Exercise Sheet 5

Submission due by 2025-07-17

Problem 1: Triangulation of Concentric Points $3 + 3 + 3 = 9$ points

Given a polygon P with n points, all lying on a common circle. The distances between any two points are pairwise distinct.

The objective of this task is to design an algorithm that finds an optimal triangulation of P (with respect to the lexicographic order of the angle vectors).

Part (a) The *length vector* is the vector of the lengths of the edges that are inserted in a triangulation of P, sorted in ascending order. Analogously to the angle vector, we order length vectors lexicographically.

Show that a triangulation is optimal if and only if it maximizes the length vector.

Part (b) The *weak dual graph* $G^\star$ of a triangulated, geometric graph G contains one vertex for each triangle in G (the outer face is not considered). Two vertices are connected in $G^\star$ if and only if the corresponding faces in G share an edge. Show that the weak dual graph of an optimal triangulation of P is a path.

Part (c) Sketch an algorithm that computes an optimal triangulation of P in $O(n)$ time.

Problem 2: MST an Delaunay-Triangulations 6 points

Given a set of points P in the plane, the *minimum spanning tree MST* is a tree of minimal total edge length that connects all points in P.

Show that the Delaunay triangulation of P contains the MST of P.

Problem 3: Foldability Test 5 points

Given a one-dimensional mountain/valley pattern G with n vertices, design an algorithm that determines whether G is flat-foldable in $O(n)$ time.

Problem 4: Creative Outlet 5 bonus points

Use origami or the fold-and-cut-method to create something cool! Take a picture of your creation and attach it to your submission.