Computational Geometry

Exercise 2

Jean-Pierre, Marcus, Wendy

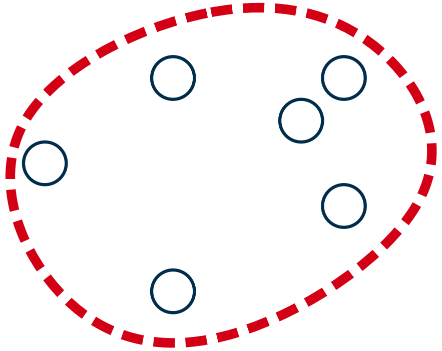
Assignment Sheet 1 – General comments

- please work in groups
- try to write clear and correct answers
  - for algorithms: always give reasoning for correctness, and running time
  - never give uncommented pseudocode, prefer natural language description
  - mention figures in the text

Assignment Sheet 1

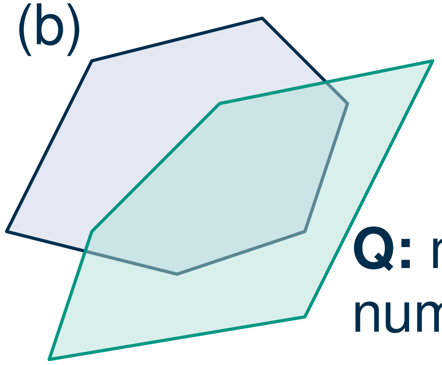
Mysterious Meteorites

(a)



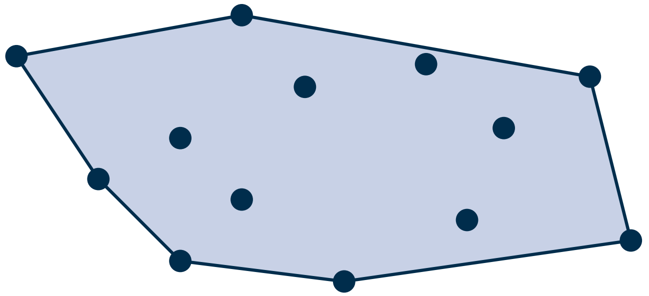
Q: minimize barrier tape

(b)



Q: maximum number of corners?

Convex Hull via Divide and Conquer



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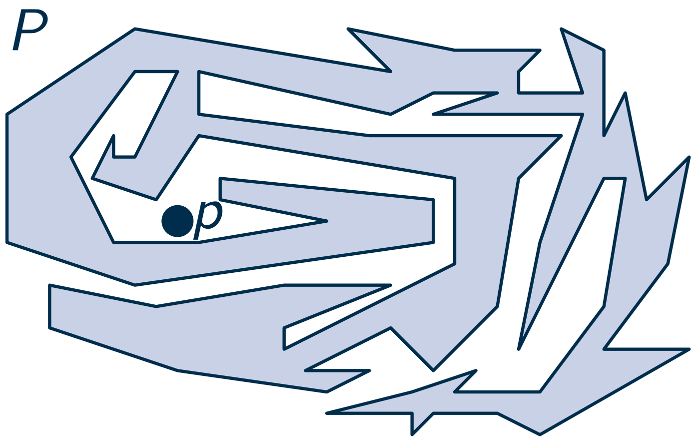
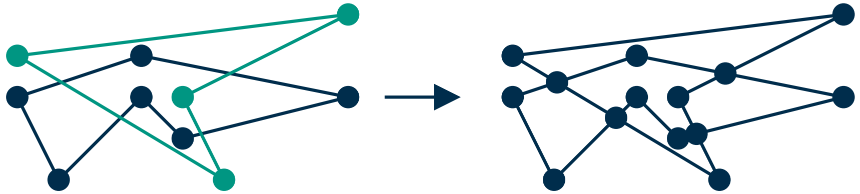


Task: Give algorithm to decide if $p \in P$.

Map Overlay



Boolean Operations

Given: Polygons P, Q

Task: compute $P \cap Q$, $P \cup Q$, and $(P \cup Q) \setminus (P \cap Q)$

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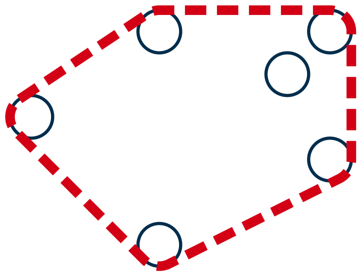

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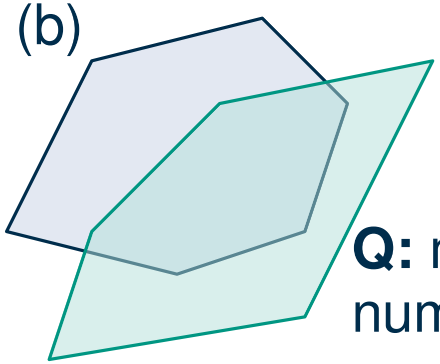

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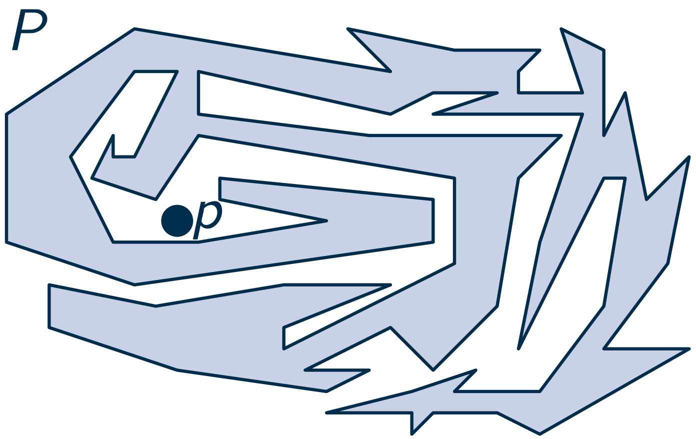

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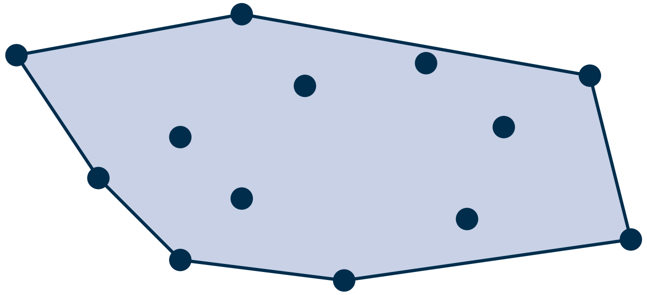

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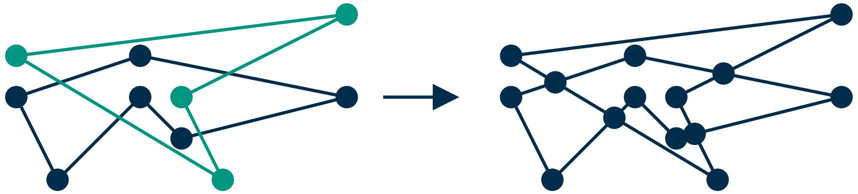

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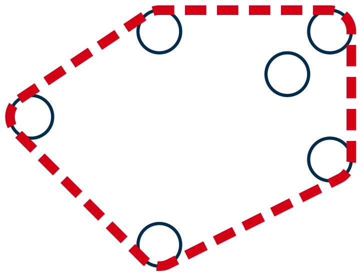

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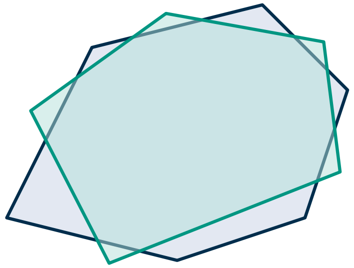

$m + n + 2\min\{n, m\}$

Q: minimize barrier tape

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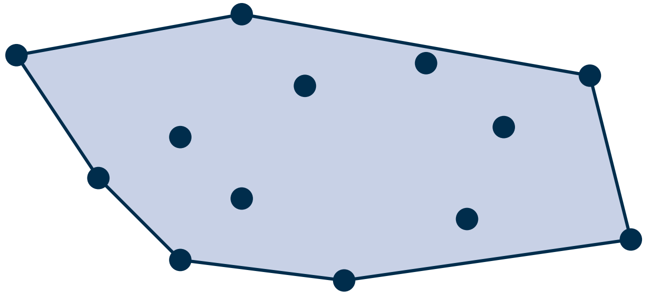

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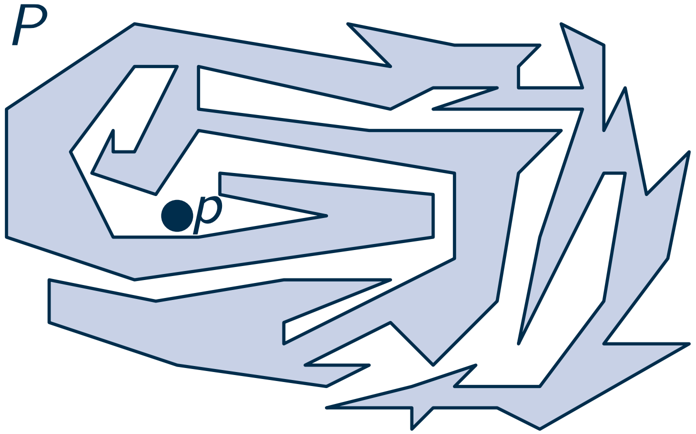

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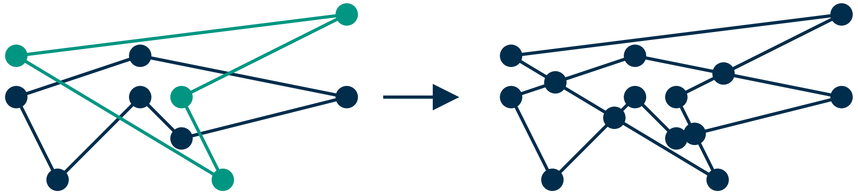

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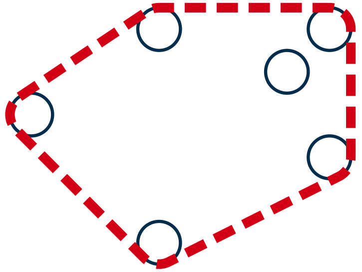

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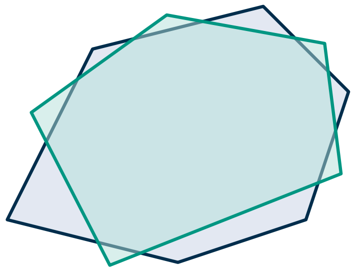

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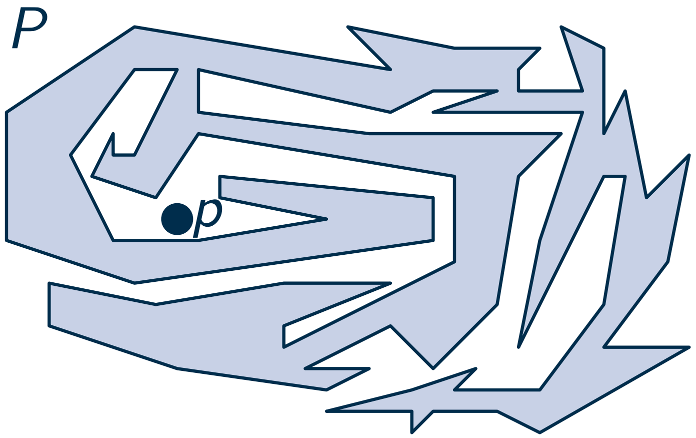

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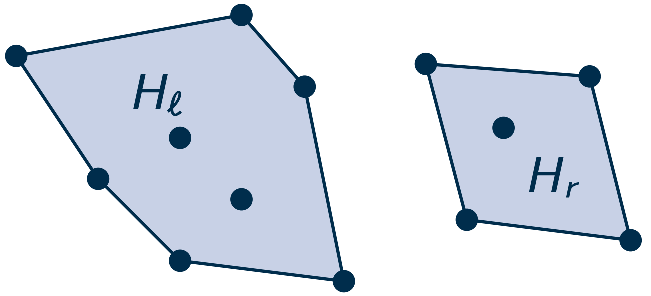

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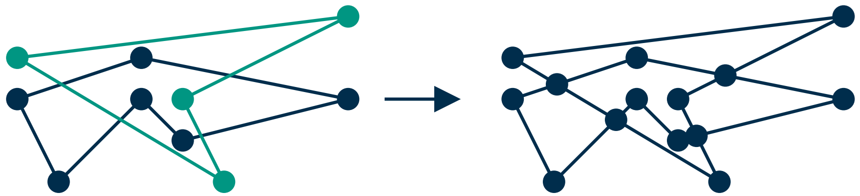

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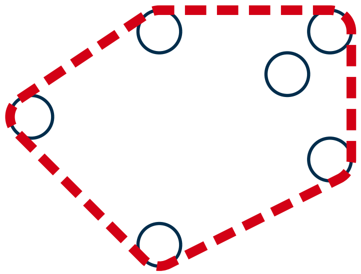

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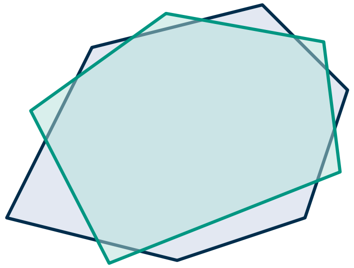

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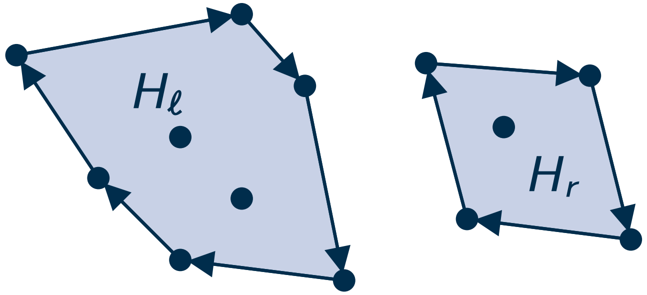
Variant 1: obtain sorting, Andrew's algorithm



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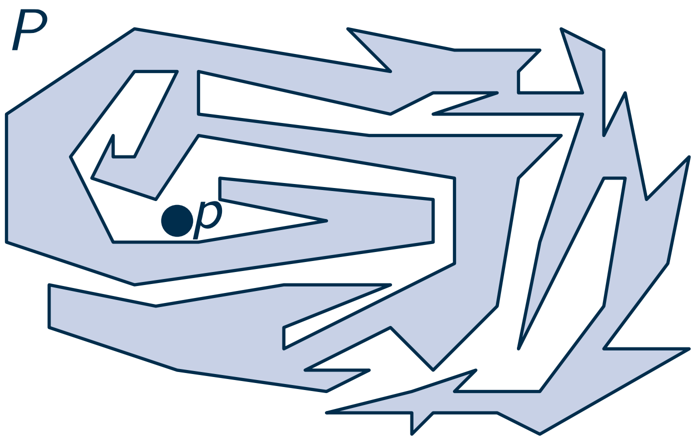

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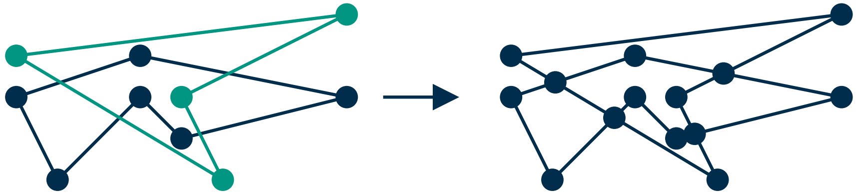

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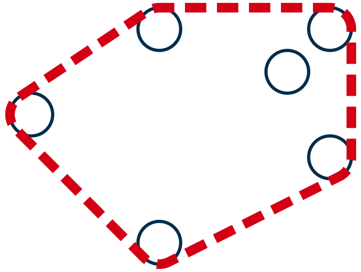

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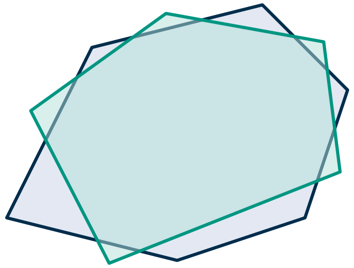

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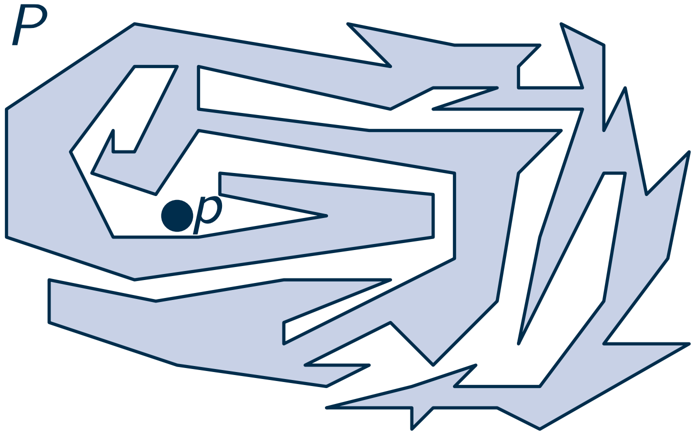

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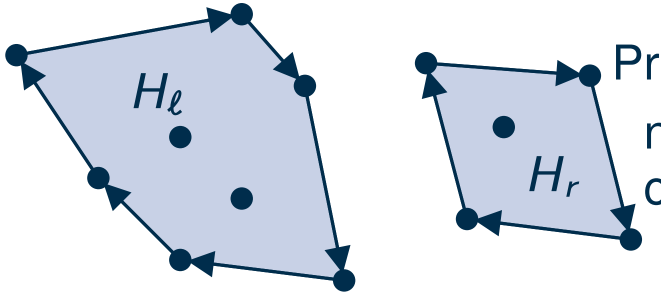
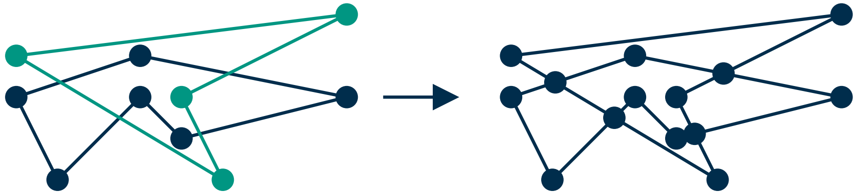


Problem:
not really div. & conquer

Map Overlay



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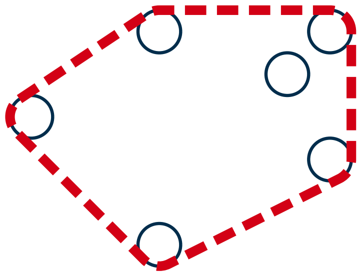

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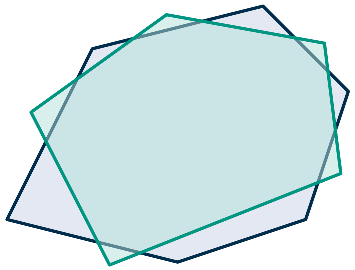

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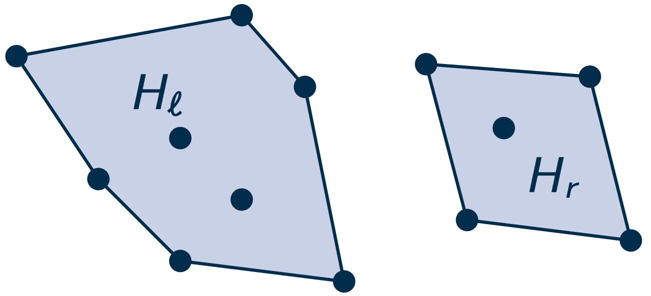

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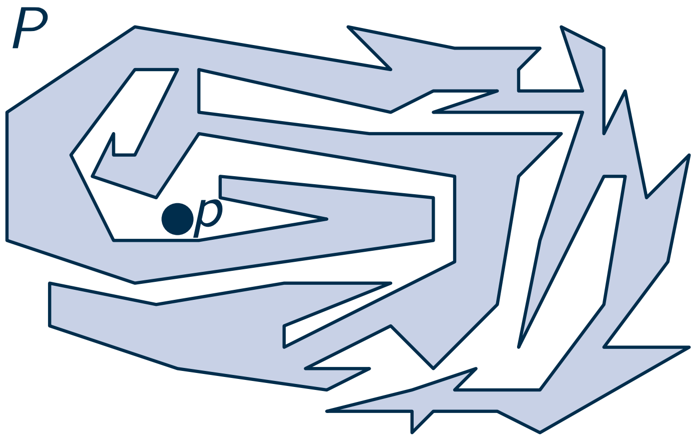

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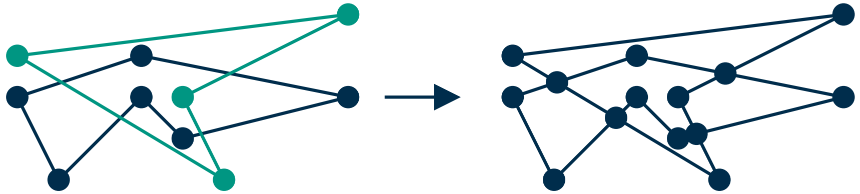

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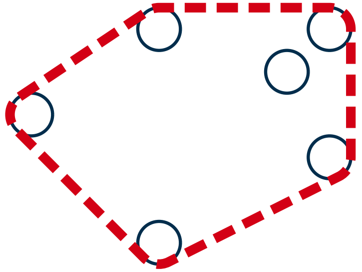

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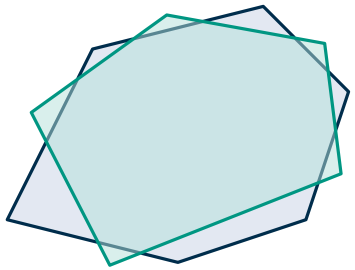

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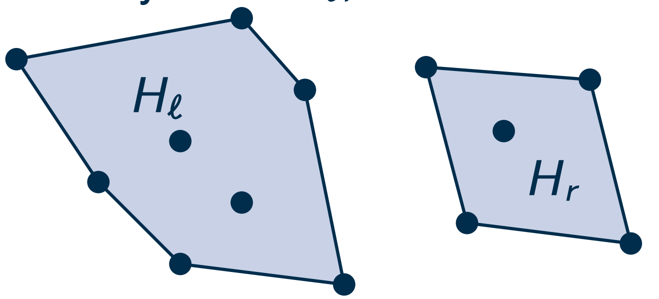

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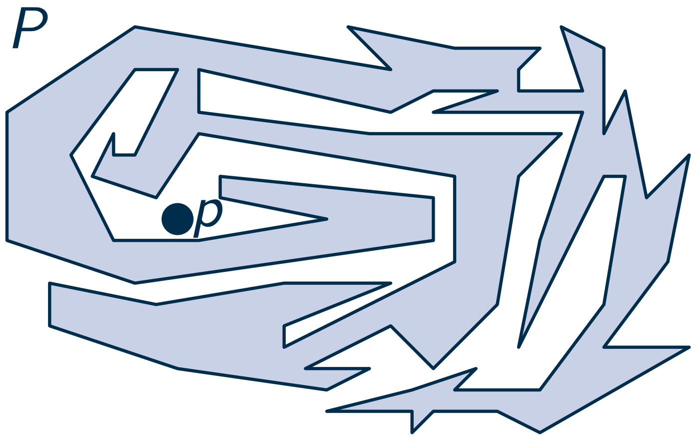

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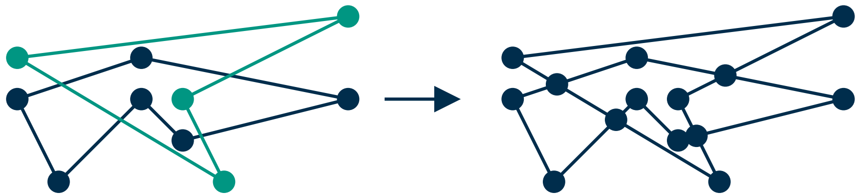

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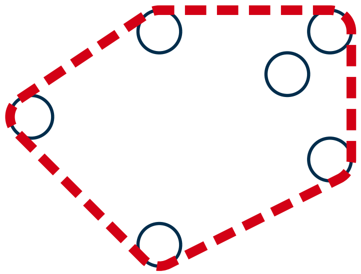

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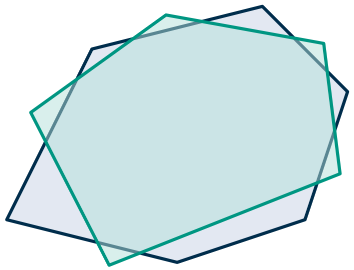

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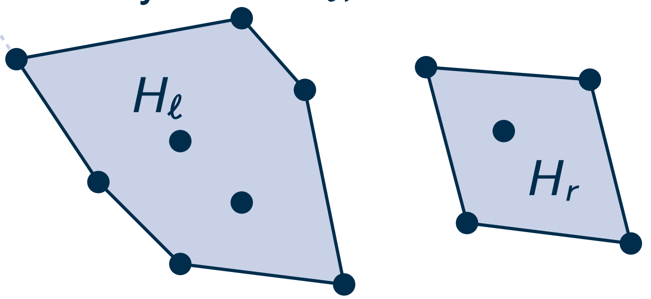

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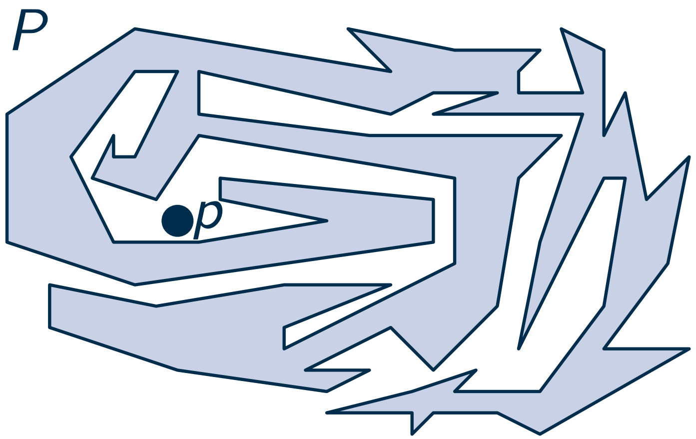

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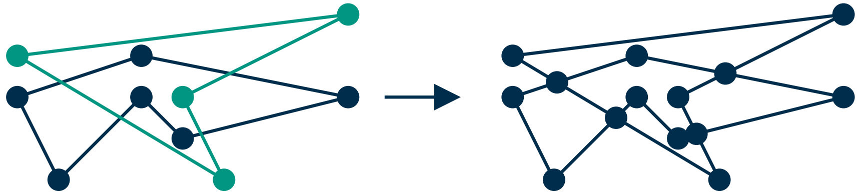

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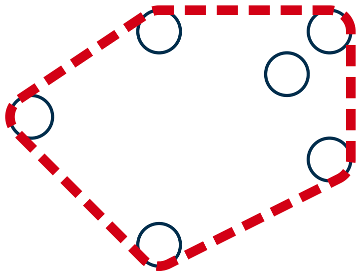

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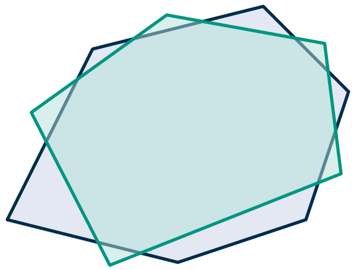

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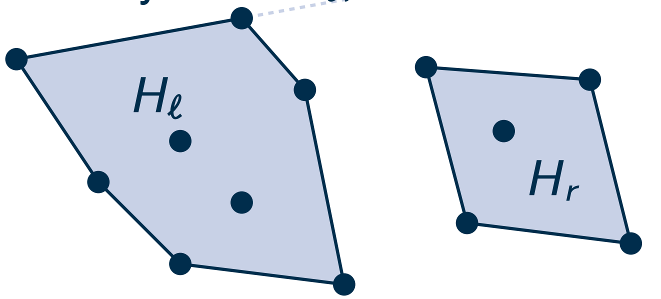

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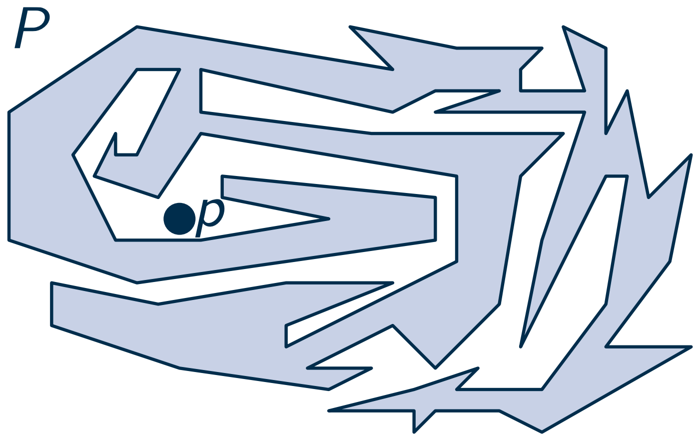

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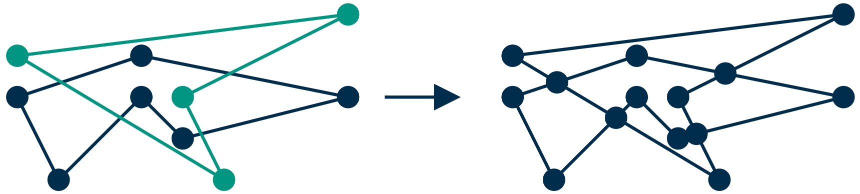

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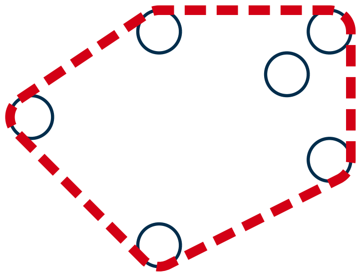

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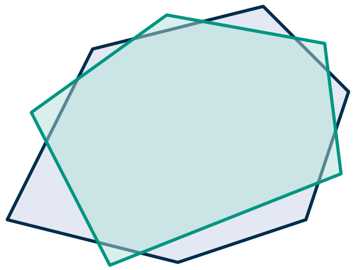

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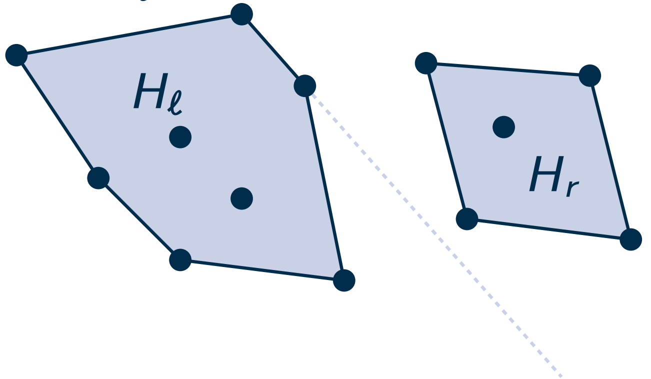

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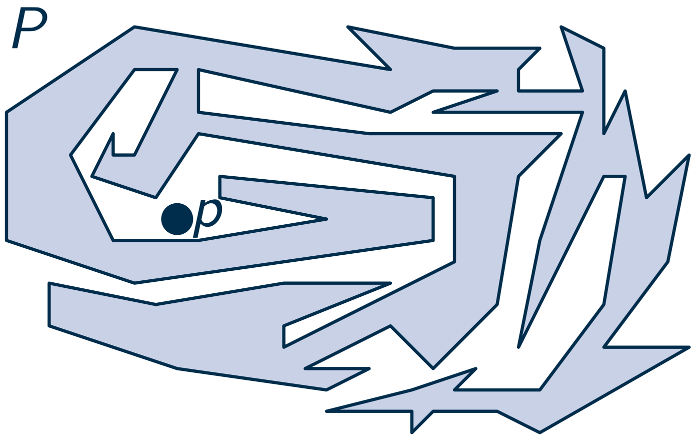

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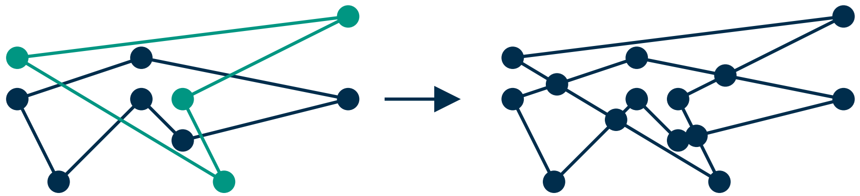

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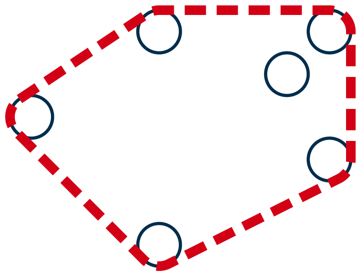

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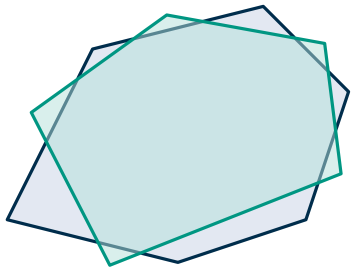

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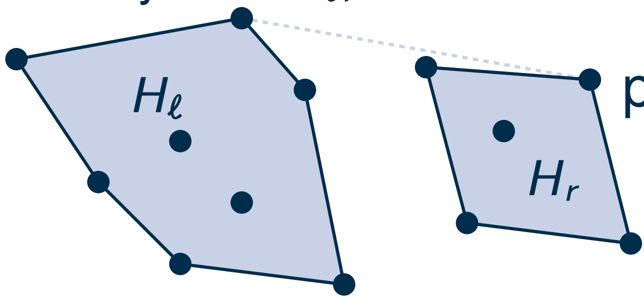

problem: $O(n \log^2 n)$

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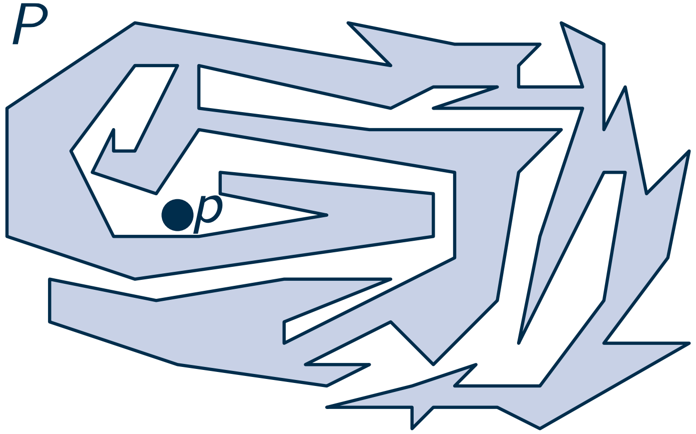
P



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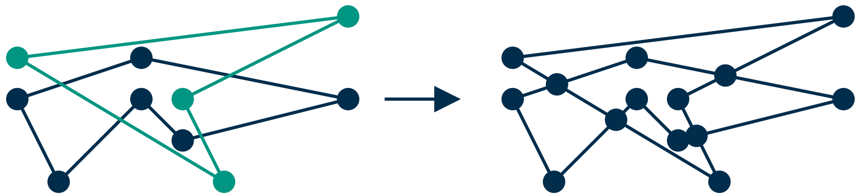

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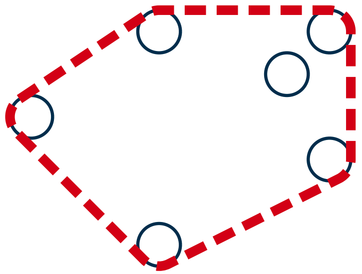

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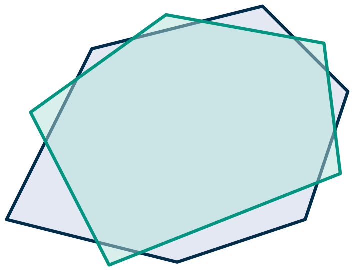

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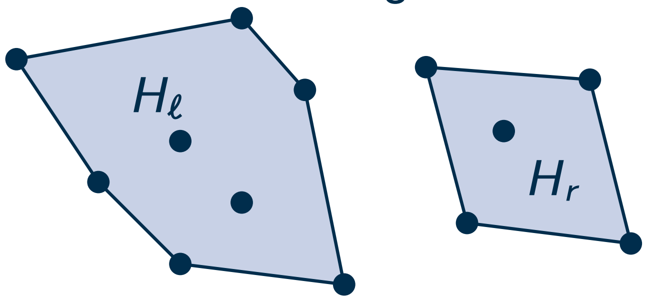
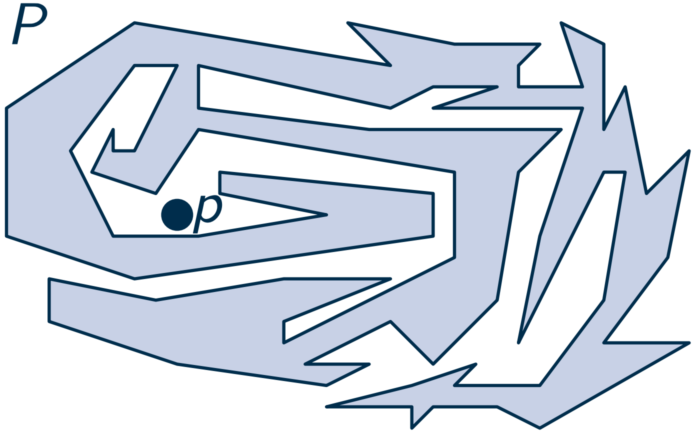


Aerial Photography

P



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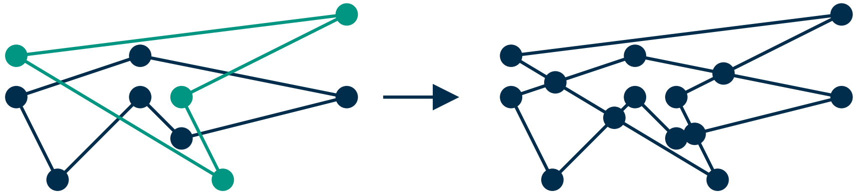

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Assignment Sheet 1

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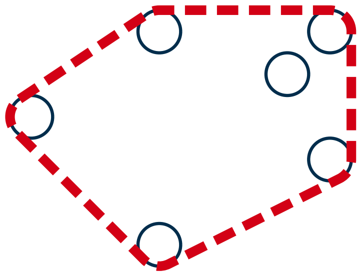

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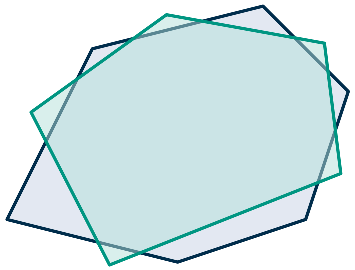

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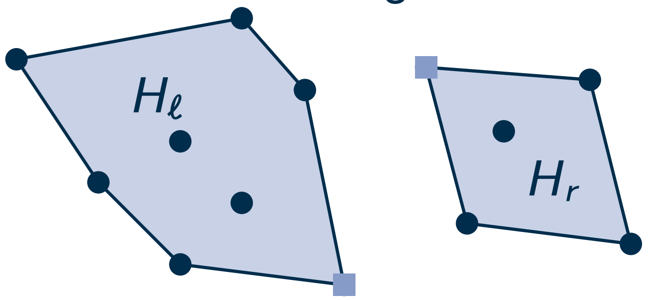

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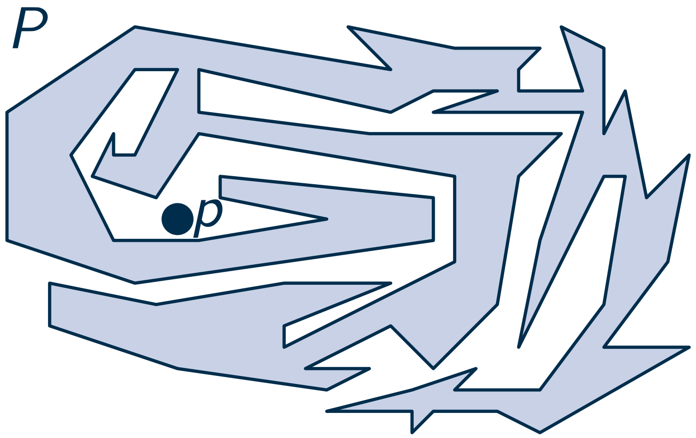

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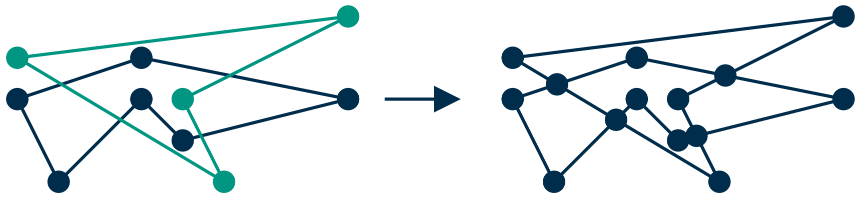

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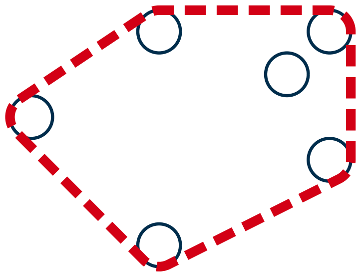

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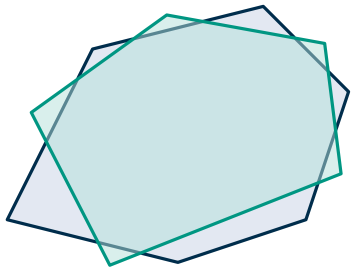

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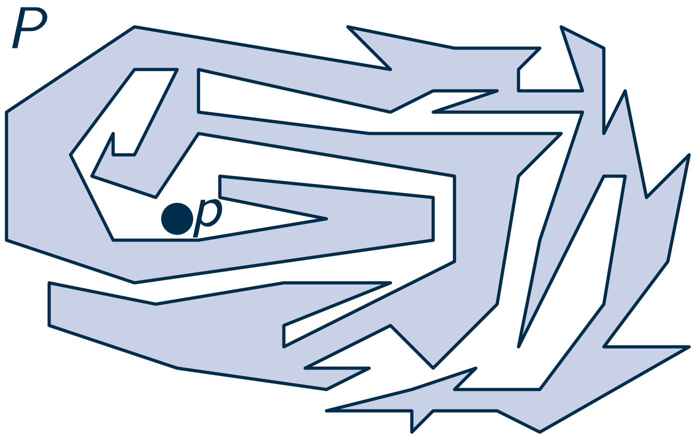

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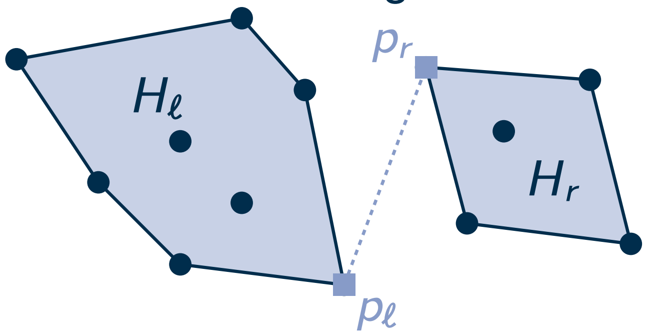

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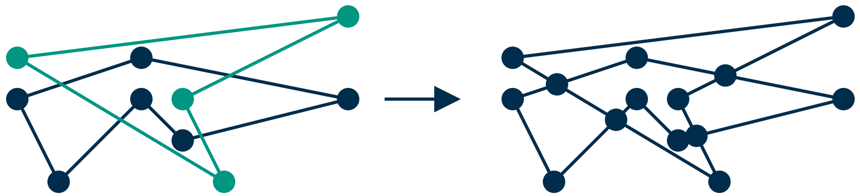

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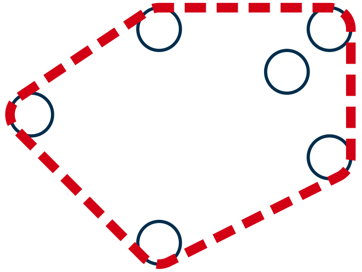

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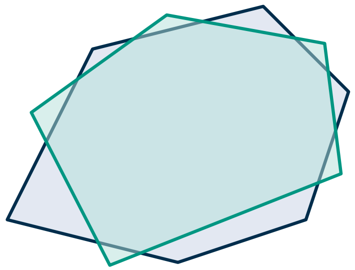

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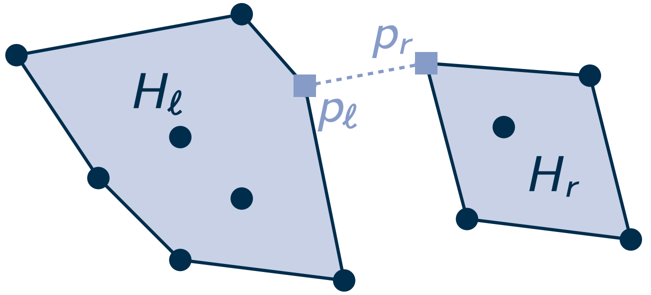

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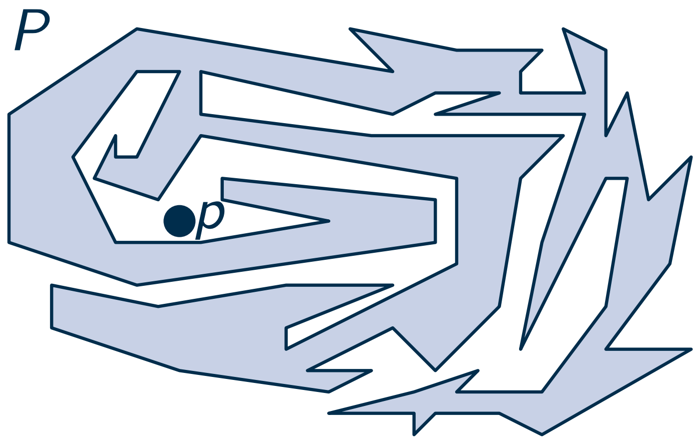

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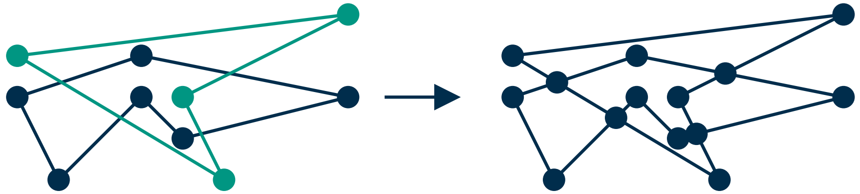

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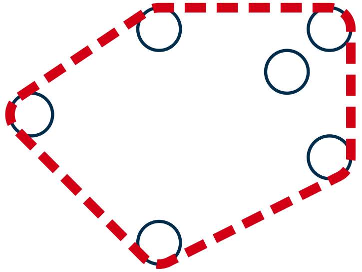

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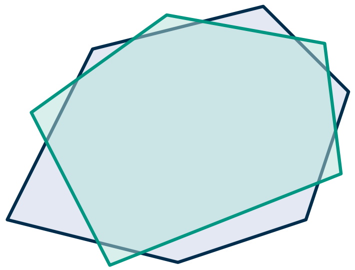

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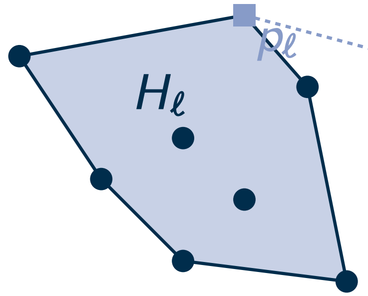

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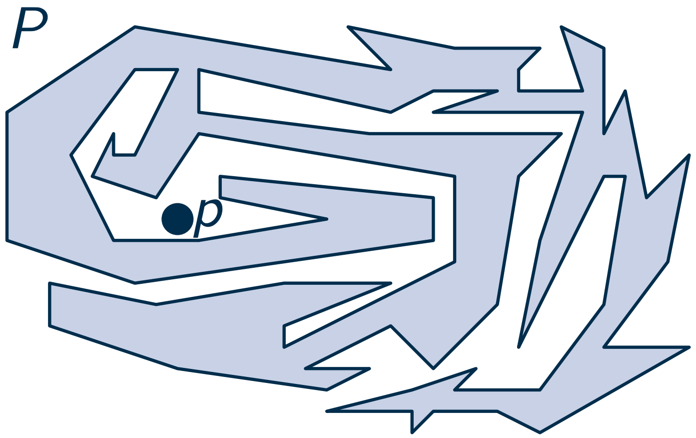

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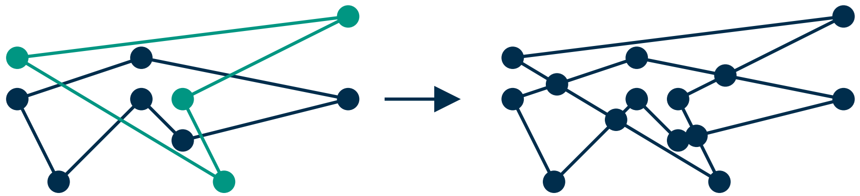

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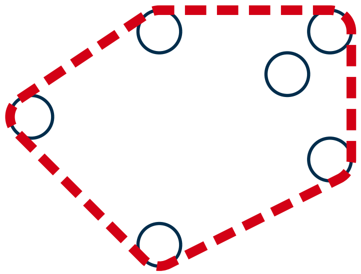

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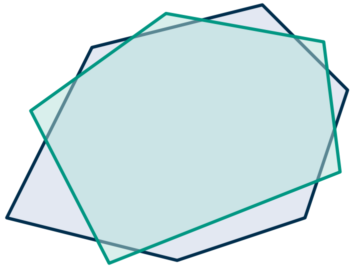

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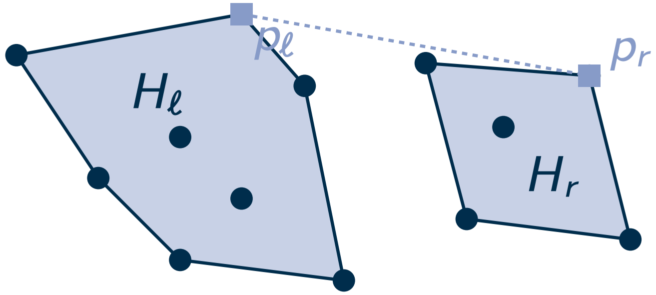

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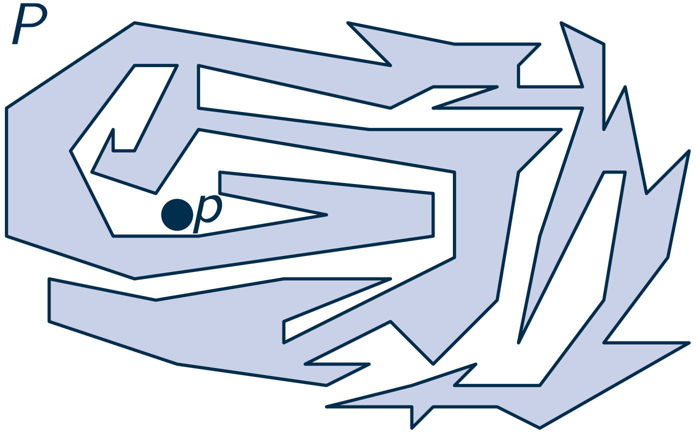

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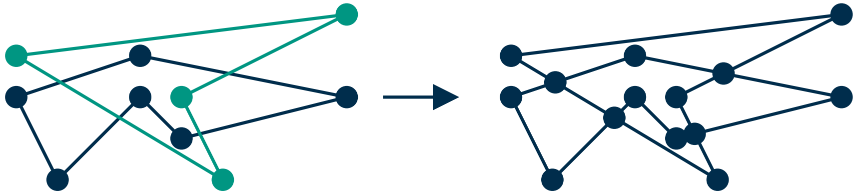

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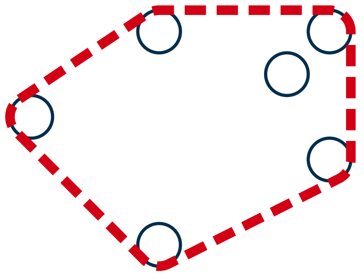

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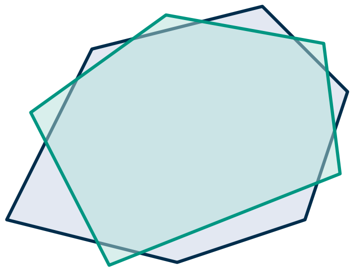

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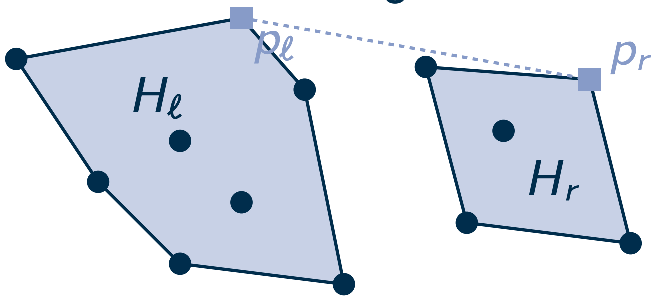

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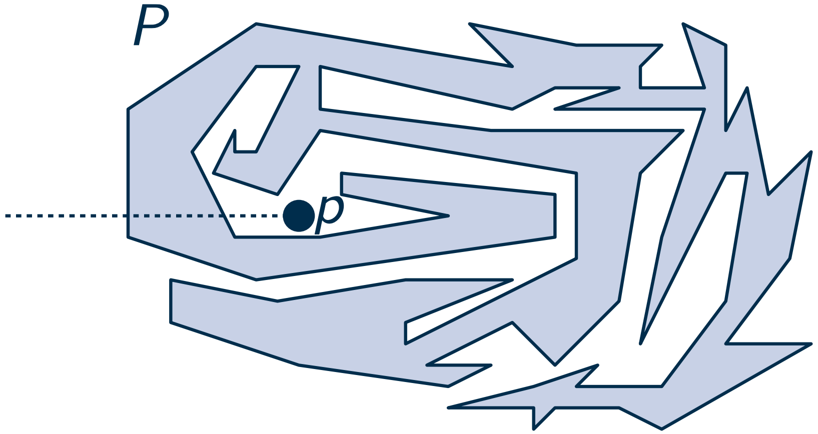
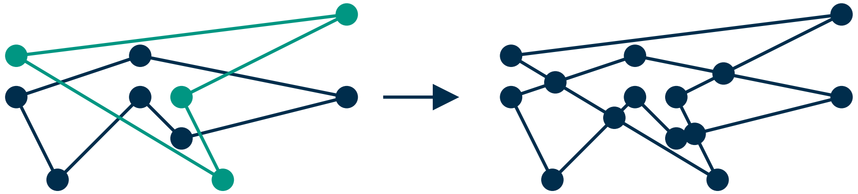


Idea: count intersections of P with ray starting at p

Map Overlay



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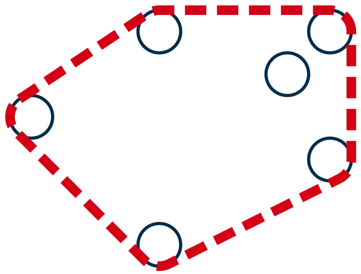

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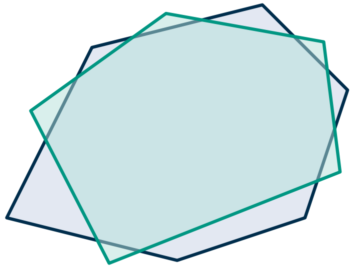

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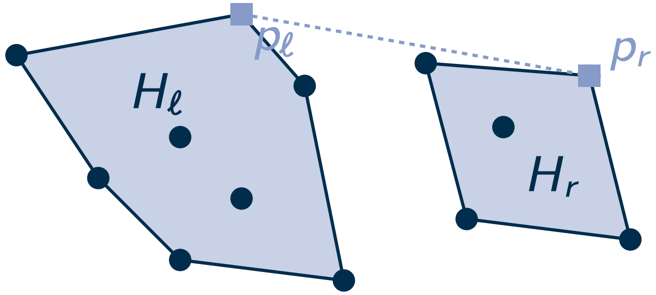

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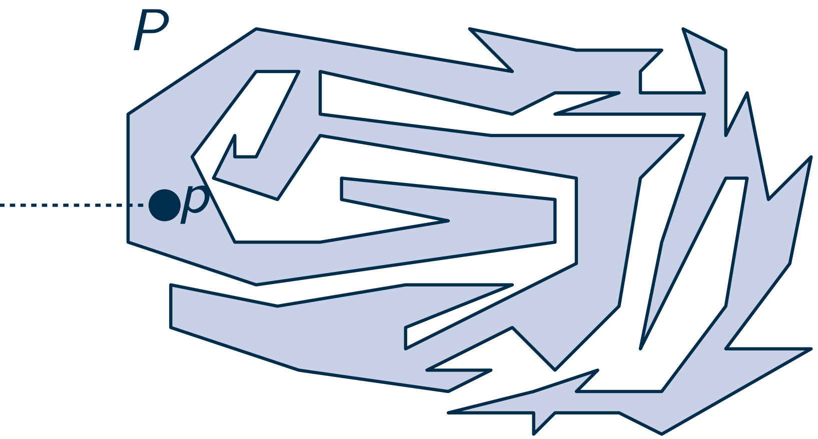

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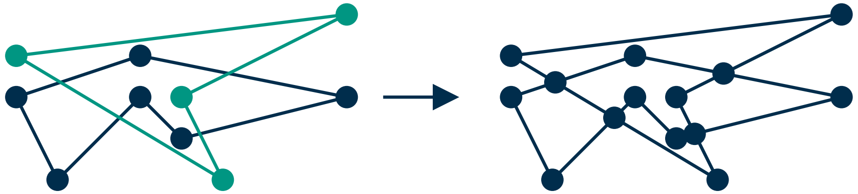

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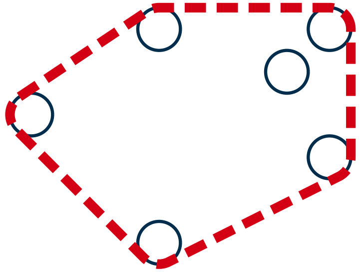

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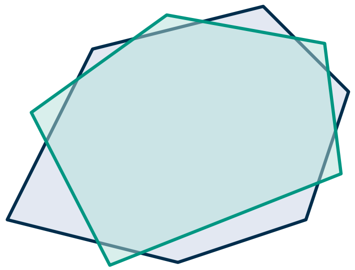

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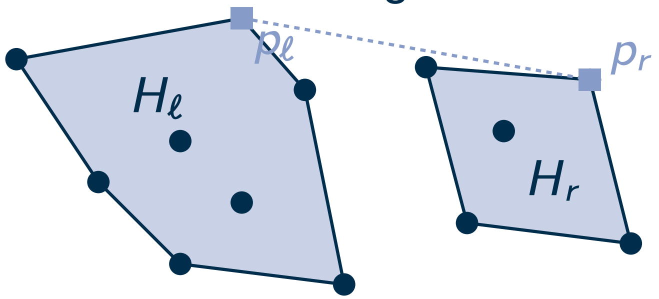

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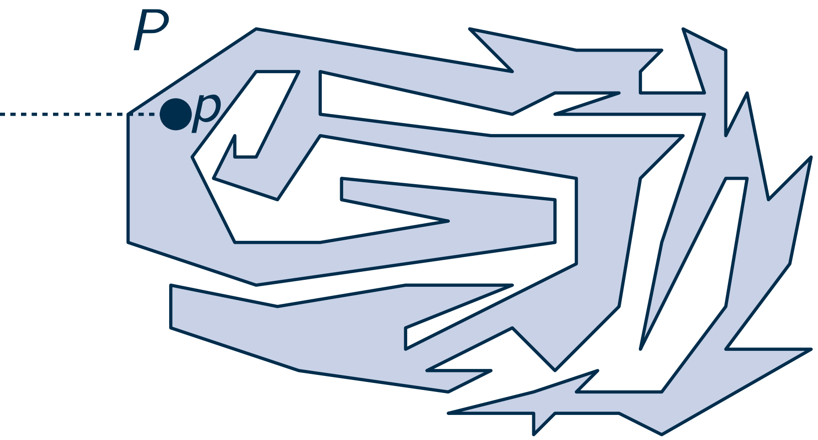

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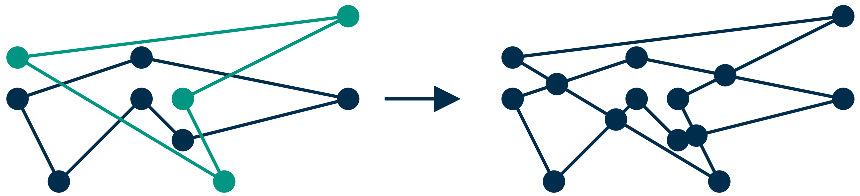

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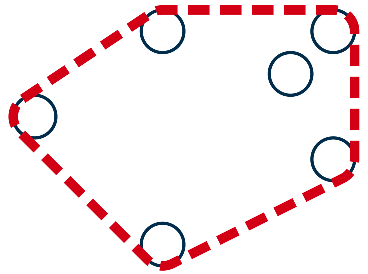

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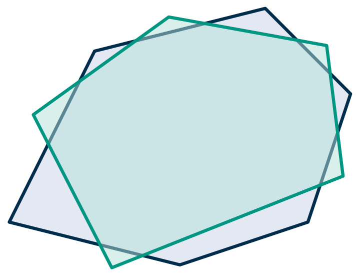

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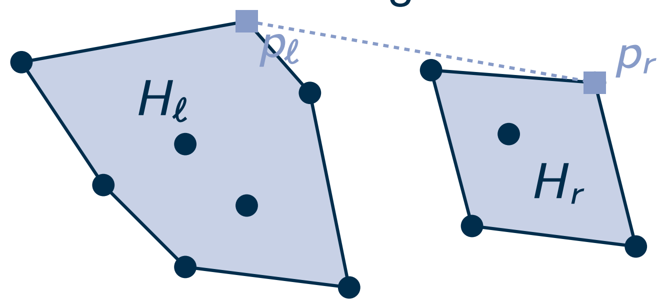

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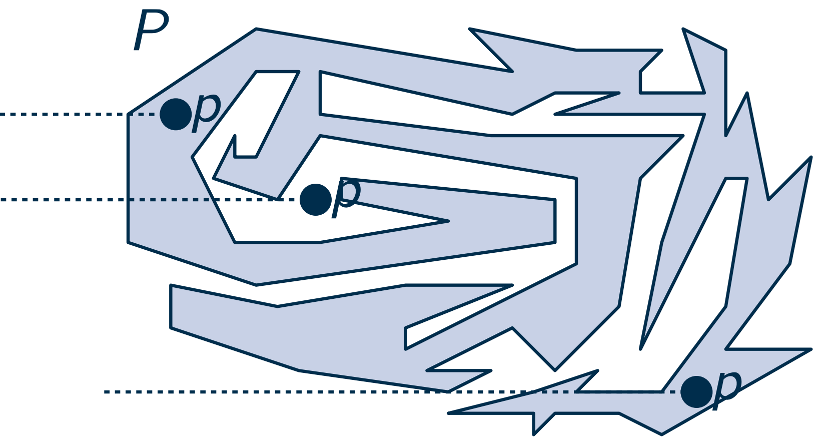

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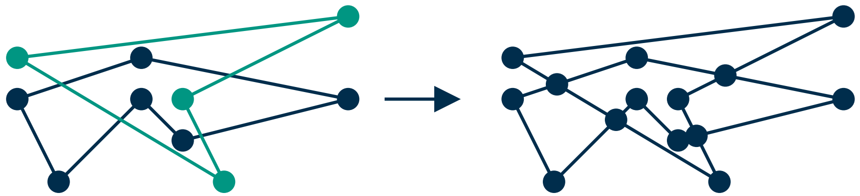

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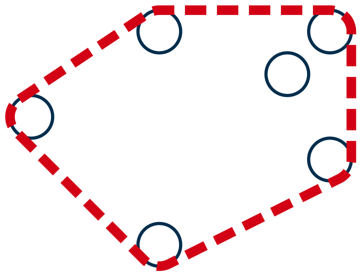

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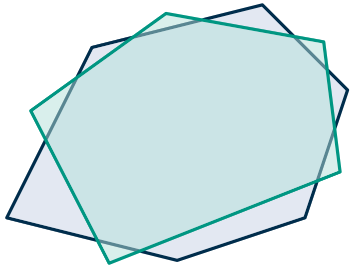

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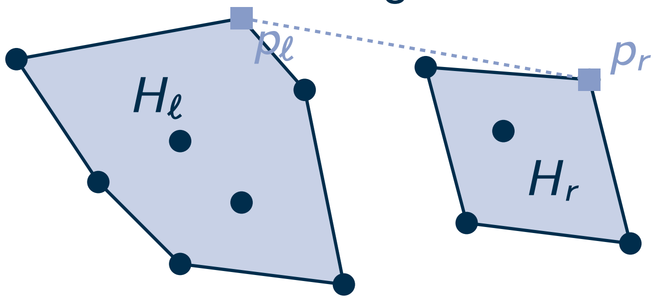

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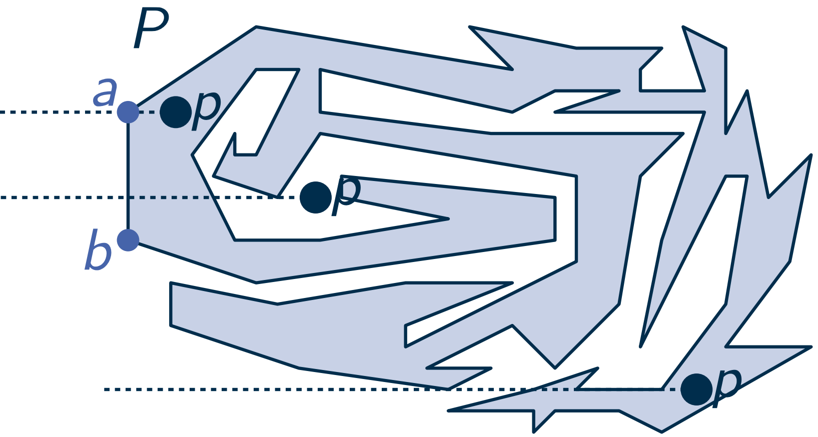

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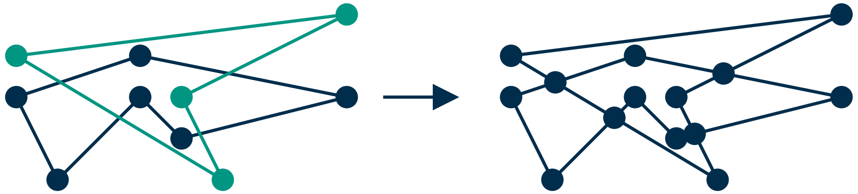

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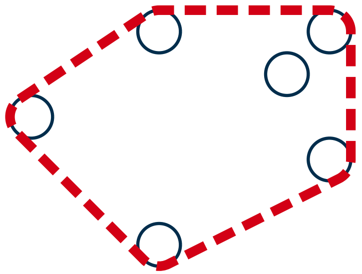

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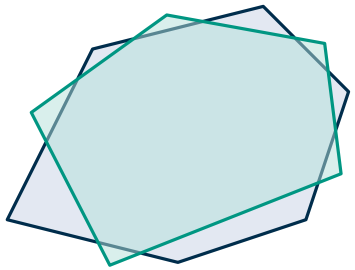

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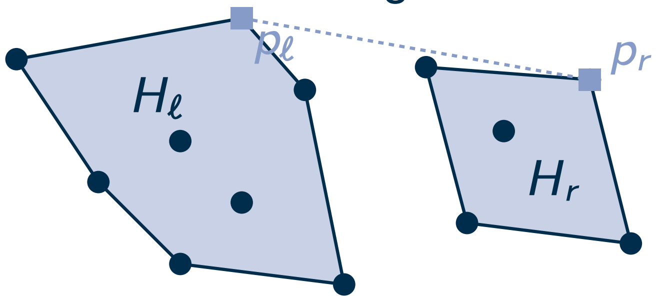

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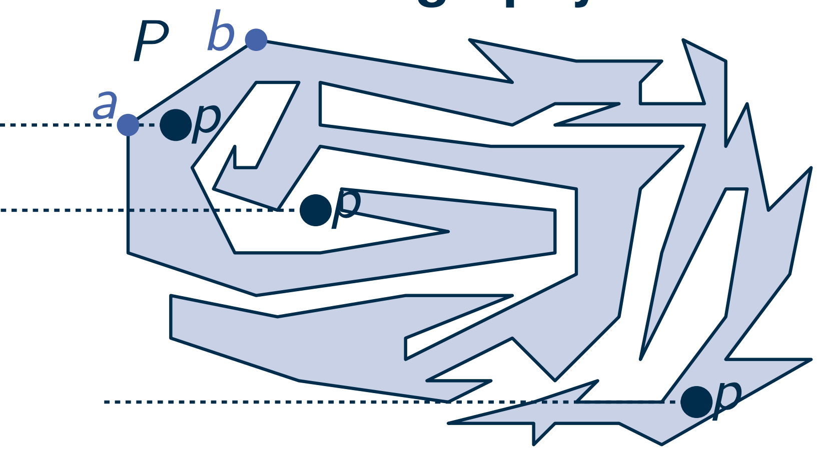

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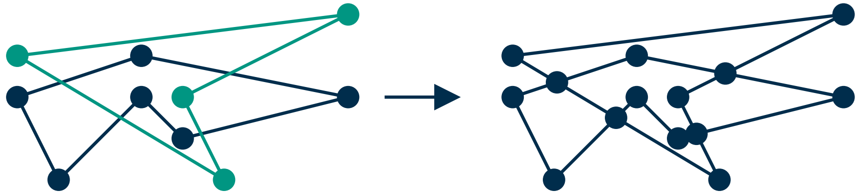

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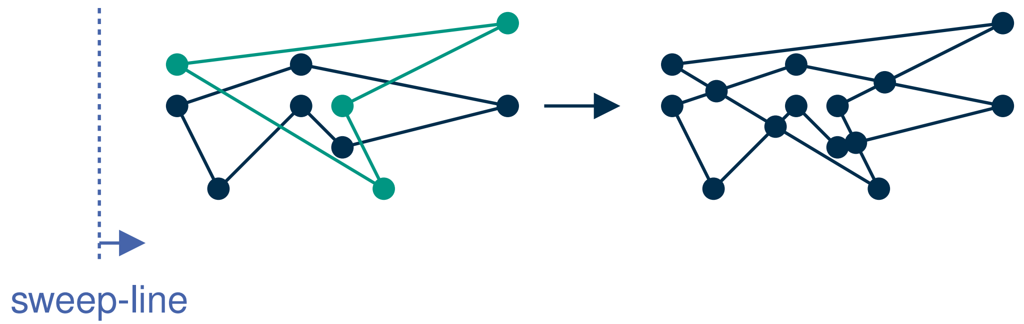

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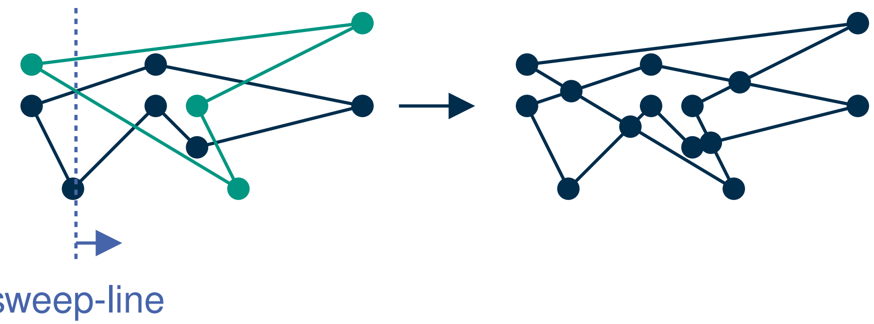

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Map Overlay

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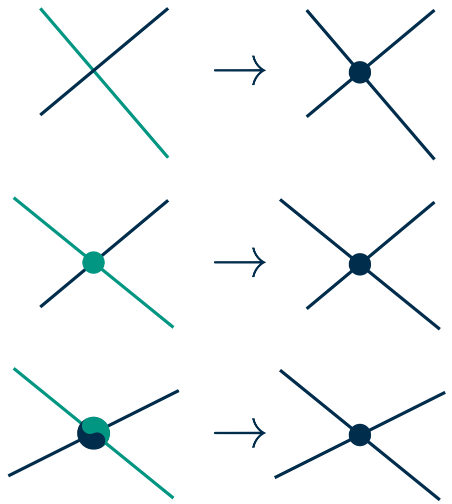
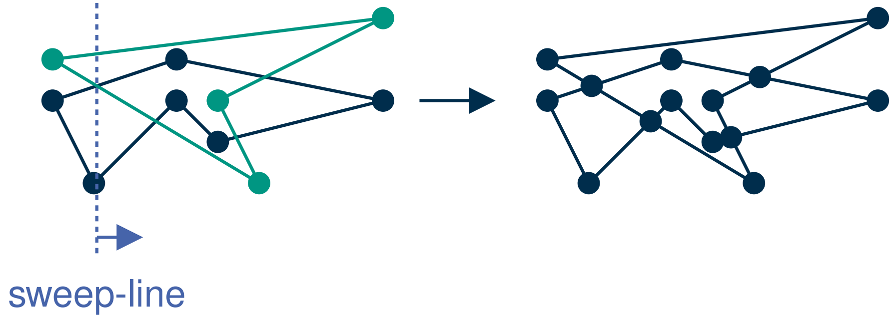
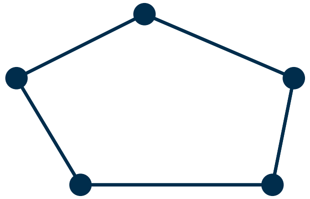




Assignment Sheet 1

Map Overlay

1. update edge pointers
2. update faces



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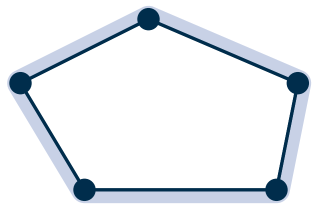
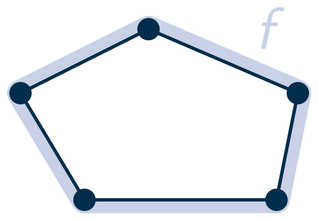


Assignment Sheet 1

Map Overlay

1. update edge pointers
2. update faces

    Find parent inner face of outer face f



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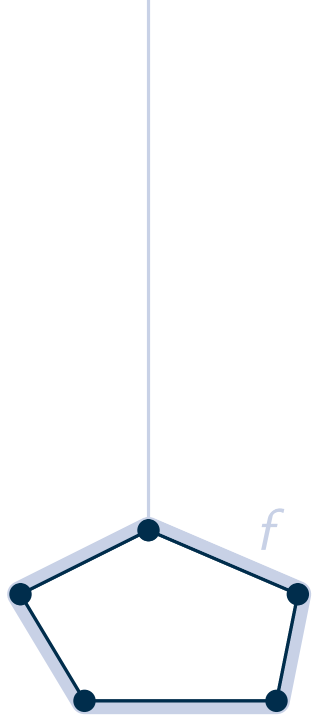

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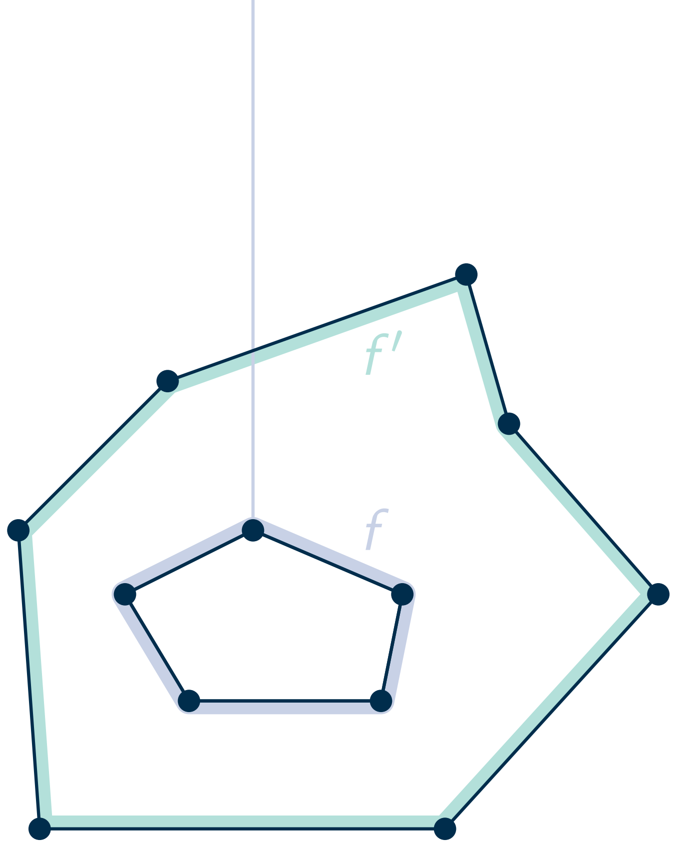

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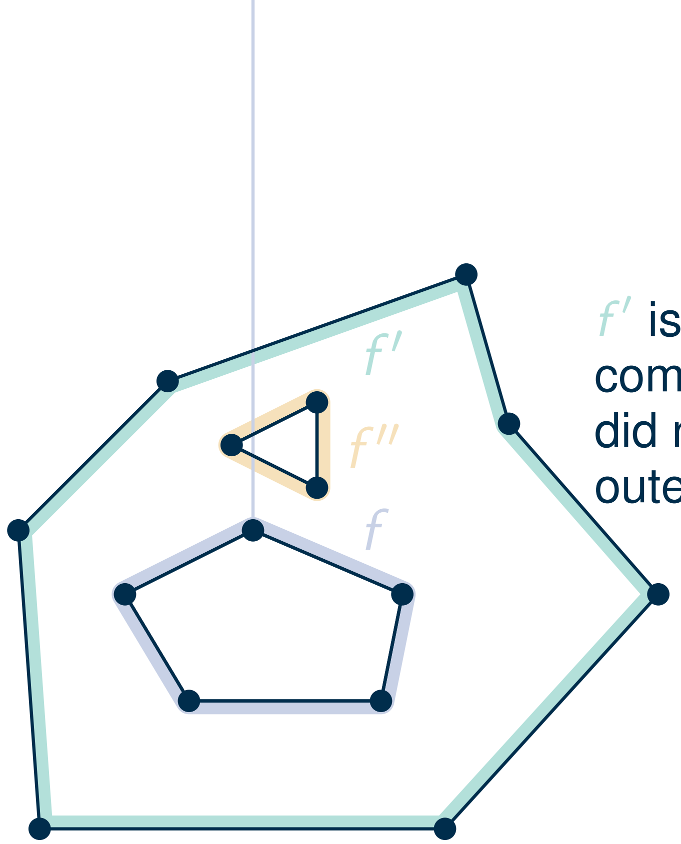


f' is inner face of first component, where ray did not intersect an outer face first

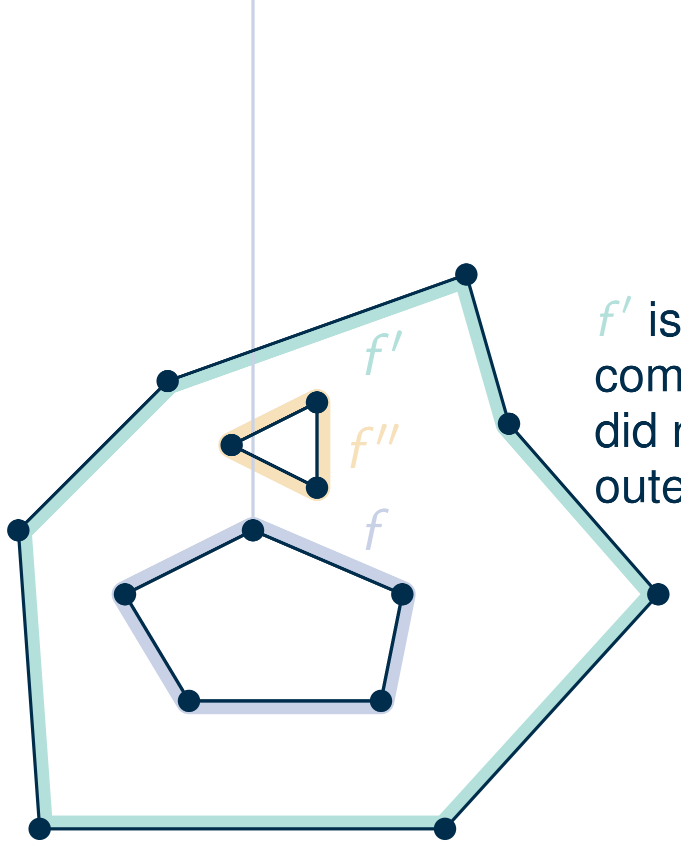
Assignment Sheet 1

Map Overlay

1. update edge pointers
2. update faces

    Find parent inner face of outer face f
3. face labels

    sweep-line



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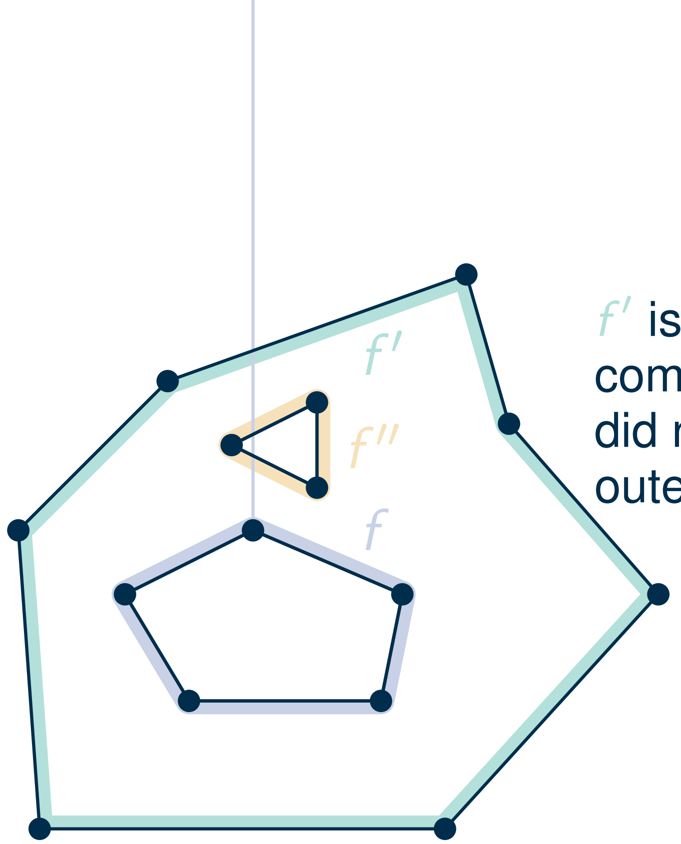


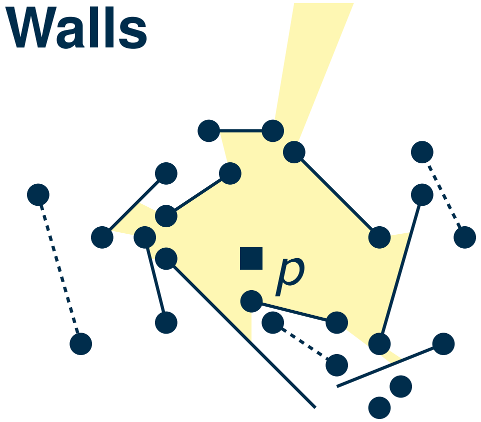
f' is inner face of first component, where ray did not intersect an outer face first

Boolean operations

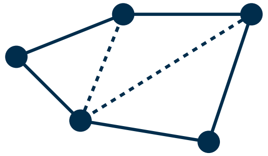
use labels of faces from map overlay

Assignment Sheet 2

Dangerous Walls



Triangulation

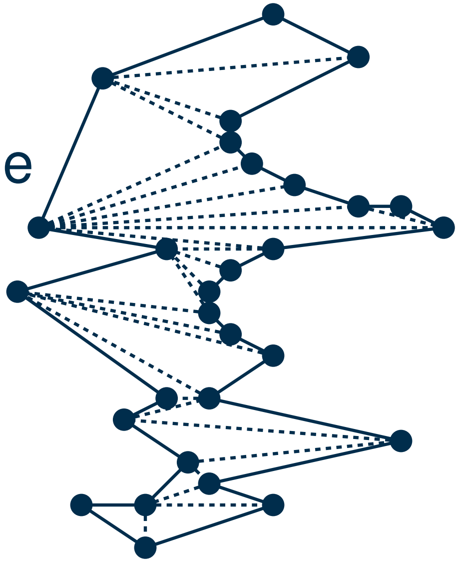


Q: how many triangles?

what if P contains holes?

y-Monotone Triangulation

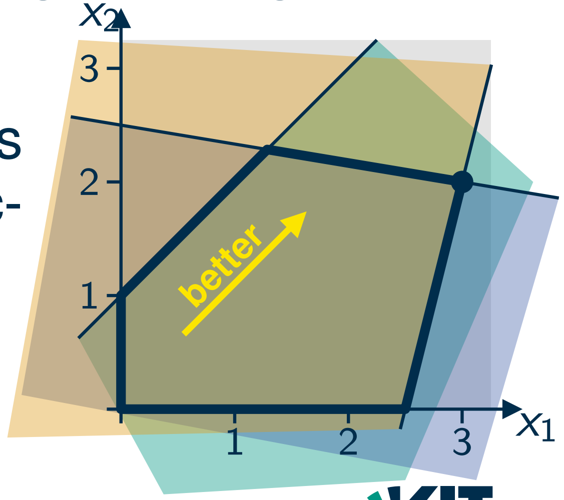
How to triangulate y-monotone Polygon in $O(n)$?



Completing 2D Linear Programming

Given: 2D LP

Decide: are there solutions with arbitrarily large objective?



Assignment Sheet 2

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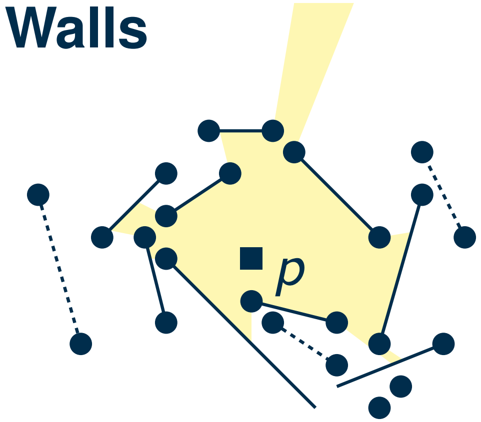

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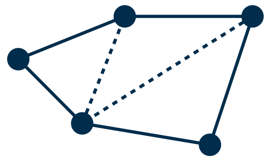

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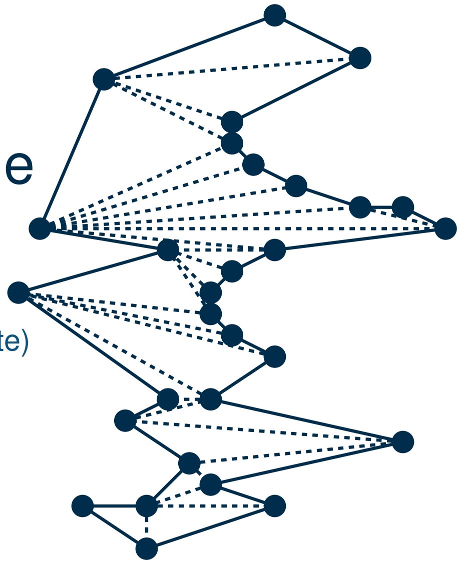
How to triangulate y-monotone Polygon in $O(n)$?

(easier: assume no two vertices have same y-coordinate)



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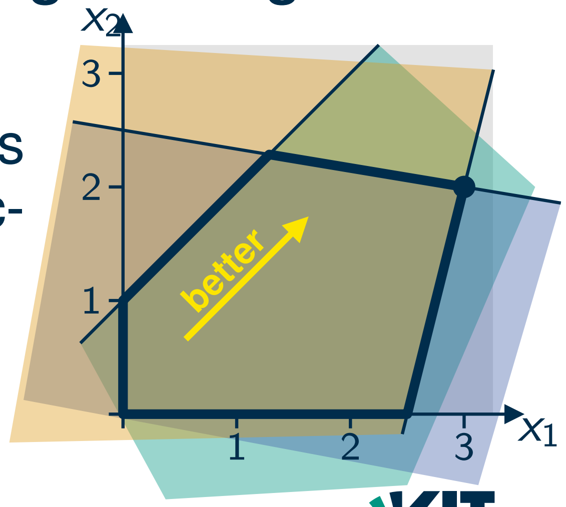
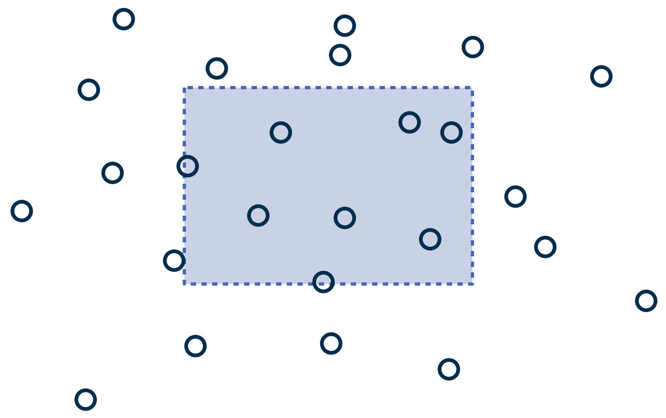


Range Trees

Problem: Range Reporting Query
Input: set of points $P \subseteq \mathbb{R}^d$, box $B = [a_1, b_1] \times \cdots \times [a_d, b_d]$
Task: compute $P \cap B$



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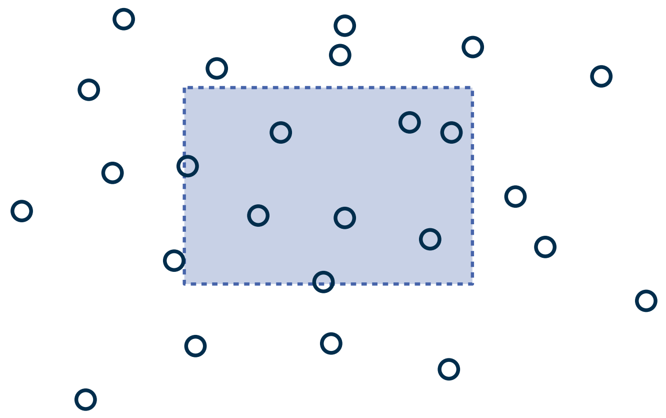

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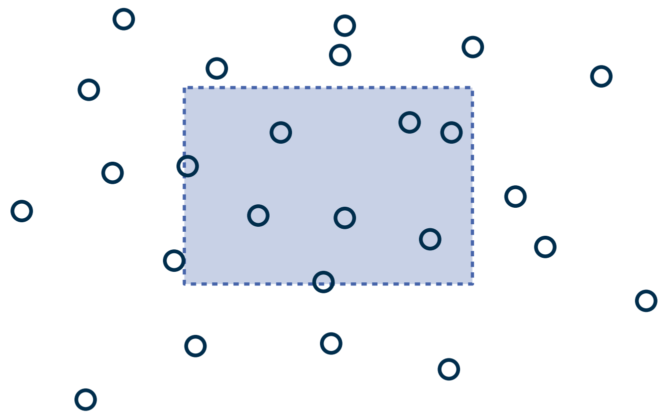

Questions:

- how to do this for $d = 1$? (running time? space?)

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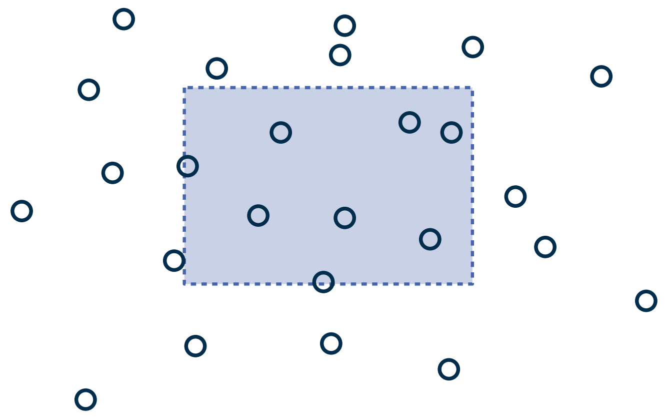

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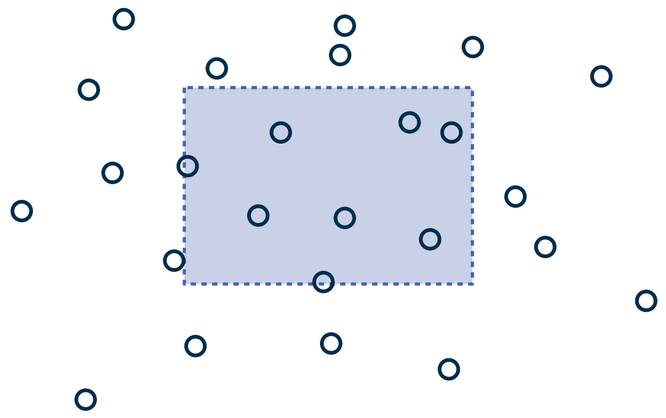

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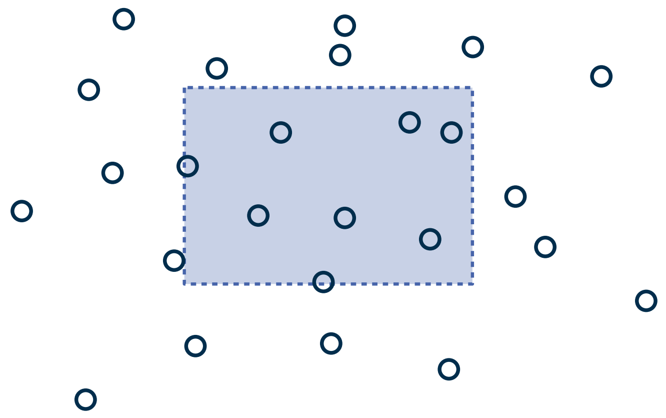

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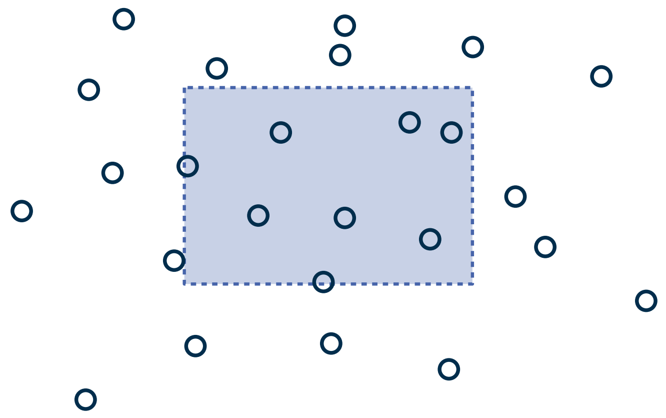

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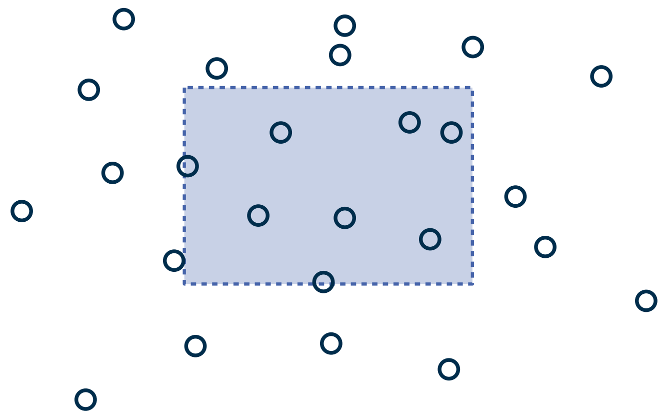

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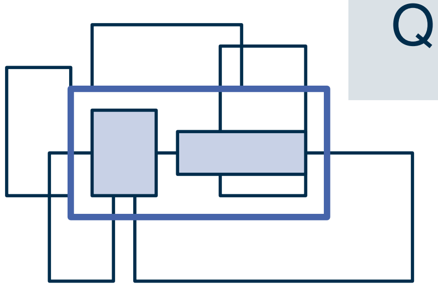

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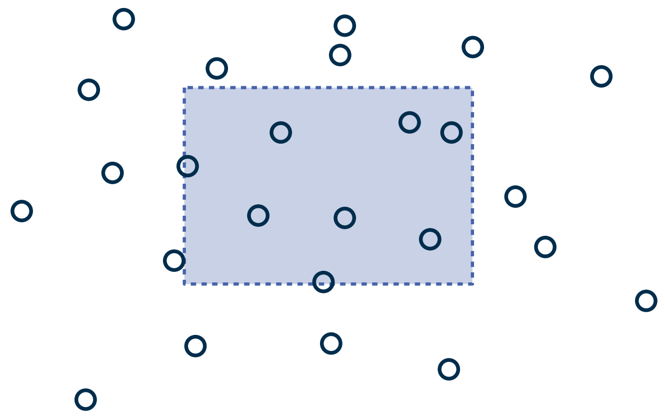

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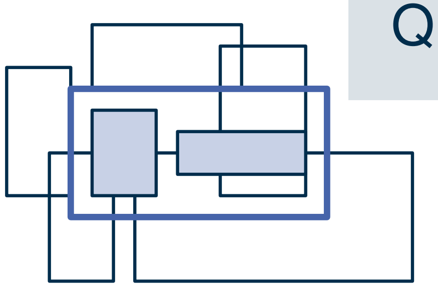

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  hint: uvtu-qvzrafvbany

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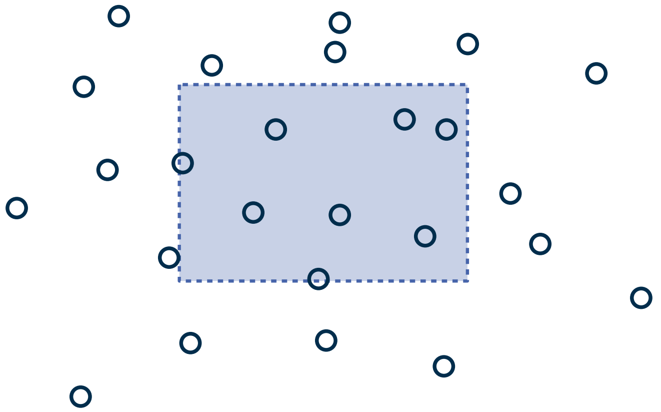

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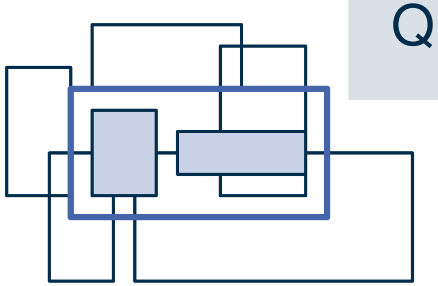

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- what about Range *Polygon* Reporting?

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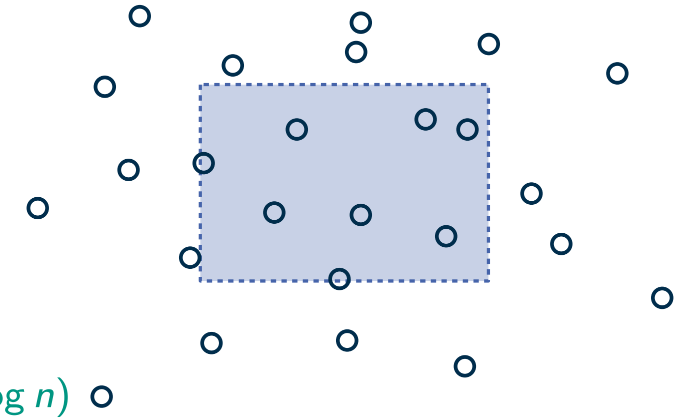

Questions:

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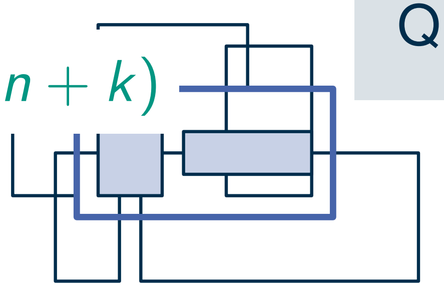
  binary search, $O(\log n)$

- how to generalize for $d \in \mathbb{N}$?
  compare with range reporting query!
- improvement for $d = 2$? $O(\log n)$
- improvement for $d = 3$?

  $O(\log^2 n)$ or $O(\log n + k)$



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  4-d queries in 4-d range tree!

- what about Range *Polygon* Reporting?