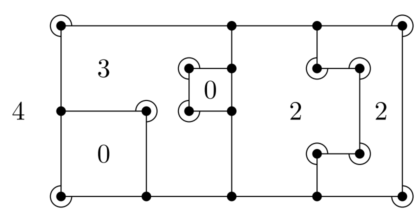
Exercise Sheet 6

Submission due by 2025-07-31

Problem 1: Bend and Behold **5 Points**

A graph is *bend-free orthogonally drawable* if it admits a planar drawing, where each edge is a horizontal or vertical line segment. The *angle complexity* of a bend-free orthogonal drawing describes the smallest number k such that each inner face contains at most k corners with angle $\frac{3}{2}\pi$ and the outer face contains at most $k + 4$ such corners.

Show that, even for (small) constant k, it is NP-hard to decide whether a given planar graph admits a bend-free orthogonal drawing with angle complexity at most k for a given planar graph.



Problem 2: Nice Decomposition **6 Points**

Given two vertical segments of lengths 2^{ℓ_L} and 2^{ℓ_R}. Additionally, let there be a set S with n non-overlapping segments between the two vertical segments. Show that one can choose k segments $s_1, s_2, \dots, s_k \in S$ with $k \in \mathcal{O}(b)$ such that (1) and (2) hold.

(1) For each $i \in \{1,2,\dots,k-1\}$ at least one of the following statements holds:

- (a) there are at most n/b segments between s_i and s_{i+1};
- (b) the left endpoints of s_i and s_{i+1} lie on a subinterval of length $2^{\ell_L - h}$;
- (c) the right endpoints of s_i and s_{i+1} lie on a subinterval of length $2^{\ell_R - h}$.

(2) There are segments $\tilde{s}_1, \tilde{s}_3, \dots$ between the two vertical segments such that:

- (a) $s_1 \prec \tilde{s}_1 \prec s_3 \prec \tilde{s}_3 \prec \dots$;
- (b) the distances between the left endpoints of the $\tilde{s}_i$ are multiples of $2^{\ell_L - h}$;
- (c) the distances between the right endpoints of the $\tilde{s}_i$ are multiples of $2^{\ell_R - h}$.

Problem 3: Geometry

$2 + 3 + 4 = 9$ Points

Part (a) "Parallels"

Draw a line through the origin and for a distance d the set of all points that are distance d from this line. Use the Poincaré Disk model for this.

Part (b) Vicious Circle

Given three non-collinear points. In the Euclidean plane, there is exactly one circle on which they lie. Prove or disprove that the same holds in the hyperbolic plane.

Part (c) Is that really true?

Given three non-collinear points A, B and C. Prove that there is no point D in the half-plane ABC^+ with $D \neq C$ such that $d(A, C) = d(A, D)$ and $d(B, C) = d(B, D)$.