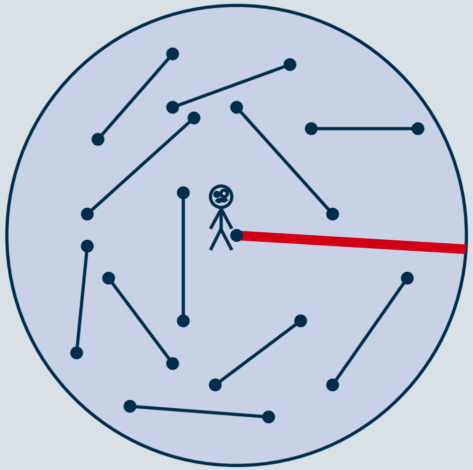
Computational Geometry

Exercise 5 *Assignment 4, 5 and Voronoi diagrams of segments*

Jean-Pierre, Marcus, Wendy

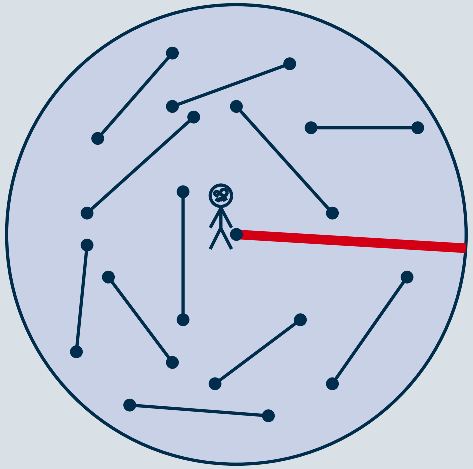
Mission Impossible + 1.5 Range Queries



query: $O(\log(n) + k)$
preprocessing: $O(n \log(n))$

Mission Impossible + 1.5 Range Queries

Persistent Rotating Sweep Line



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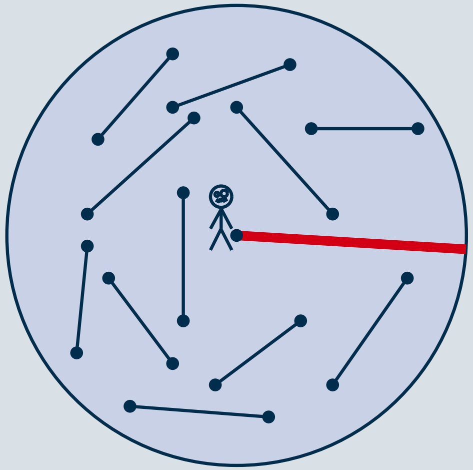
- walls that intersect sweep line
- sort by distance to center



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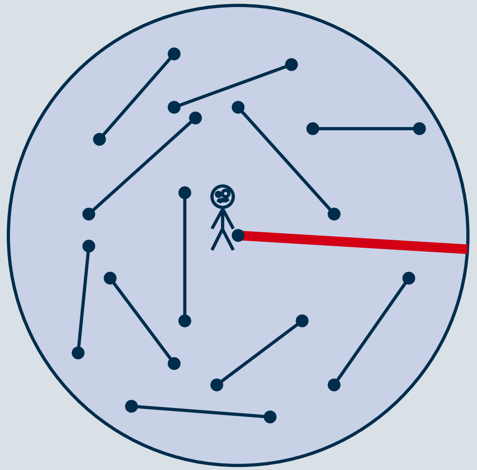
timestamp?

- data structure: persistent binary search tree



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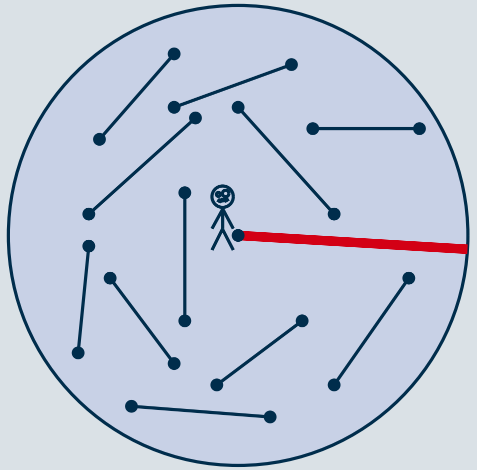
at $\alpha = 0$?



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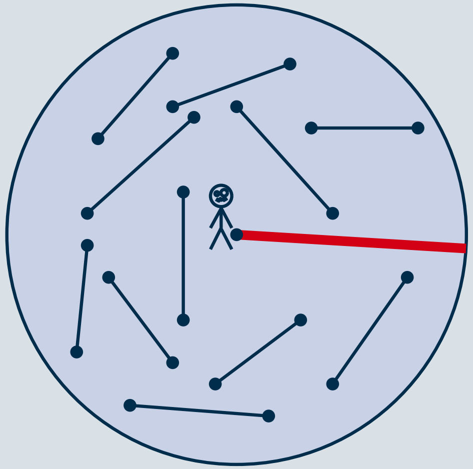

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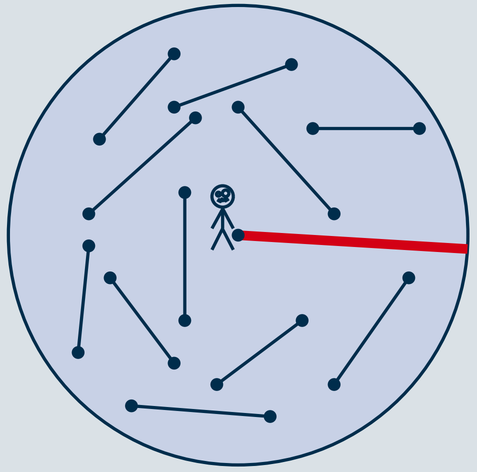

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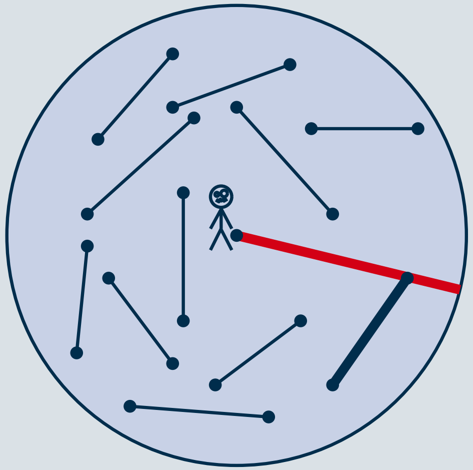

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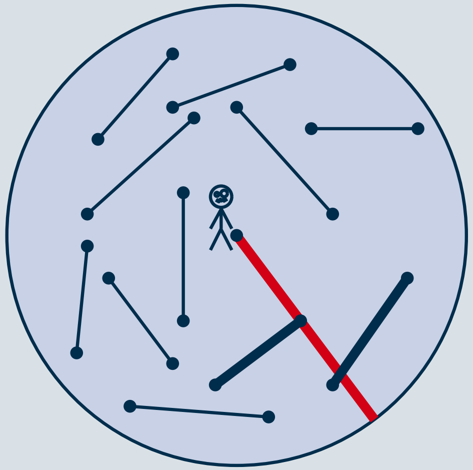

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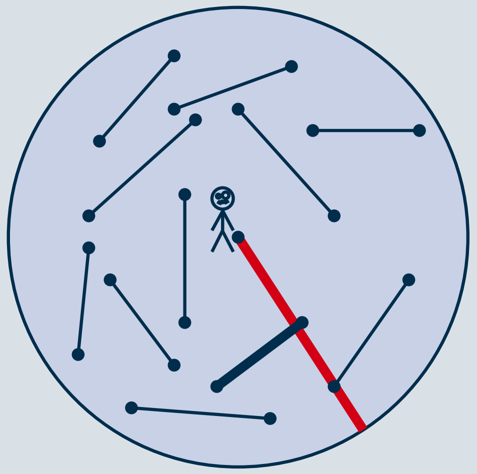

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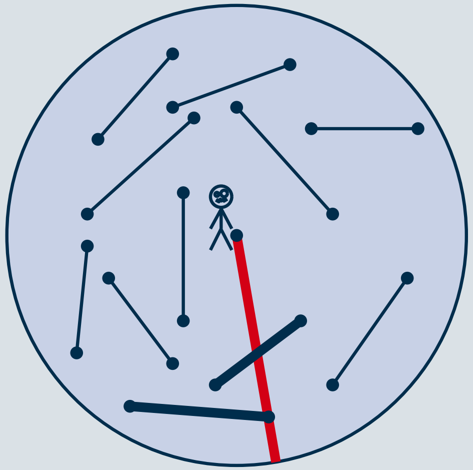

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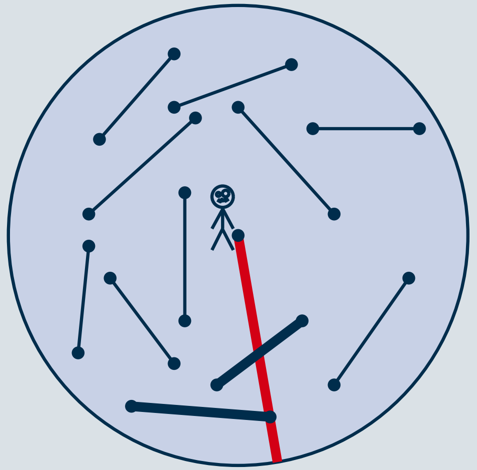

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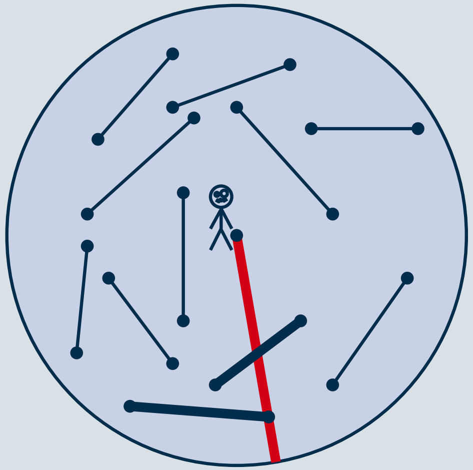
at $\alpha = 0$?

Events

- start point: add wall
- end point: remove wall

Queries

- ask for time $= \alpha$
- output all walls currently in sweep line state



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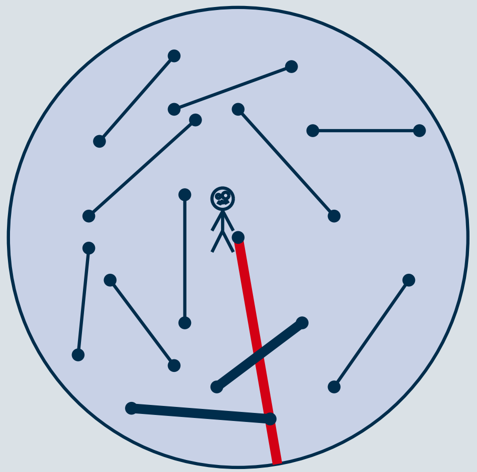
Queries

- ask for time $= \alpha$

Why $O(\log(n) + k)$?

- output all walls currently in sweep line state



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Mission Impossible + 1.5 Range Queries

Persistent Rotating Sweep Line

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timestamp?

at $\alpha = 0$?

Events

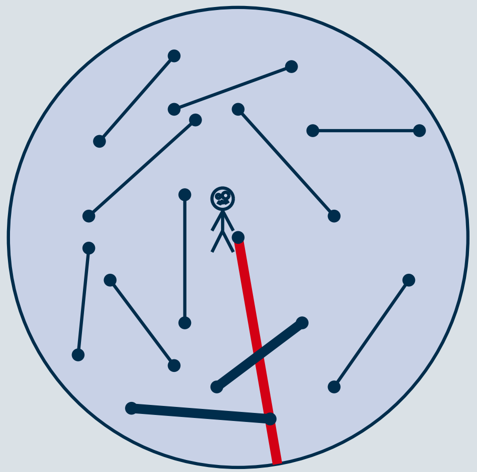
- start point: add wall
- end point: remove wall

But wait! What about space?

Queries

- ask for time $= \alpha$
- output all walls currently in sweep line state

Why $O(\log(n) + k)$?



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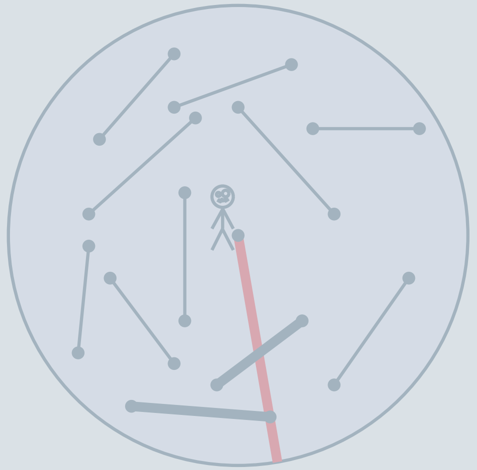
Why $O(\log(n) + k)$?

- output all walls currently in sweep line state



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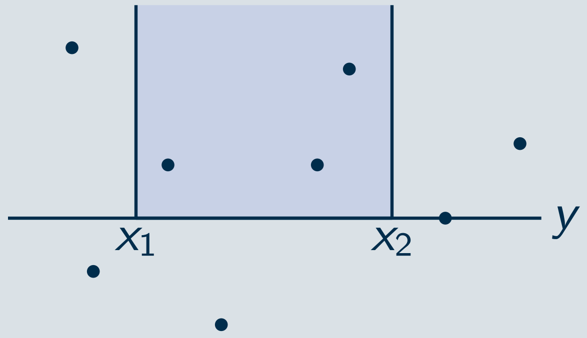


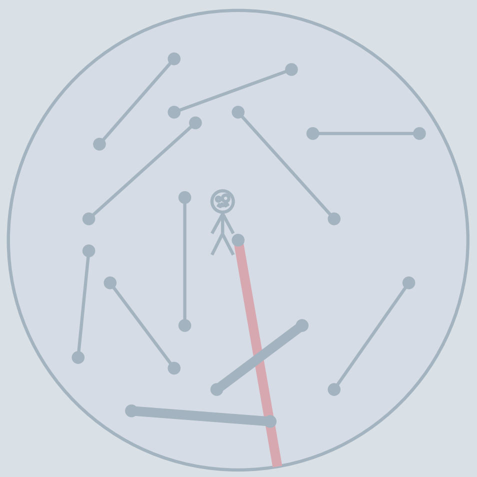
Mission Impossible + 1.5 Range Queries

Persistent ~~Rotating~~ Sweep Line

- ~~walls~~ points that ~~intersect~~ are above sweep line
- sort by distance to center

timestamp?

- data structure: persistent binary search tree



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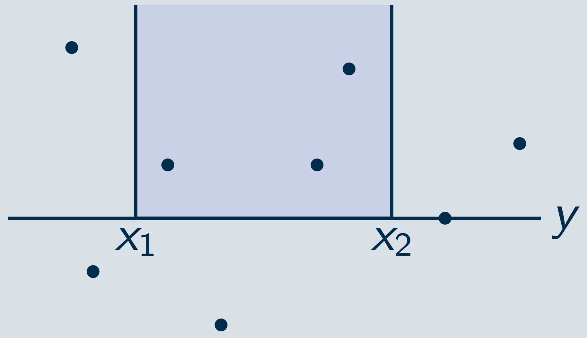

Events

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Why $O(\log(n) + k)$?

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Mission Impossible + 1.5 Range Queries

Persistent ~~Rotating~~ Sweep Line

- ~~walls~~ points that ~~intersect~~ are above sweep line
- sort by ~~distance to center~~ x-coordinate
- data structure: persistent binary search tree

timestamp?

Events

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- end point: remove wall

Queries

- ask for time $= \alpha$
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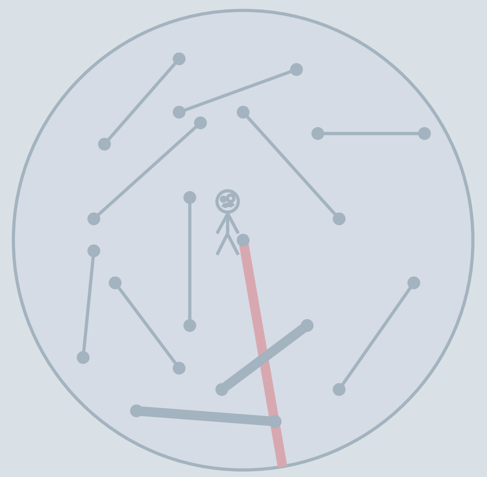
Why $O(\log(n) + k)$?



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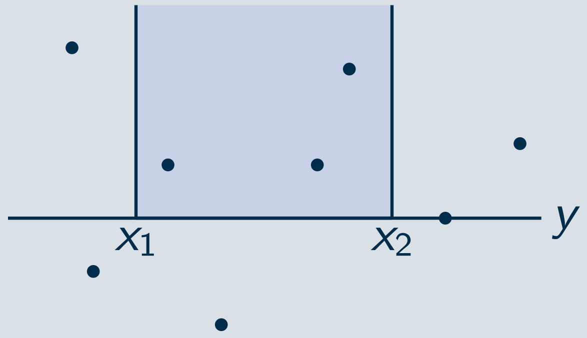

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timestamp?

Events

- start point: add ~~wall~~ point
- ~~end point: remove wall~~

Queries

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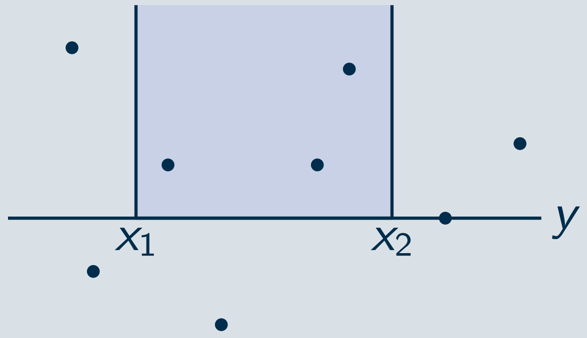

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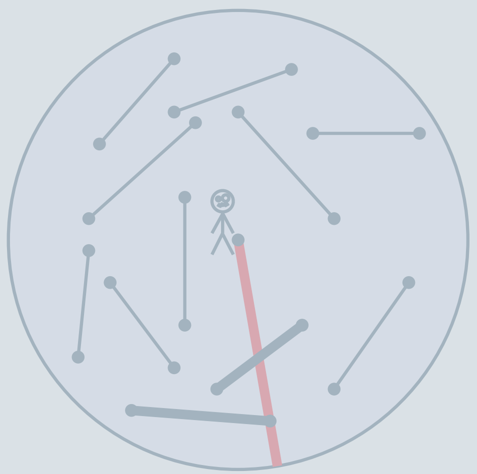

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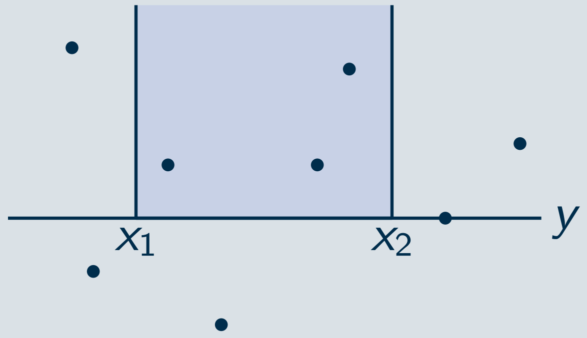

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timestamp?

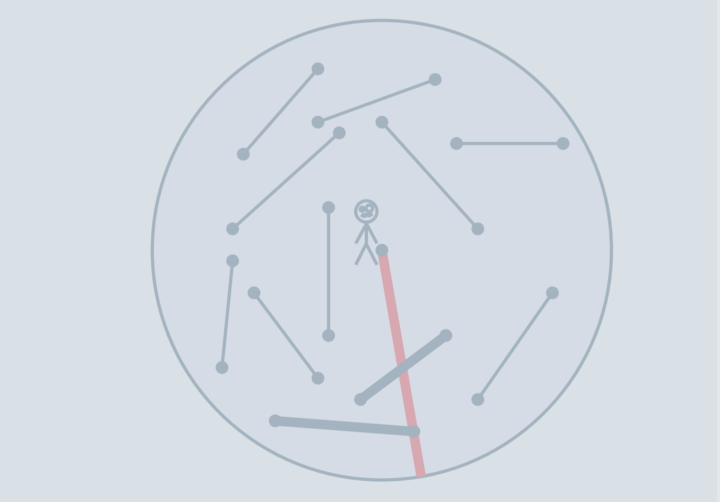
Events

- start point: add ~~wall~~ point
- ~~end point: remove wall~~

Queries

- ask for time $=$ ~~α~~ y
- ~~output all walls currently in sweep line state~~ find x_1, output all between x_1 and x_2

Why $O(\log(n) + k)$?



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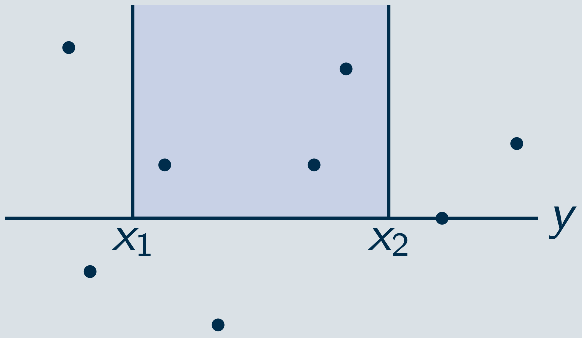
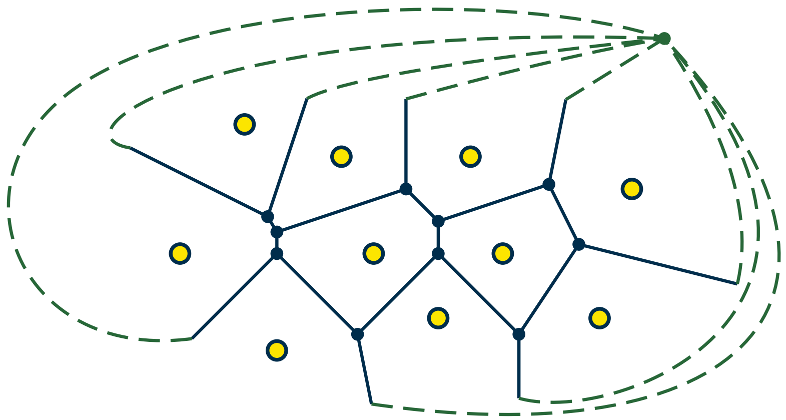


Voronoi Diagram

Size of Voronoi Diagram



- add vertex to collect loose edges

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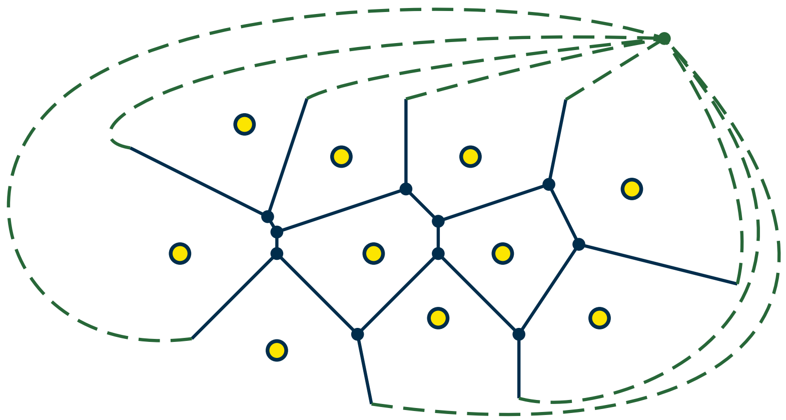

- add vertex to collect loose edges
- n faces

Voronoi Diagram

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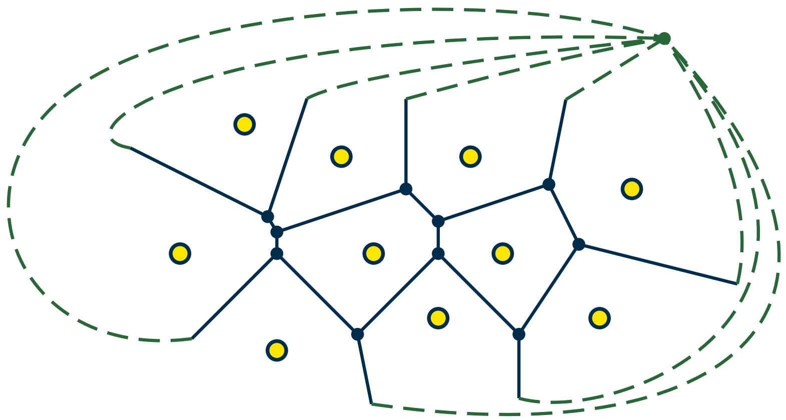

- add vertex to collect loose edges
- n faces
- $v + 1$ vertices, with degree ≥ 3

Voronoi Diagram

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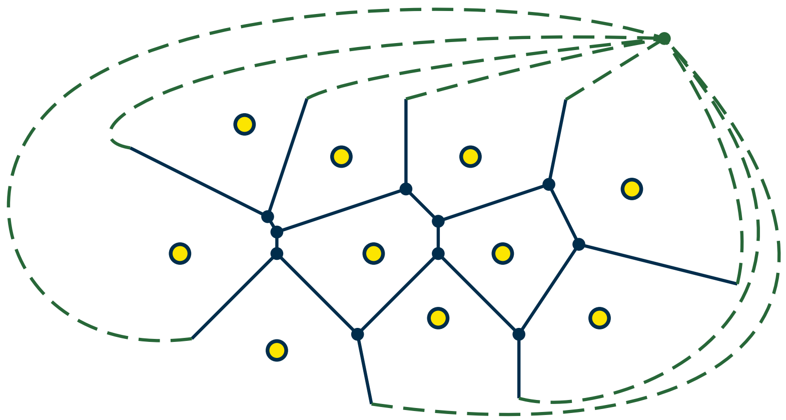

- add vertex to collect loose edges
- n faces
- $v + 1$ vertices, with degree ≥ 3
- m edges with $2m \geq 3(v + 1)$

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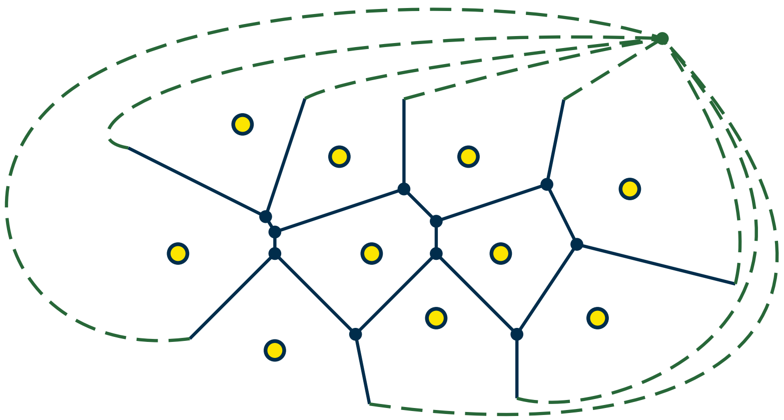

- add vertex to collect loose edges
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- $v + 1$ vertices, with degree ≥ 3
- m edges with $2m \geq 3(v + 1)$
- use Euler's formula

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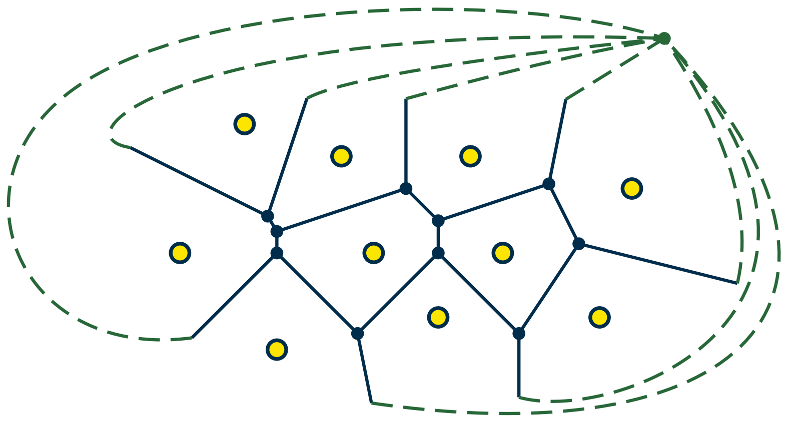
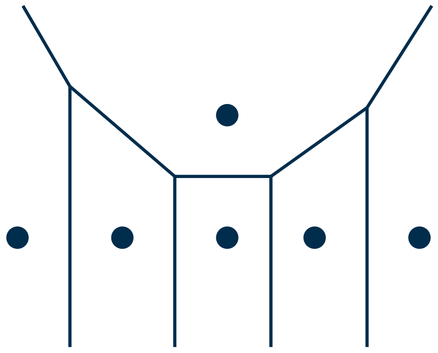


$\Omega(n \log(n))$ lower bound



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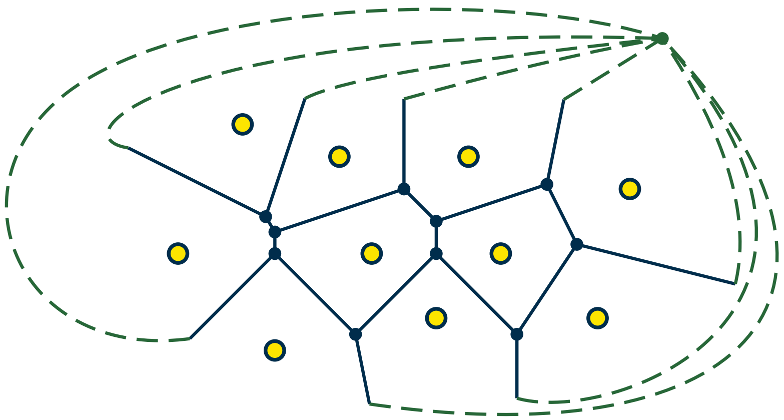

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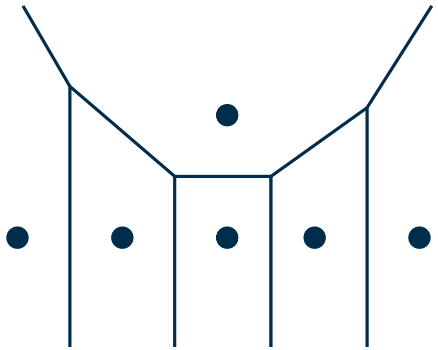
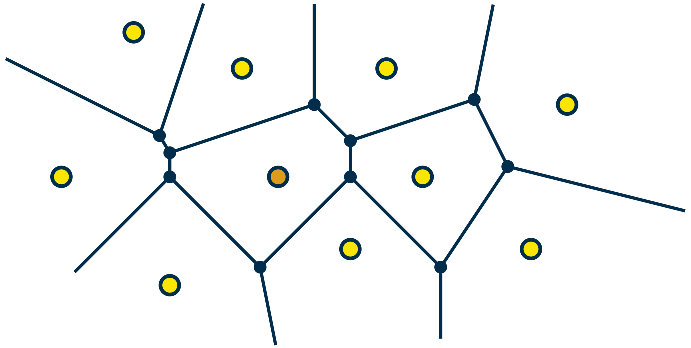


Find closest point



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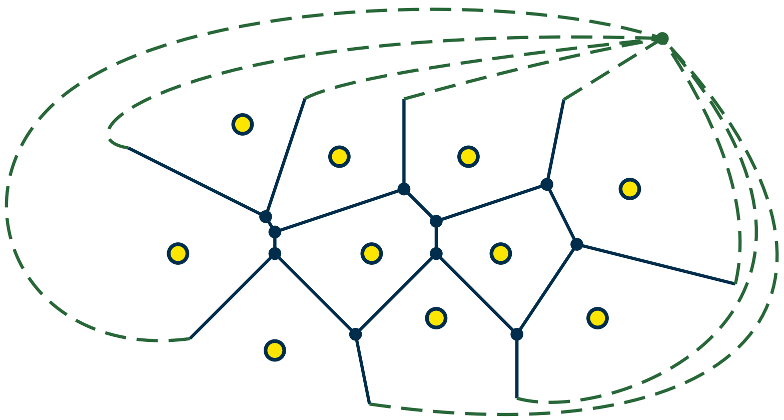

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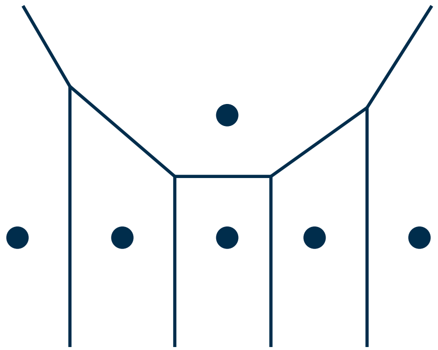

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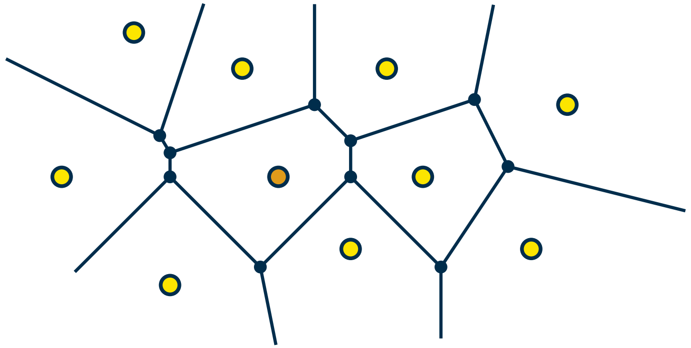

why?

- closest point is in neighbor cell

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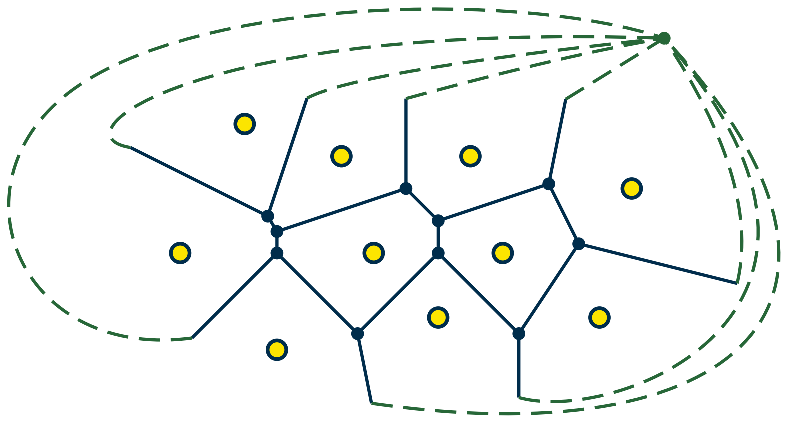

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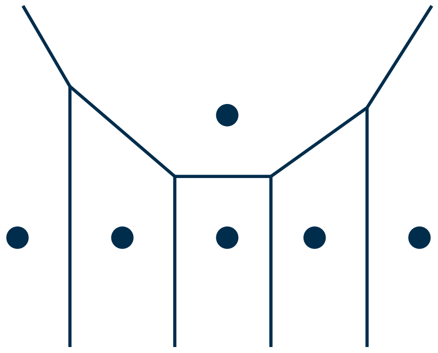

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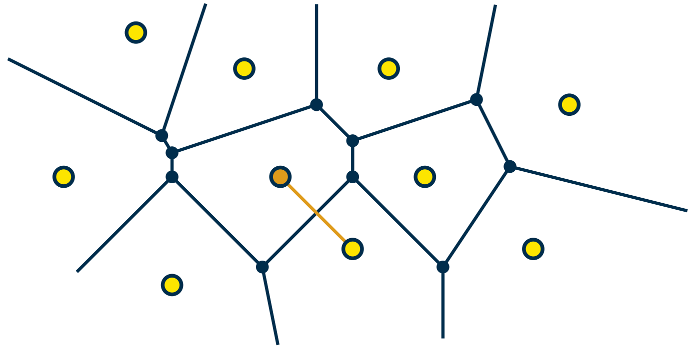

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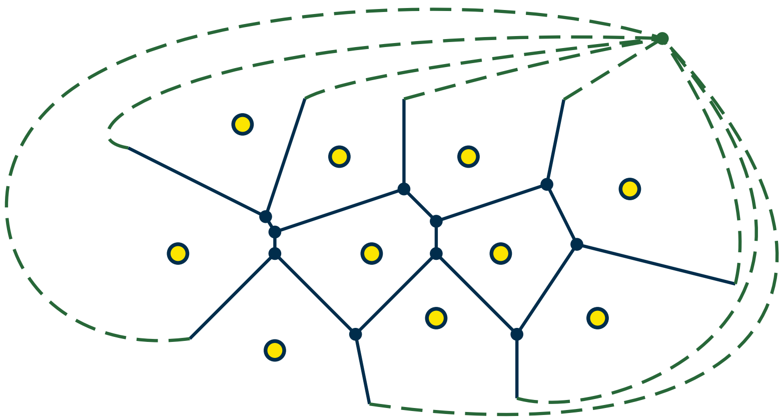

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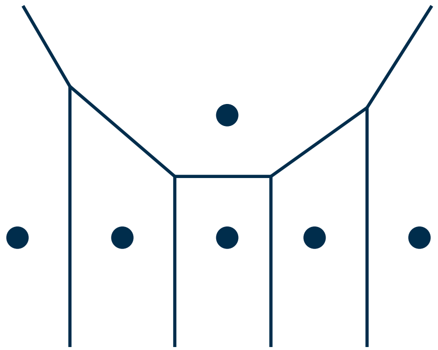

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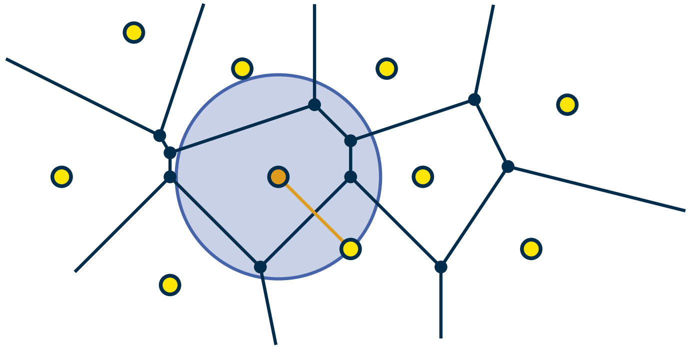

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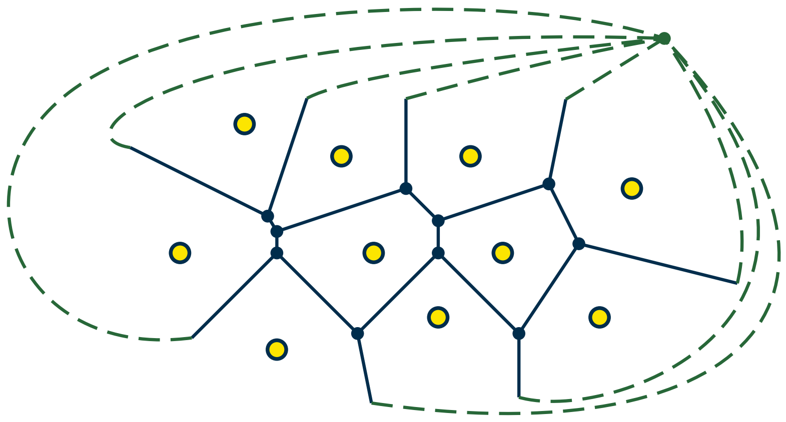

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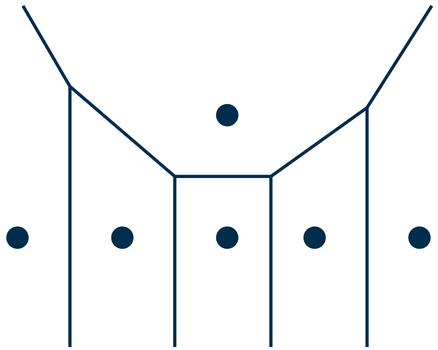
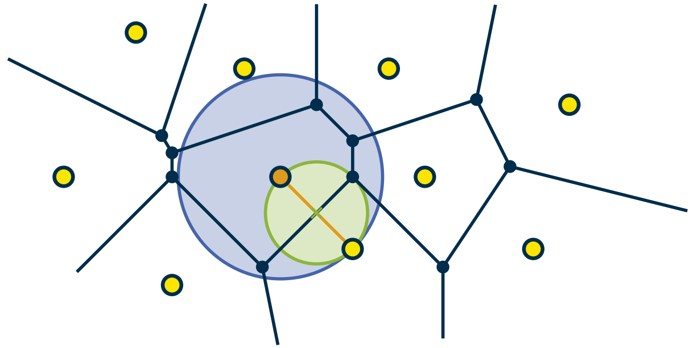


Find closest point

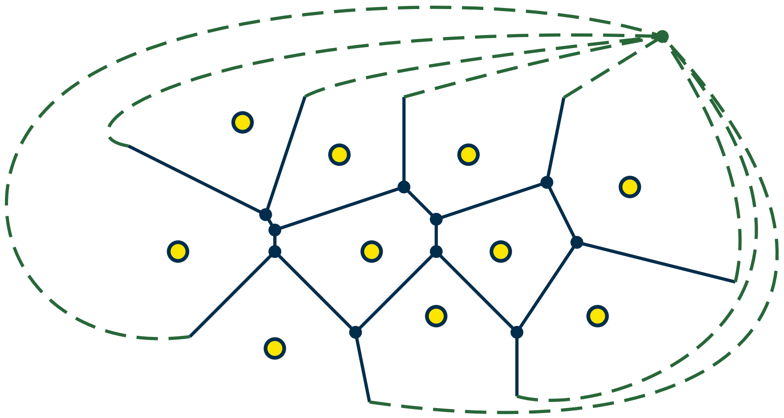


why?

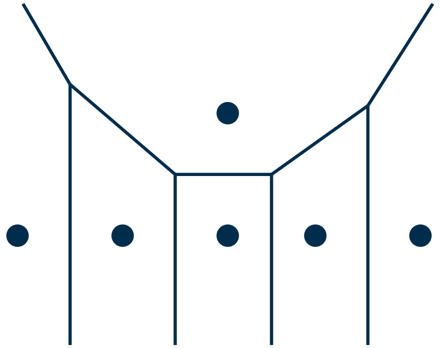
- closest point is in neighbor cell

Voronoi Diagram

Size of Voronoi Diagram



$\Omega(n \log(n))$ lower bound



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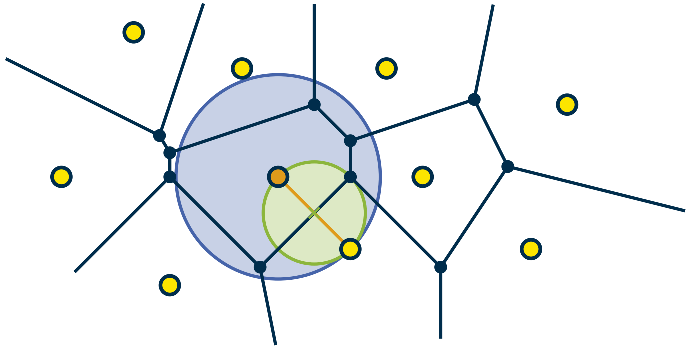

- add vertex to collect loose edges
- n faces
- $v + 1$ vertices, with degree ≥ 3
- m edges with $2m \geq 3(v + 1)$
- use Euler's formula

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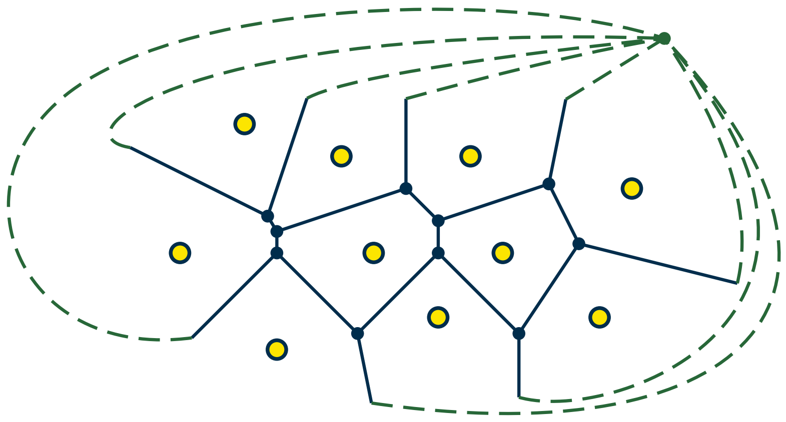

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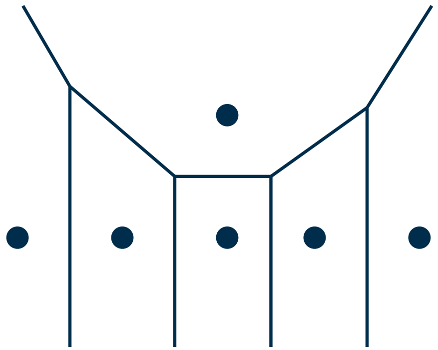

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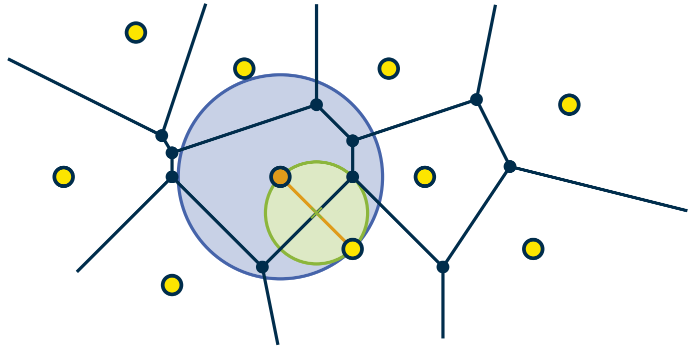

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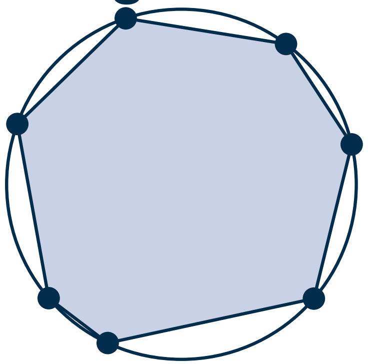
- closest point is in neighbor cell
- mid point lies on edge of cell
- for each point: try all neighbors in VD

Running time?

Assignment 5

Triangulation of Concentric Points



find optimal triangulation
(smallest angle vector)

Assignment 5

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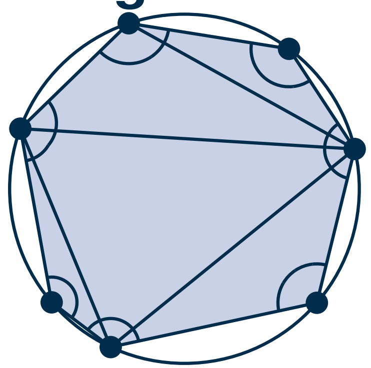

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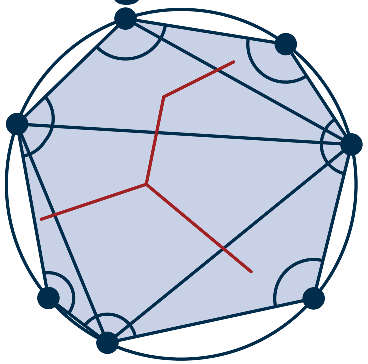

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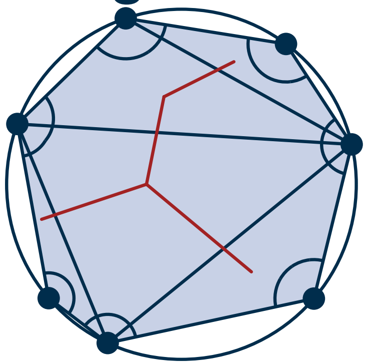

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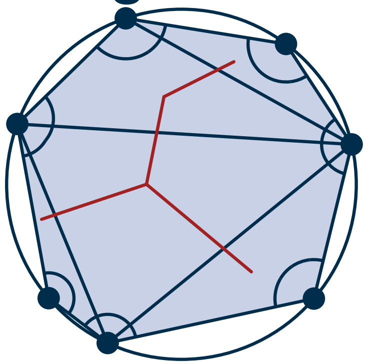

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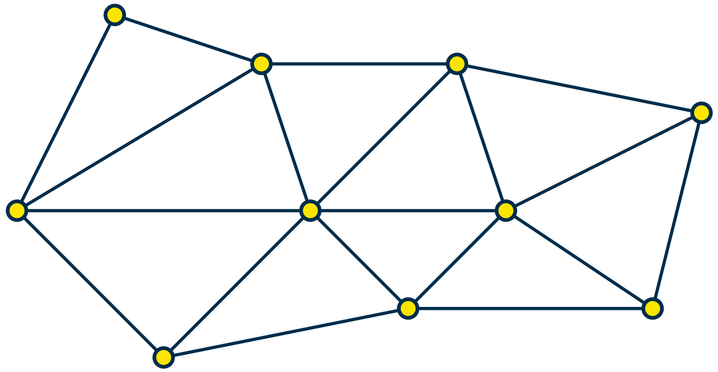

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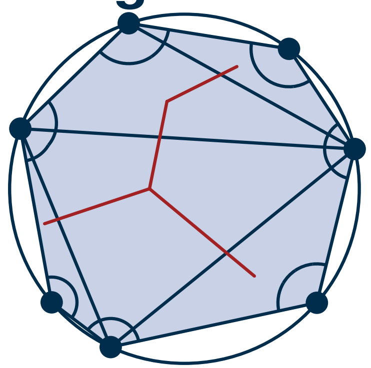

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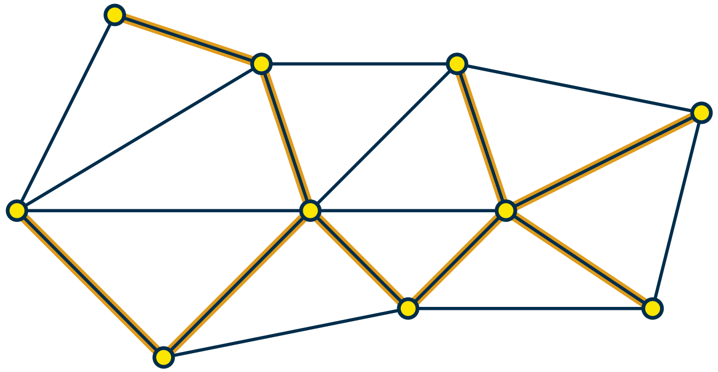

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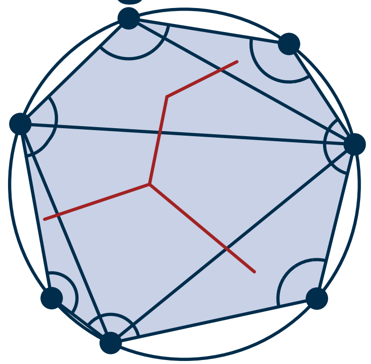

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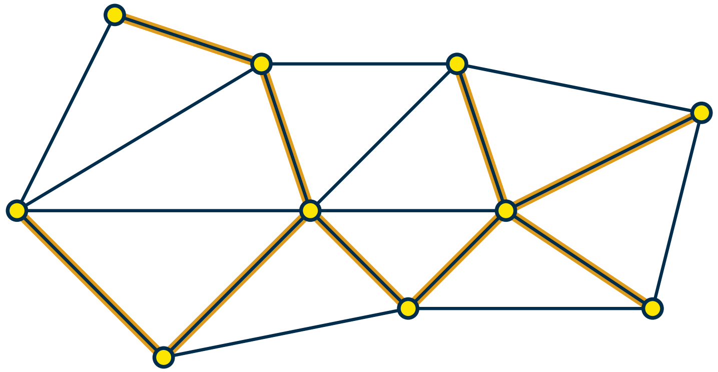

Foldability of Mountain/Valley Patterns

- test foldability in $O(n)$

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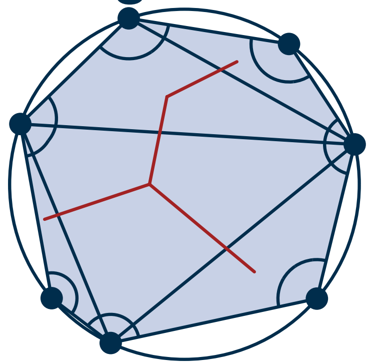

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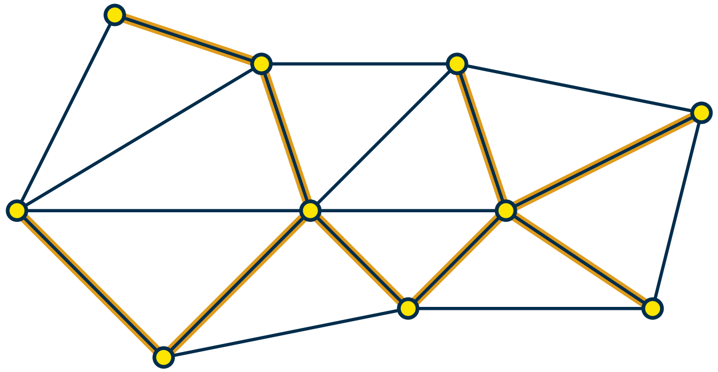
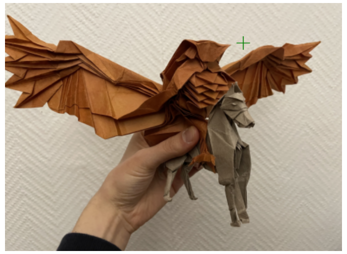
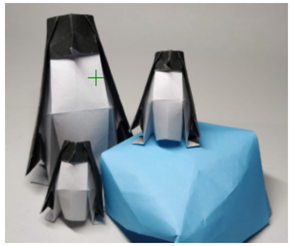


Creative Outlet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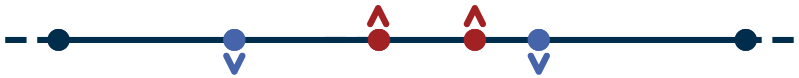

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Blocking Doors

- doors are weird and can block each other

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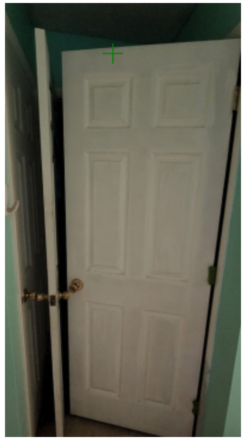

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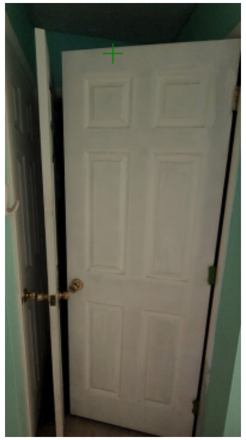

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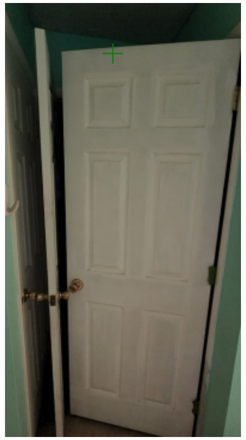

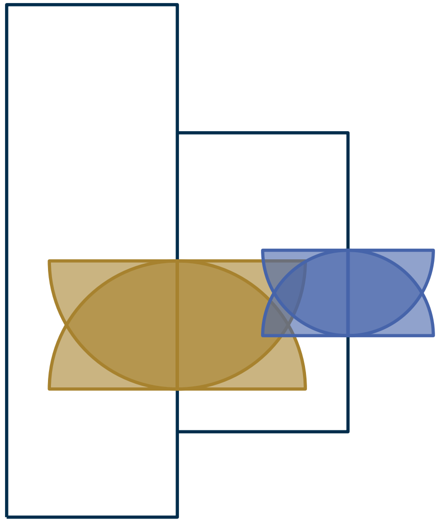

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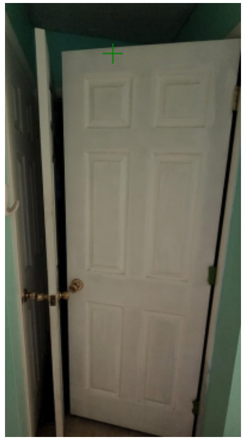


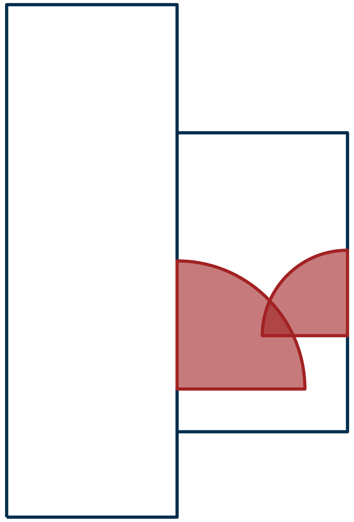

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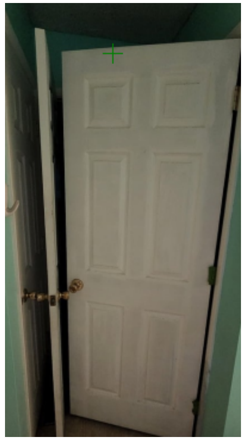

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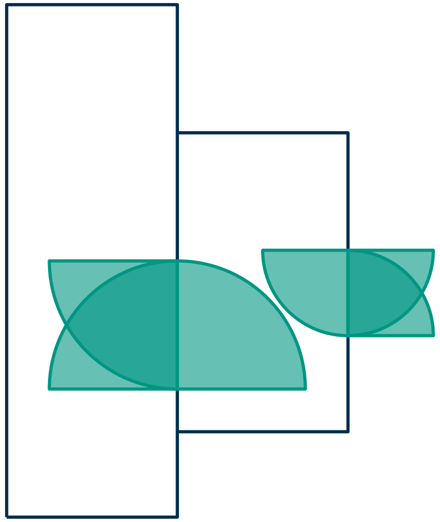

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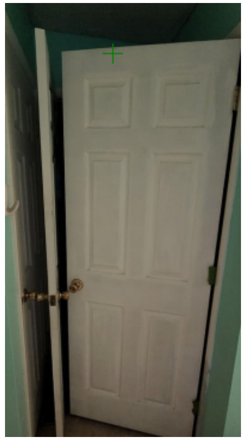

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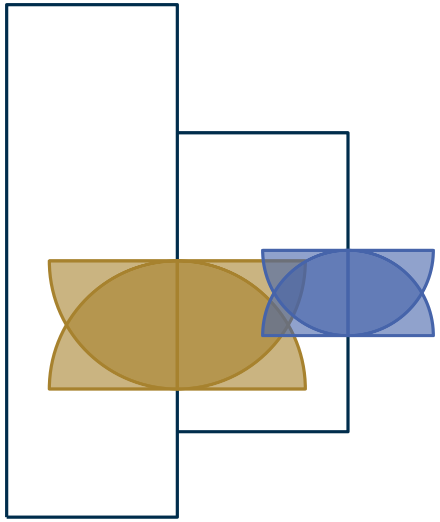

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Build gadgets!

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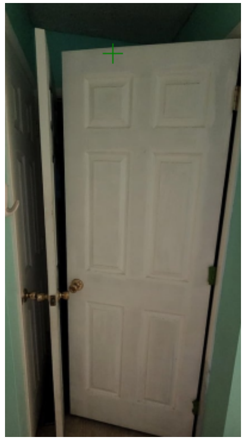

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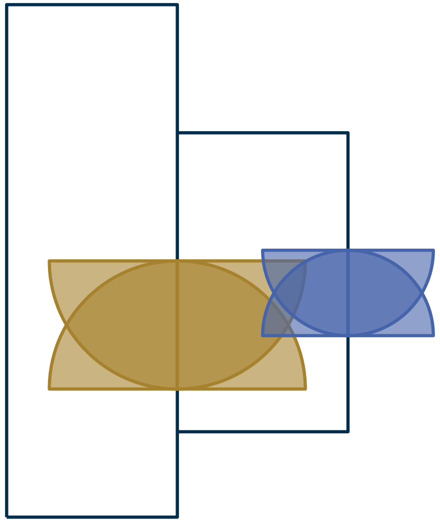

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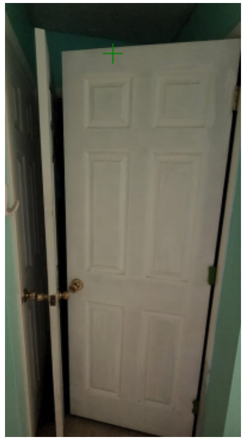

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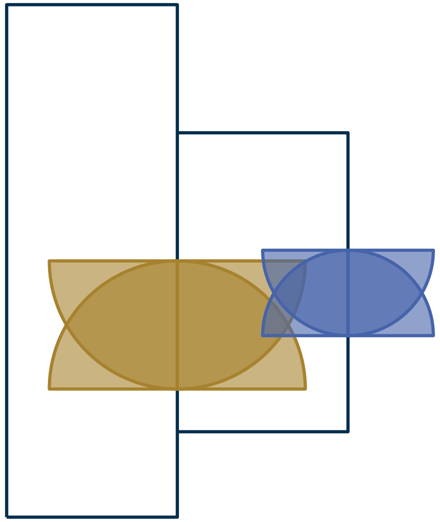

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- Complexity if the side of the hinge is given for each door?

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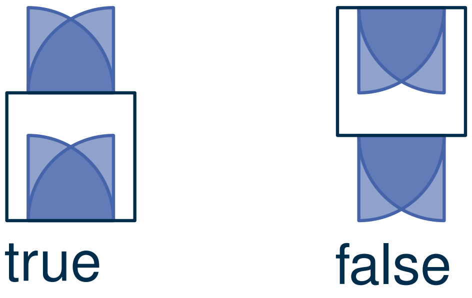

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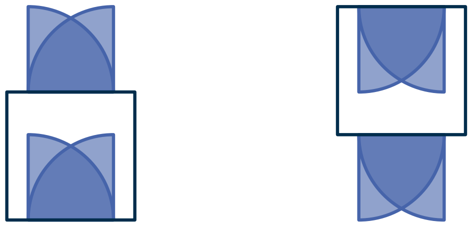

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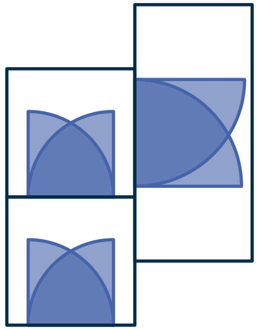
Transport



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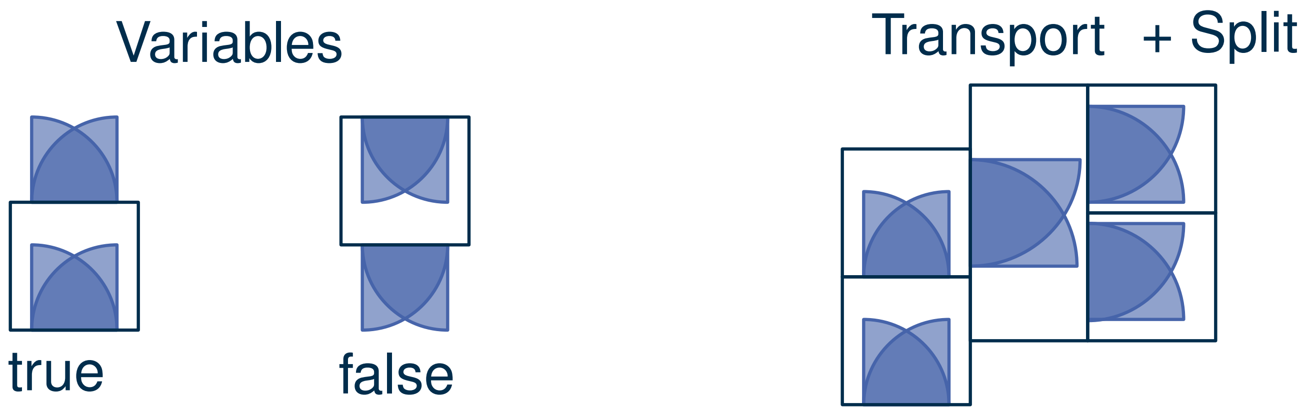

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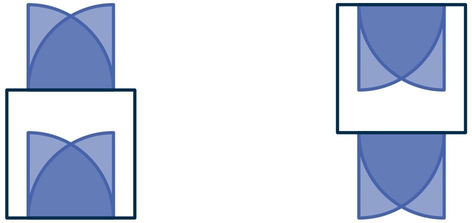

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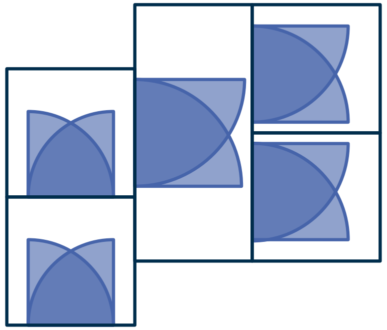
Transport + Split



Clauses

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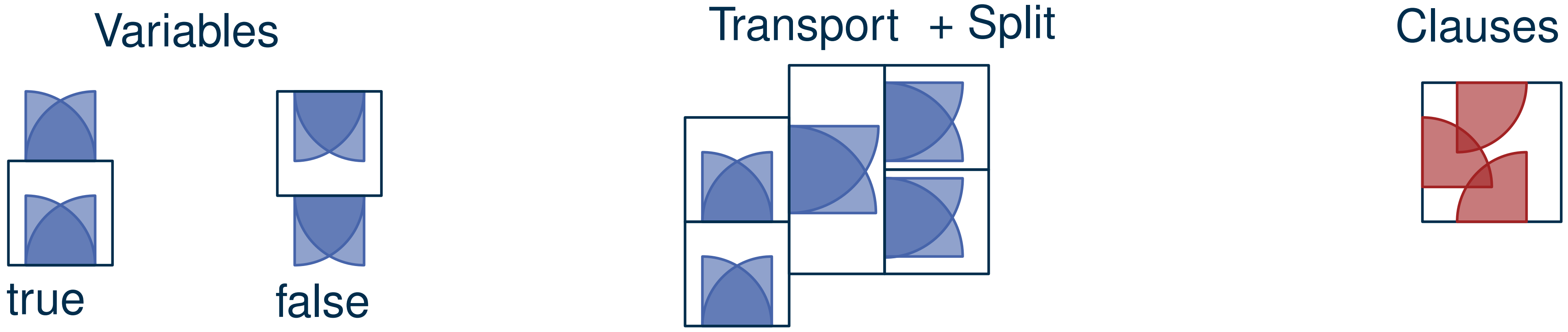

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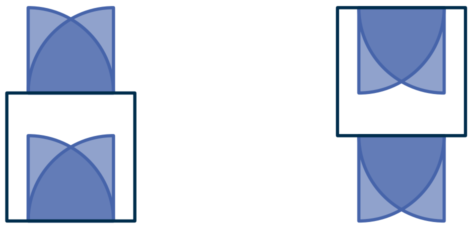

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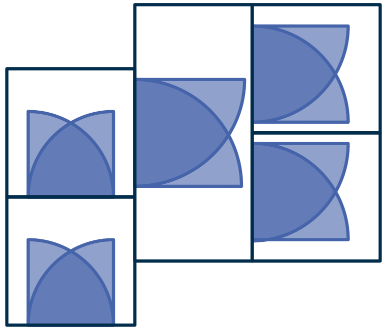

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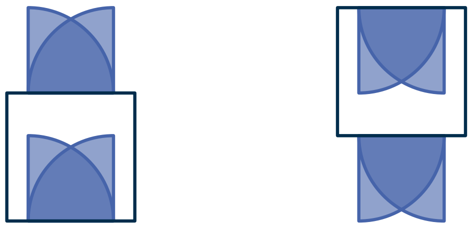

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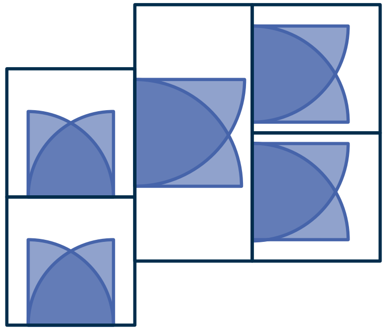

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