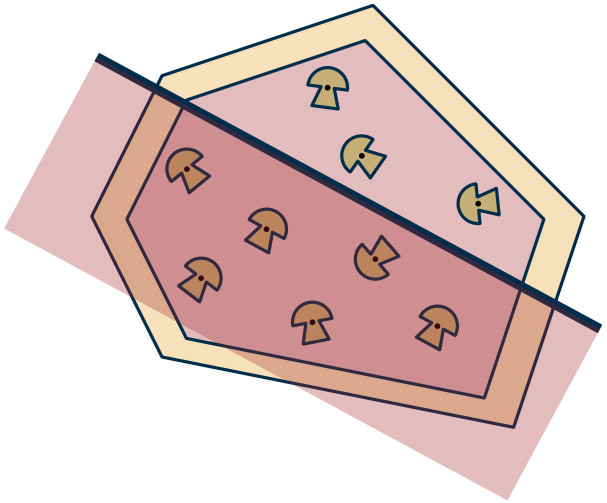
Computational Geometry

Exercise 4 *Assignment 3, 4 and Voronoi diagrams of segments*

Jean-Pierre, Marcus, Wendy

Assignment 3

Pizza-Guillotine



- At which points does the blade cut the pizza polygon?
- Which ingredients are cooked?

- $\mathcal{O}(n \log n)$ precompute, $\mathcal{O}(\log n)$ query

Assignment 3

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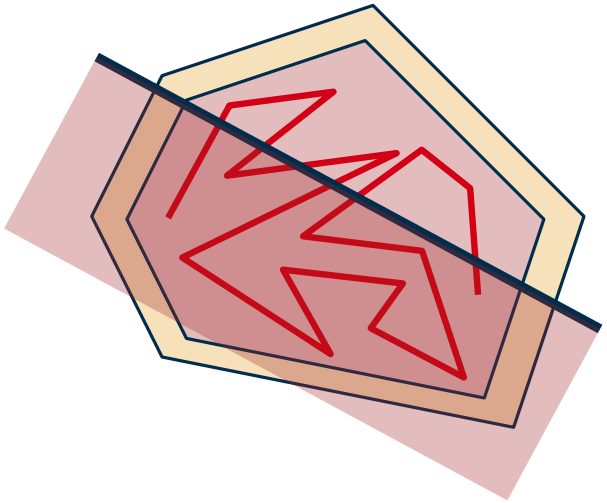

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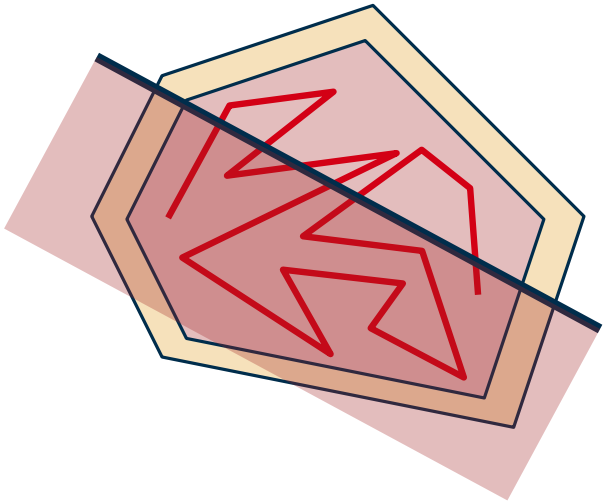

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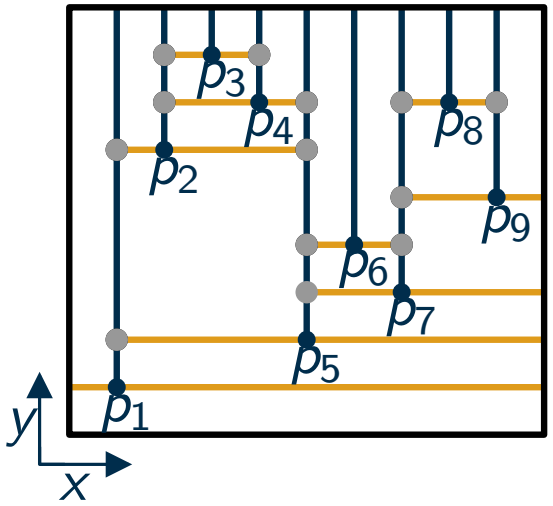

Given the points sorted by x, calculate the geometric graph in $\mathcal{O}(n)$

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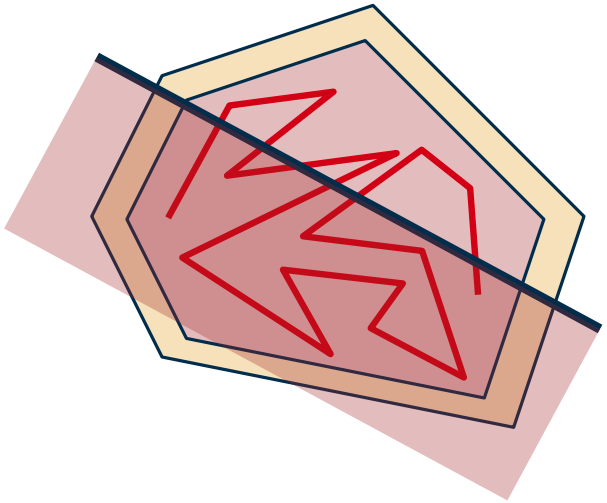

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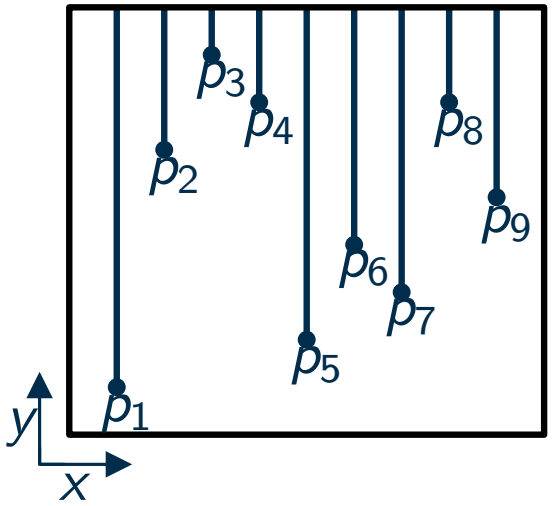

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- process points from left to right

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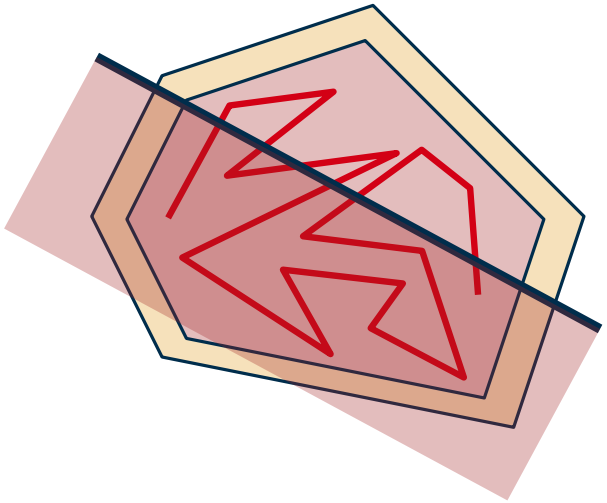

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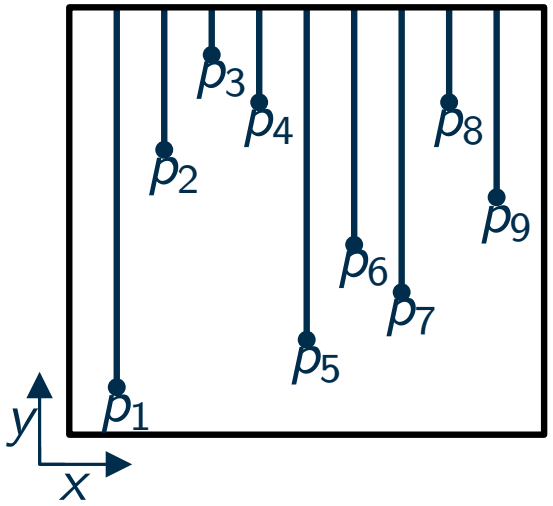

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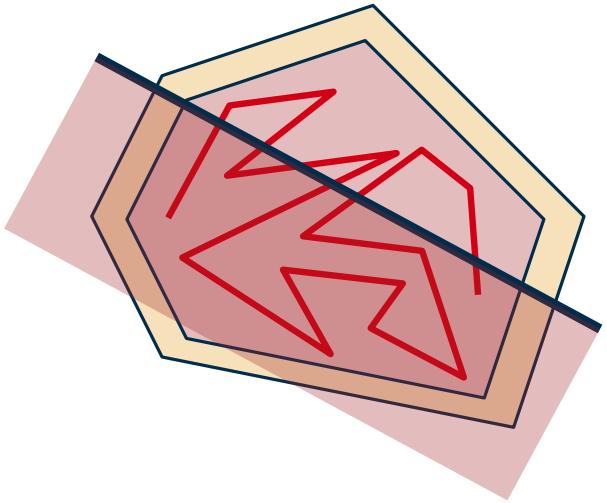

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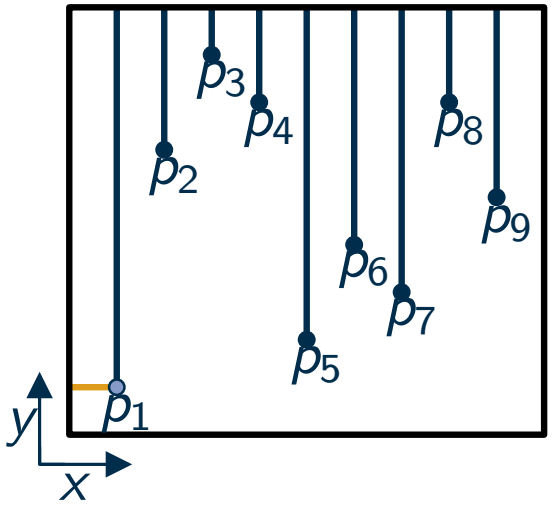

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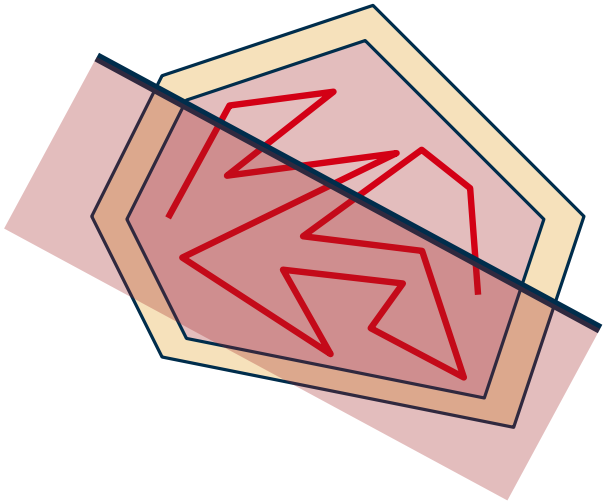

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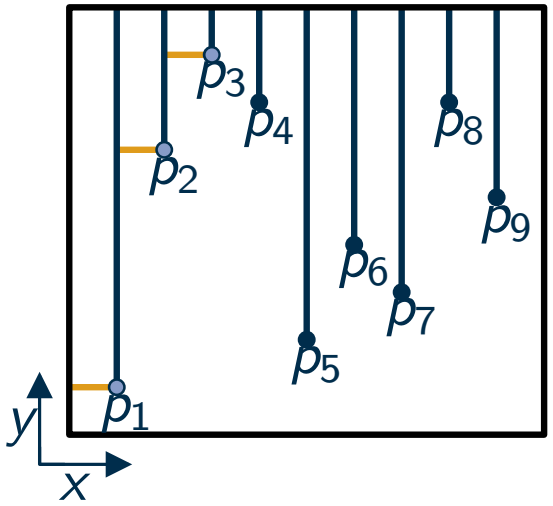

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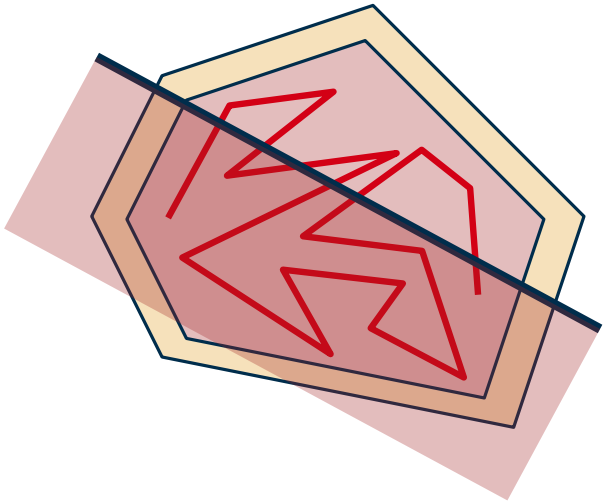

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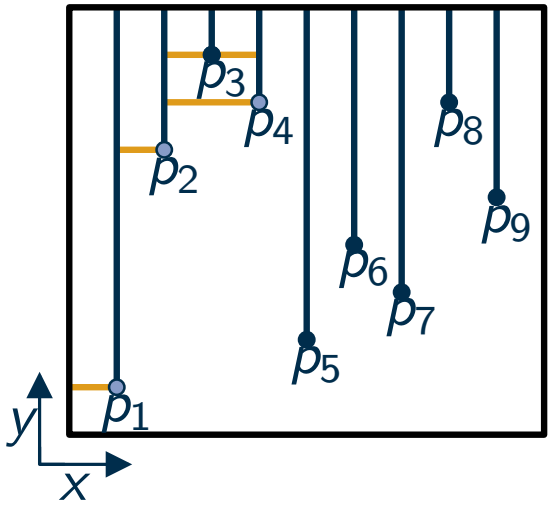

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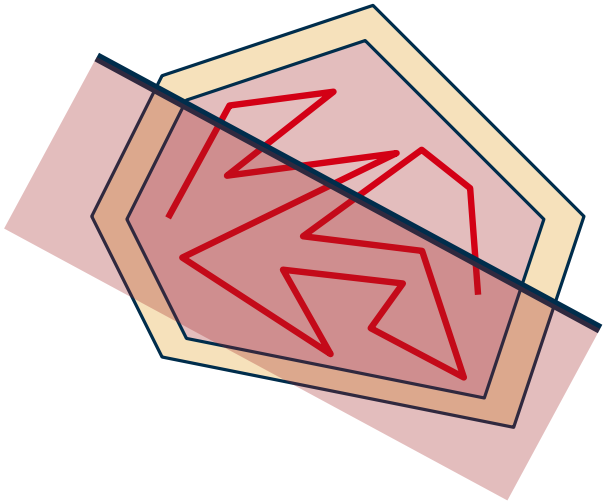

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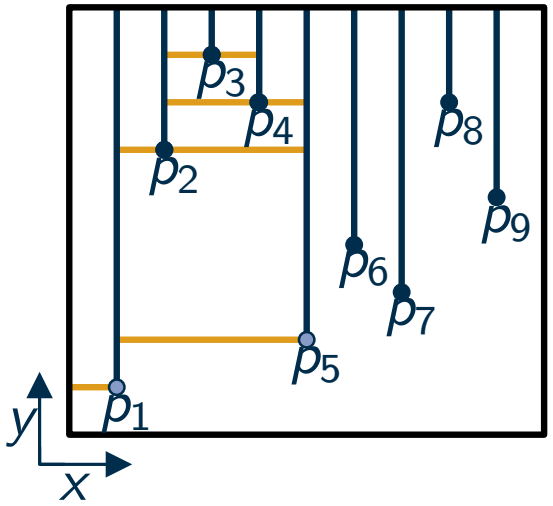

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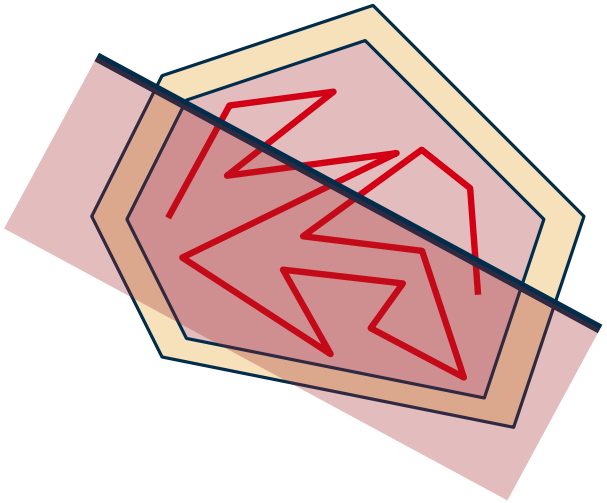

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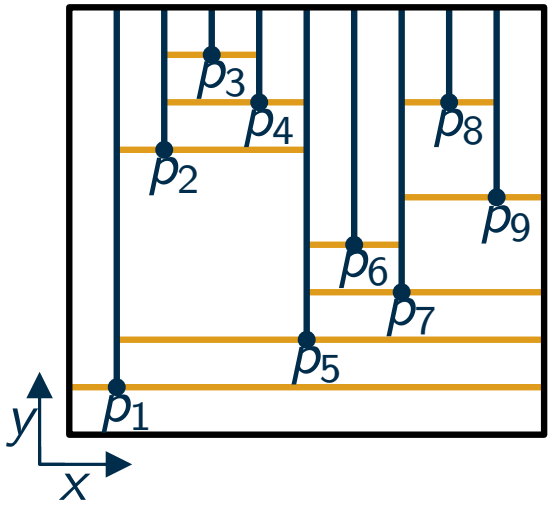

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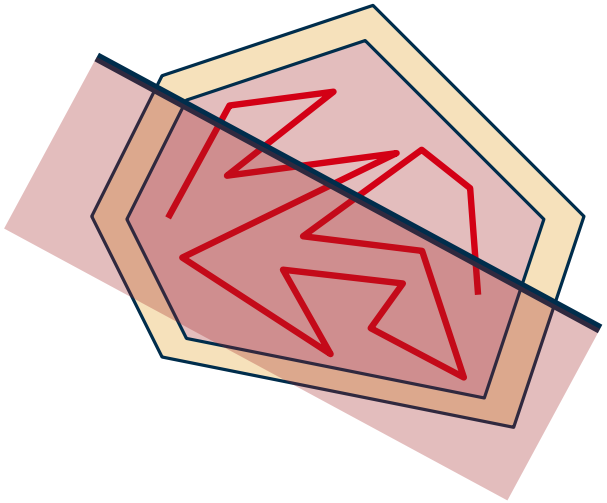

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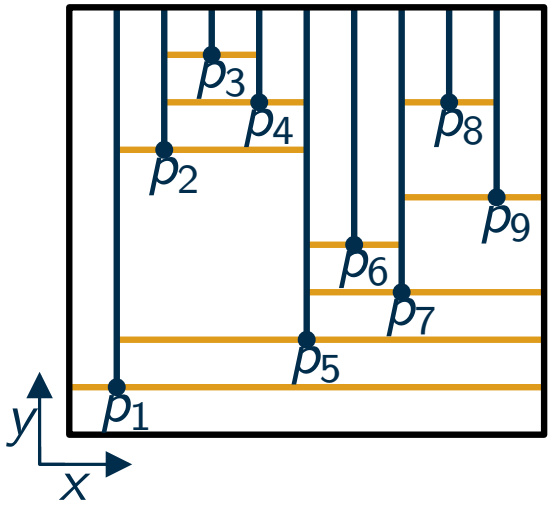

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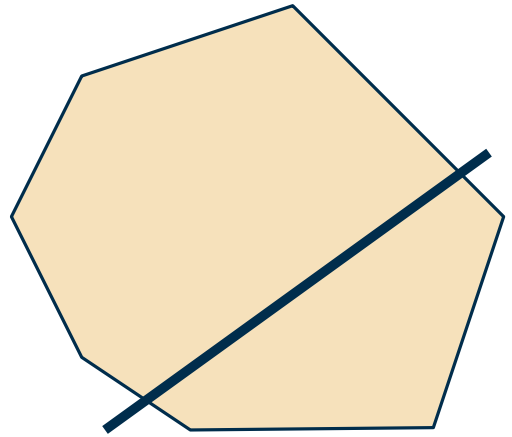
Invariant? Correctness? Runtime?

Pizza Guillotine I

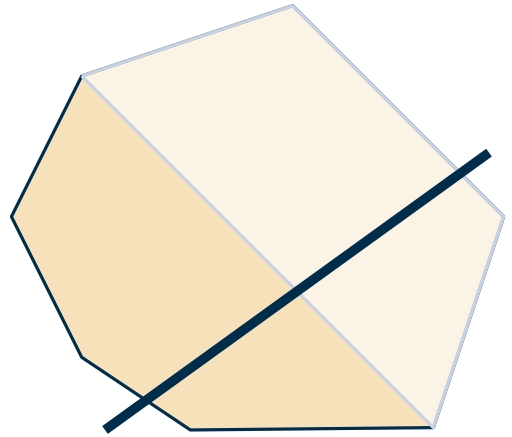
a) Find intersection of line with convex polygon



Pizza Guillotine I

a) Find intersection of line with convex polygon

Step 1: split polygon such that each part contains one intersection



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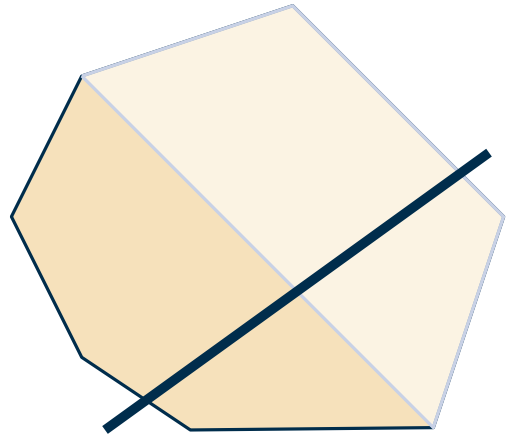

Step 2: find intersection for each part

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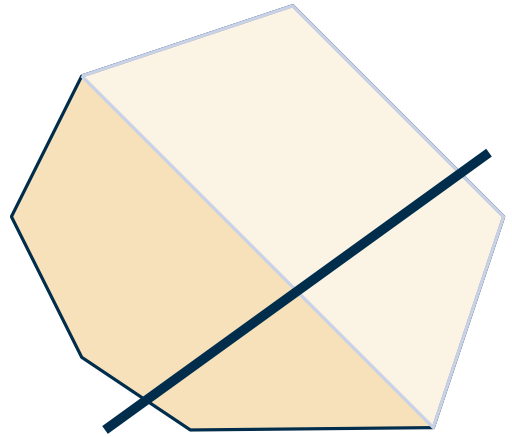

Step 2: find intersection for each part

- use binary search over vertices

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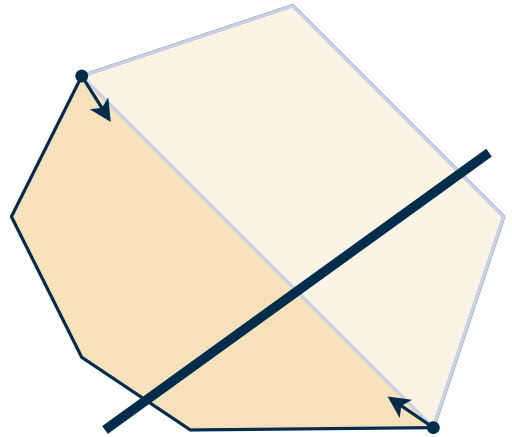

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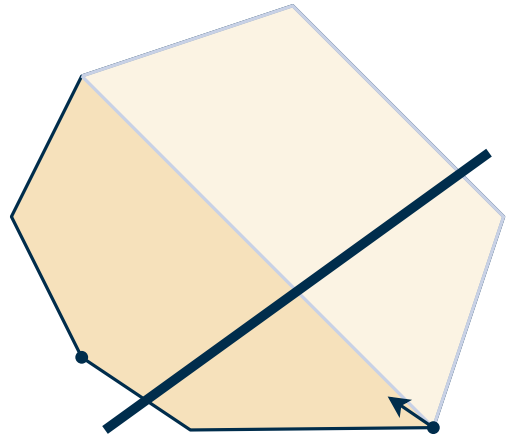

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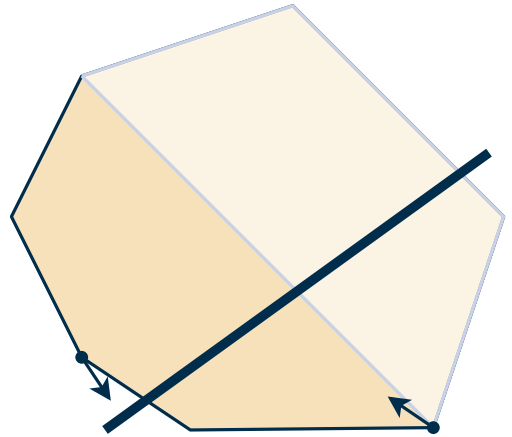

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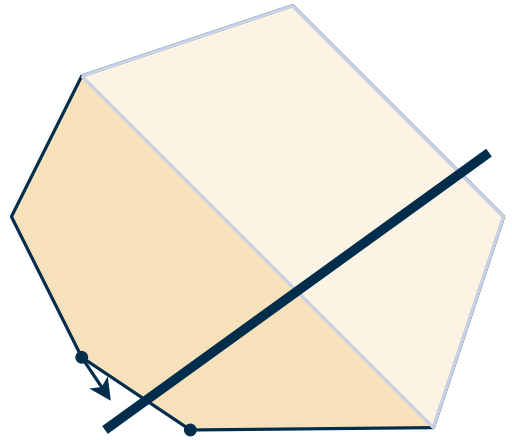

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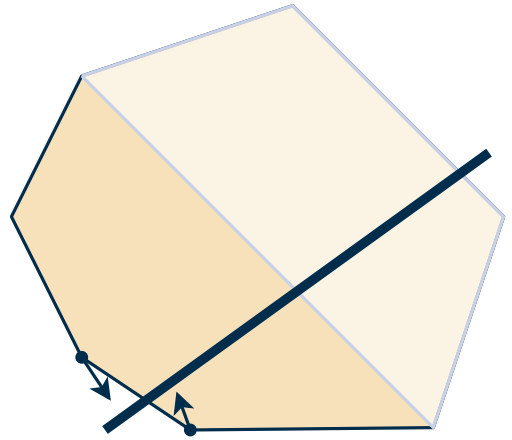

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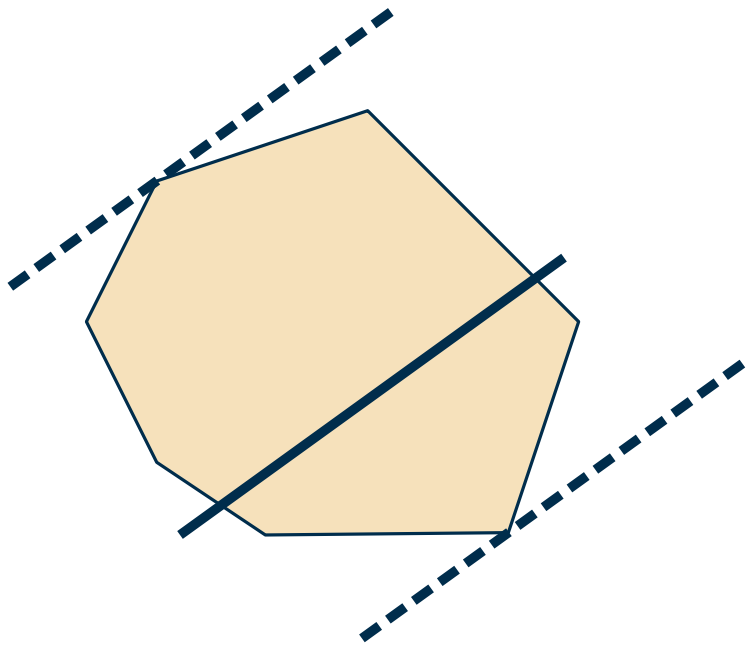
FIND TANGENTS PARALLEL TO LINE



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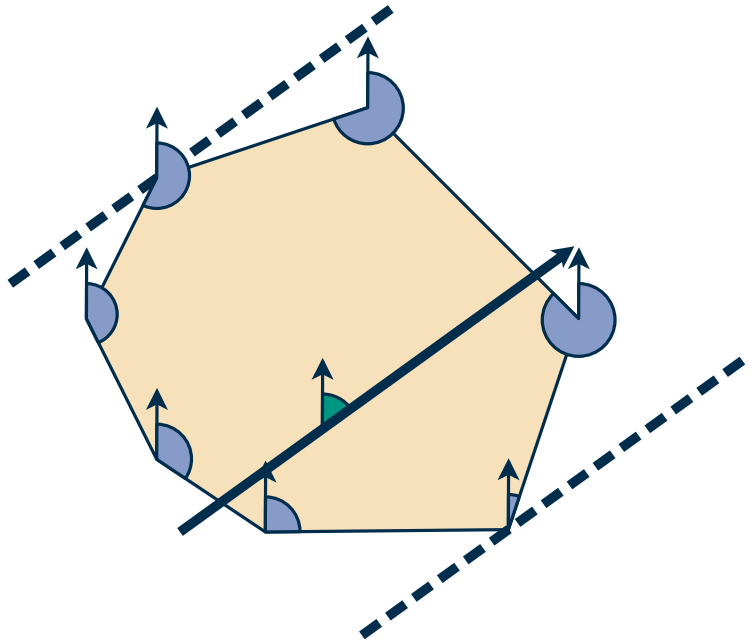
FIND TANGENTS PARALLEL TO LINE

- find edge with closest y-axis-angle to oriented line



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a) Find intersection of line with convex polygon

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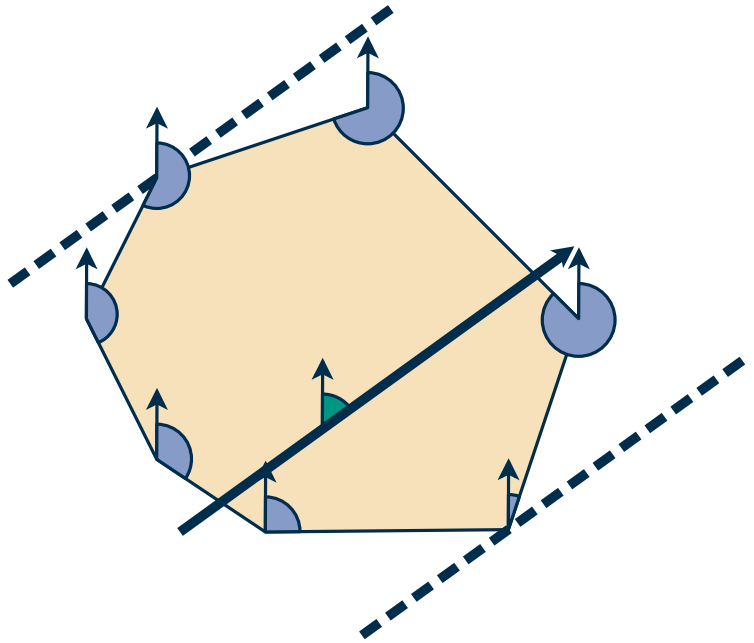
FIND TANGENTS PARALLEL TO LINE

- find edge with closest y-axis-angle to oriented line
- binary search, monotone property: y-axis-angle sorted along polygon



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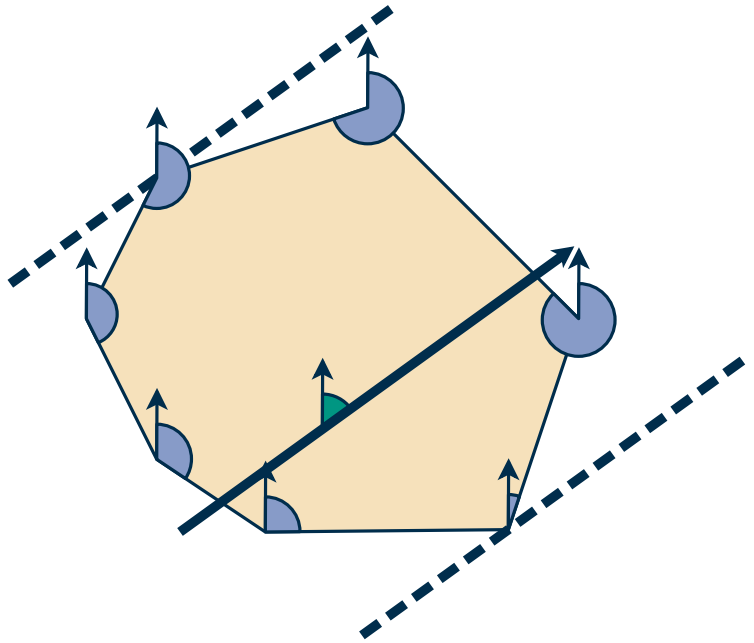
FIND TANGENTS PARALLEL TO LINE

- find edge with closest y-axis-angle to oriented line
- binary search, monotone property: y-axis-angle sorted along polygon
- try both orientations



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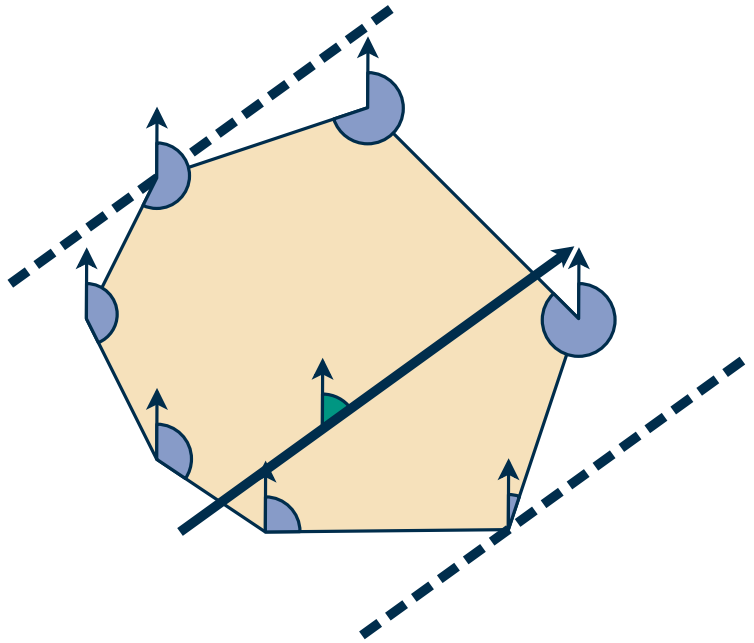

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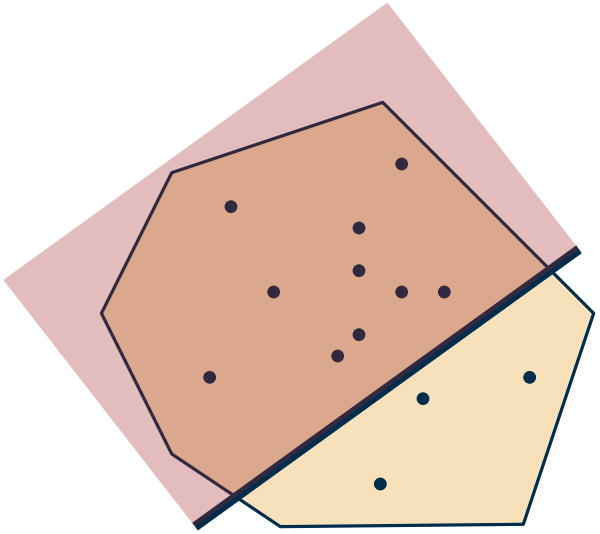
Edge cases? Runtime?

Pizza Guillotine II

b) Output all points in polygon of one side of line



Pizza Guillotine II

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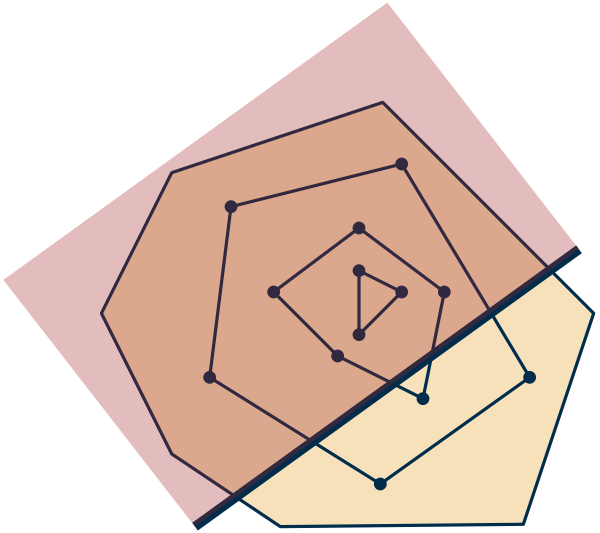
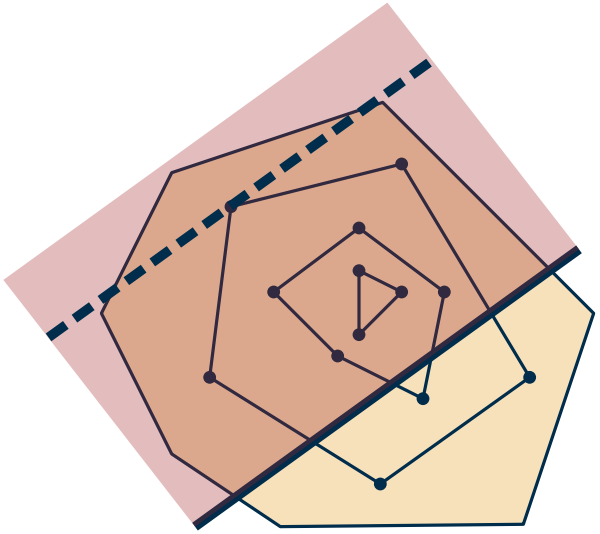


Pizza Guillotine II

b) Output all points in polygon of one side of line

Outermost convex hull

- FIND TANGENTS PARALLEL TO LINE on correct side

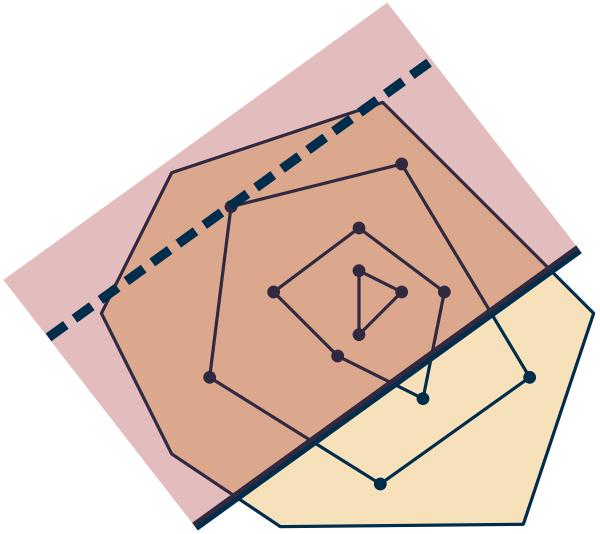


Pizza Guillotine II

b) Output all points in polygon of one side of line

Outermost convex hull

- FIND TANGENTS PARALLEL TO LINE on correct side
- linearly output all points on correct side



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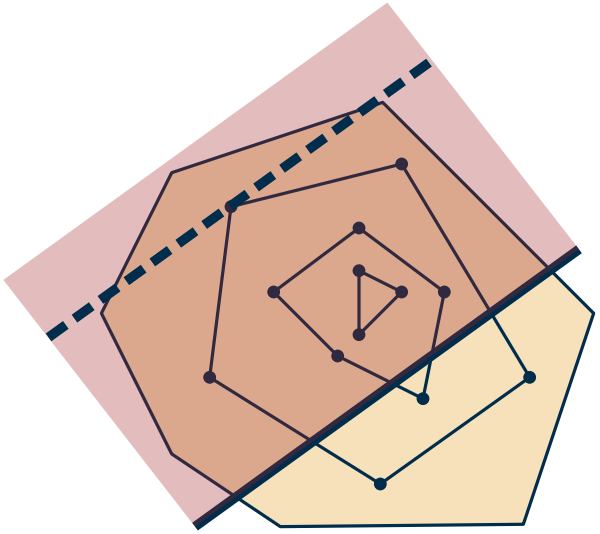
Next convex hull



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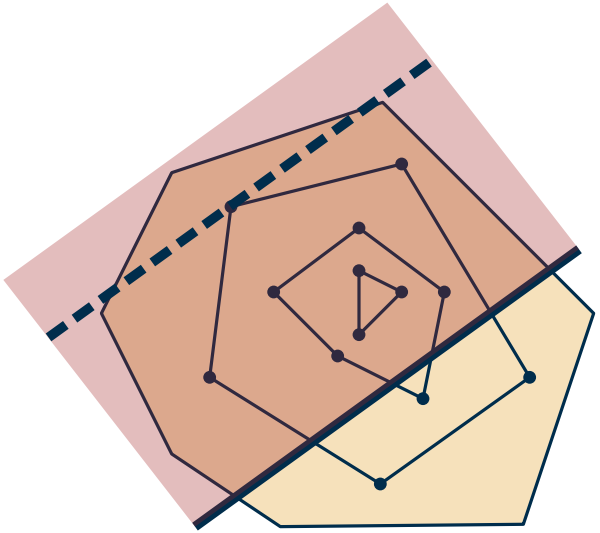
fractional cascading magic 🥳



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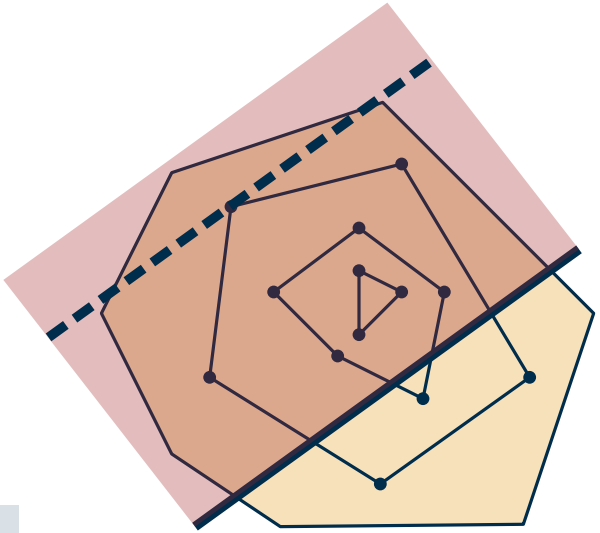

Next convex hull

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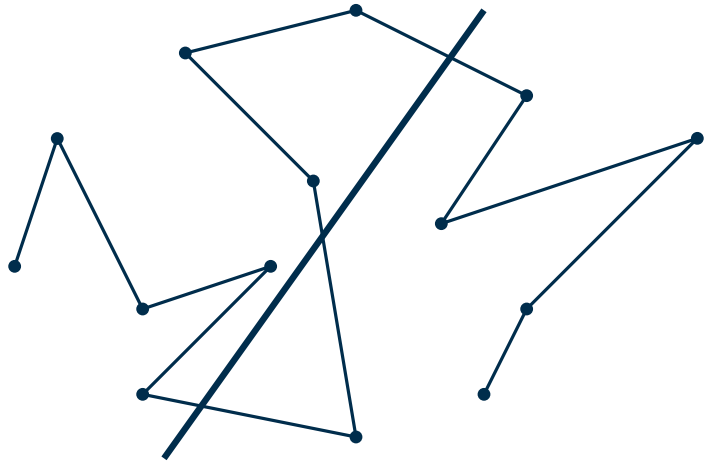
Why does this work? How to preprocess? Runtime?

Pizza Guillotine III

c) Output all intersections of line with a polygon chain

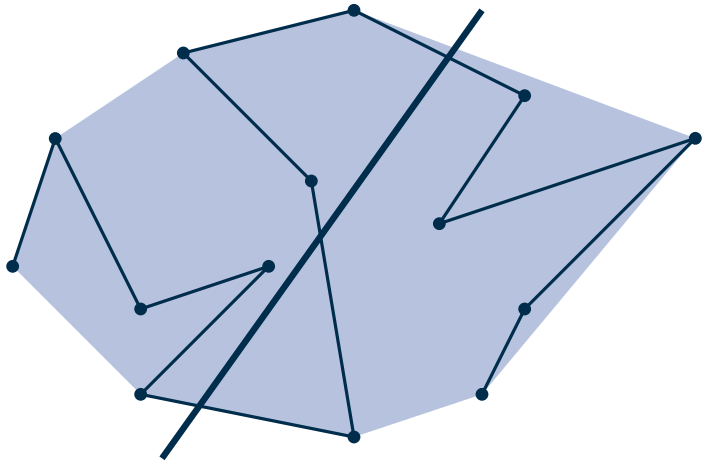


Pizza Guillotine III

c) Output all intersections of line with a polygon chain

Preprocessing

- determine convex hulls recursively and divide path in two parts



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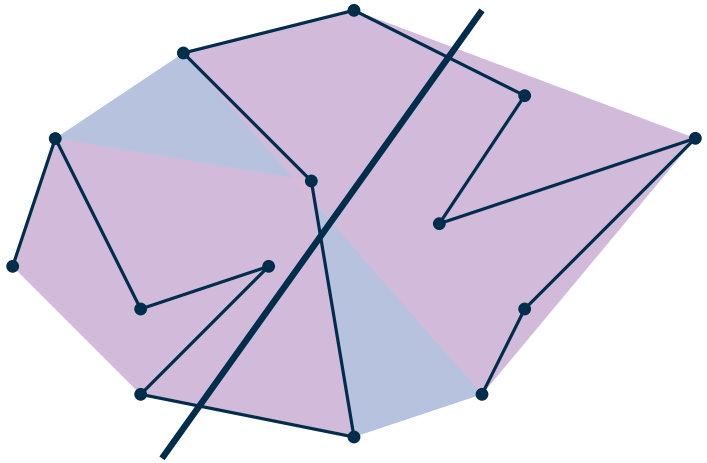

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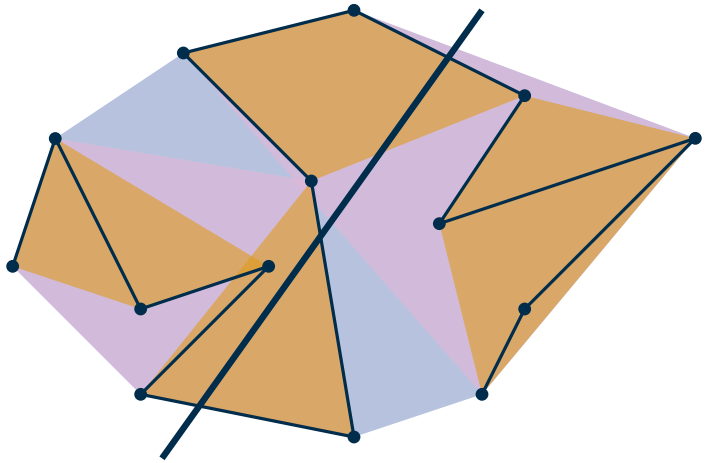
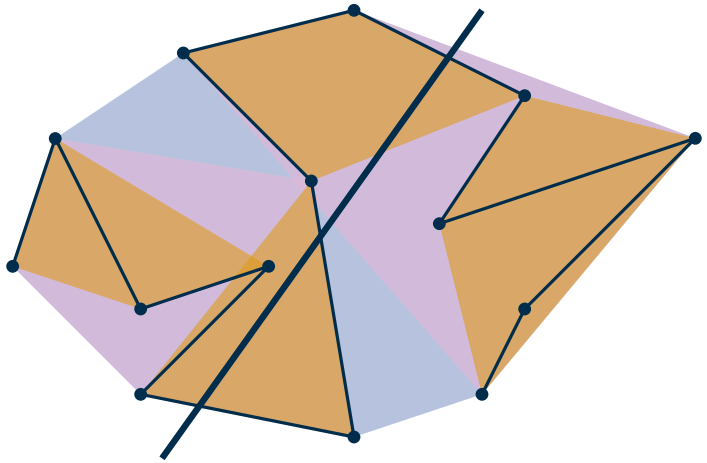


Pizza Guillotine III

c) Output all intersections of line with a polygon chain

Preprocessing

- determine convex hulls recursively and divide path in two parts
- build binary tree from convex hulls



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Preprocessing

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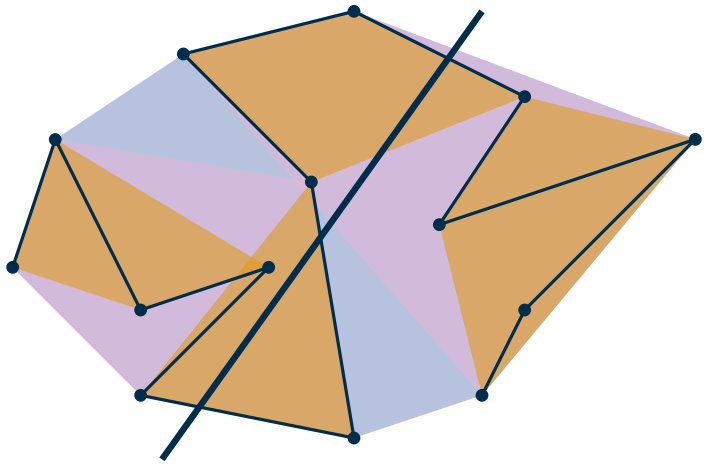
Query

- traverse nodes in binary tree that are intersected



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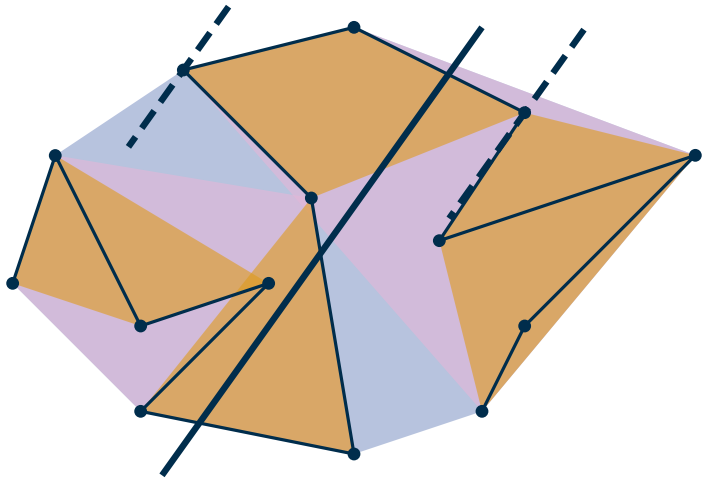
- intersected if line between parallel tangents (use FIND TANGENTS PARALLEL TO LINE)



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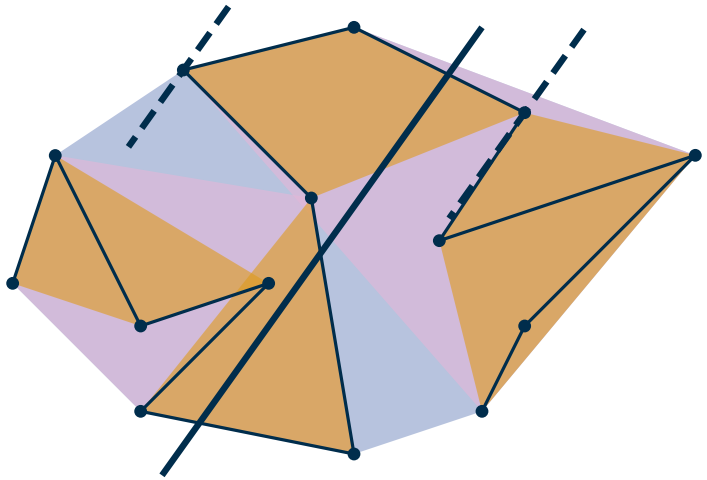
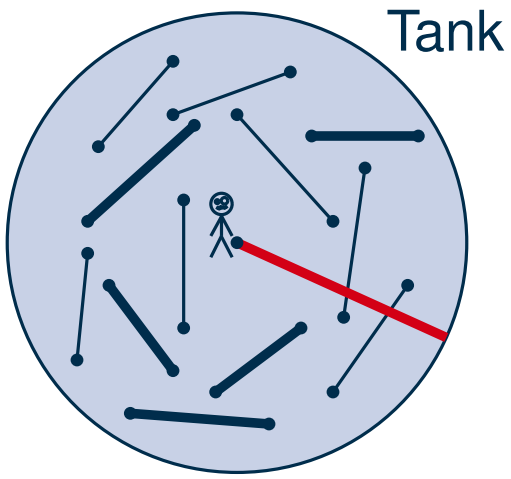
more fractional cascading magic 🥳



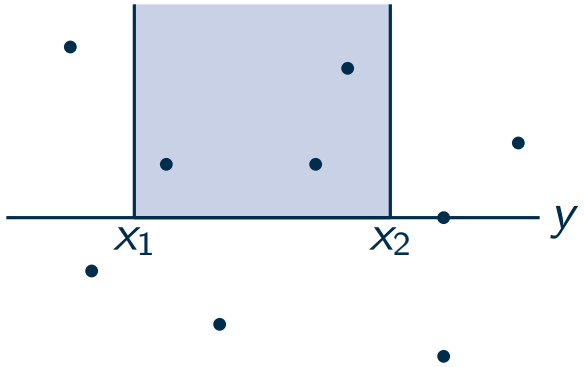
Runtime?

Assignment 4

Mission Impossible

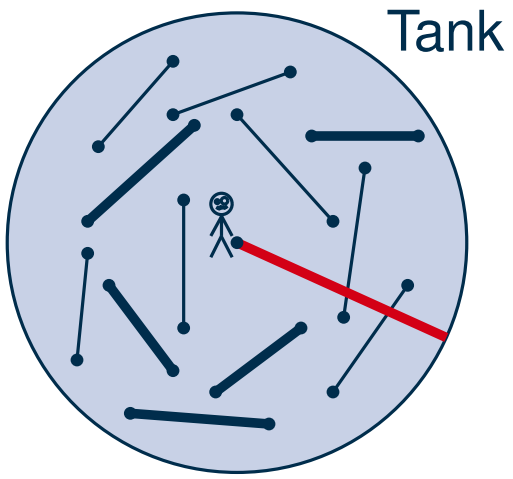


1.5 Range Queries



Assignment 4

Mission Impossible



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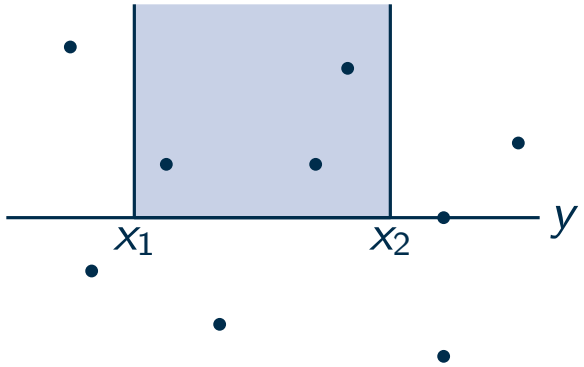
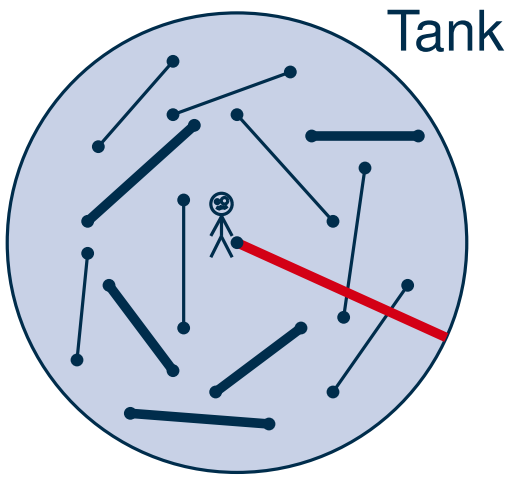


- output something in $O(\log(n) + k)$
- preprocessing time $O(n \log(n))$

Assignment 4

Mission Impossible



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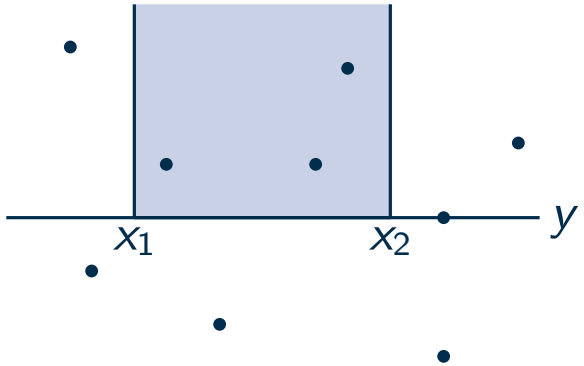
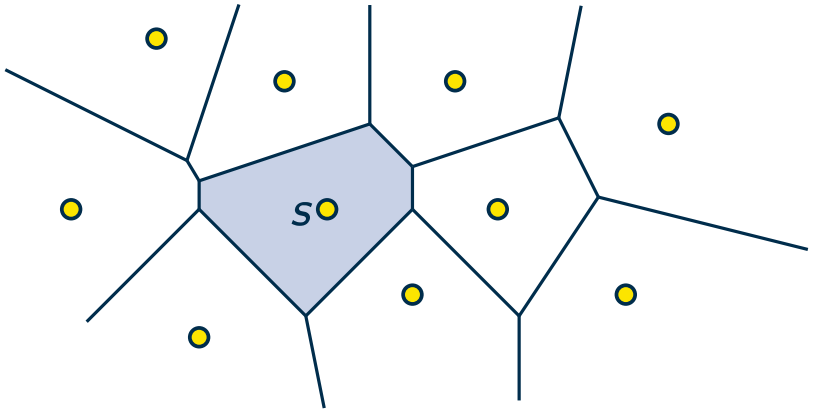


Voronoi diagram



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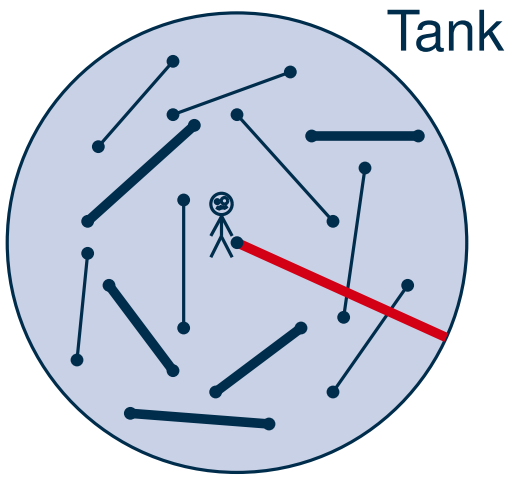
Assignment 4

Mission Impossible



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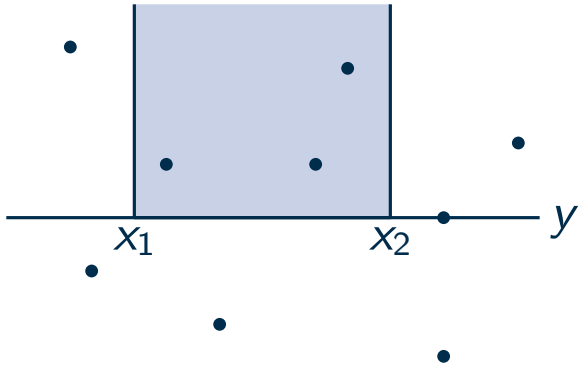

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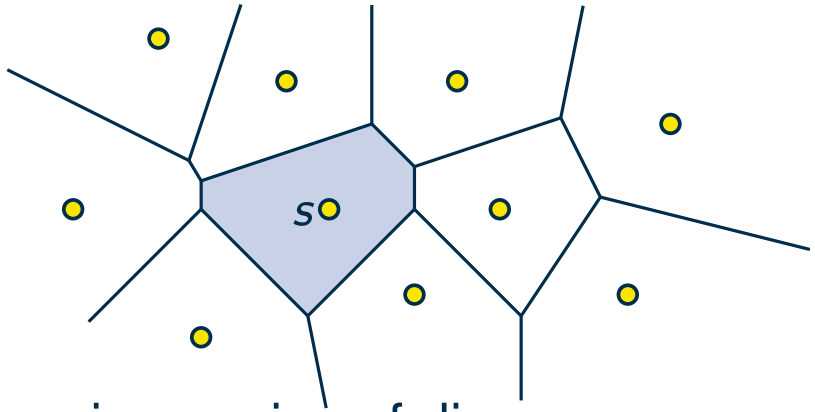

- maximum size of diagram

- output something in $O(\log(n) + k)$
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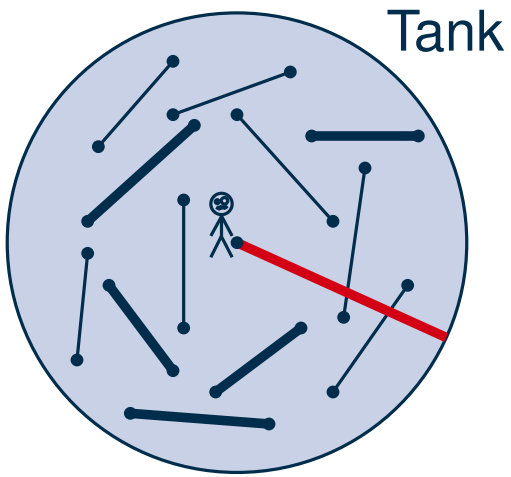
Assignment 4

Mission Impossible



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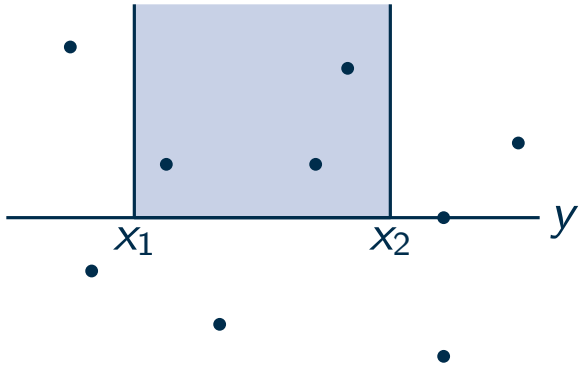

- output something in $O(\log(n) + k)$
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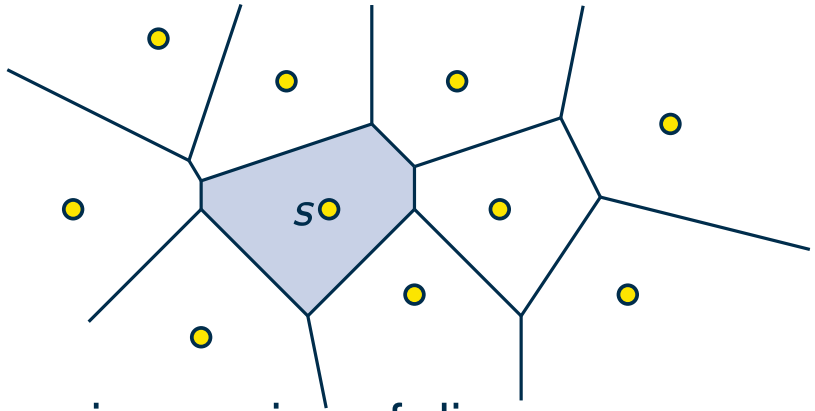
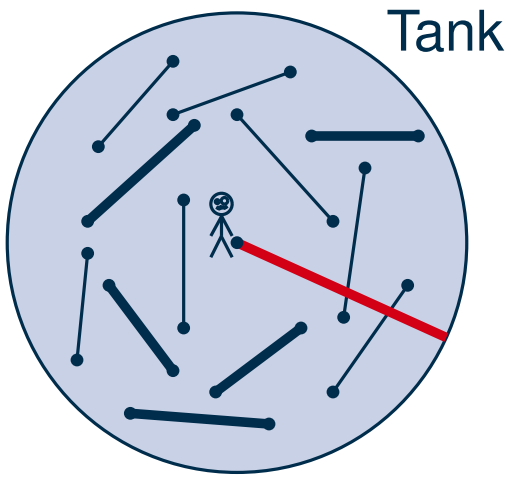
Voronoi diagram



- maximum size of diagram
- lower bound for algorithm

Assignment 4

Mission Impossible



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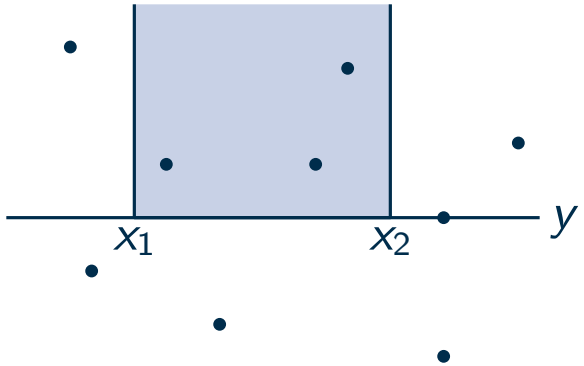

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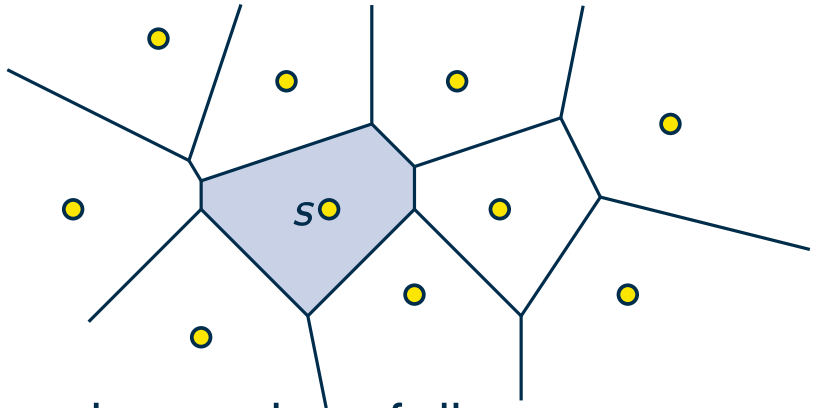


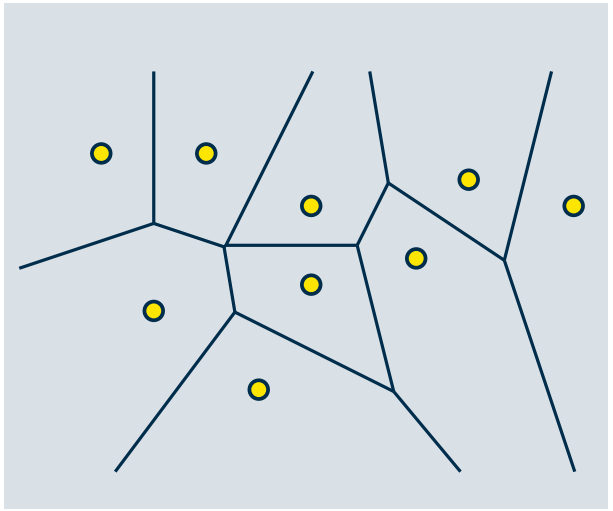
- maximum size of diagram
- lower bound for algorithm

Closest Point

- find closest point for each point in $O(n \log(n))$

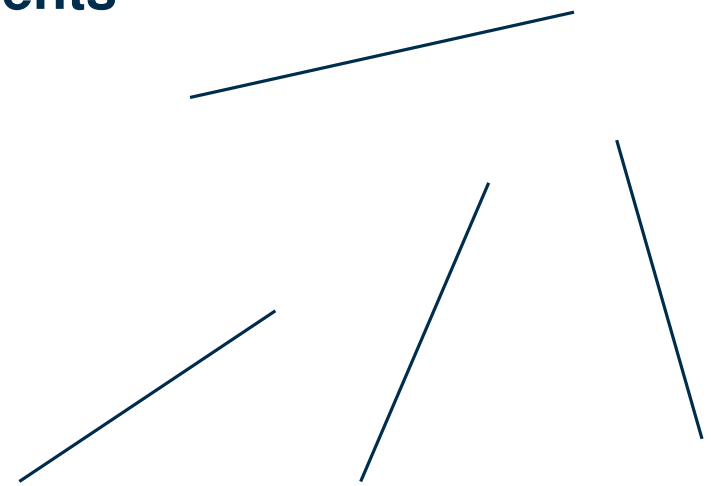
Voronoi Diagram of Segments

Points

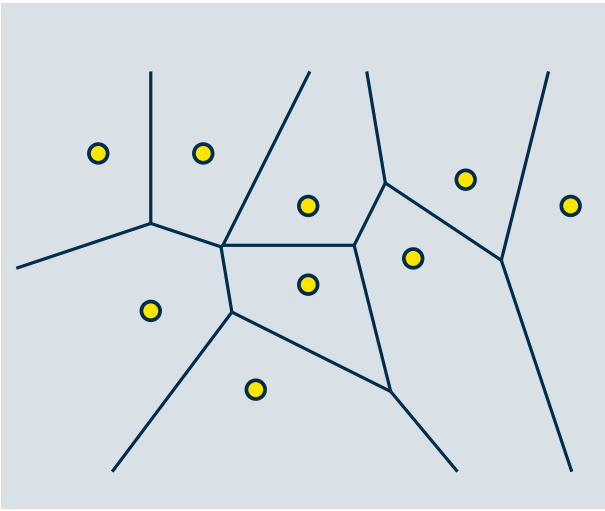


Voronoi Diagram of Segments

Segments



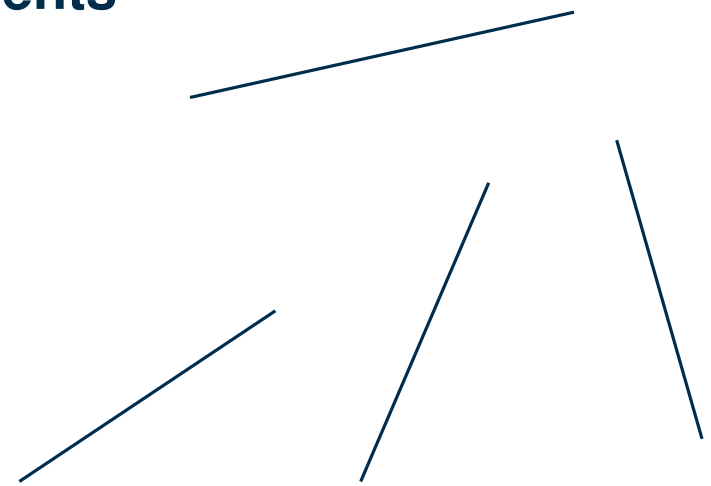
Points



What does VD of segments even mean?

Voronoi Diagram of Segments

Segments



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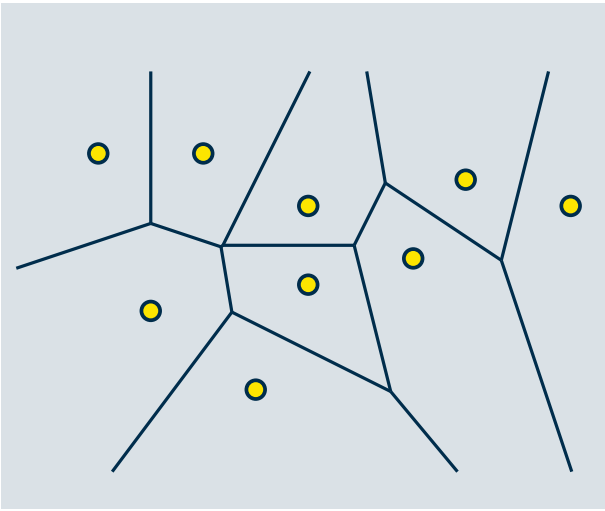
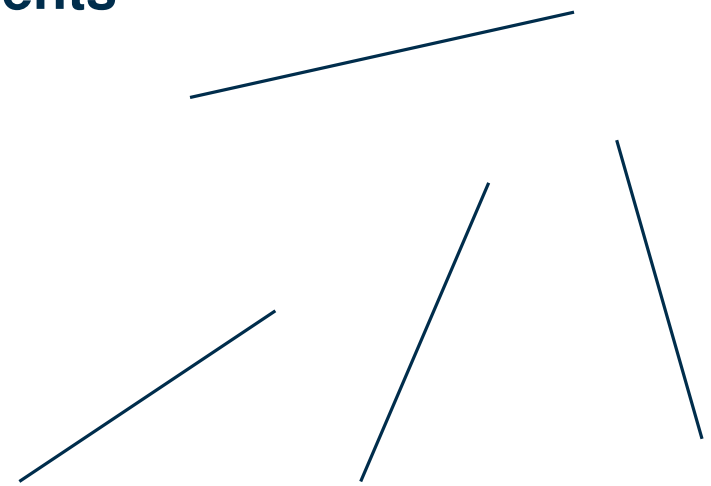


What does VD of segments even mean?

Which points are vertices of degree 3?

Voronoi Diagram of Segments

Segments



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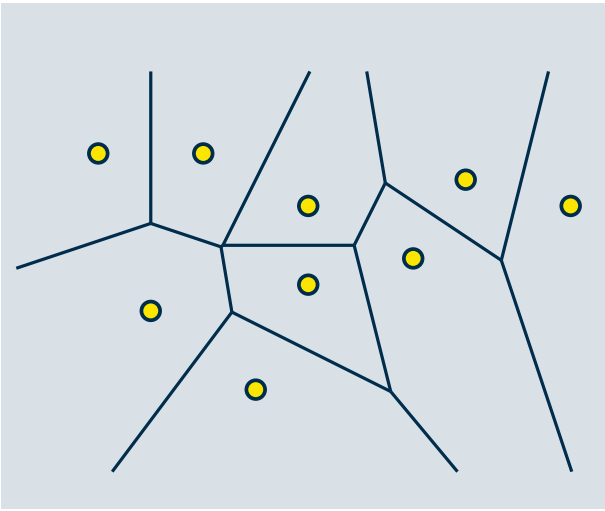

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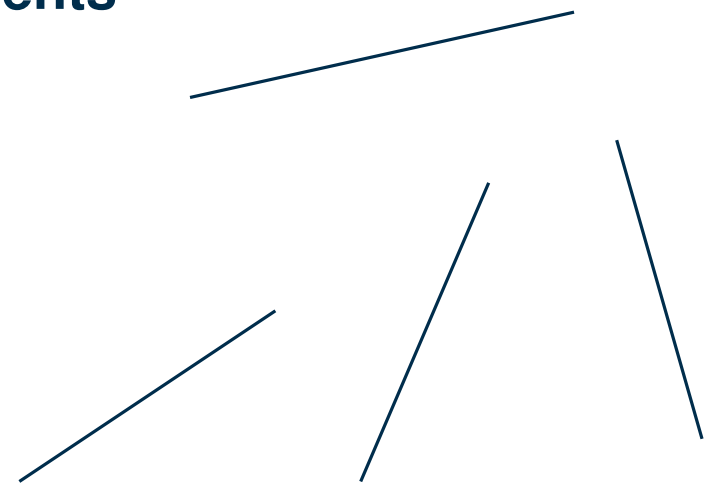

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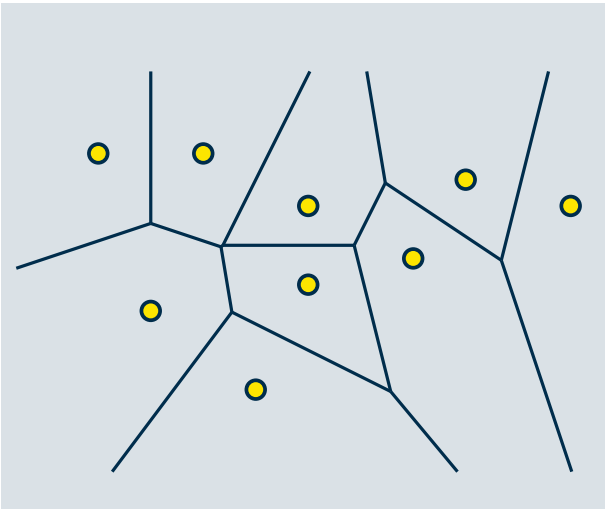

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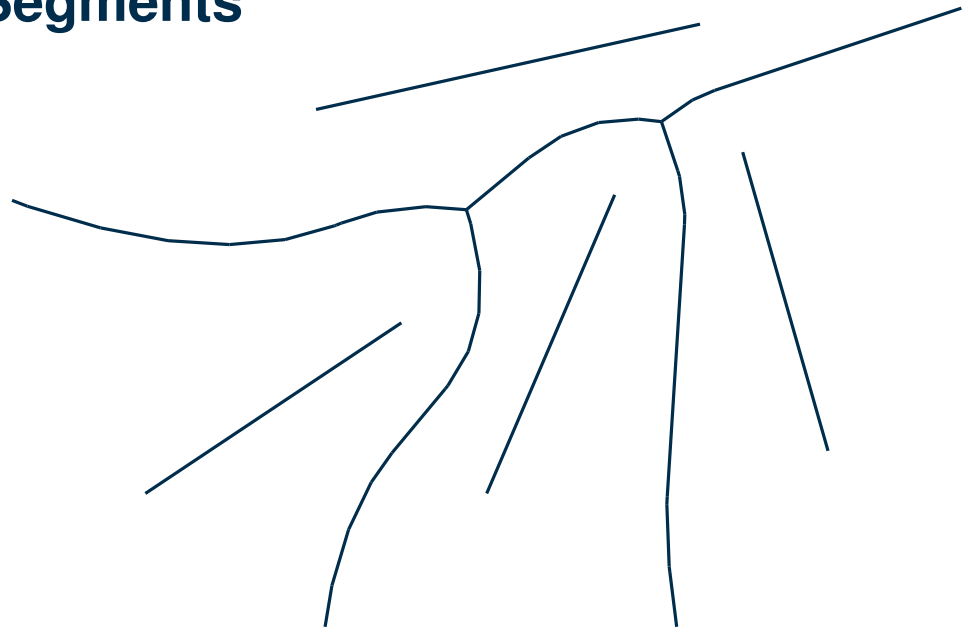
Voronoi Diagram of Segments

Segments



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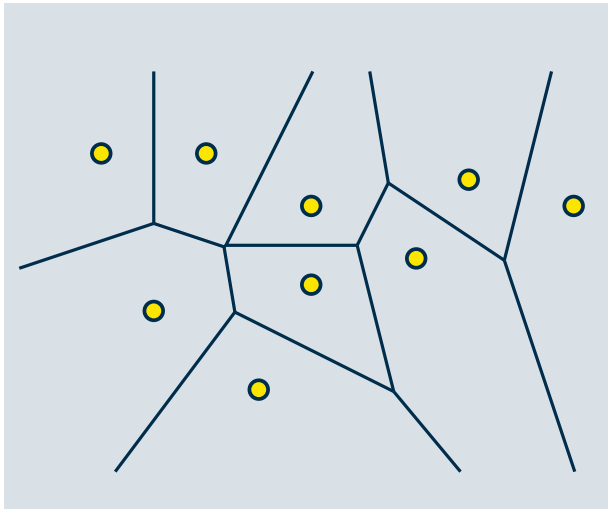

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https://onlineumfrage.kit.edu/evasys/online.php?p=LCCHE

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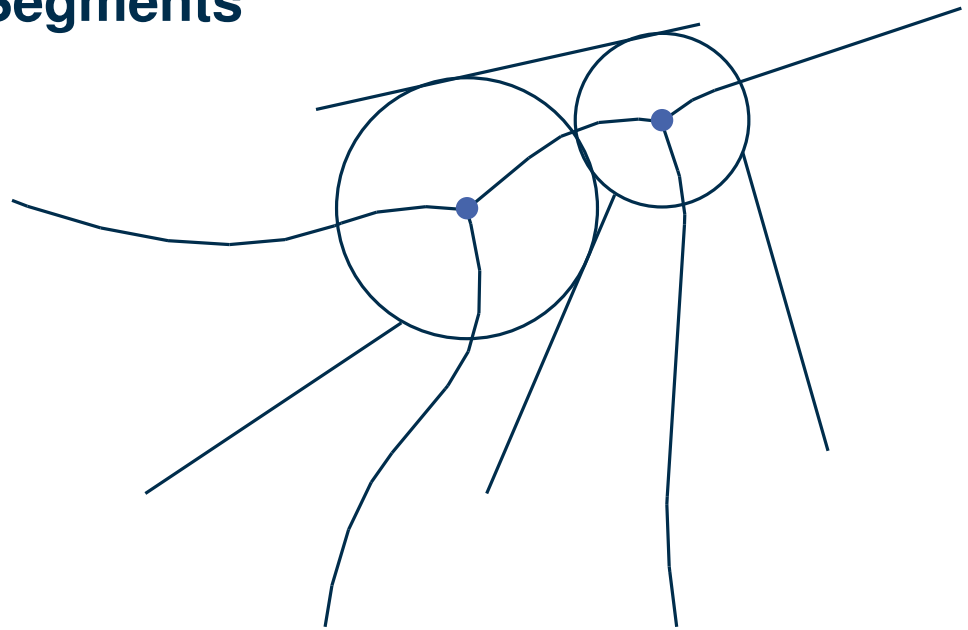

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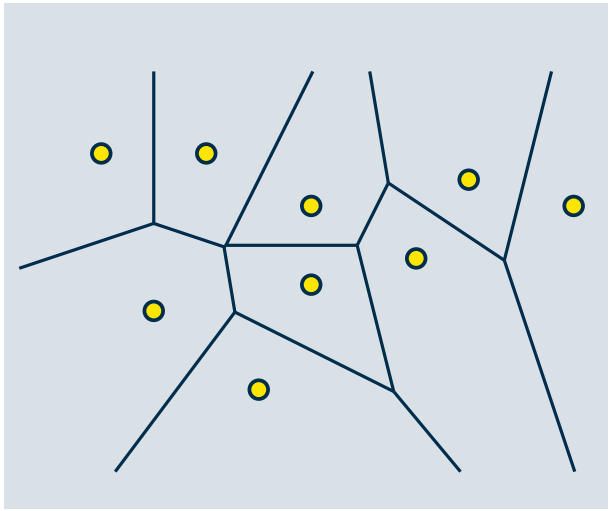

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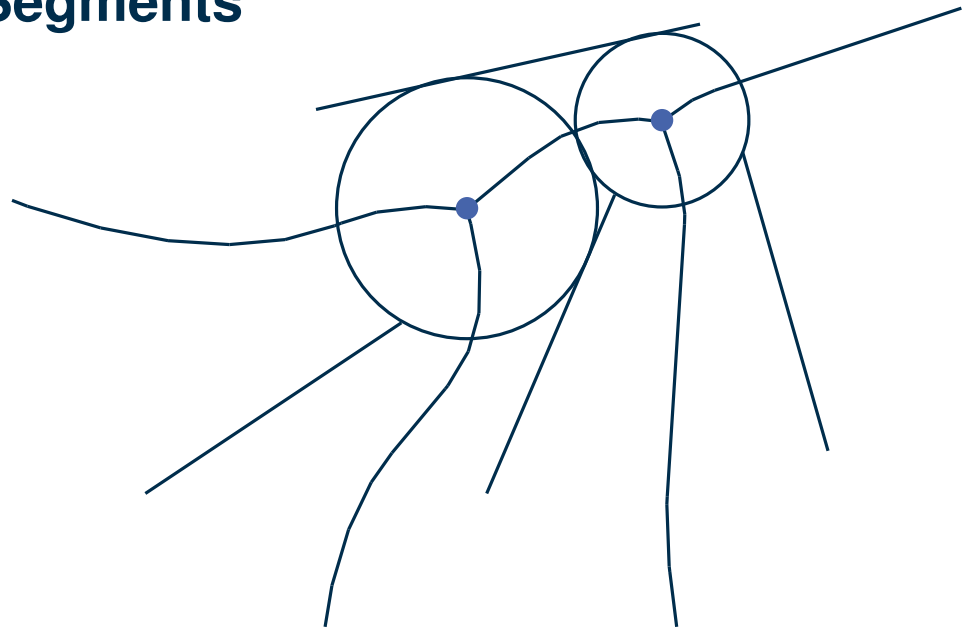

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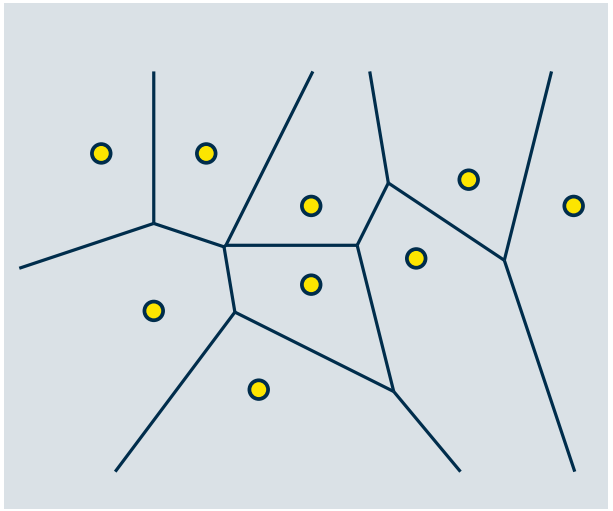

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