Exercise Sheet 1

Submission due by 2025-05-15

Organizational Info

- You have *two weeks* to work on the exercise problems[1].
- Please work in pairs or groups of three if possible.
- Submit your solutions as a *well-formatted* PDF via email to `algogeom_abgaben.lists.kit.edu`

Problem 1: Mysterious Meteorites **5 + 5 points**

Part (a) Last night, n meteorites impacted the ground. Surprisingly, all resulting craters are perfect circles with exactly the same radius. Because of this, a research team wants to cordon off the area to investigate undisturbed. They aim to create a connected restricted zone using as little barrier tape as possible. Describe a *simple and efficient* algorithm that, given n circles of radius r, computes the restricted zone. Briefly argue why your algorithm is correct.

Part (b) Meteorites have struck again—this time only two. Interestingly, both meteorites had highly unusual physical properties, resulting in craters shaped as convex polygons. Since the meteorites struck very close to each other, the craters overlap, and the resulting shape is fairly complex.

Due to complaints about excessive use of space last time, the research team now wants to cordon off only the exact outline of the craters. They are bringing barrier tape and posts (to redirect the tape). Unfortunately, the exact shape of the combined crater is unknown. However, astronomical observations revealed that the individual craters would have n and m vertices, respectively. How many posts should the research team bring in the worst case?

Problem 2: Convex Hull via Divide and Conquer **3 + 2 points**

Part (a) Let P_1 and P_2 be two non-overlapping convex polygons with a total of n vertices. Describe a linear-time algorithm that computes the convex hull of $P_1 \cup P_2$.

Part (b) Describe a divide and conquer algorithm that computes the convex hull of n points in $O(n \log n)$ time.

[1] Exception: for this sheet you have *three* weeks, due to Easter and Labour Day.

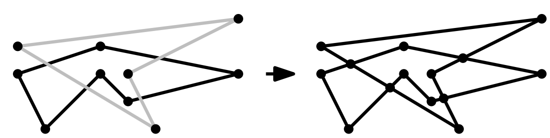
Problem 3: Aerial Photography **5 points**

You have bought a camera drone and driven out into the wilderness to take some nice photos. (Un)fortunately, it is not permitted to fly in nature reserves. Thus, you want to carefully check whether flying is allowed at your chosen spot.

The nature reserve that might be relevant for your location (point p) is a polygon $P = (p_1, p_2, \dots, p_n)$. Give an efficient algorithm that determines whether you are inside, on the boundary, or outside the nature reserve. Have you considered all edge cases?

Problem 4: Map Overlay **8 points**

In this task, the overlay G of two geometric graphs G_1 and G_2 is to be computed. G is again a geometric graph, which partitions the plane into the faces induced by the edges of both G_1 *and* G_2.



Part (a) Describe an efficient algorithm that takes as input two geometric graphs represented as doubly-connected edge lists and outputs their overlay, also as a doubly-connected edge list. You may ignore faces for now, i.e., `face(e)`, `parent(f)`, and `children(f)` can be omitted.

Part (b) Given the result of **part (a)**, describe an algorithm that determines the missing entries `face(e)`, `parent(f)`, and `children(f)` to finalize the doubly-connected edge list.

Part (c) Now suppose the input graphs G_1 and G_2 have their faces labeled. Each face in the output graph G is contained in one face of G_1 and one face of G_2. Describe an algorithm that labels each face in G with the labels of the corresponding faces from G_1 and G_2.

Problem 5: Boolean Operations on Polygons **2 points**

Given two polygons P_1 and P_2, describe an algorithm that computes the *intersection*, the *union*, and the *symmetric difference* of P_1 and P_2.