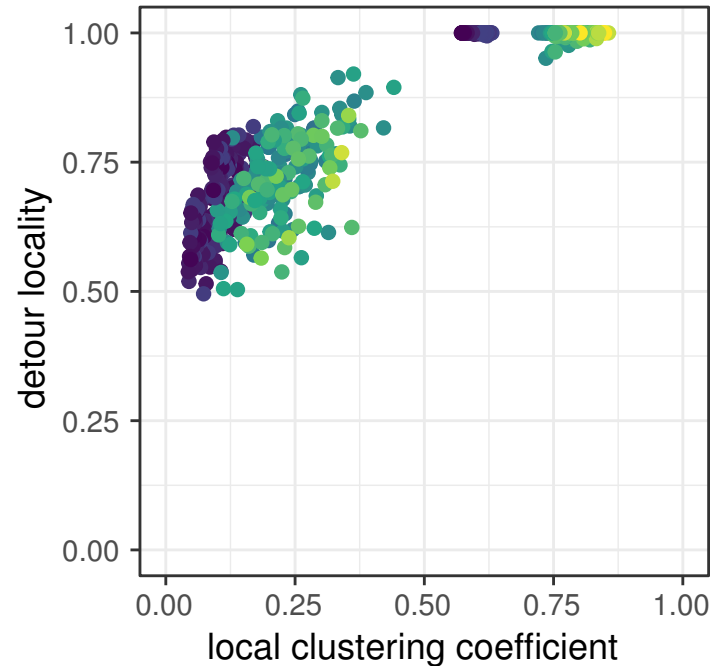
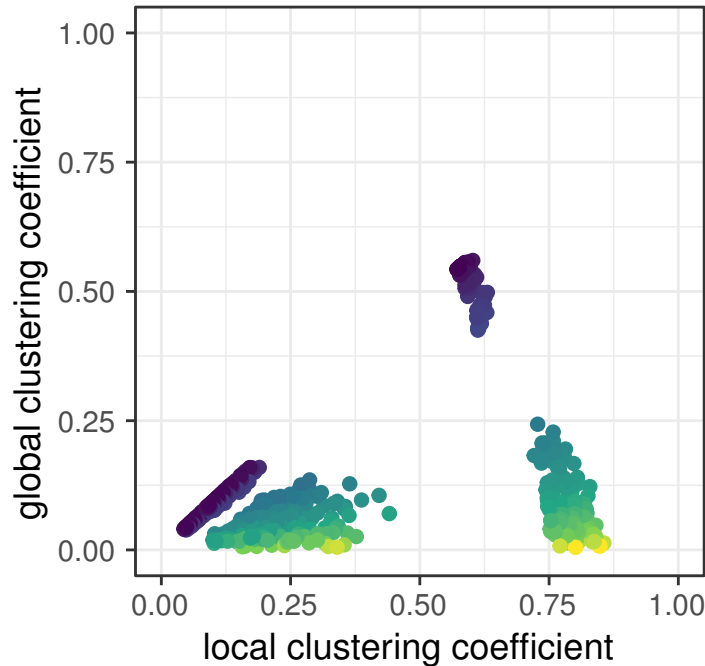


Global Clustering Coefficient

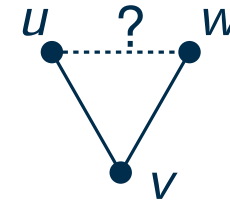


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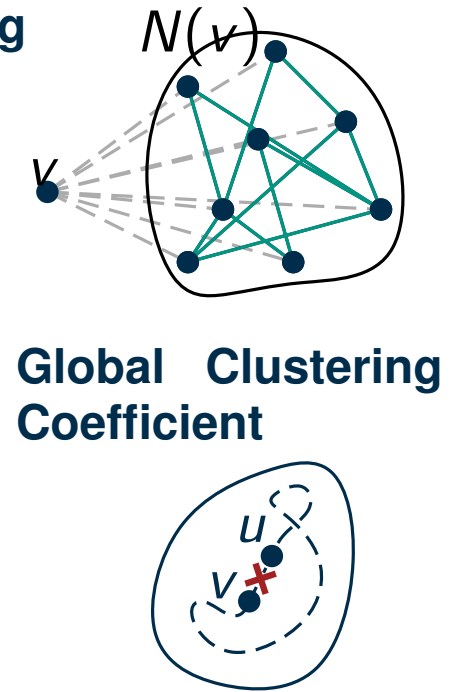
Clustering Coefficient Correlations



**Local Clustering Coefficient**

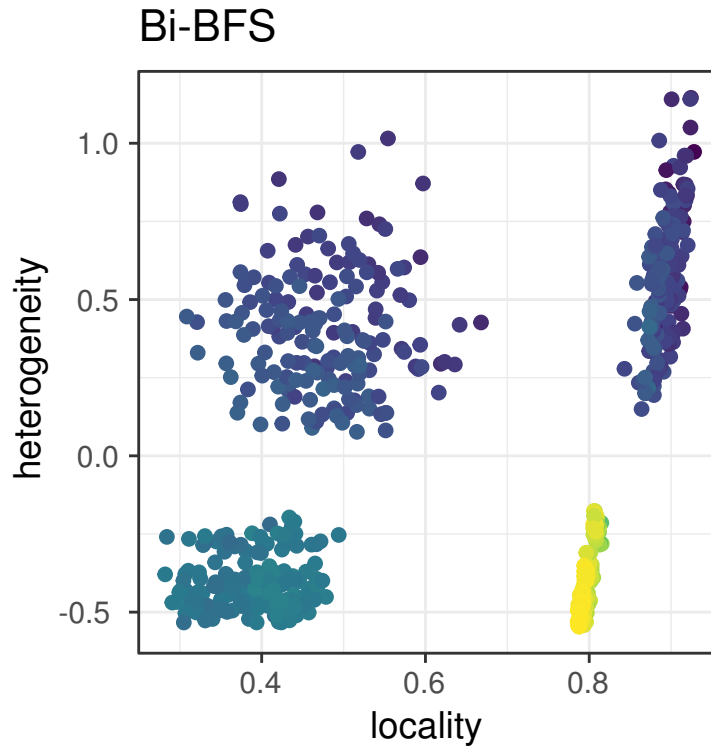


**Detour Locality**

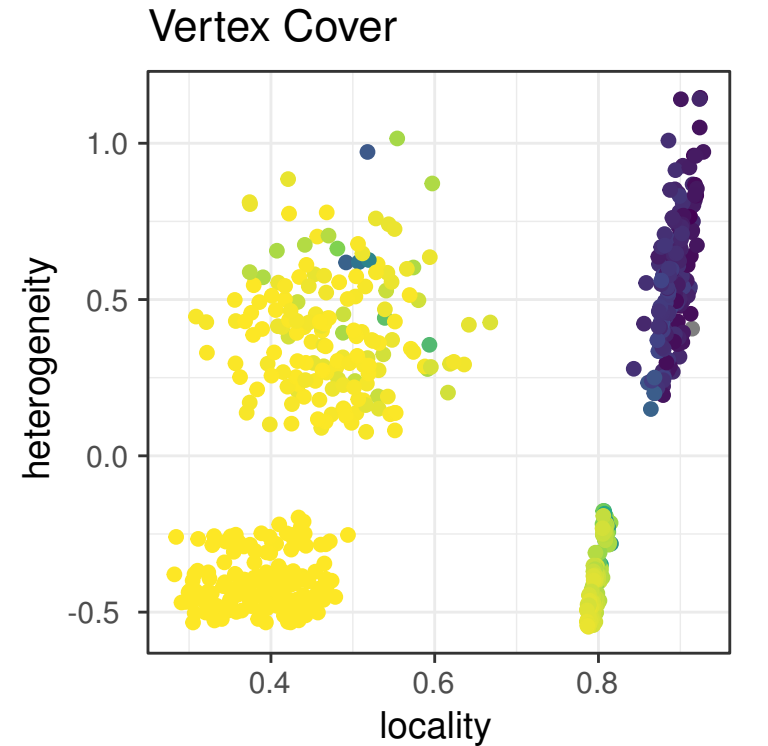
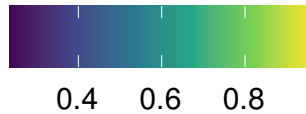


- global clustering coefficient does not work on heterogeneous graphs
- $\text{Locality} = \frac{1}{2}(\text{local} + \text{detour})$

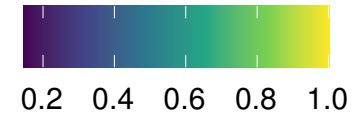
# Heterogeneity + Locality



Bi-BFS  
search exponent

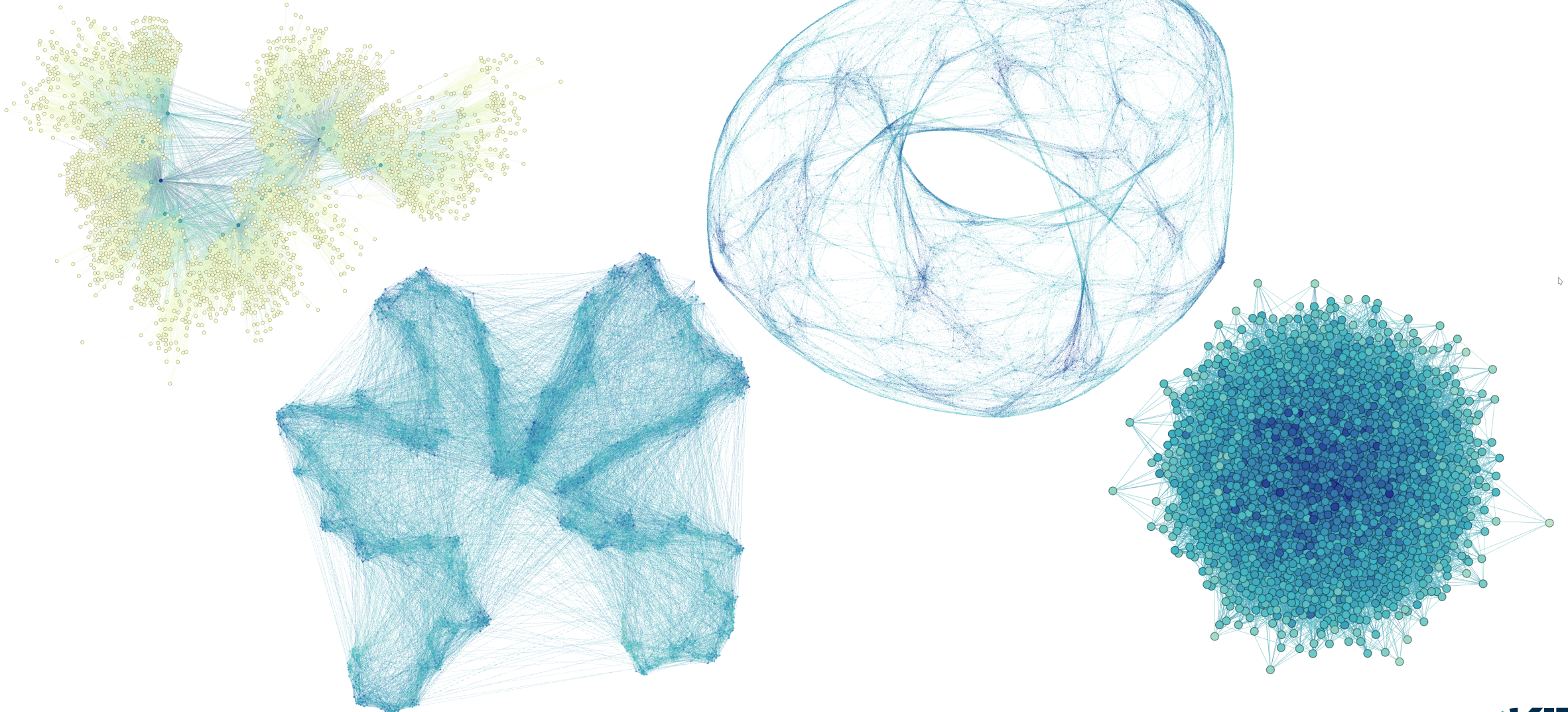


Vertex Cover  
reduce exponent



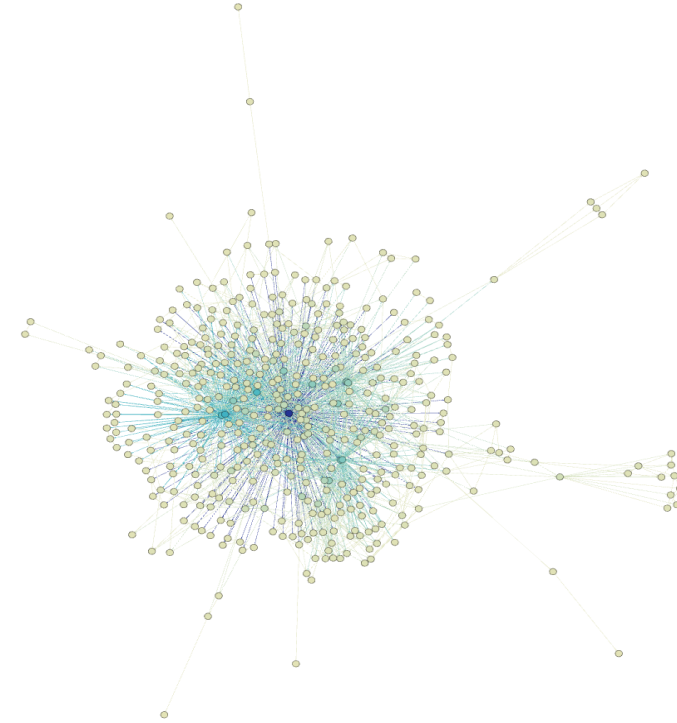


# Network Science



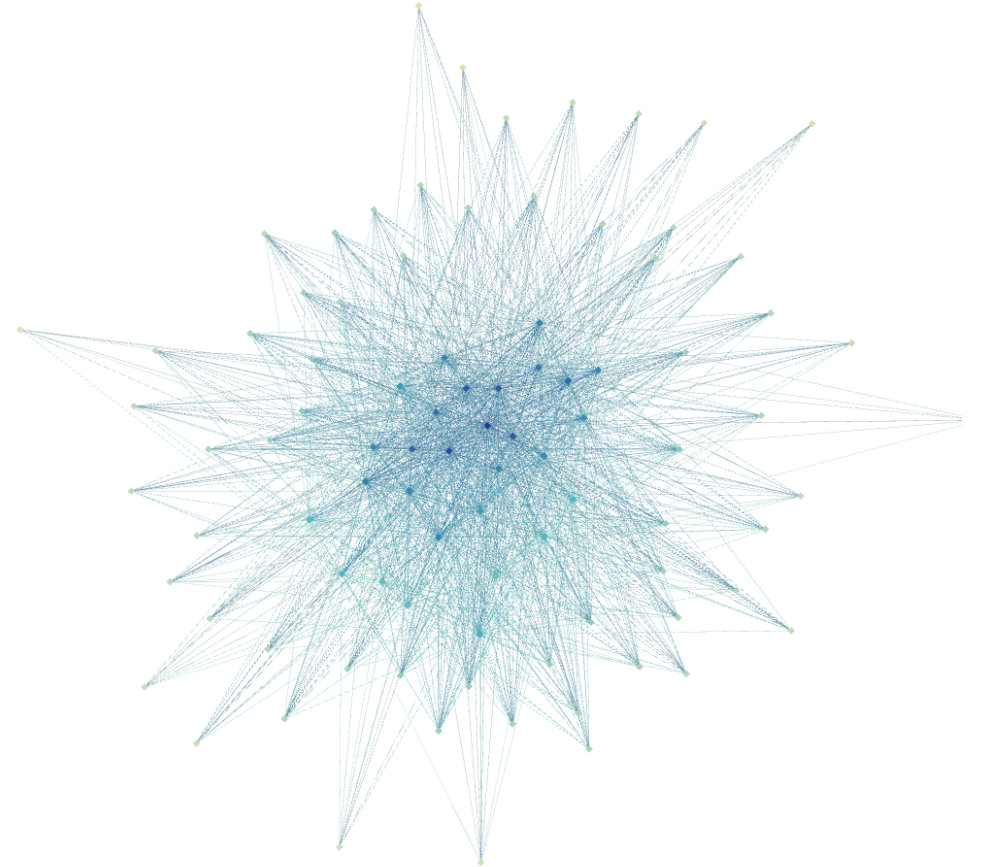
# Examples for Real-World Networks

## ■ bio-celegans



# Examples for Real-World Networks

- bio-celegans
- bn-macaque-rhesus\_cerebral-cortex\_1



# Examples for Real-World Networks

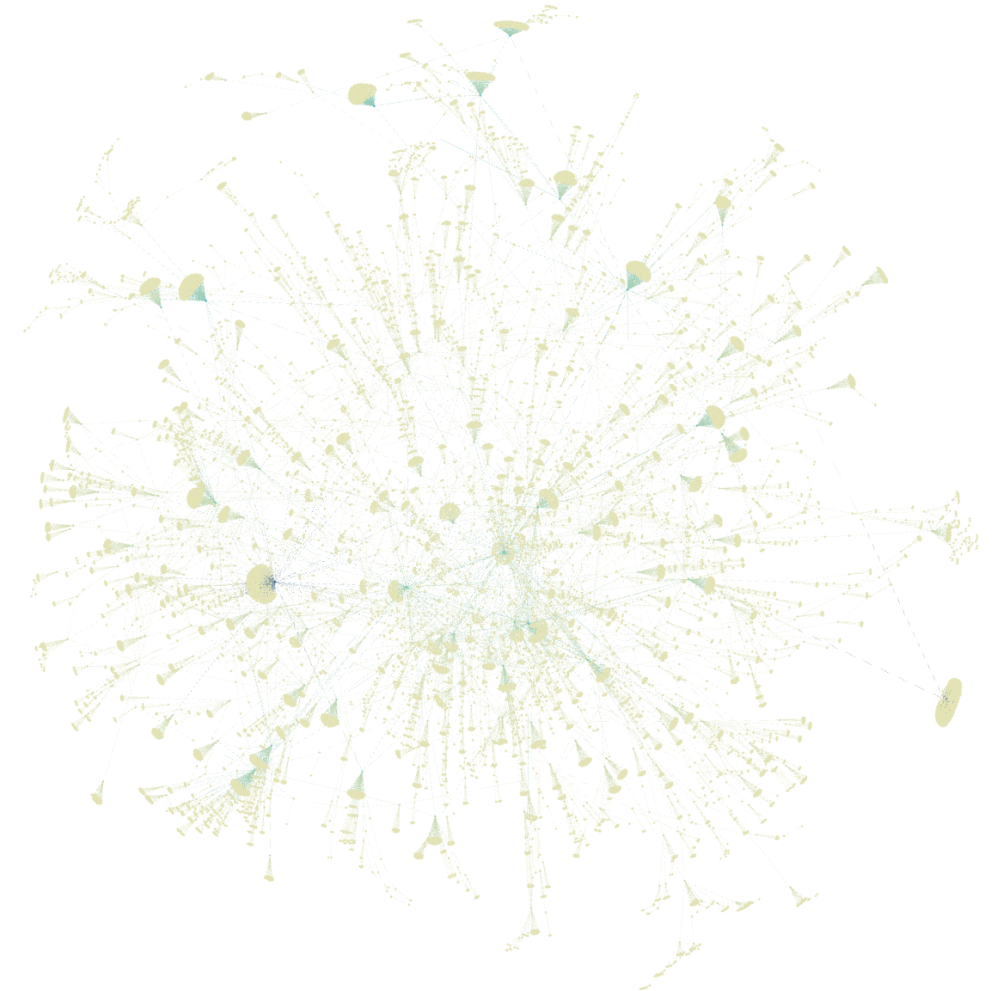
- bio-celegans
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- opsahl-powergrid





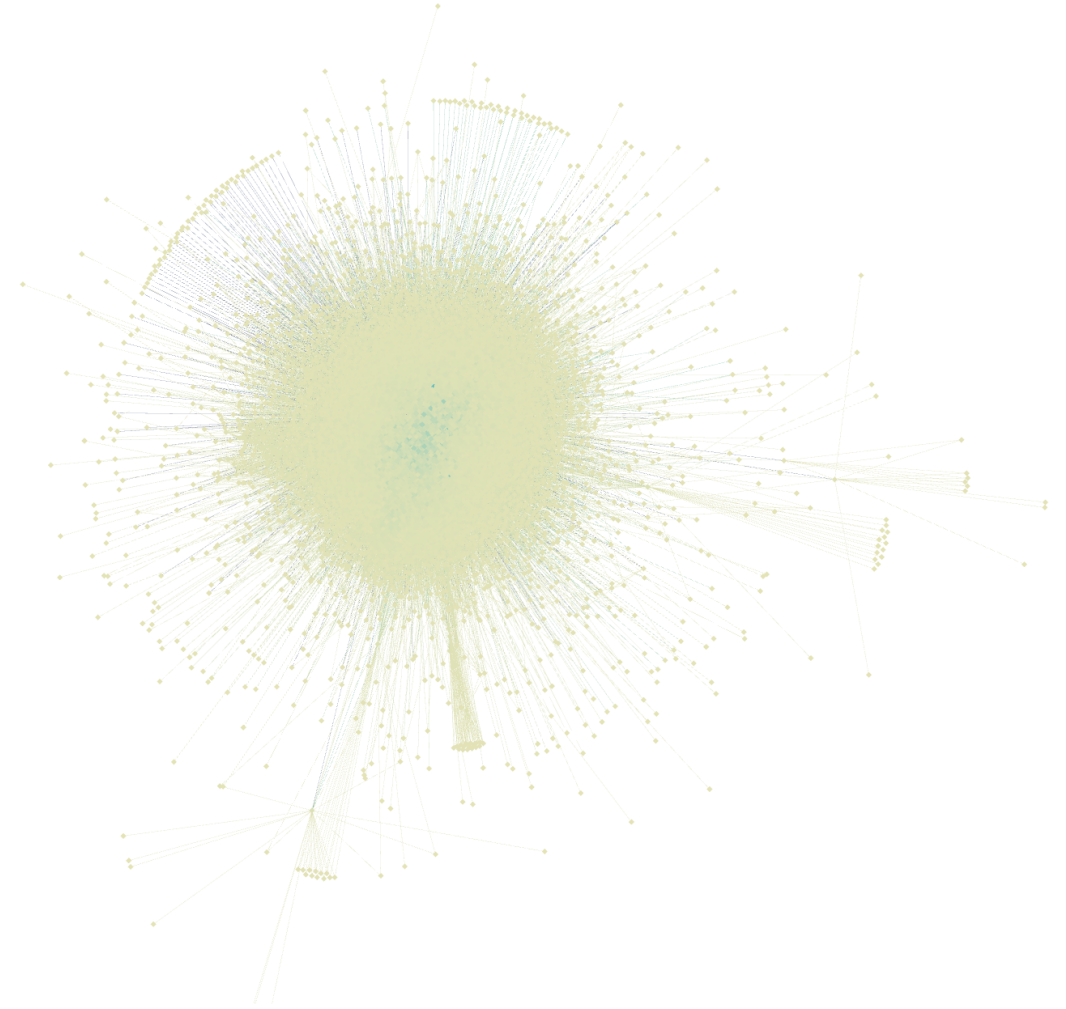
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- opsahl-powergrid
- econ-poli-large



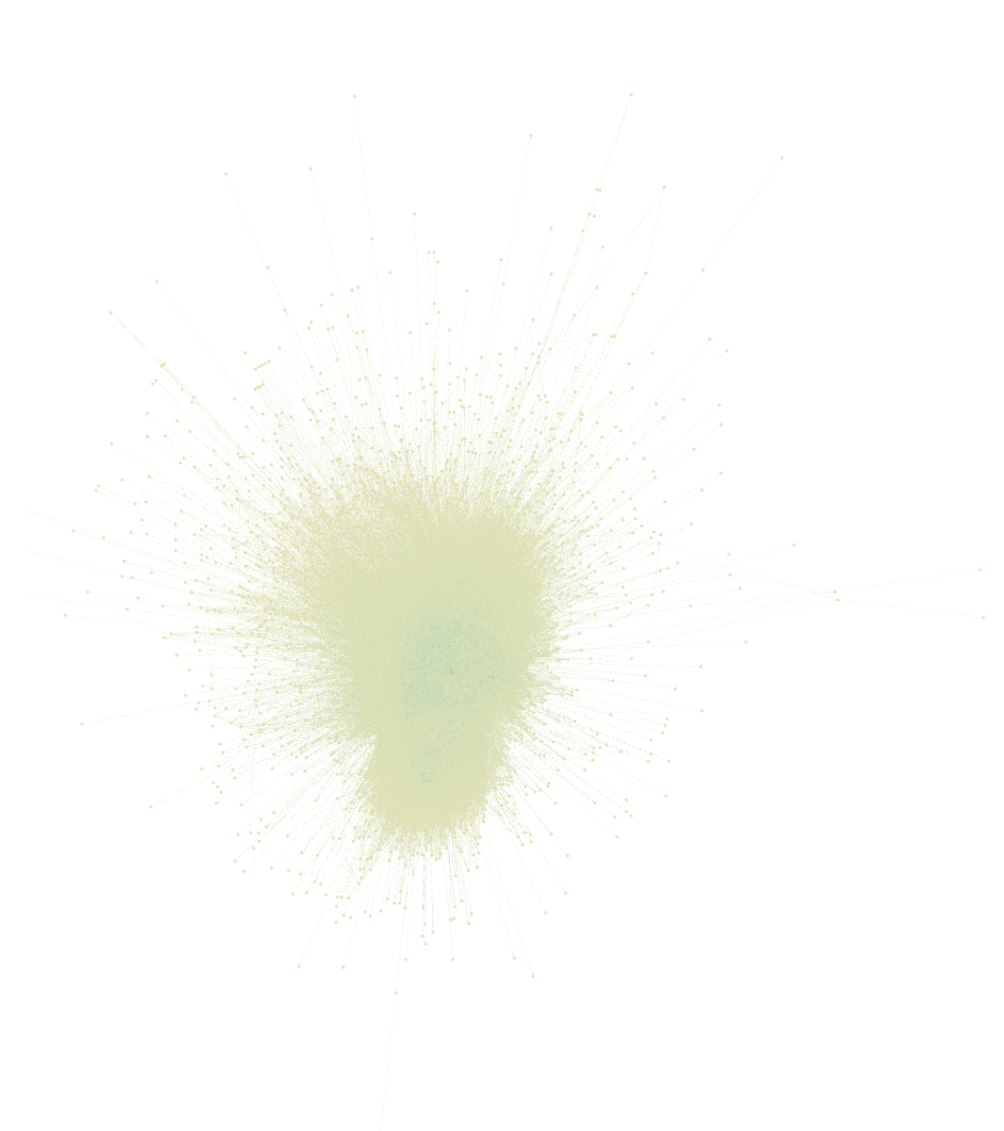
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- `bio-celegans`
- `bn-macaque-rhesus_cerebral-cortex_1`
- `opsahl-powergrid`
- `econ-poli-large`
- `bio-grid-yeast`



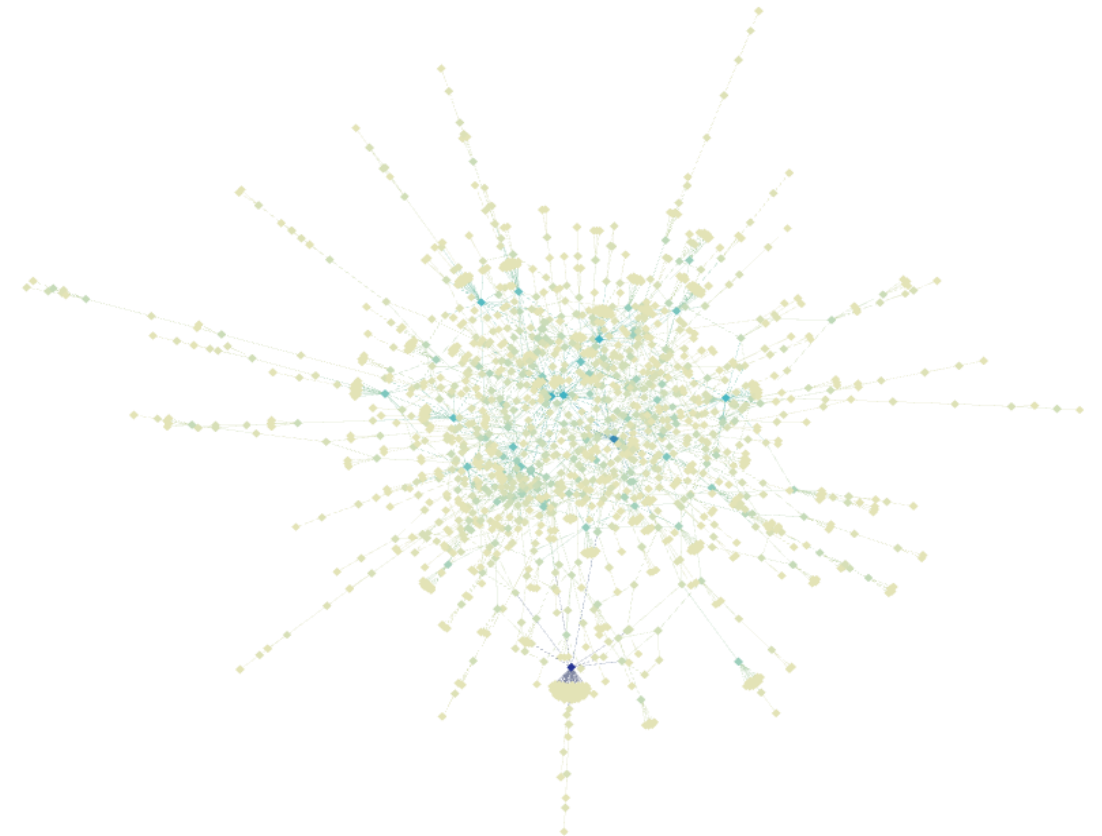
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- `socfb-Yale4`



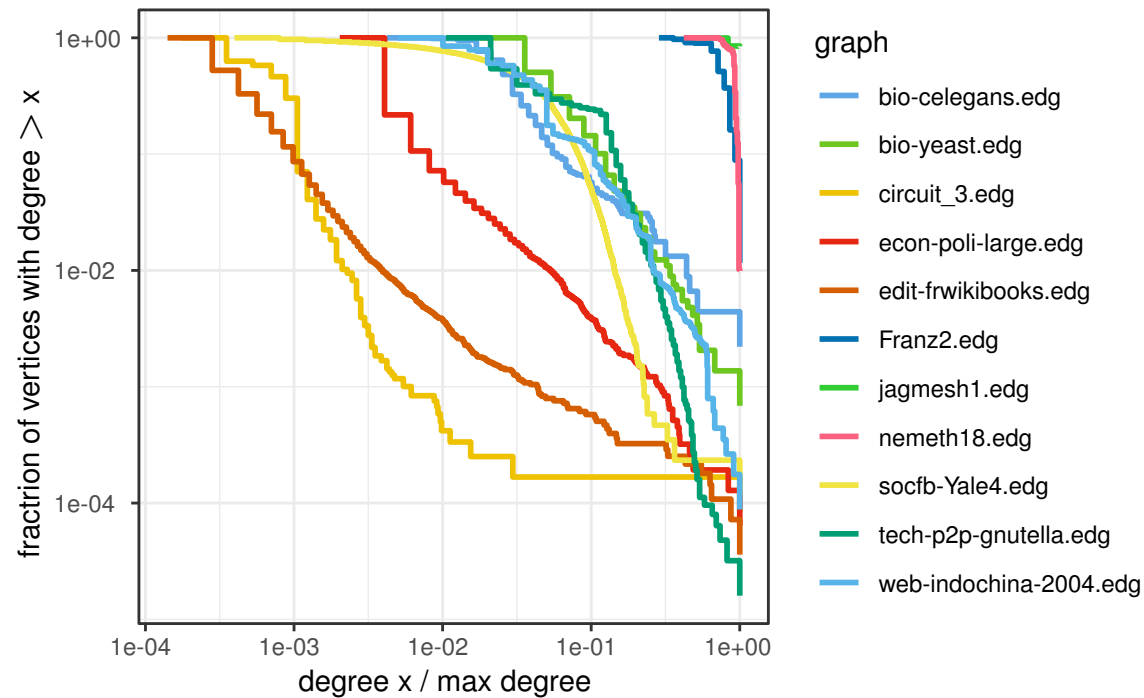
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- bio-grid-yeast
- socfb-Yale4
- bio-yeast-protein-inter

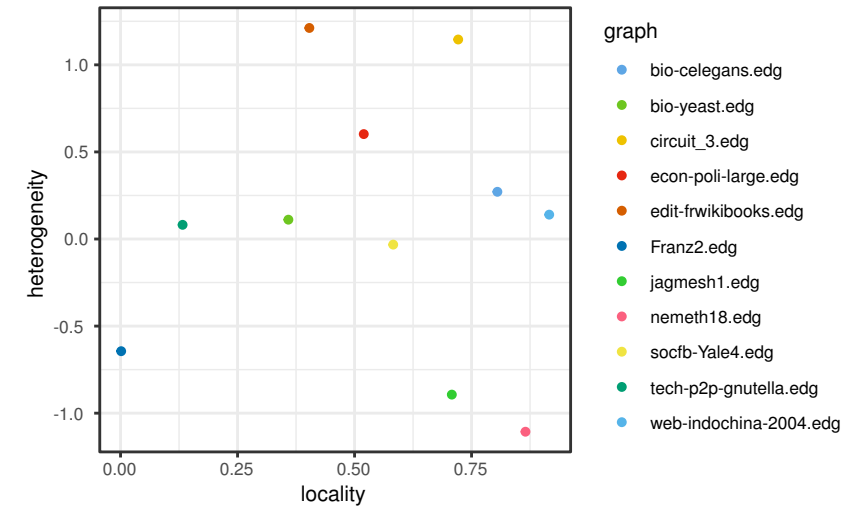
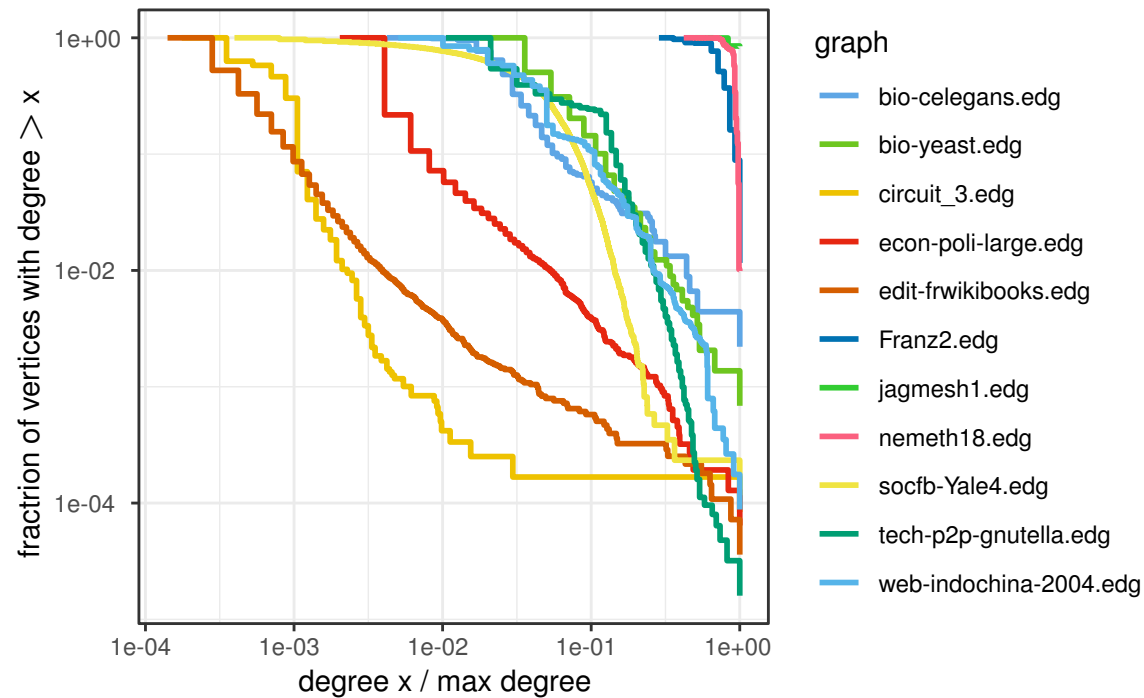




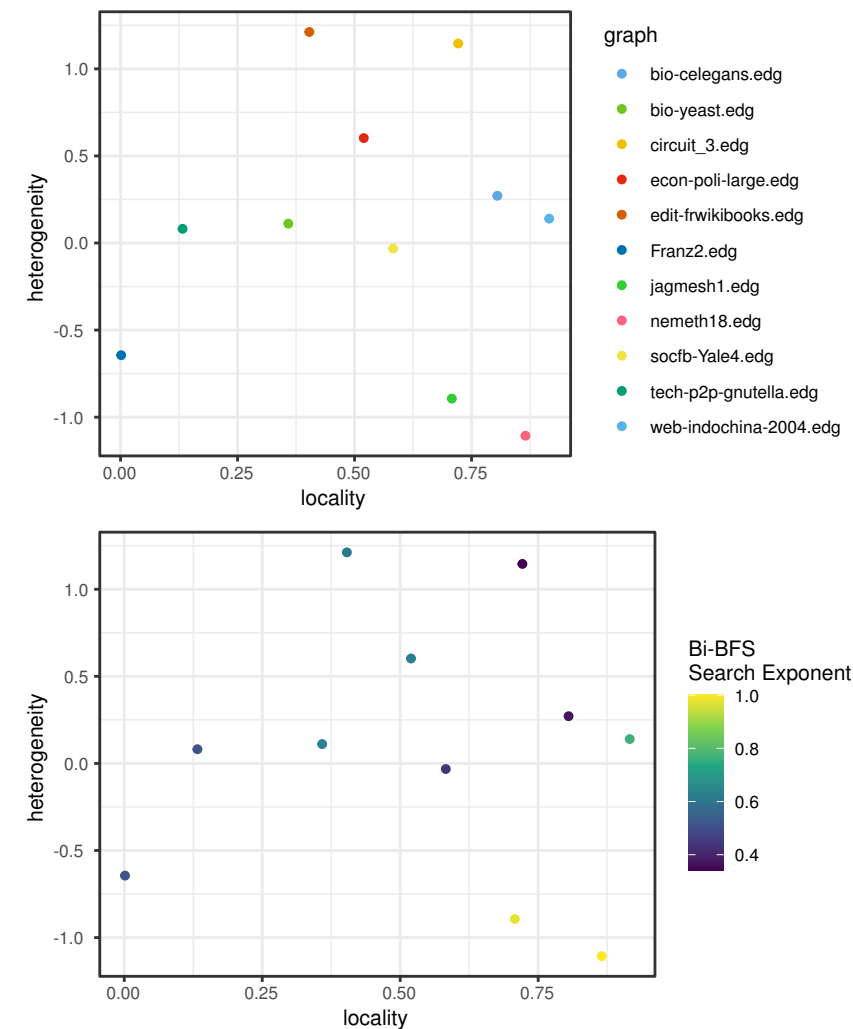
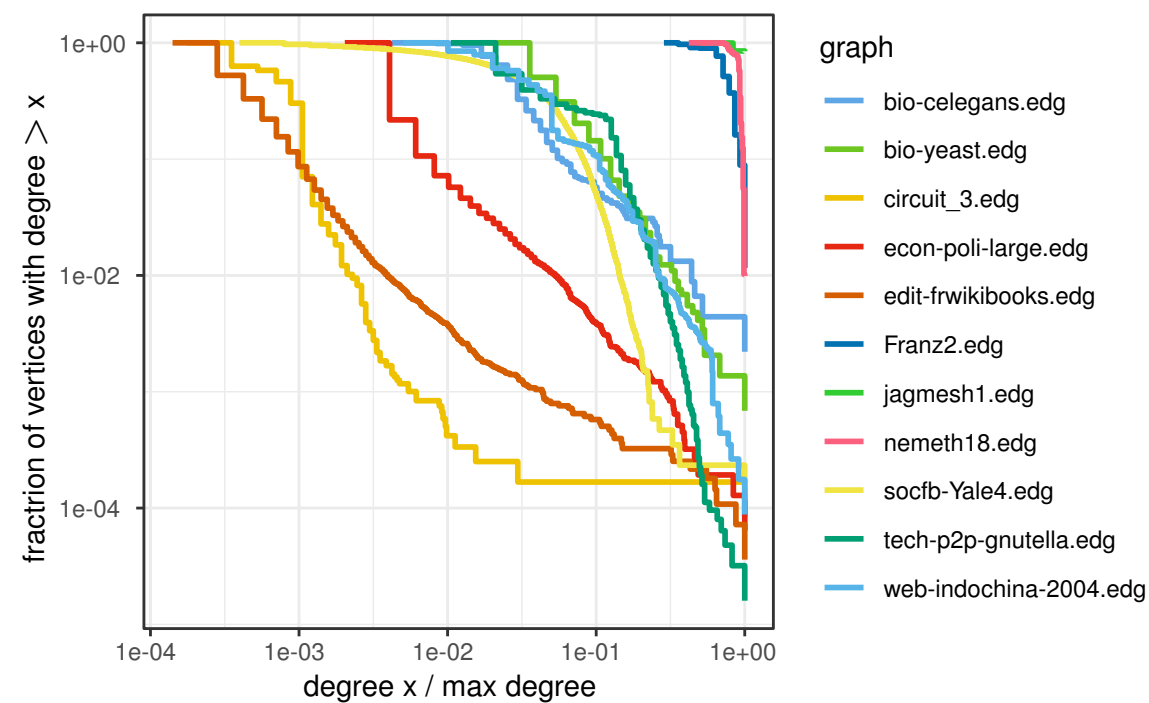
# Examples for Real-World Networks



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# Examples for Real-World Networks



# Complex Networks

**Keyword:** complex network, scale-free network

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**Three Characteristics:**

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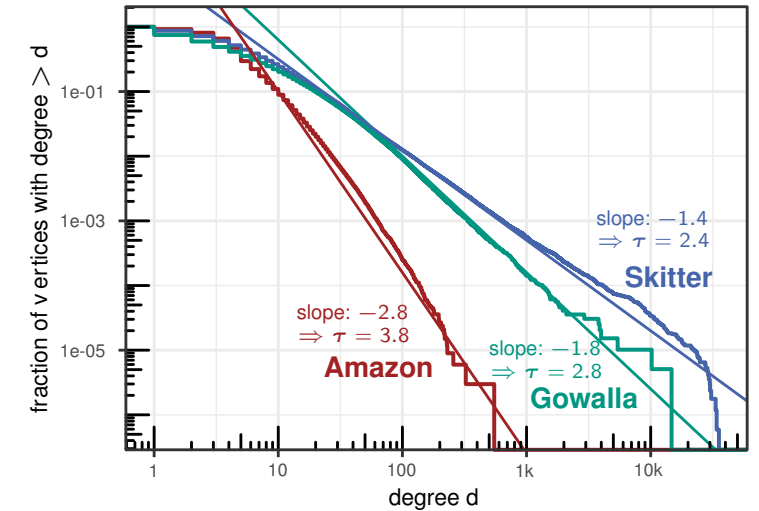
- heterogeneous degree distribution

# Complex Networks

**Keyword:** complex network, scale-free network

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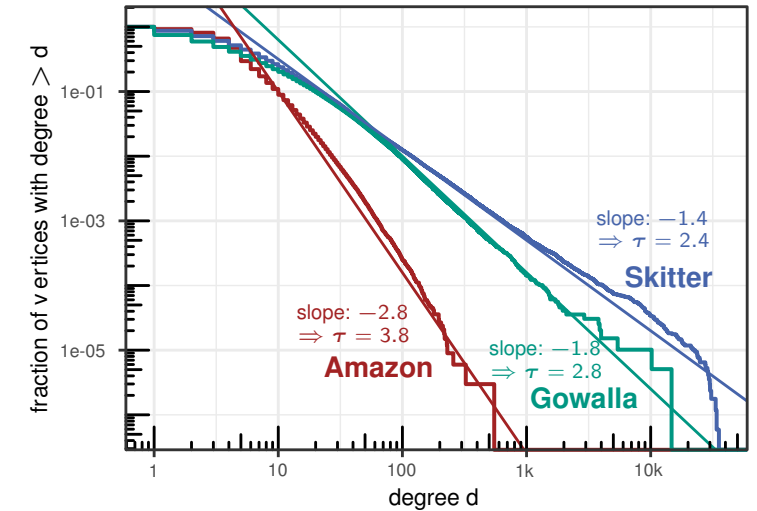


# Complex Networks

**Keyword:** complex network, scale-free network

## Three Characteristics:

- heterogeneous degree distribution
- short distances / „small-world“



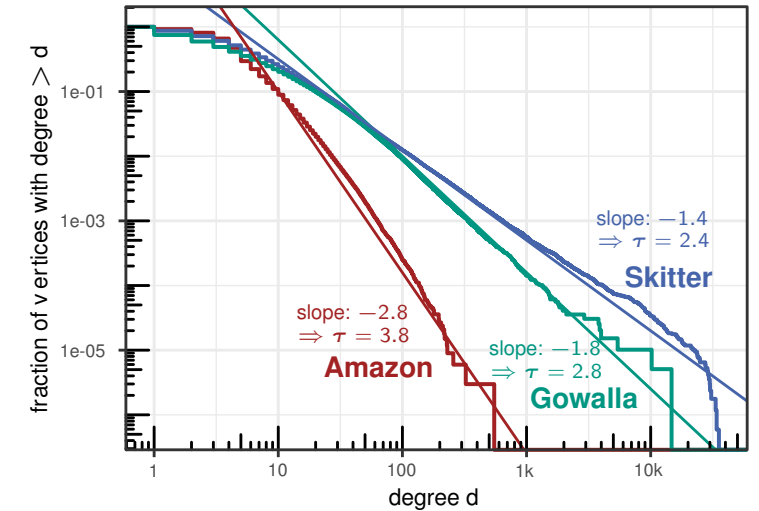


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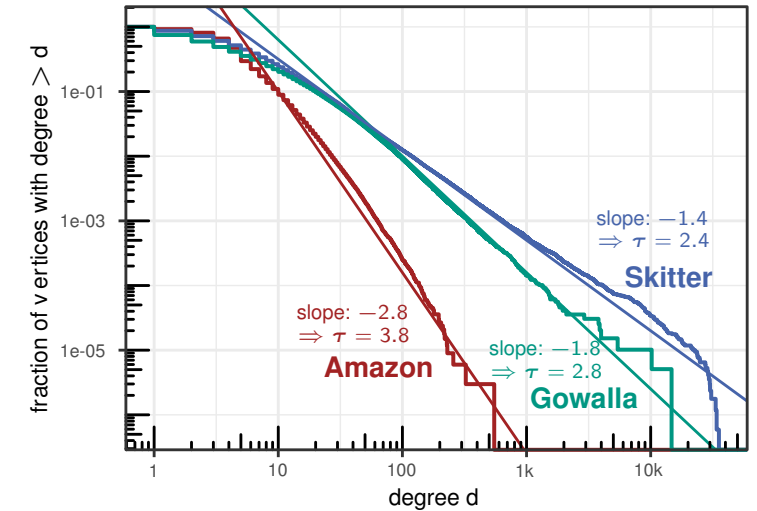
six-degrees of ...

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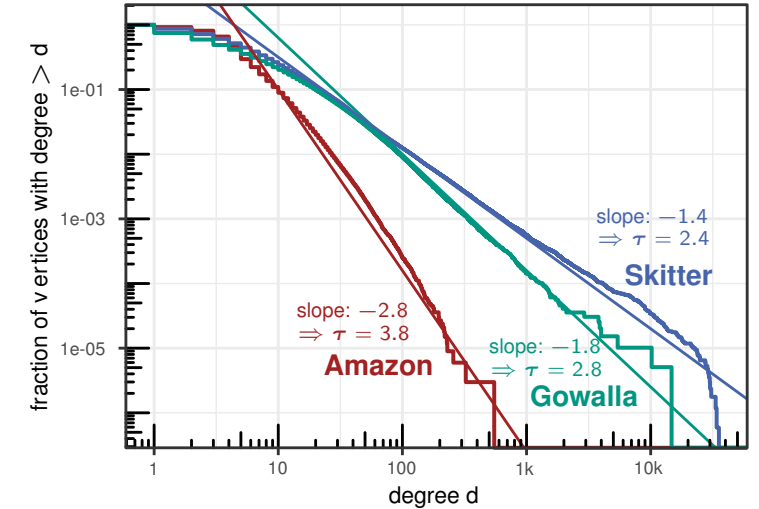
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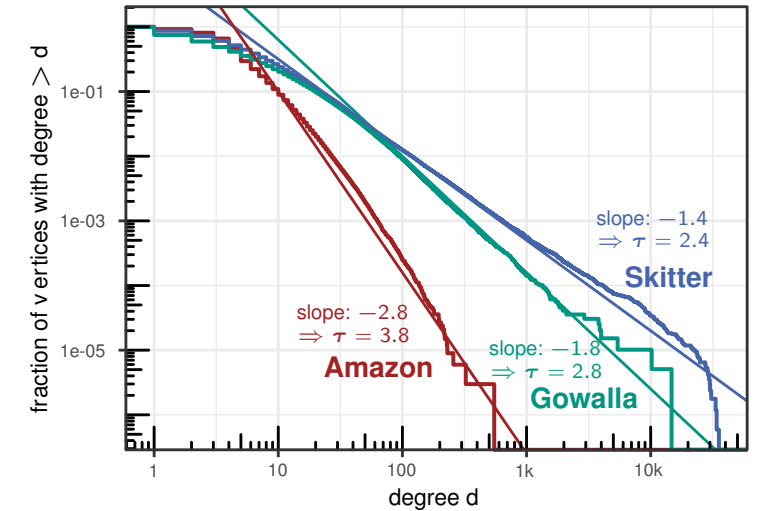
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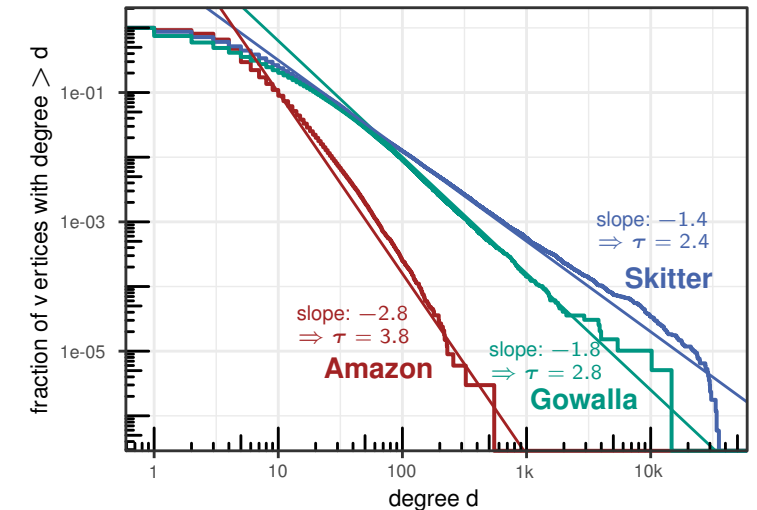
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- ... Wikipedia
- ... Kevin Bacon

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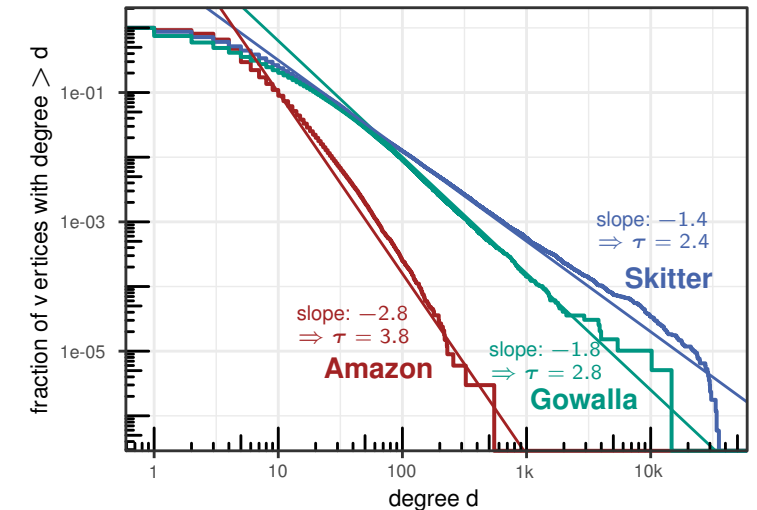
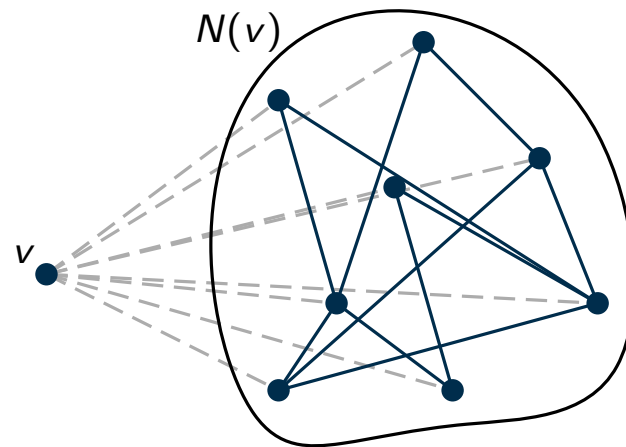
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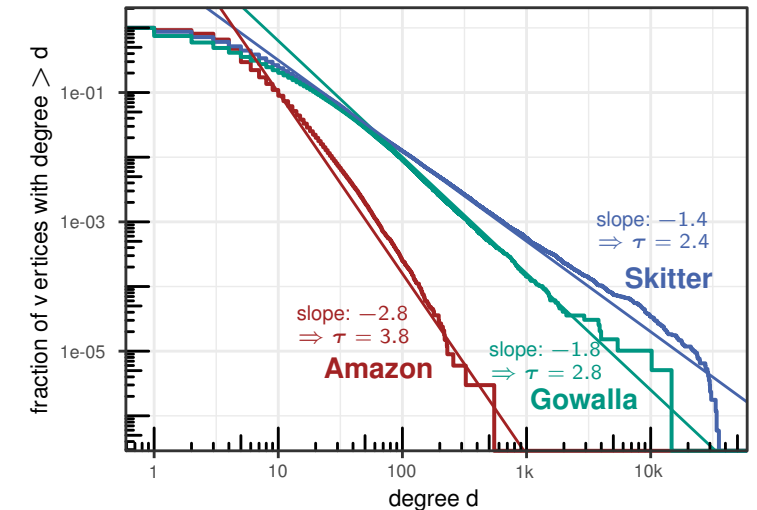
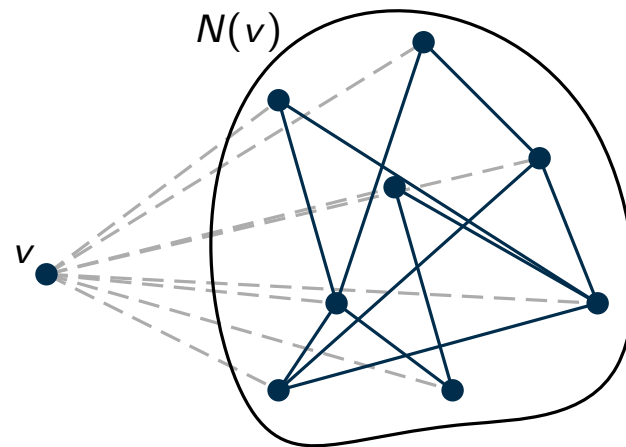
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goal: explain / model



six-degrees of ...

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# Models for Complex Networks

**Goal:** model and explain characteristics

**Three characteristics:**

- heterogeneous degrees
- short distances / „small-world“
- high locality / clustering

1959

1923 / 1999

2002

1998

2010

2019



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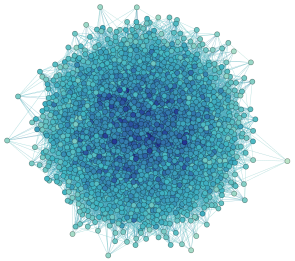
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## Erdős–Rényi model



# Models for Complex Networks

**Goal:** model and explain characteristics

## Three characteristics:

ER

1959

1923 / 1999

2002

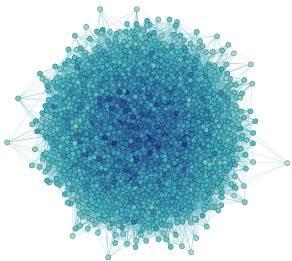
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- heterogeneous degrees
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## Erdős–Rényi model



# Models for Complex Networks

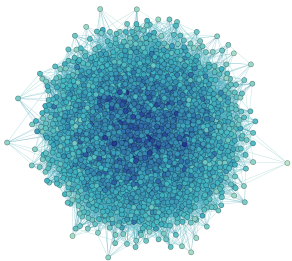
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ER 1959	Pref. Attach. / Barabási-Albert 1923 / 1999	2002	1998	2010	2019
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## Erdős–Rényi model



# Models for Complex Networks

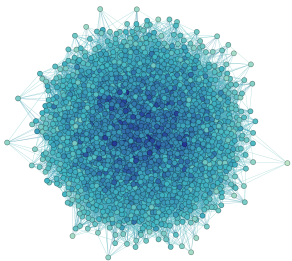
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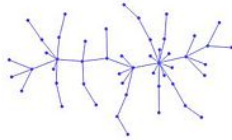
ER	Pref. Attach. / Barabási-Albert						
1959	1923 / 1999	2002	1998	2010	2019		

### Erdős–Rényi model



### Preferential Attachment

iteratively add vertices, choose edges with probability proportional to current degree

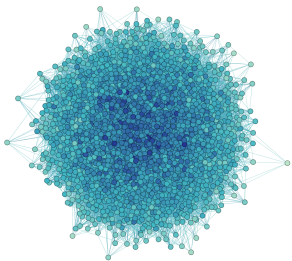


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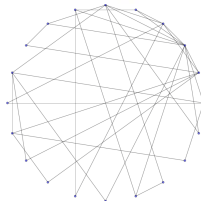
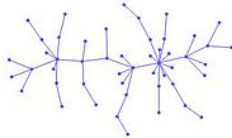
Three characteristics:	ER 1959	Pref. Attach. / Barabási-Albert 1923 / 1999	2002	1998	2010	2019
■ heterogeneous degrees		✓				
■ short distances / „small-world“	✓	✓				
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**Erdős–Rényi model**



## Preferential Attachment

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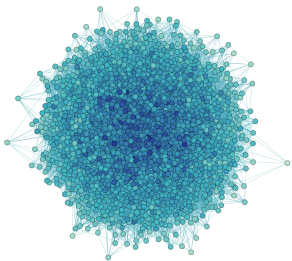


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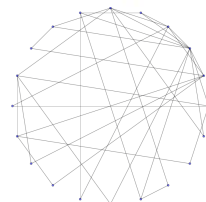
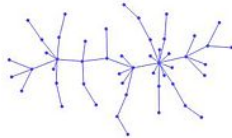
Three characteristics:	ER 1959	Pref. Attach. / Barabási-Albert 1923 / 1999	Chung-Lu 2002	1998	2010	2019
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■ short distances / „small-world“	✓	✓				
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**Erdős–Rényi model**



## Preferential Attachment

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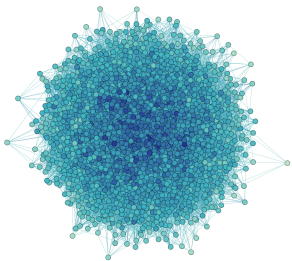
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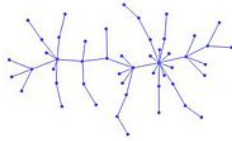
	ER 1959	Pref. Attach. / Barabási-Albert 1923 / 1999	Chung-Lu 2002	1998	2010	2019
■ heterogeneous degrees		✓				
■ short distances / „small-world“	✓	✓				
■ high locality / clustering						

### Erdős–Rényi model

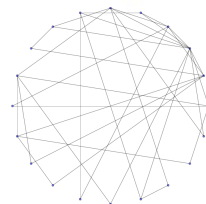


### Preferential Attachment

iteratively add vertices, choose edges with probability proportional to current degree



### Chung-Lu / Configuration model

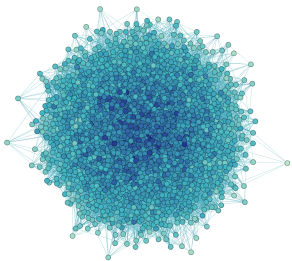


# Models for Complex Networks

**Goal:** model and explain characteristics

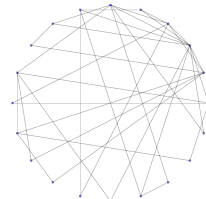
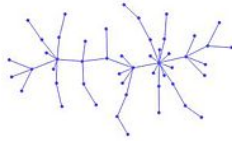
Three characteristics:	ER 1959	Pref. Attach. / Barabási-Albert 1923 / 1999	Chung-Lu 2002	1998	2010	2019
■ heterogeneous degrees		✓				
■ short distances / „small-world“	✓	✓				
■ high locality / clustering						

**Erdős–Rényi model**



## Preferential Attachment

iteratively add vertices, choose edges with probability proportional to current degree



## Chung-Lu / Configuration model

vertices with weights  $w_i$  (following power-law distribution);

$$\Pr [\{e_i, e_j\} \in E] \sim \frac{w_i \cdot w_j}{W}$$

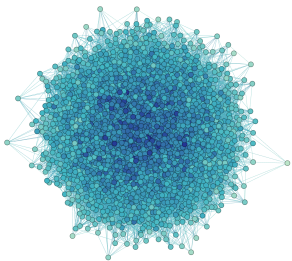


# Models for Complex Networks

**Goal:** model and explain characteristics

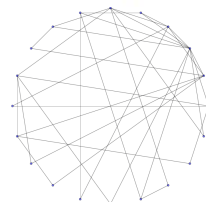
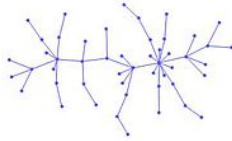
Three characteristics:	ER 1959	Pref. Attach. / Barabási-Albert 1923 / 1999	Chung-Lu 2002	1998	2010	2019
■ heterogeneous degrees		✓	✓			
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**Erdős–Rényi model**



## Preferential Attachment

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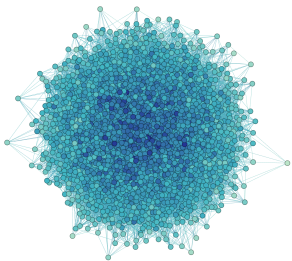
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# Models for Complex Networks

**Goal:** model and explain characteristics

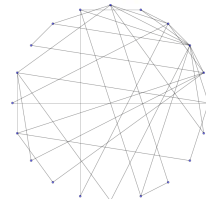
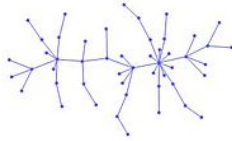
Three characteristics:	ER 1959	Pref. Attach. / Barabási-Albert 1923 / 1999	Chung-Lu 2002	Watts-Strogatz model 1998	2010	2019
■ heterogeneous degrees		✓	✓			
■ short distances / „small-world“	✓	✓	✓			
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**Erdős–Rényi model**



## Preferential Attachment

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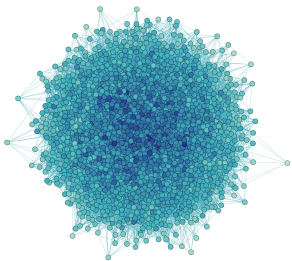
# Models for Complex Networks

**Goal:** model and explain characteristics

## Three characteristics:

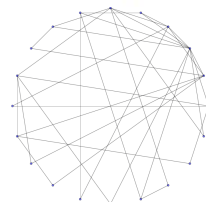
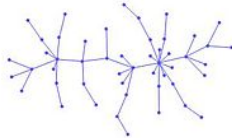
	ER 1959	Pref. Attach. / Barabási-Albert 1923 / 1999	Chung-Lu 2002	Watts-Strogatz model 1998	2010	2019
■ heterogeneous degrees		✓	✓			
■ short distances / „small-world“	✓	✓	✓			
■ high locality / clustering						

### Erdős–Rényi model

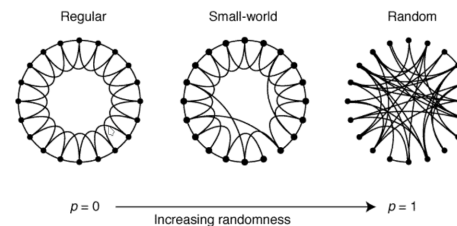


### Preferential Attachment

iteratively add vertices, choose edges with probability proportional to current degree



### Watts–Strogatz model



### Chung-Lu / Configuration model

vertices with weights  $w_i$  (following power-law distribution);

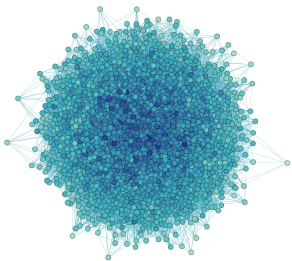
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# Models for Complex Networks

**Goal:** model and explain characteristics

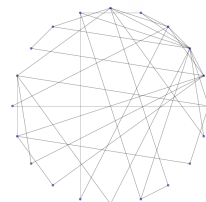
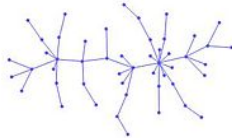
Three characteristics:	ER 1959	Pref. Attach. / Barabási-Albert 1923 / 1999	Chung-Lu 2002	Watts-Strogatz model 1998	2010	2019
■ heterogeneous degrees		✓	✓			
■ short distances / „small-world“	✓	✓	✓	✓		
■ high locality / clustering				✓		

**Erdős–Rényi model**

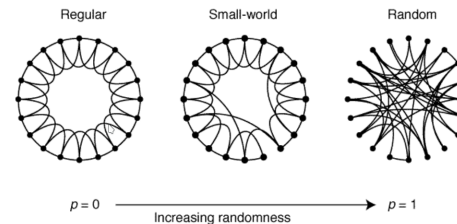


## Preferential Attachment

iteratively add vertices, choose edges with probability proportional to current degree



## Watts–Strogatz model



## Chung-Lu / Configuration model

vertices with weights  $w_i$  (following power-law distribution);

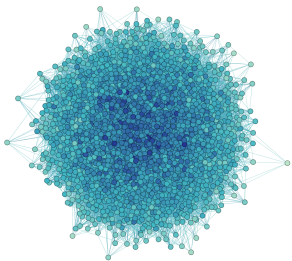
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# Models for Complex Networks

**Goal:** model and explain characteristics

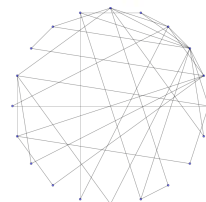
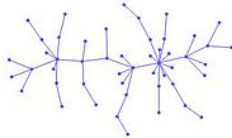
Three characteristics:	ER 1959	Pref. Attach. / Barabási-Albert 1923 / 1999	Chung-Lu 2002	Watts-Strogatz model 1998	GRG 2010	2019
■ heterogeneous degrees		✓	✓			
■ short distances / „small-world“	✓	✓	✓	✓		
■ high locality / clustering				✓		

**Erdős–Rényi model**

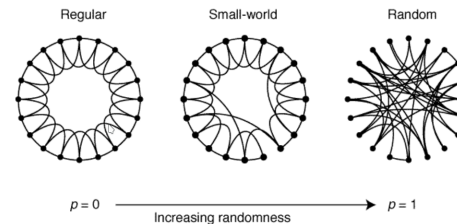


## Preferential Attachment

iteratively add vertices, choose edges with probability proportional to current degree



## Watts–Strogatz model



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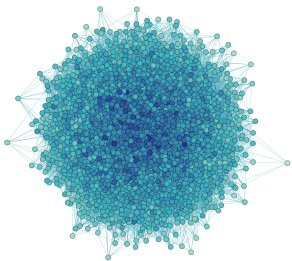
# Models for Complex Networks

**Goal:** model and explain characteristics

## Three characteristics:

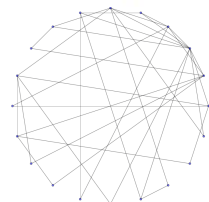
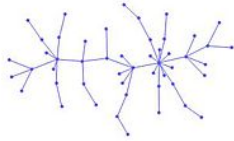
	ER 1959	Pref. Attach. / Barabási-Albert 1923 / 1999	Chung-Lu 2002	Watts-Strogatz model 1998	GRG 2010	2019
■ heterogeneous degrees		✓	✓			
■ short distances / „small-world“	✓	✓	✓	✓		
■ high locality / clustering				✓		

### Erdős–Rényi model



### Preferential Attachment

iteratively add vertices, choose edges with probability proportional to current degree

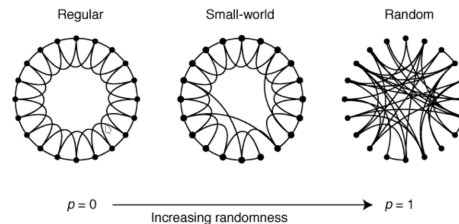


### Chung-Lu / Configuration model

vertices with weights  $w_i$  (following power-law distribution);

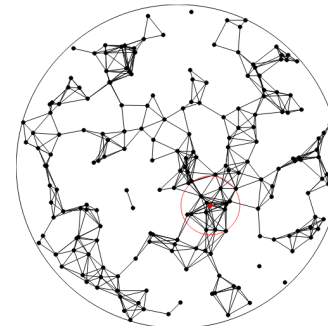
$$\Pr [\{e_i, e_j\} \in E] \sim \frac{w_i \cdot w_j}{W}$$

### Watts–Strogatz model



### Geometric Random Graph

sample vertices uniformly in geometry, connect if distance below threshold



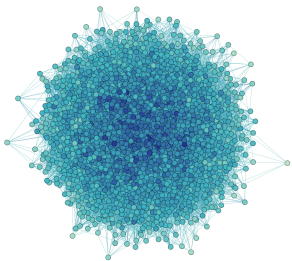
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**Goal:** model and explain characteristics

## Three characteristics:

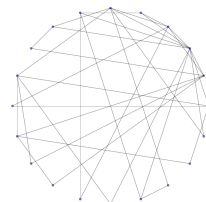
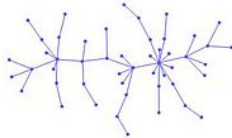
	ER 1959	Pref. Attach. / Barabási-Albert 1923 / 1999	Chung-Lu 2002	Watts-Strogatz model 1998	GRG 2010	2019
■ heterogeneous degrees		✓	✓			
■ short distances / „small-world“	✓	✓	✓	✓		
■ high locality / clustering				✓	✓	

### Erdős–Rényi model



### Preferential Attachment

iteratively add vertices, choose edges with probability proportional to current degree

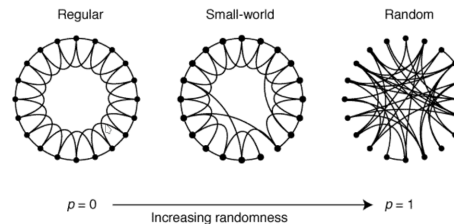


### Chung-Lu / Configuration model

vertices with weights  $w_i$  (following power-law distribution);

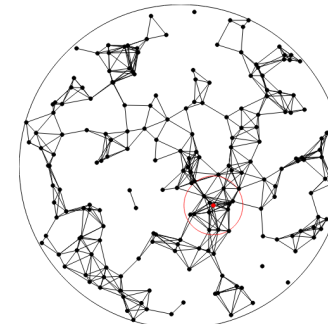
$$\Pr [\{e_i, e_j\} \in E] \sim \frac{w_i \cdot w_j}{W}$$

### Watts–Strogatz model



### Geometric Random Graph

sample vertices uniformly in geometry, connect if distance below threshold





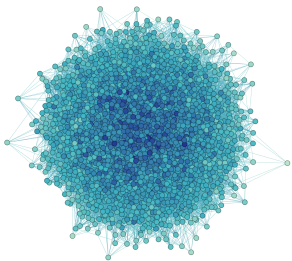
# Models for Complex Networks

**Goal:** model and explain characteristics

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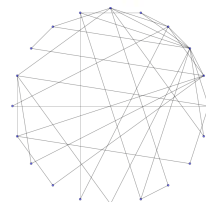
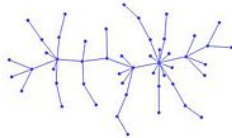
	ER 1959	Pref. Attach. / Barabási-Albert 1923 / 1999	Chung-Lu 2002	Watts-Strogatz model 1998	GRG	HRG 2010	2019
■ heterogeneous degrees		✓	✓				
■ short distances / „small-world“	✓	✓	✓	✓			
■ high locality / clustering				✓	✓		

### Erdős–Rényi model

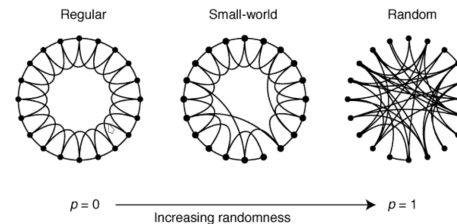


### Preferential Attachment

iteratively add vertices, choose edges with probability proportional to current degree



### Watts–Strogatz model



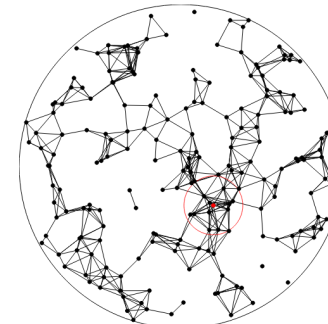
### Chung-Lu / Configuration model

vertices with weights  $w_i$  (following power-law distribution);

$$\Pr [\{e_i, e_j\} \in E] \sim \frac{w_i \cdot w_j}{W}$$

### Geometric Random Graph

sample vertices uniformly in geometry, connect if distance below threshold





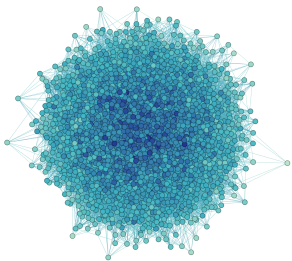
# Models for Complex Networks

**Goal:** model and explain characteristics

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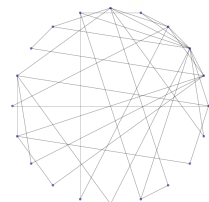
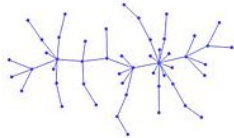
	ER 1959	Pref. Attach. / Barabási-Albert 1923 / 1999	Chung-Lu 2002	Watts-Strogatz model 1998	GRG	HRG 2010	2019
■ heterogeneous degrees		✓	✓				
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### Erdős–Rényi model



### Preferential Attachment

iteratively add vertices, choose edges with probability proportional to current degree

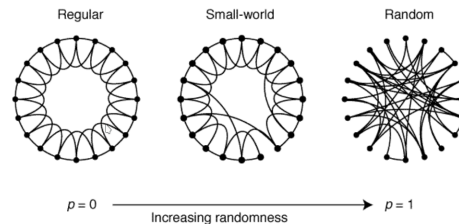


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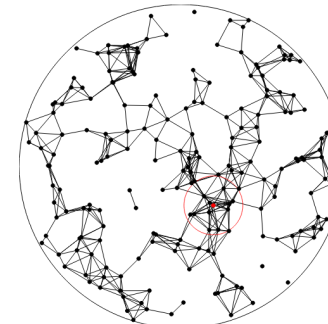
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### Watts–Strogatz model



### Geometric Random Graph (Hyperbolic)

sample vertices uniformly in geometry, connect if distance below threshold



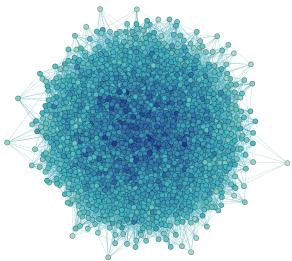
# Models for Complex Networks

**Goal:** model and explain characteristics

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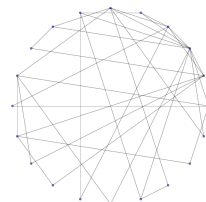
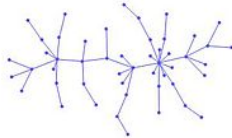
	ER 1959	Pref. Attach. / Barabási-Albert 1923 / 1999	Chung-Lu 2002	Watts-Strogatz model 1998	GRG 2010	HRG 2019
■ heterogeneous degrees		✓	✓			
■ short distances / „small-world“	✓	✓	✓	✓		
■ high locality / clustering				✓	✓	

### Erdős–Rényi model



### Preferential Attachment

iteratively add vertices, choose edges with probability proportional to current degree

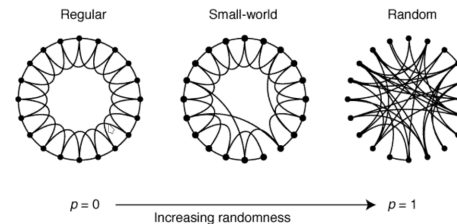


### Chung-Lu / Configuration model

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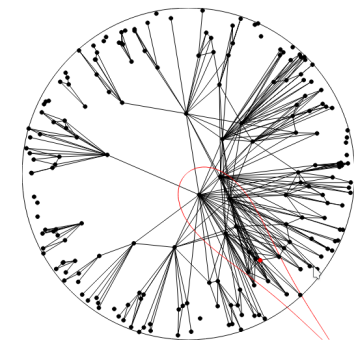
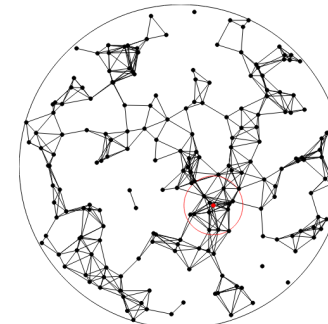
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### Watts–Strogatz model



### Geometric Random Graph (Hyperbolic)

sample vertices uniformly in geometry, connect if distance below threshold

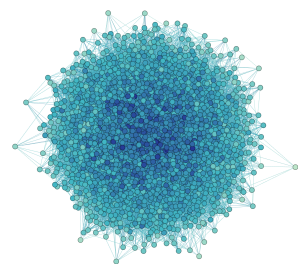


# Models for Complex Networks

**Goal:** model and explain characteristics

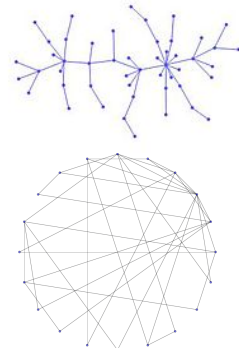
Three characteristics:	ER 1959	Pref. Attach. / Barabási-Albert 1923 / 1999	Chung-Lu 2002	Watts-Strogatz model 1998	GRG	HRG 2010	2019
■ heterogeneous degrees		✓	✓			✓	
■ short distances / „small-world“	✓	✓	✓	✓		✓	
■ high locality / clustering				✓	✓	✓	

**Erdős–Rényi model**

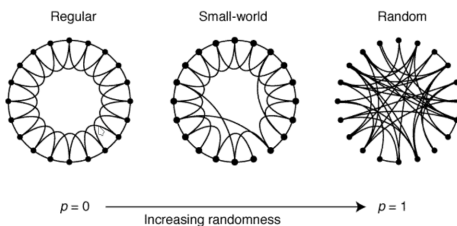


## Preferential Attachment

iteratively add vertices, choose edges with probability proportional to current degree



## Watts–Strogatz model



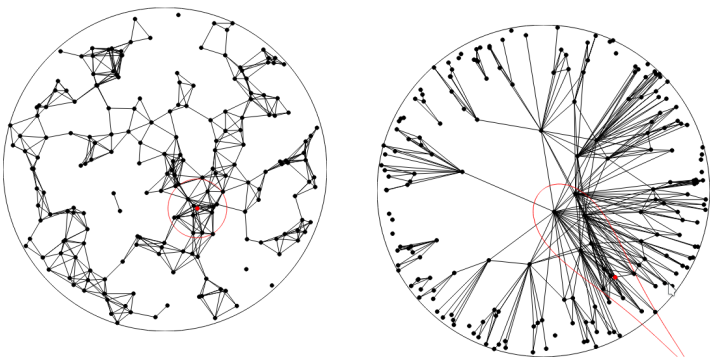
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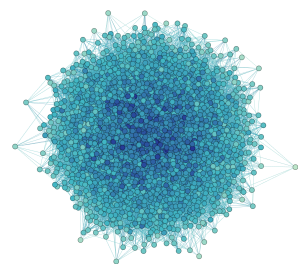


# Models for Complex Networks

**Goal:** model and explain characteristics

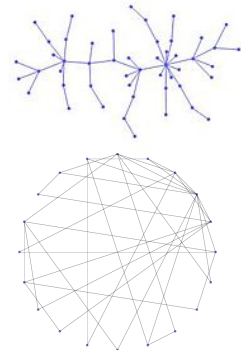
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■ short distances / „small-world“	✓	✓	✓	✓		✓	
■ high locality / clustering				✓	✓	✓	

Erdős–Rényi model

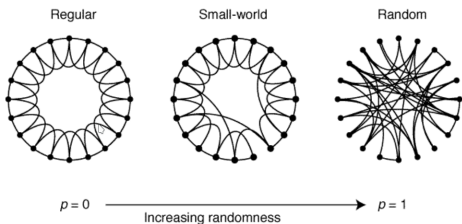


Preferential Attachment

iteratively add vertices, choose edges with probability proportional to current degree



Watts–Strogatz model

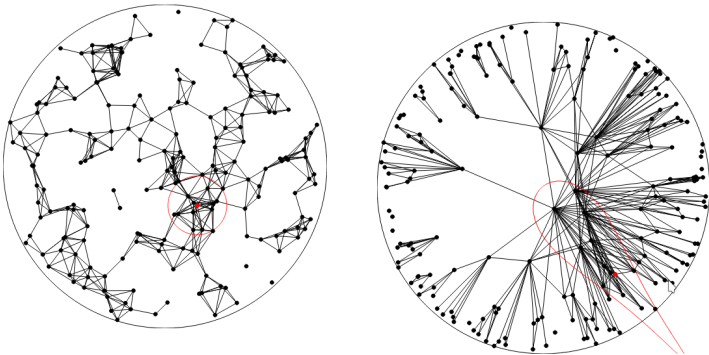


Chung-Lu / Configuration model

vertices with weights  $w_i$  (following power-law distribution);  
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Geometric Random Graph (Hyperbolic)

sample vertices uniformly in geometry, connect if distance below threshold



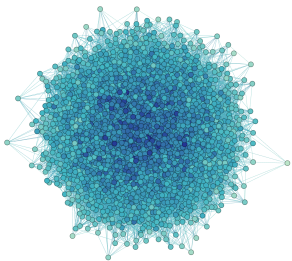
# Models for Complex Networks

**Goal:** model and explain characteristics

## Three characteristics:

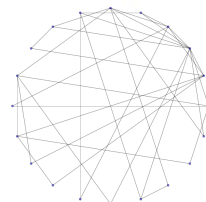
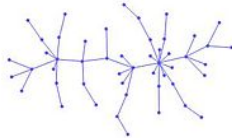
	ER 1959	Pref. Attach. / Barabási-Albert 1923 / 1999	Chung-Lu 2002	Watts-Strogatz model 1998	GRG	HRG 2010	GIRG 2019
■ heterogeneous degrees		✓	✓			✓	
■ short distances / „small-world“	✓	✓	✓	✓		✓	
■ high locality / clustering				✓	✓	✓	

### Erdős–Rényi model

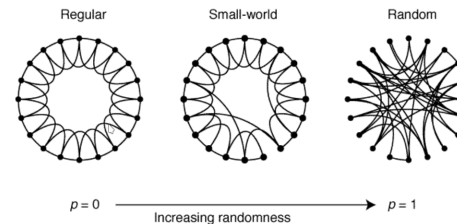


### Preferential Attachment

iteratively add vertices, choose edges with probability proportional to current degree



### Watts–Strogatz model



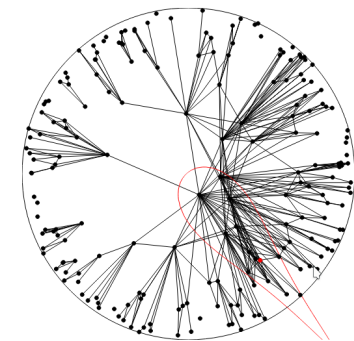
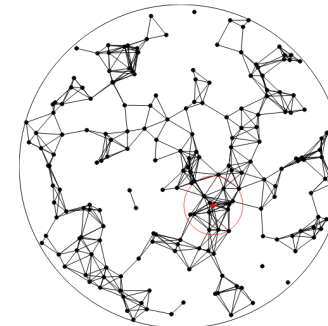
### Chung-Lu / Configuration model / IRG

vertices with weights  $w_i$  (following power-law distribution);

$$\Pr [\{e_i, e_j\} \in E] \sim \frac{w_i \cdot w_j}{W}$$

### Geometric Random Graph (Hyperbolic)

sample vertices uniformly in geometry, connect if distance below threshold



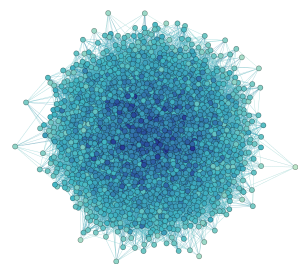


# Models for Complex Networks

**Goal:** model and explain characteristics

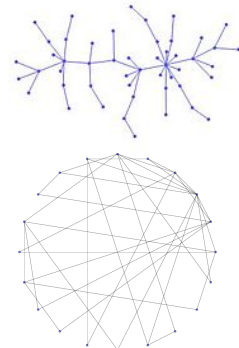
Three characteristics:	ER 1959	Pref. Attach. / Barabási-Albert 1923 / 1999	Chung-Lu 2002	Watts-Strogatz model 1998	GRG	HRG 2010	GIRG 2019
■ heterogeneous degrees		✓	✓			✓	
■ short distances / „small-world“	✓	✓	✓	✓		✓	
■ high locality / clustering				✓	✓	✓	

**Erdős–Rényi model**

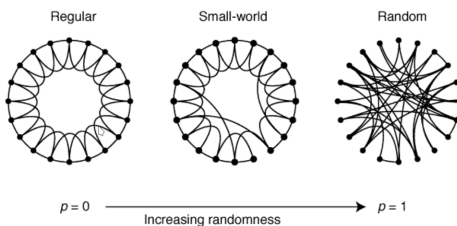


## Preferential Attachment

iteratively add vertices, choose edges with probability proportional to current degree



## Watts–Strogatz model



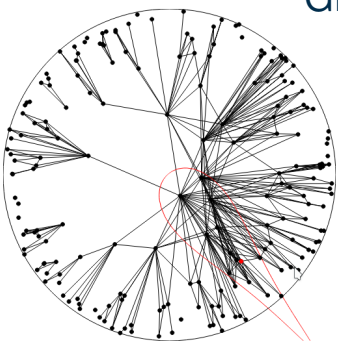
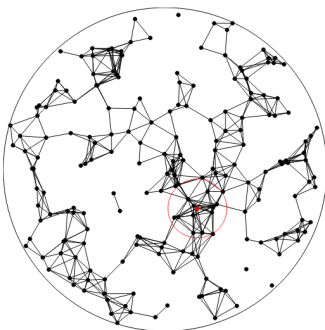
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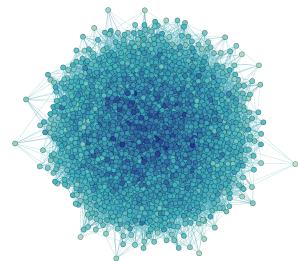
**GIRG**  
GRG + IRG

# Models for Complex Networks

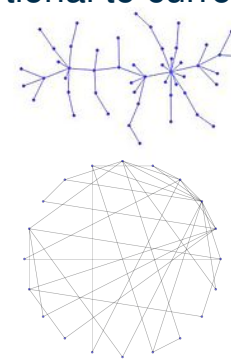
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**Erdős–Rényi model**




**Preferential Attachment**  
iteratively add vertices, choose edges with probability proportional to current degree

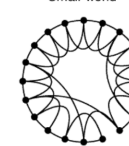


**Watts–Strogatz model**


Regular



Small-world

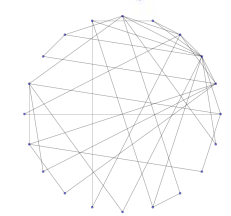


Random

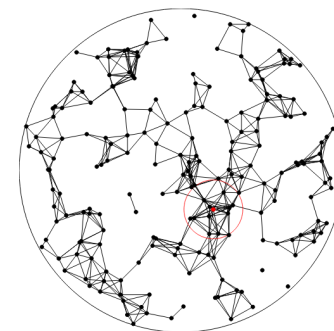


$p = 0$  — Increasing randomness —  $p = 1$

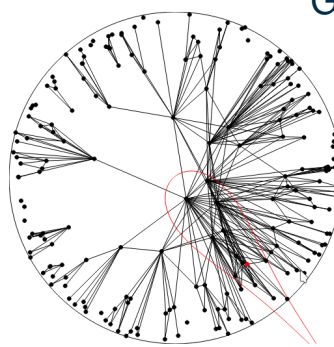
**Chung-Lu / Configuration model / IRG**  
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**Geometric Random Graph (Hyperbolic)**  
sample vertices uniformly in geometry, connect if distance below threshold



**GIRG**  
GRG + IRG



# Sheet 3

## Generated Graphs

## Real-World Graphs



# Sheet 3

## Generated Graphs

- select multiple random models to generate graphs
- can you determine, how we generated the initial test set?

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External Validity of Average-Case Analyses



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External Validity of Average-Case Analyses



- How fast are Bi-BFS and VC on the new graphs?
- What is the heterogeneity and locality of the new graphs?
- How do graphs with high heterogeneity and low locality look like?