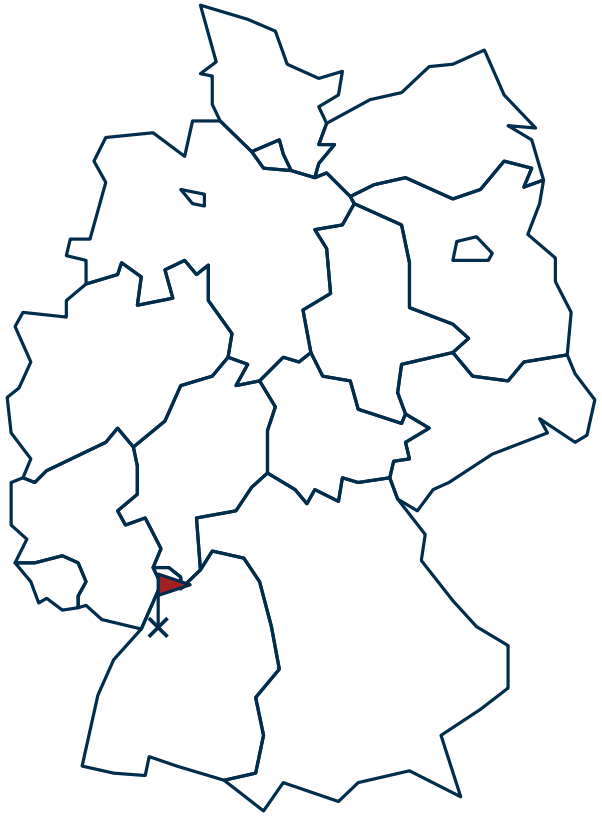
Computational Geometry

Point Location & Persistence – Where am I? And When?

Thomas Bläsius

Where am I?

Problem: Point Location
In which face of a geometric graph does a given point p lie?



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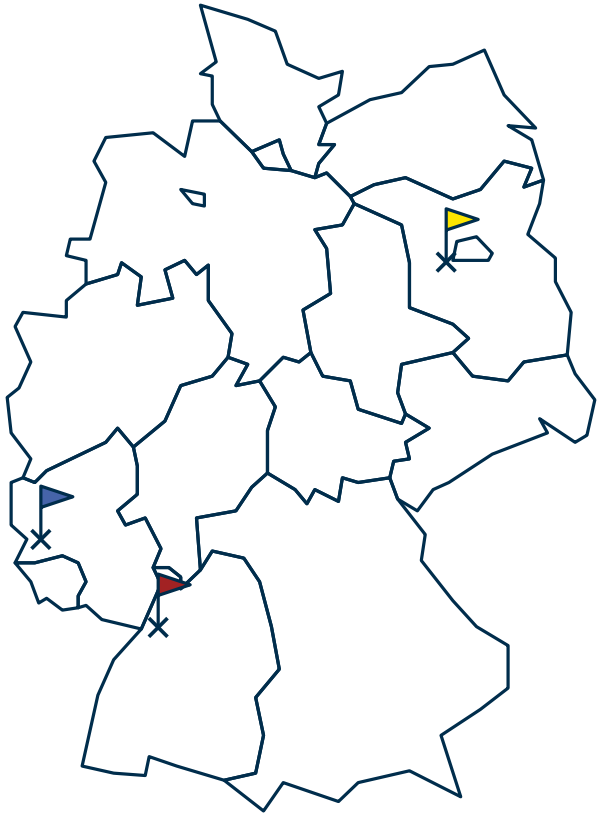

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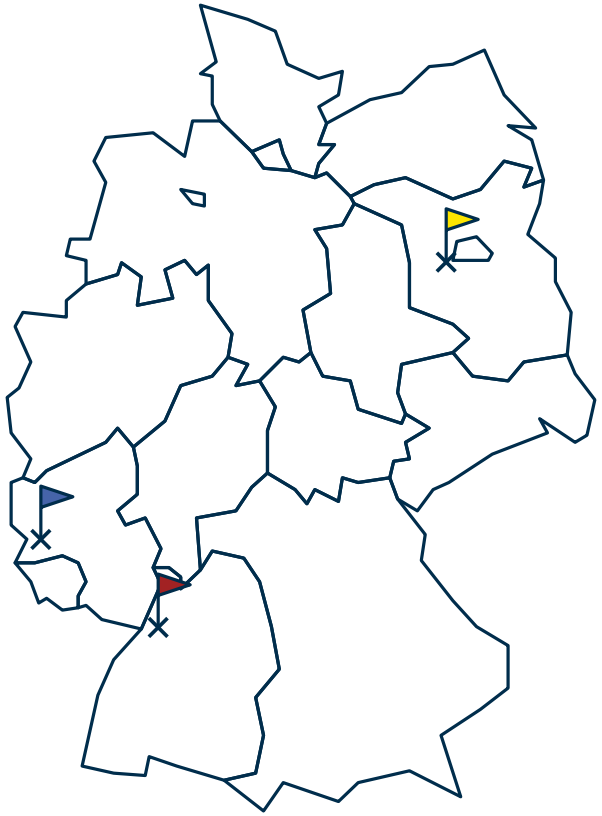

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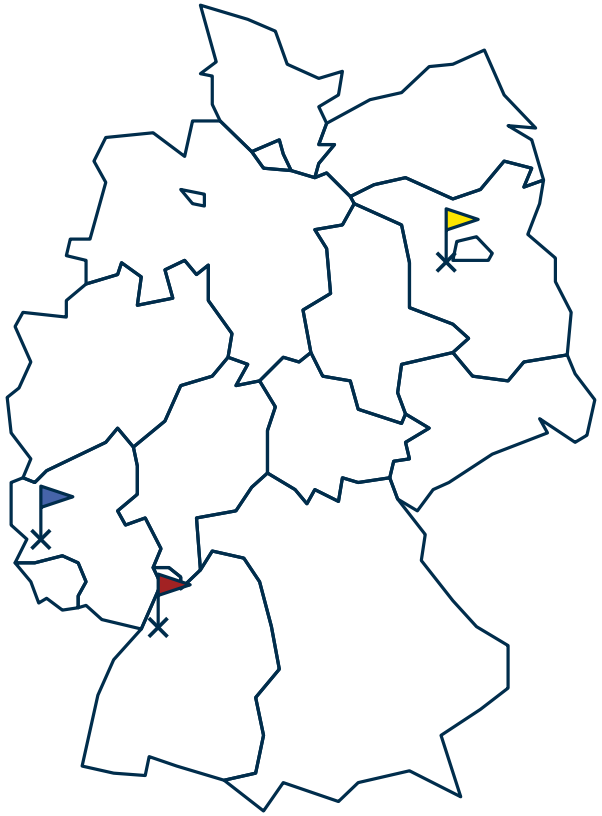

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What are possible applications?

Data Structures And Time Travel

What Can We Typically Do With A Data Structure?

- queries to the data structure in its current state
- applying an operation $\rightarrow$ new state

Example: Priority Queue

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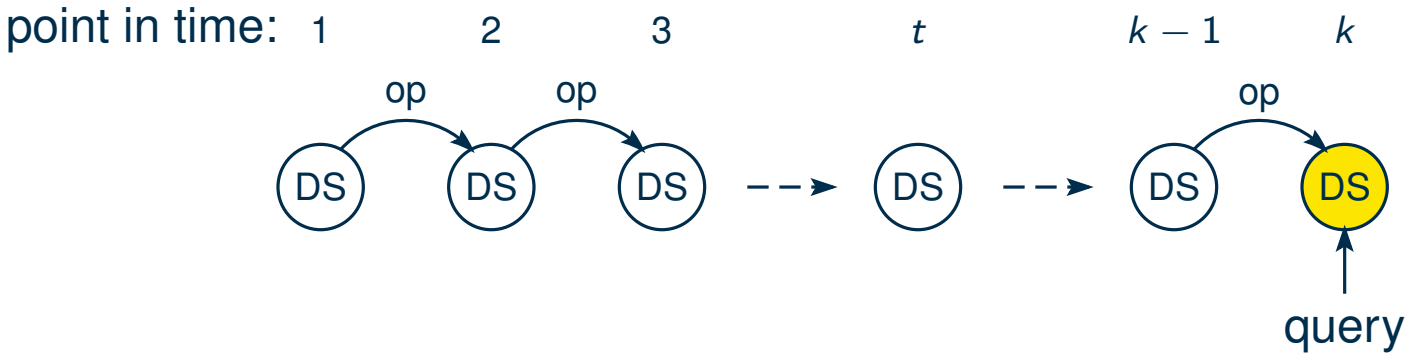

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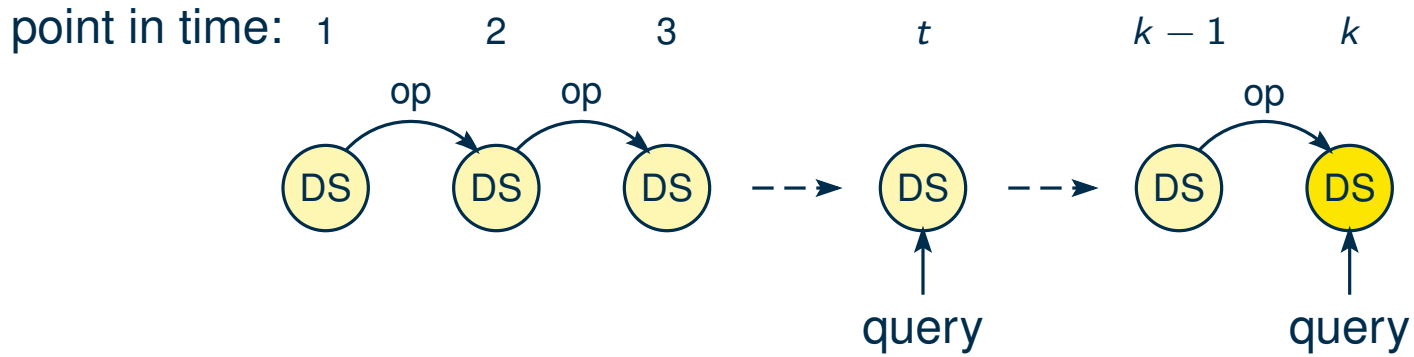

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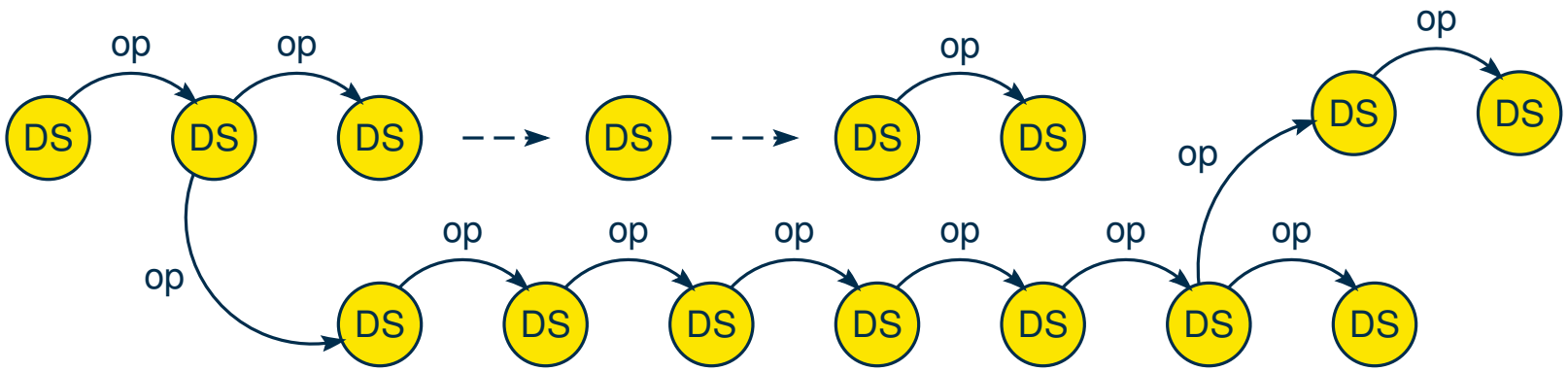

Partial Persistence

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Full Persistence

- also allow operations in the past
- time is then no longer linear but branches

Pointer Machine Data Structures

The Pointer Machine Model

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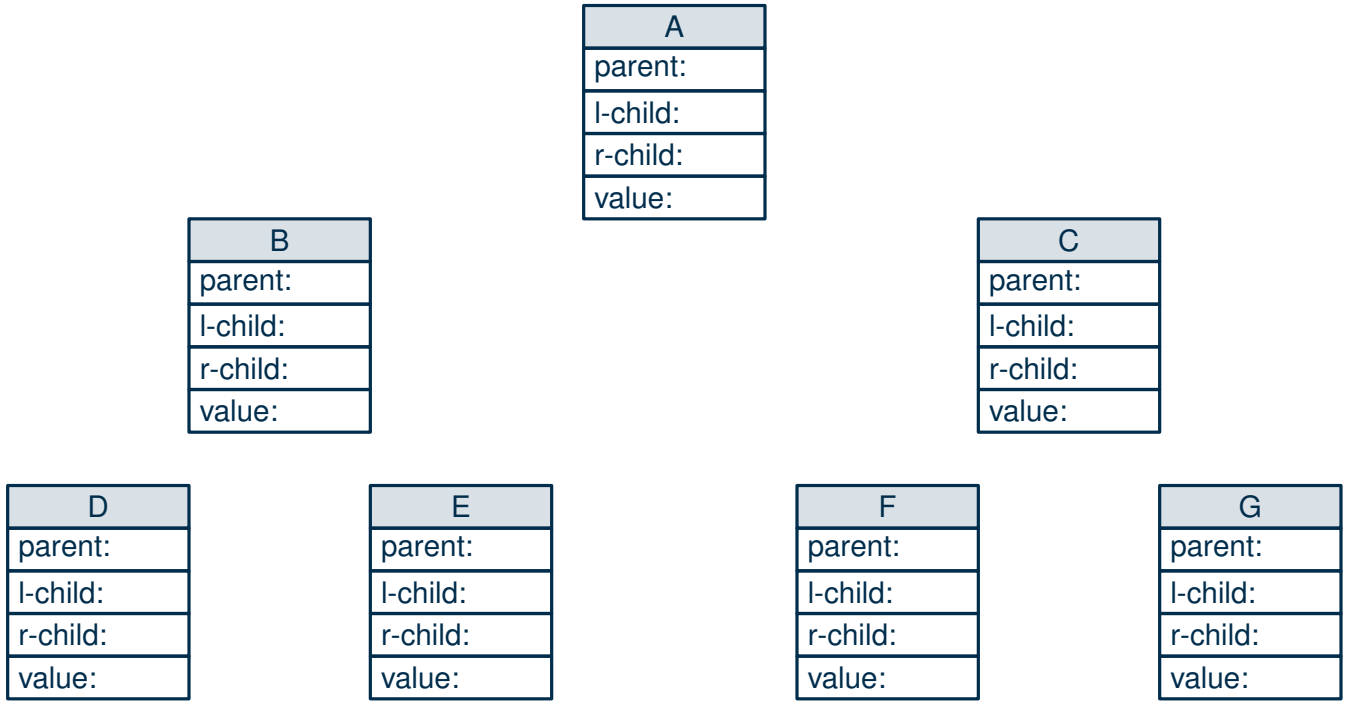
The Pointer Machine Model

- constant number of different node types, each with a constant number of fields

Example: Binary Search Tree



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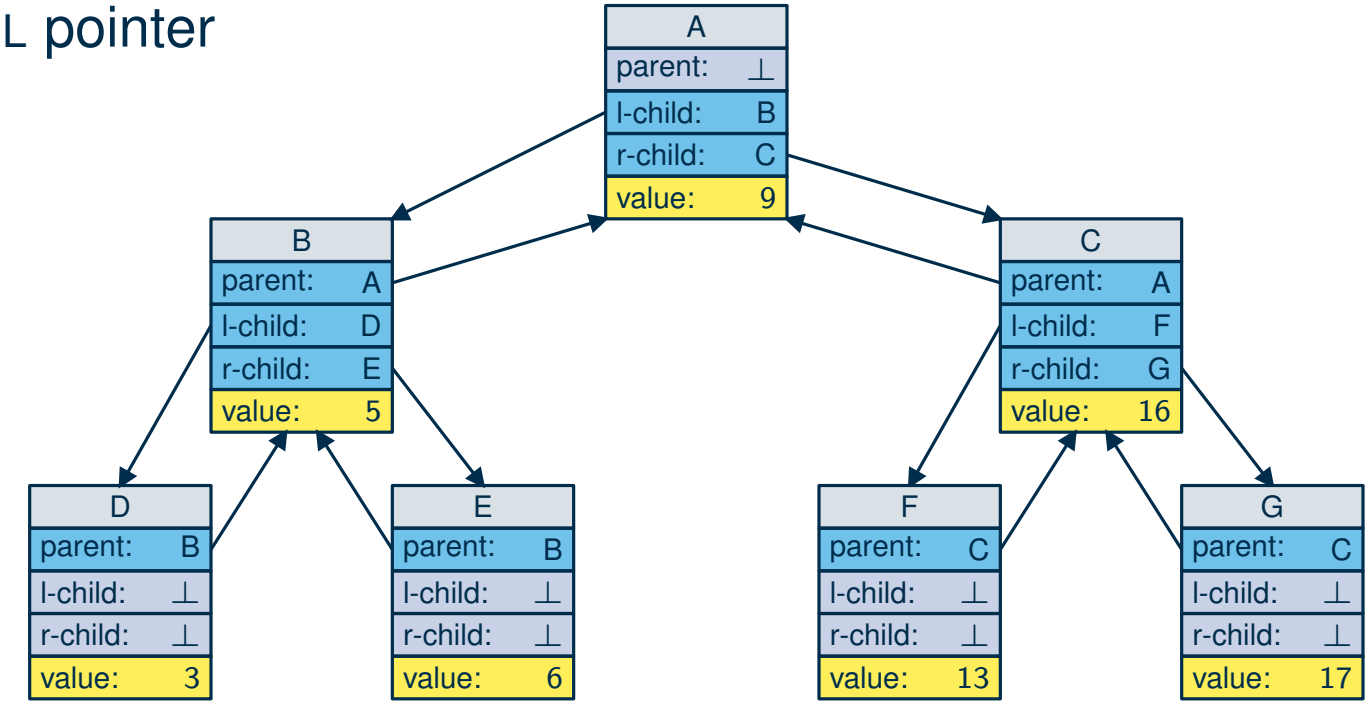

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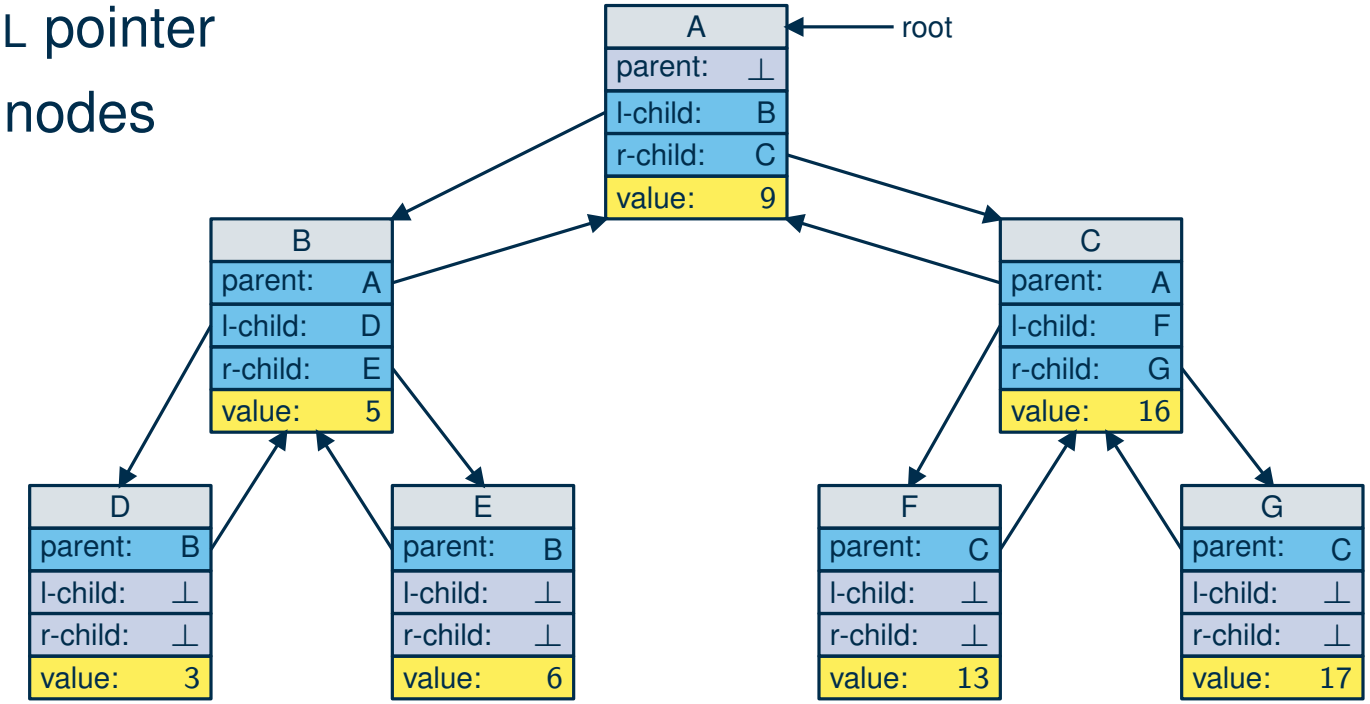

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- a directed graph with constant out-degree
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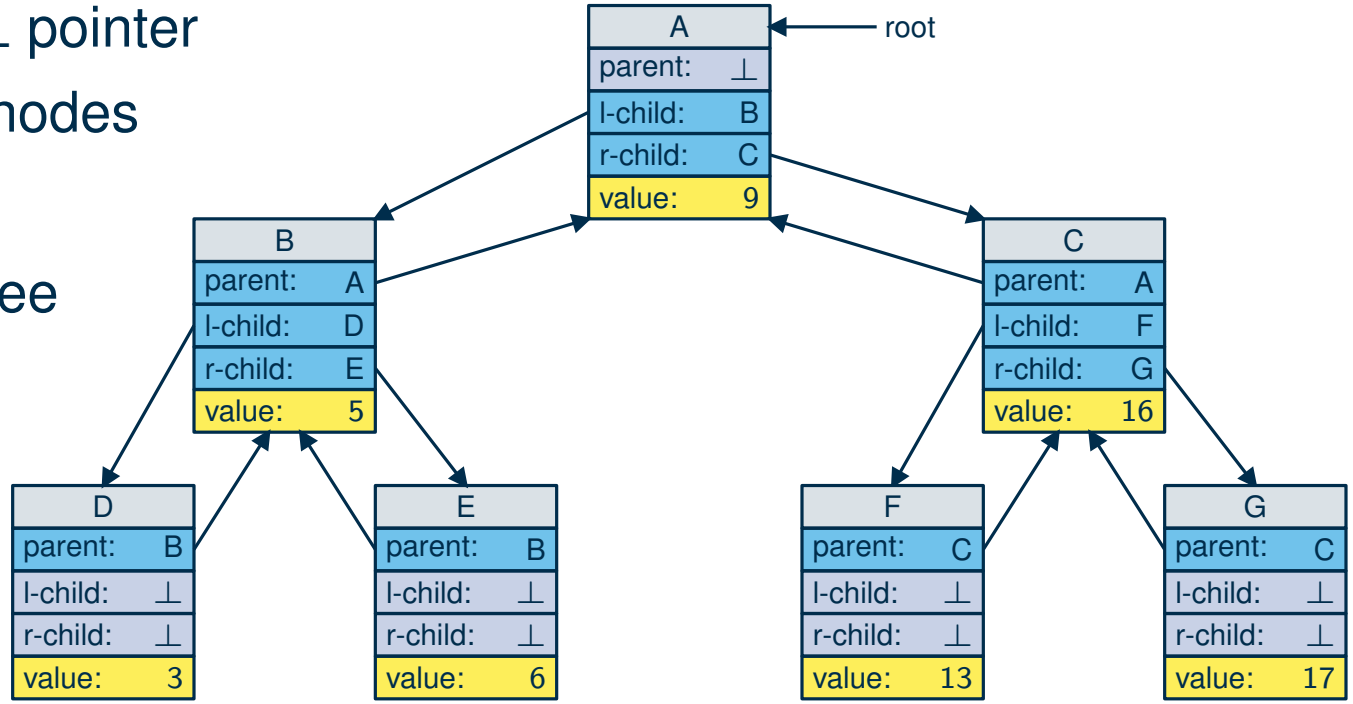

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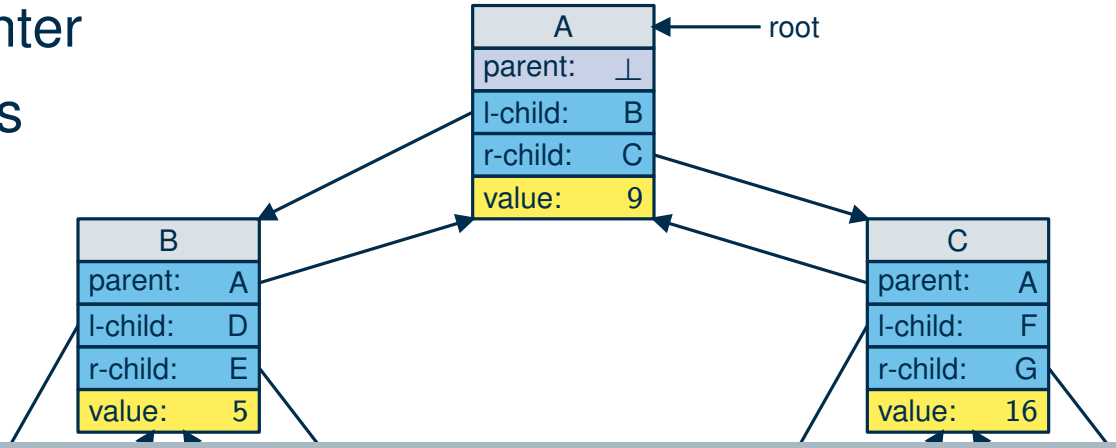

Theorem **(partial persistence for everyone)**
Every data structure in the pointer machine model with constant in-degree can be made partially persistent with (amortized) constant overhead.

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Timestamps

- is incremented after each complete operation (not after queries)
- entry nodes before: constant number of pointers to entry nodes
- now: for every timestamp a constant number of pointers to entry nodes

(e.g., constant number of arrays with one pointer per timestamp)

Nodes With A History

Atomic Operation

Example Operation

- operation: C.value = 1
- current timestamp: 28

C	
parent:	A
l-child:	F
r-child:	G
value:	16

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- idea: every node stores its own diff in an additional “mod” field

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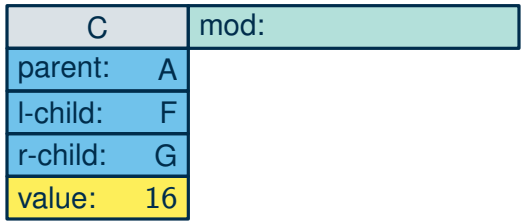

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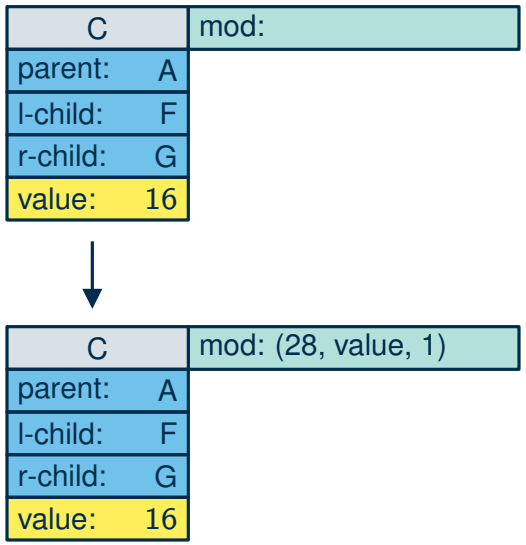

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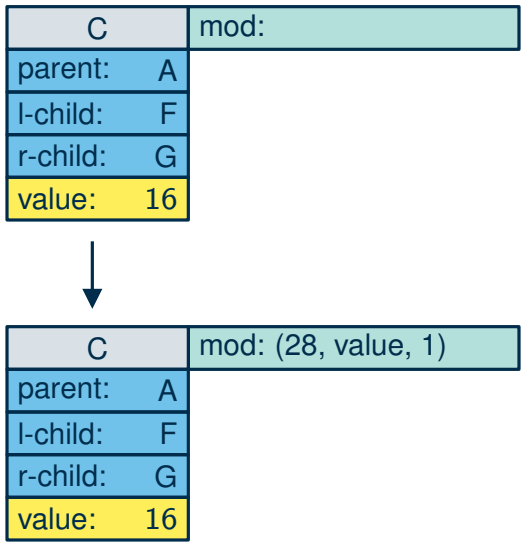

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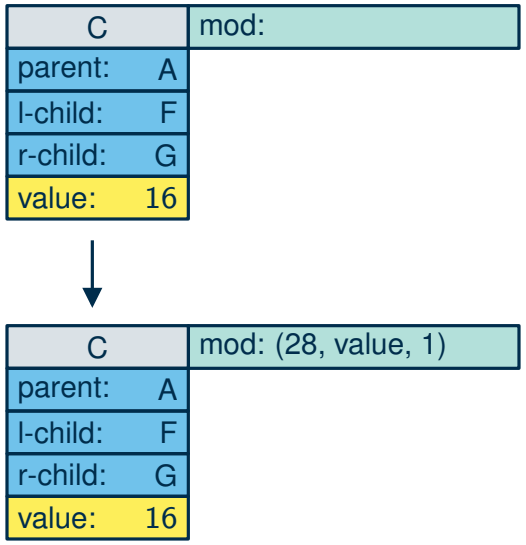

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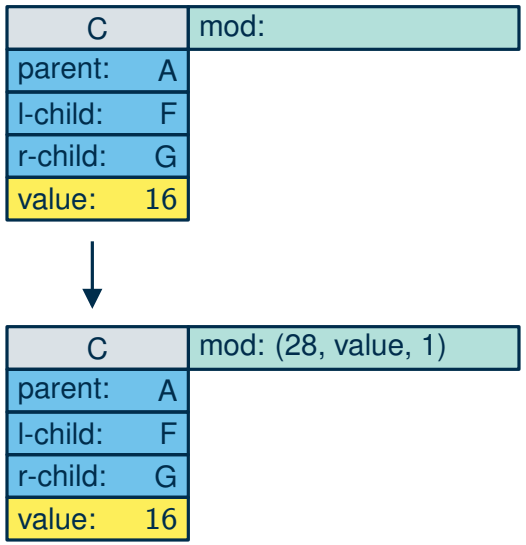

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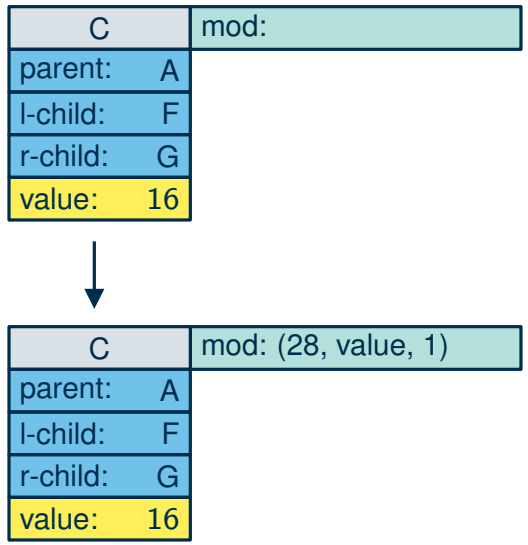

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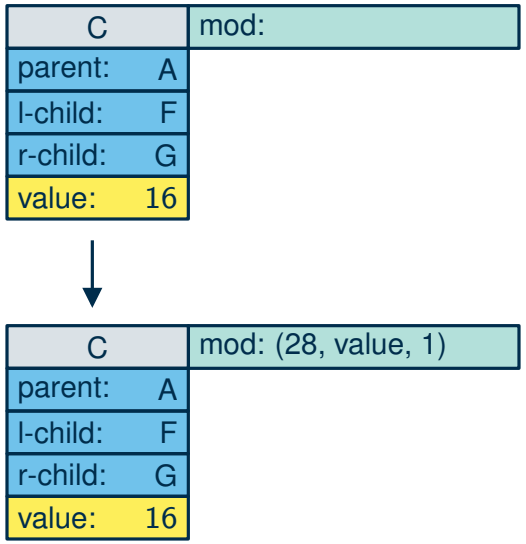

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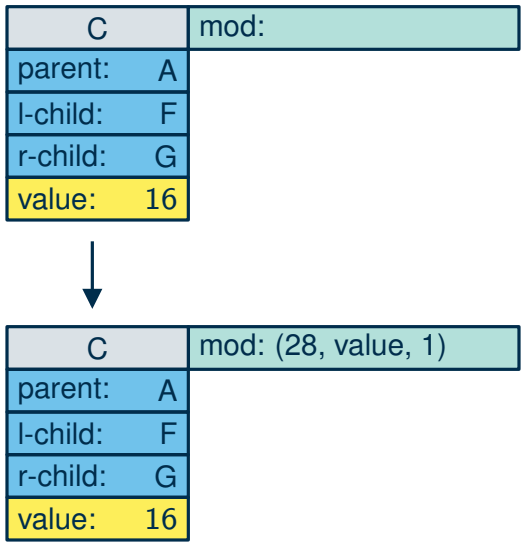

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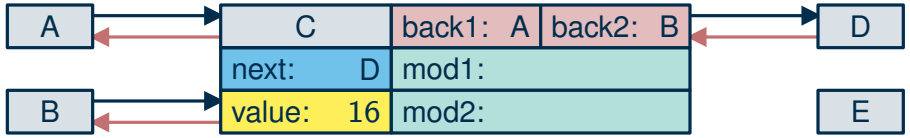

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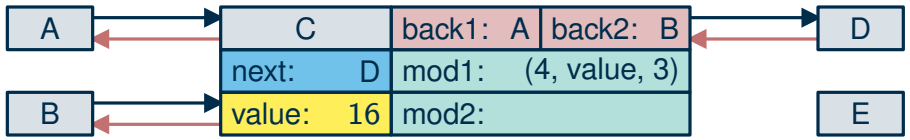

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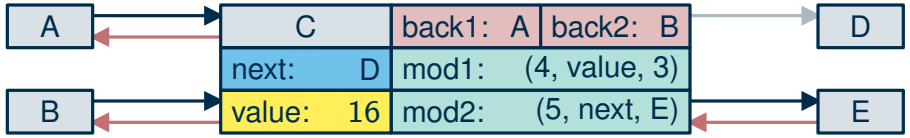

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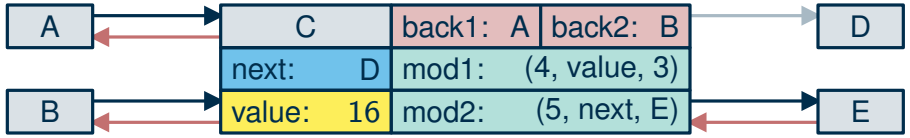

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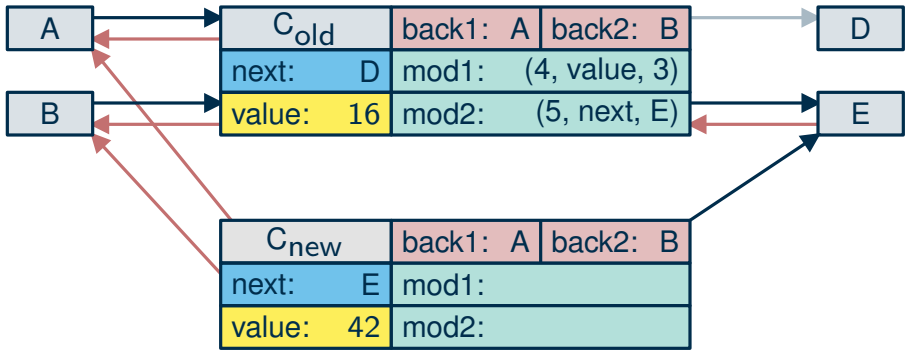

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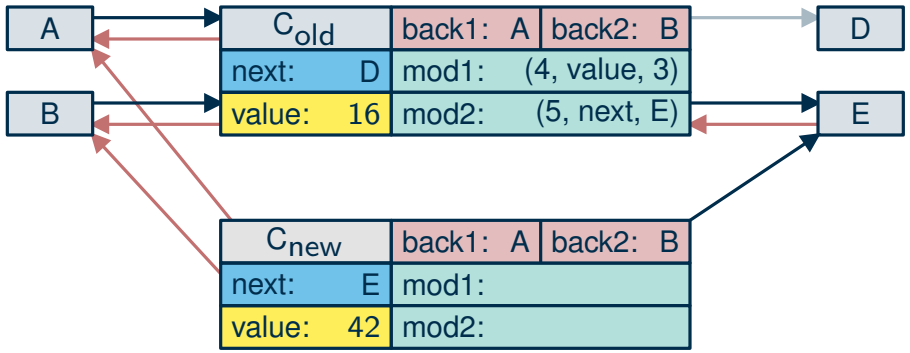

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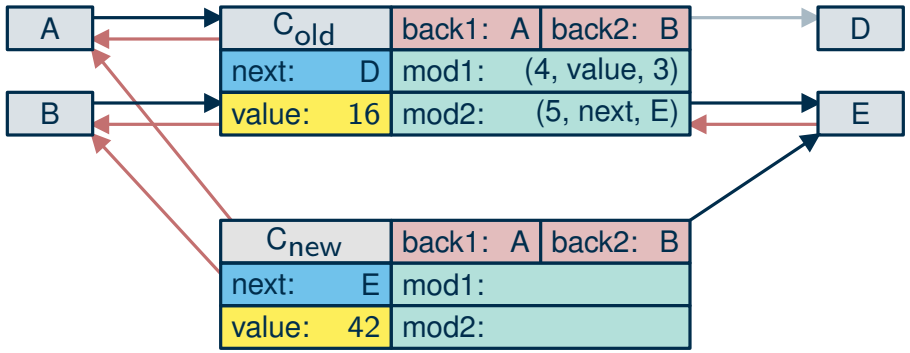

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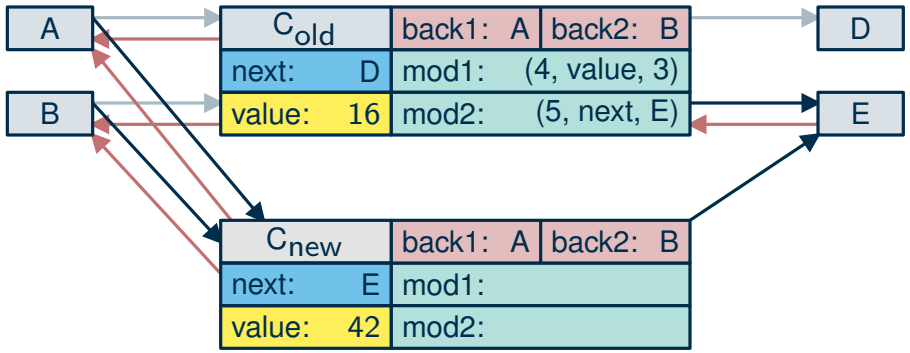

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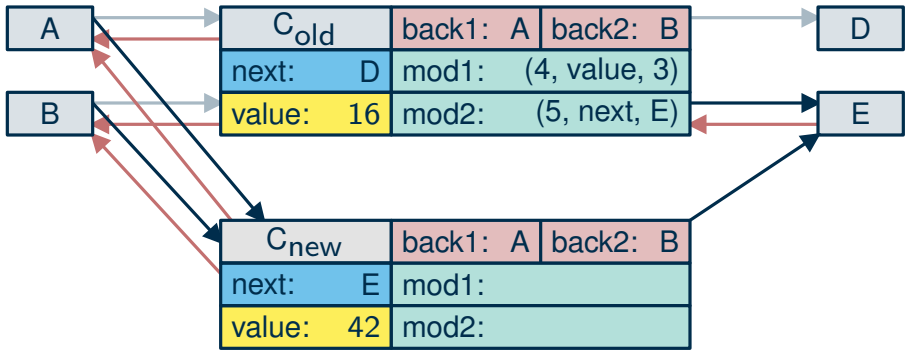

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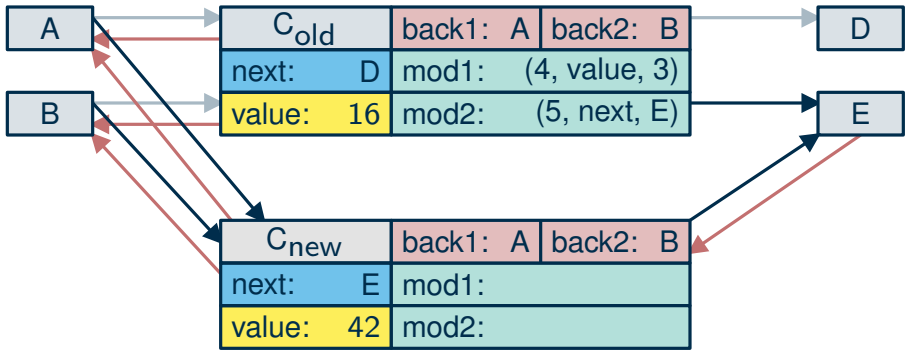

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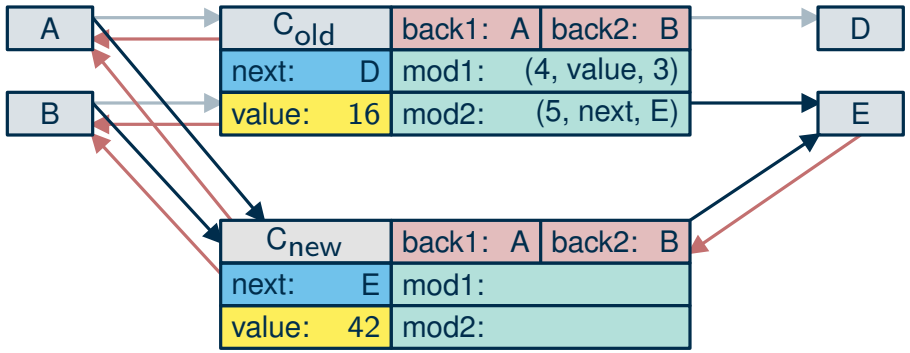

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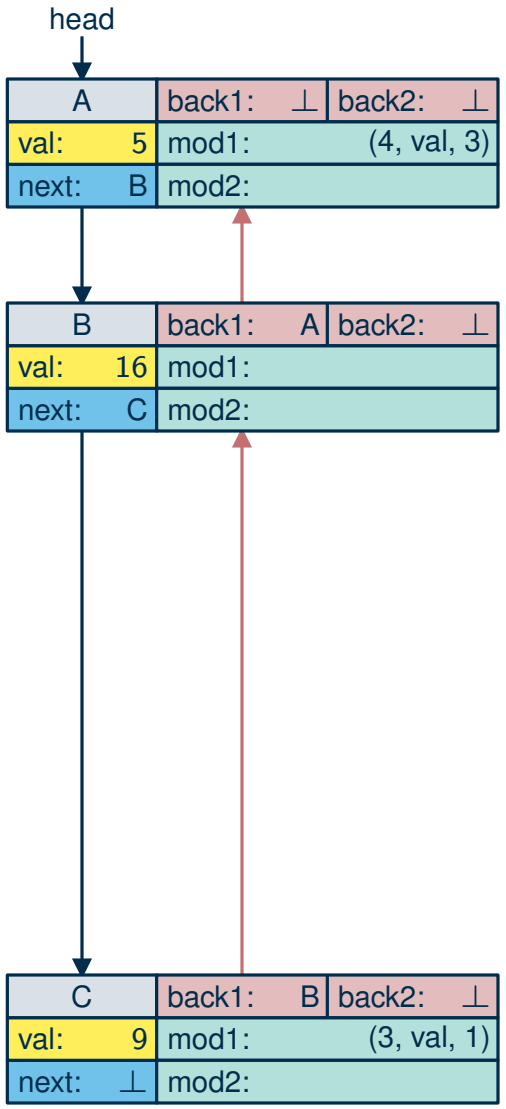
- remove back-pointers to v_{old}, add back-pointers to v_{new}
  - note: back-pointers are only needed for the present time

this creates new atomic operations:

A.next $= C_{new}$ | B.next $= C_{new}$

this recursion may cascade!

Example: Singly Linked List



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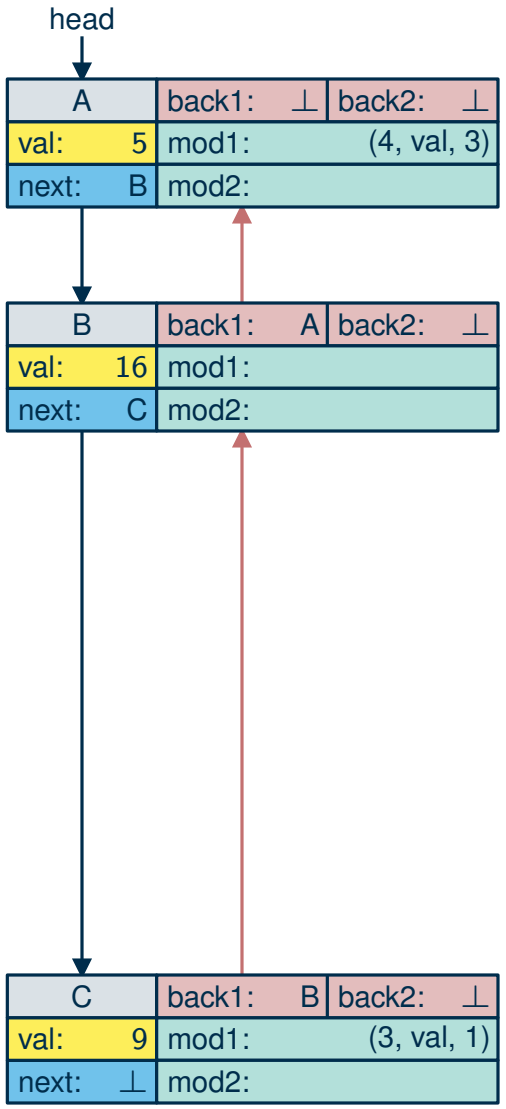

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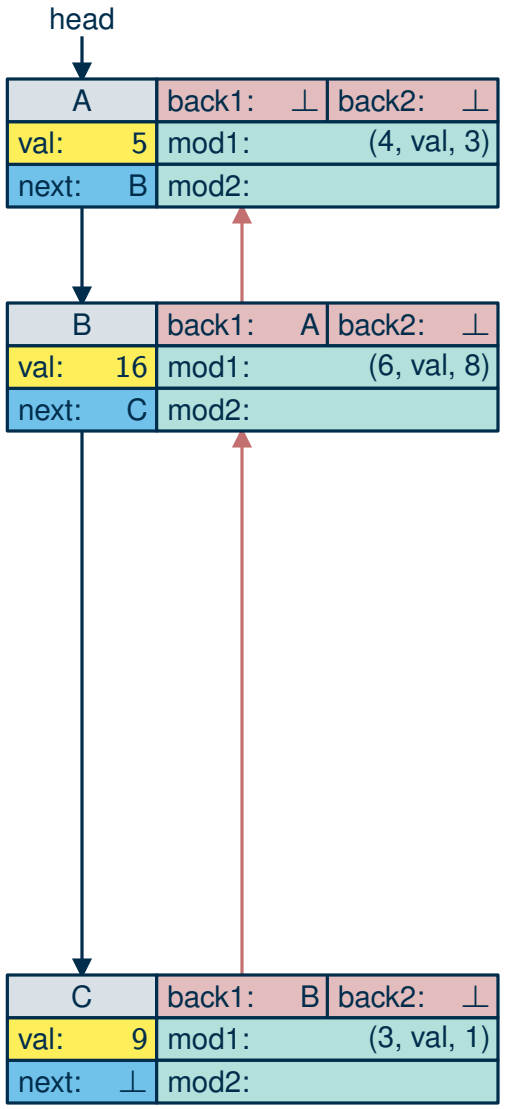

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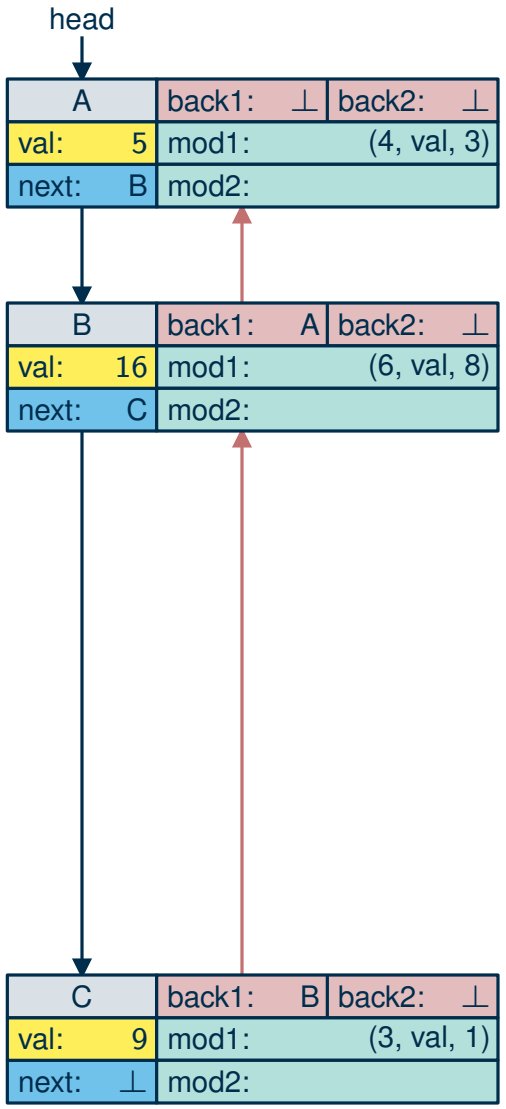

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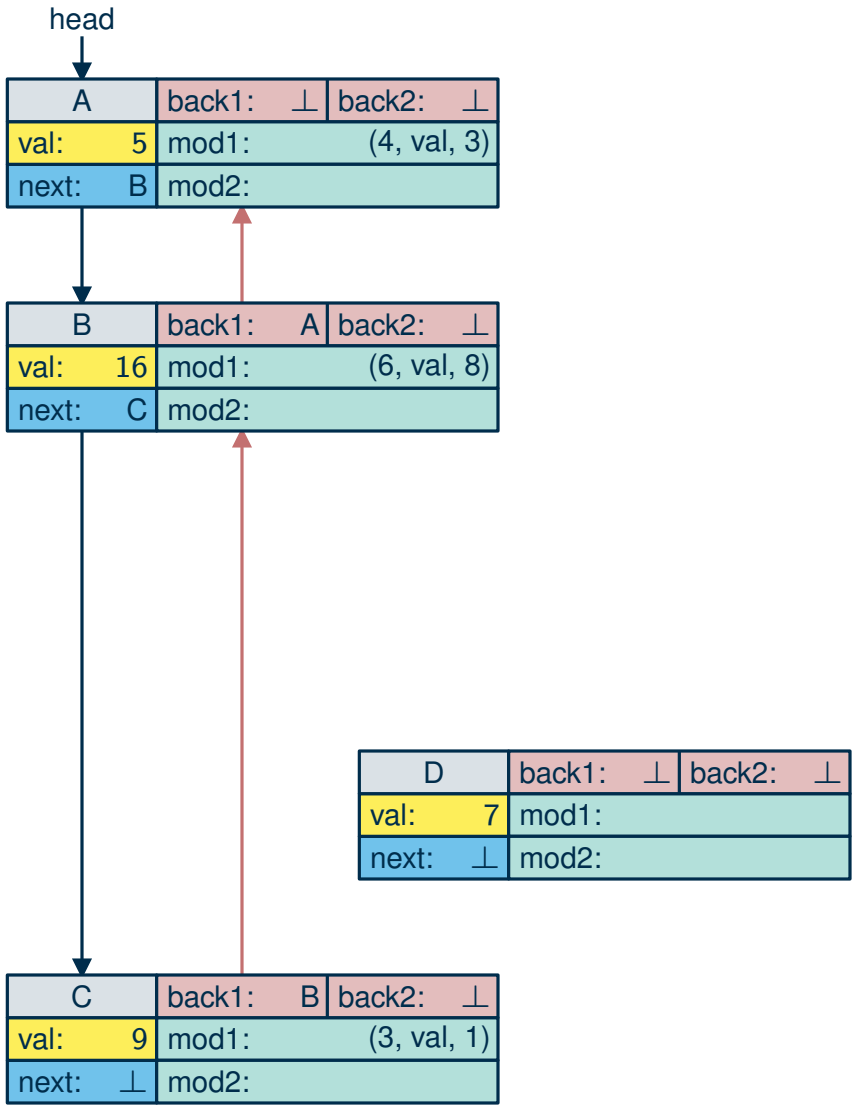

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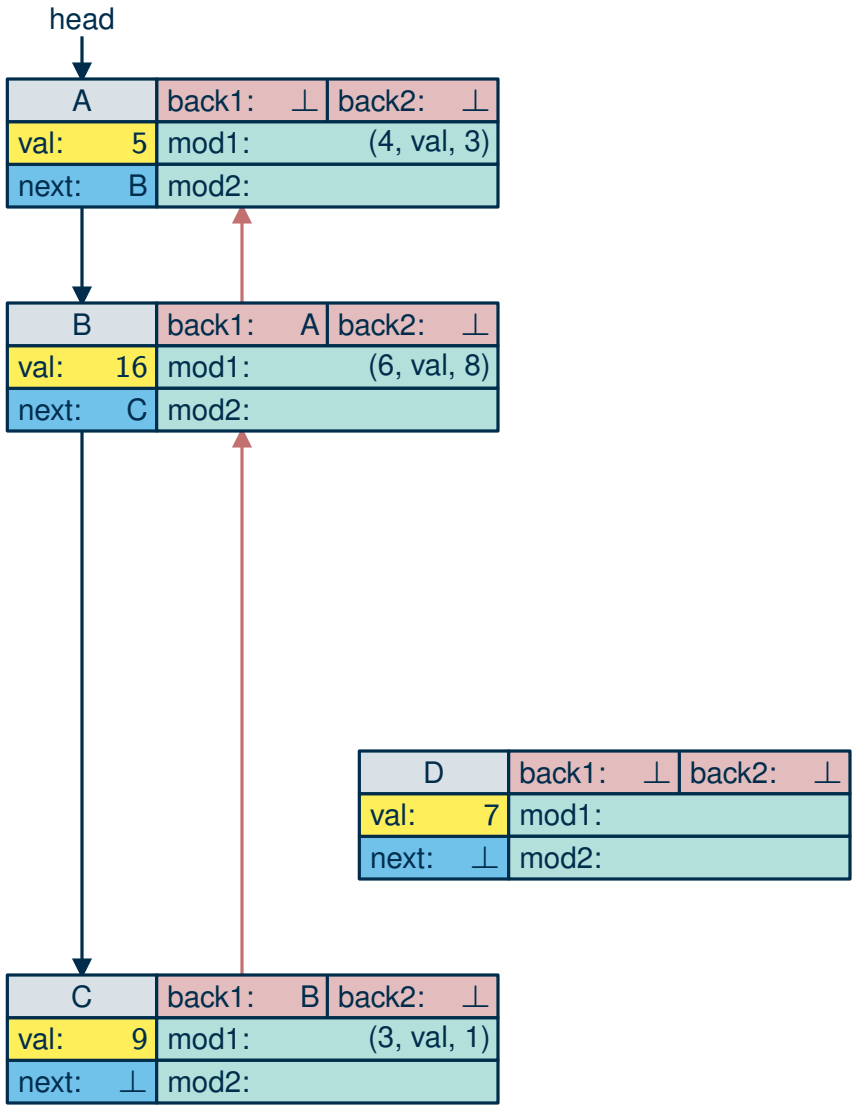

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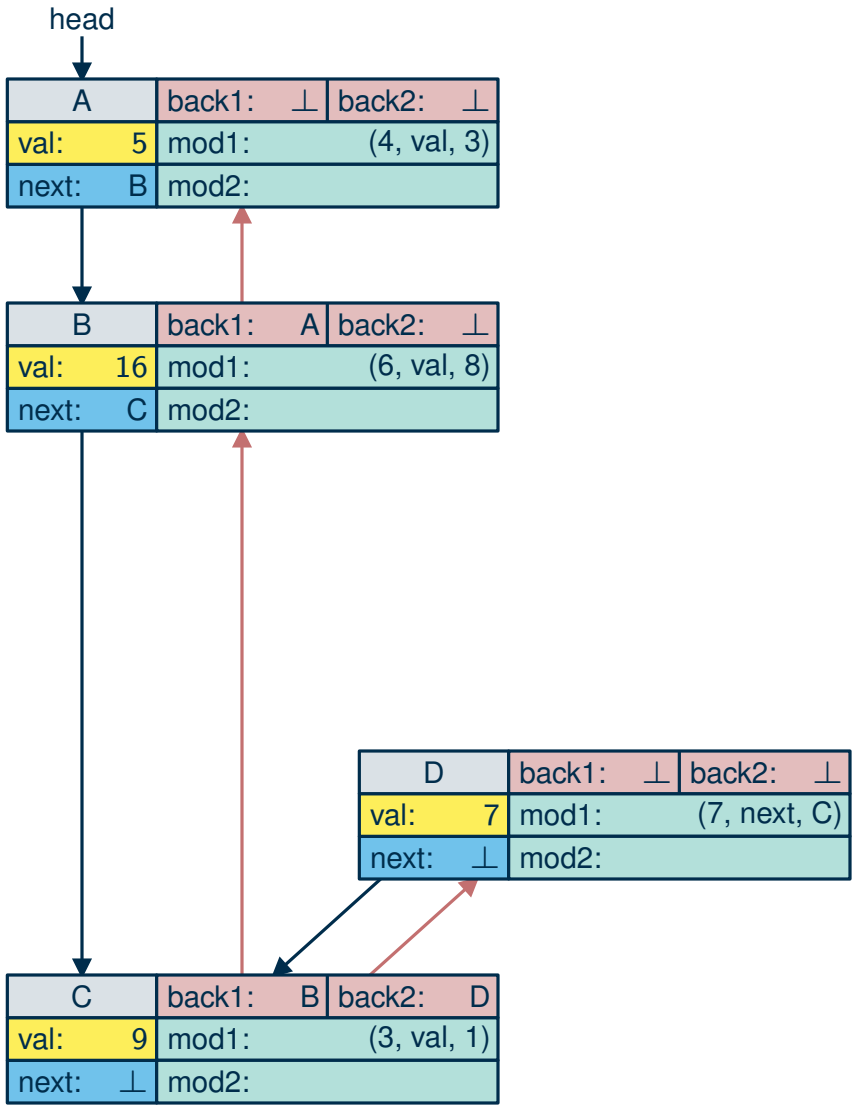

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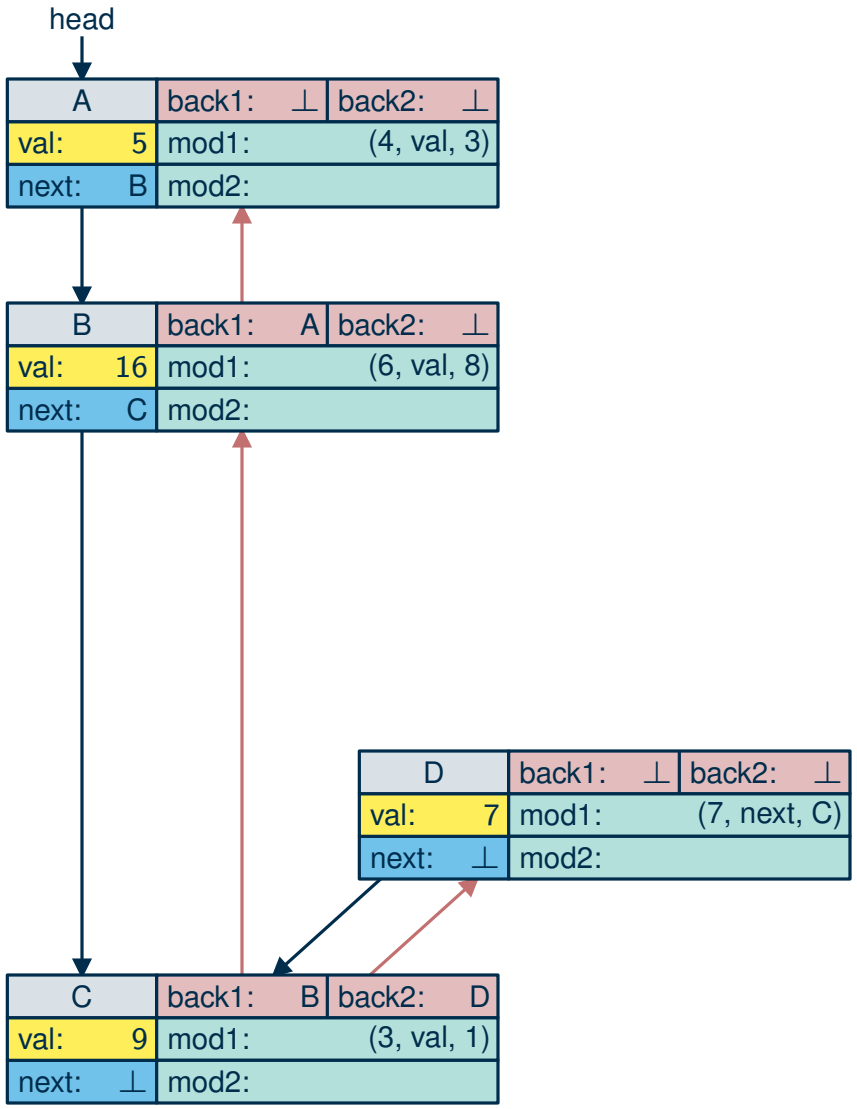

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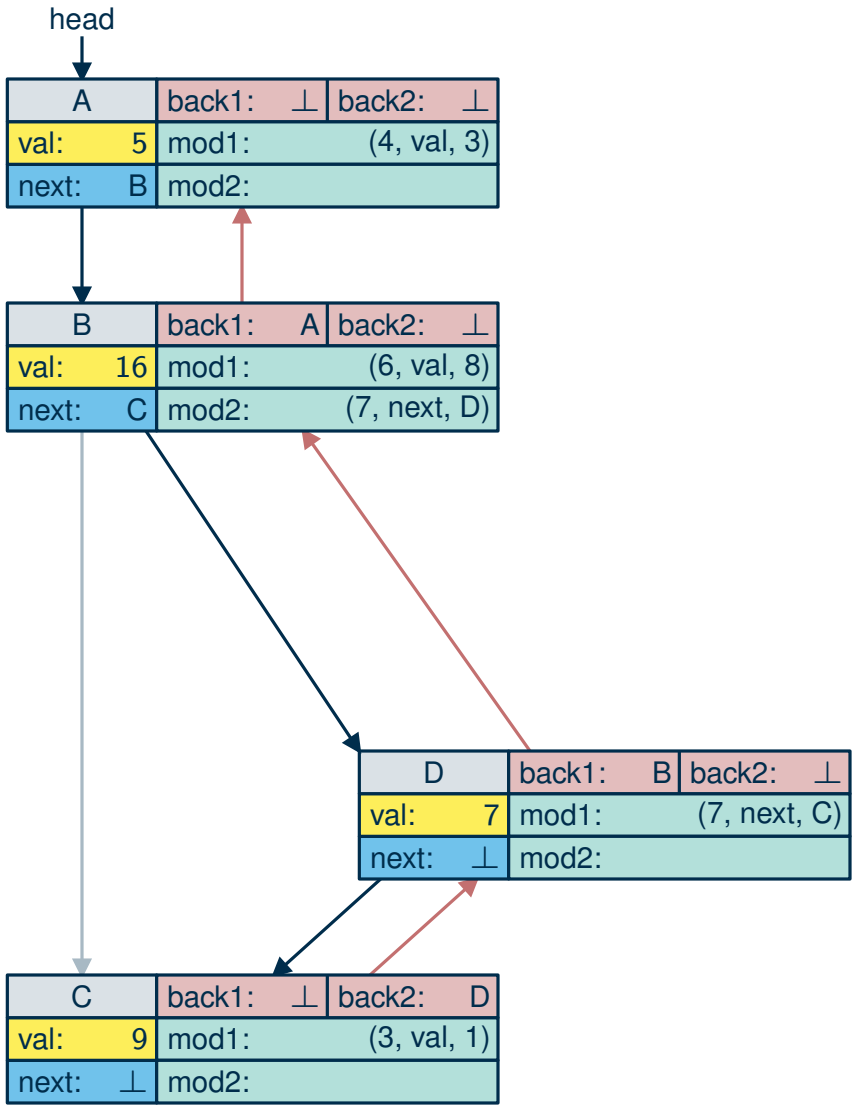

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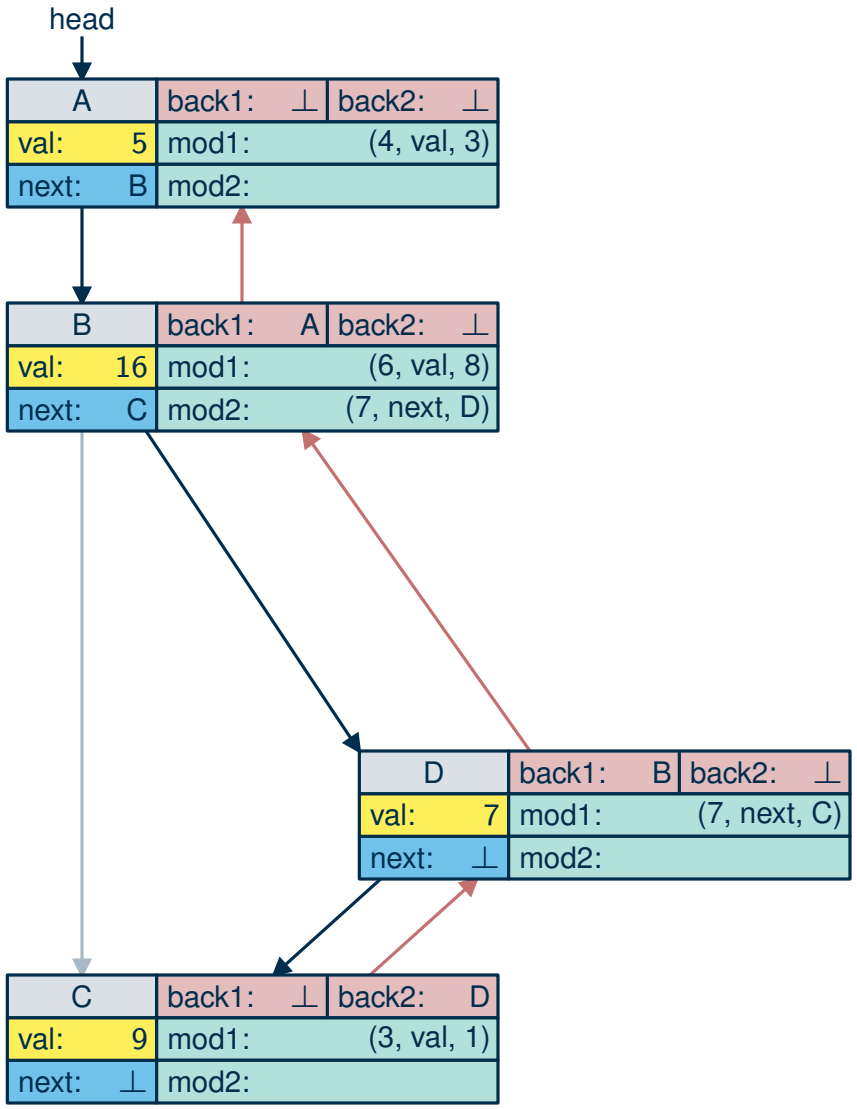

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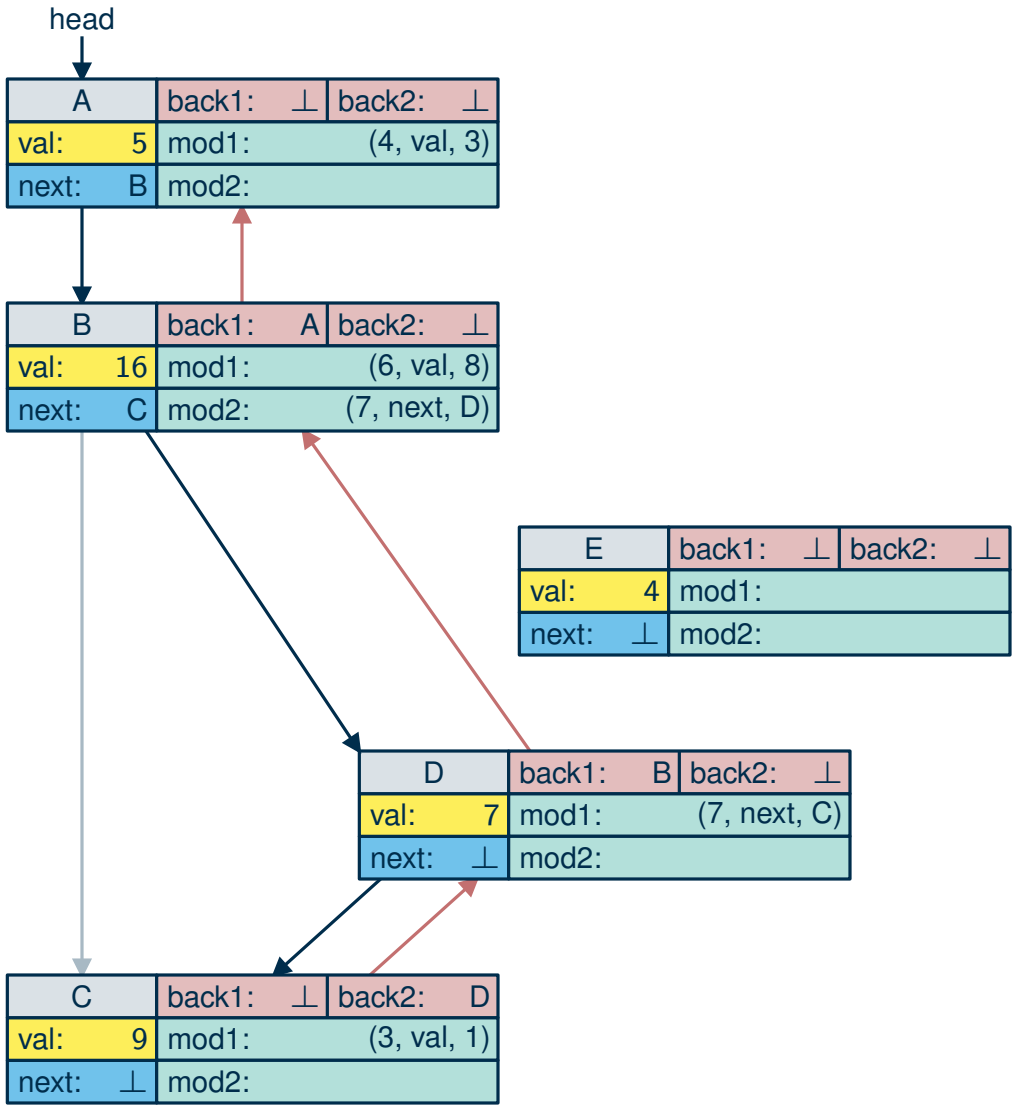

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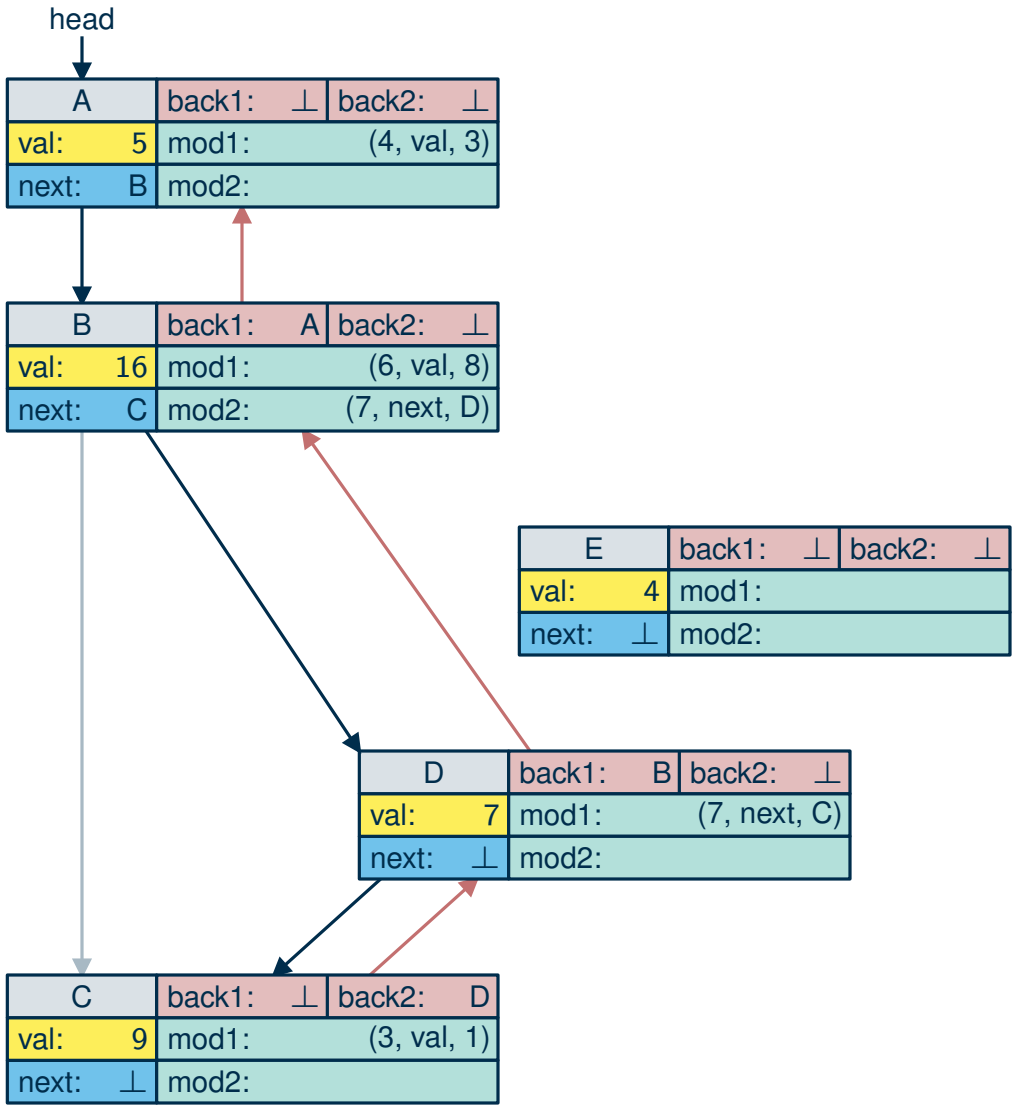

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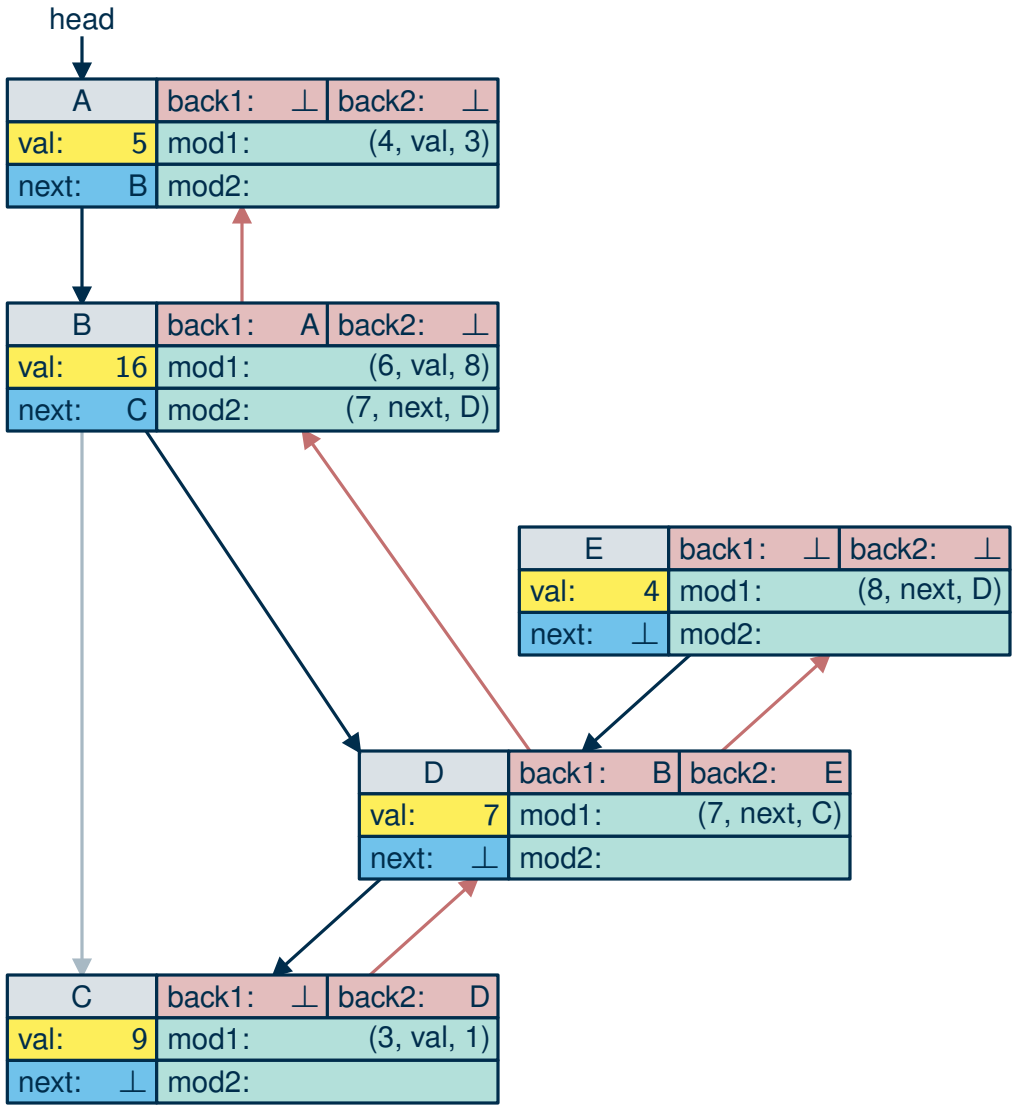

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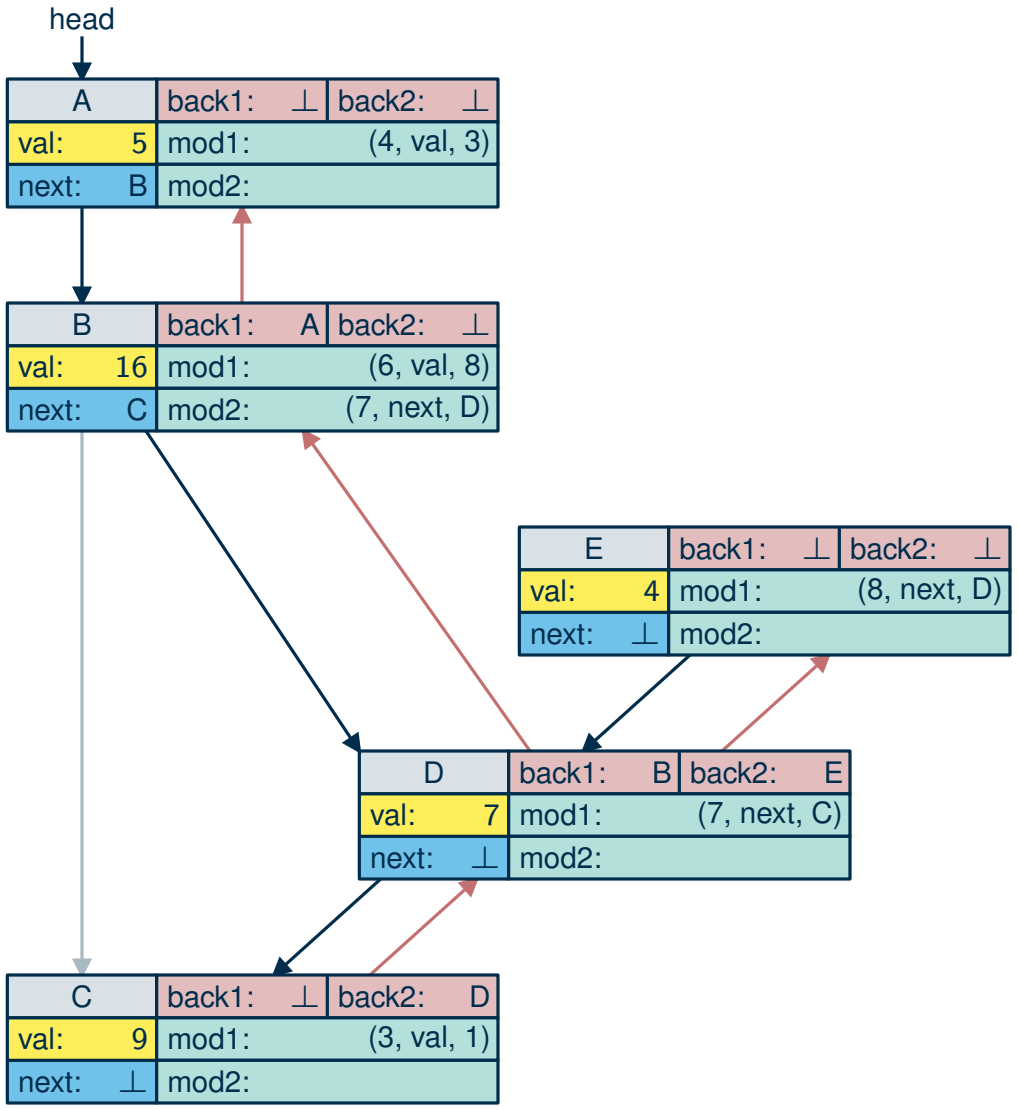

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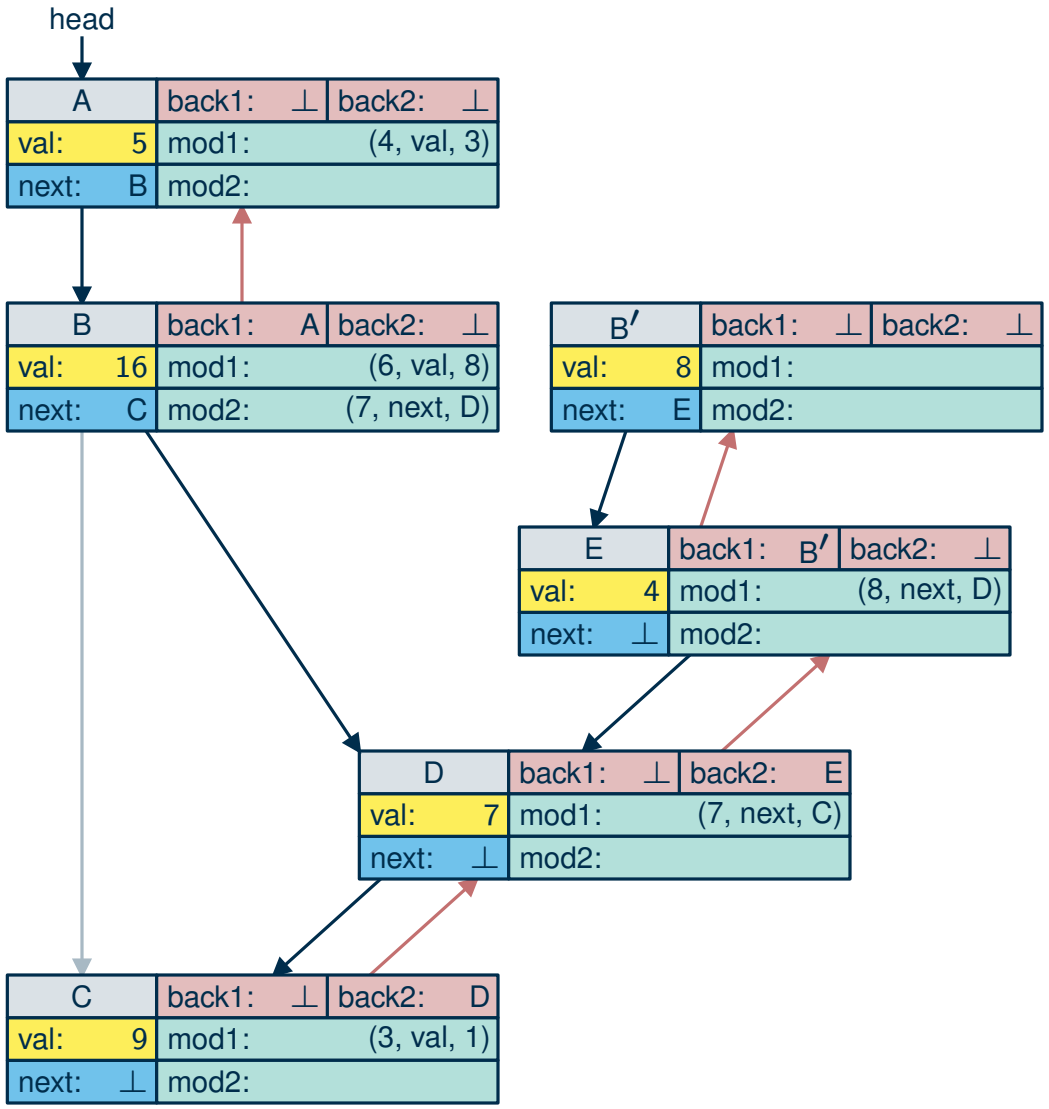

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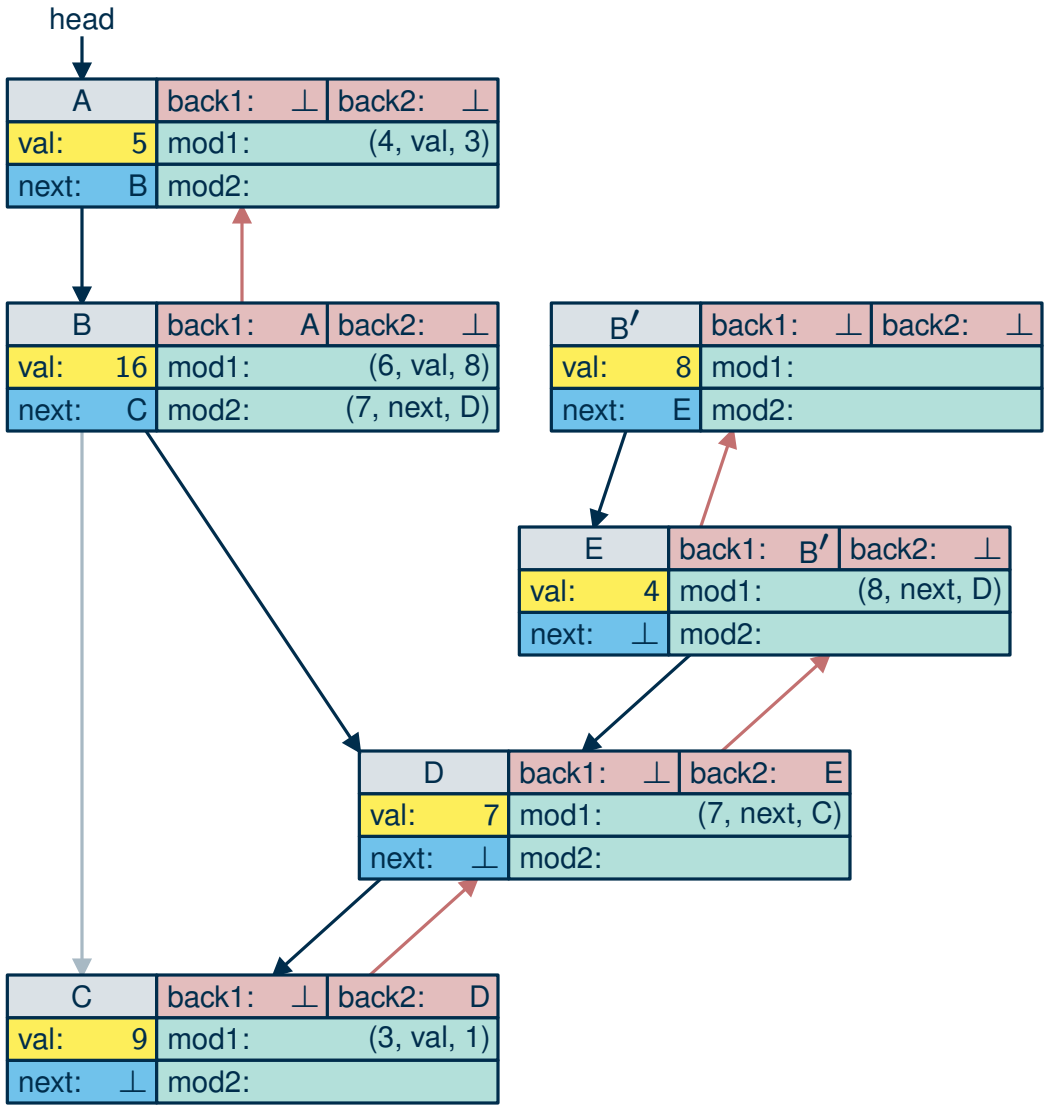

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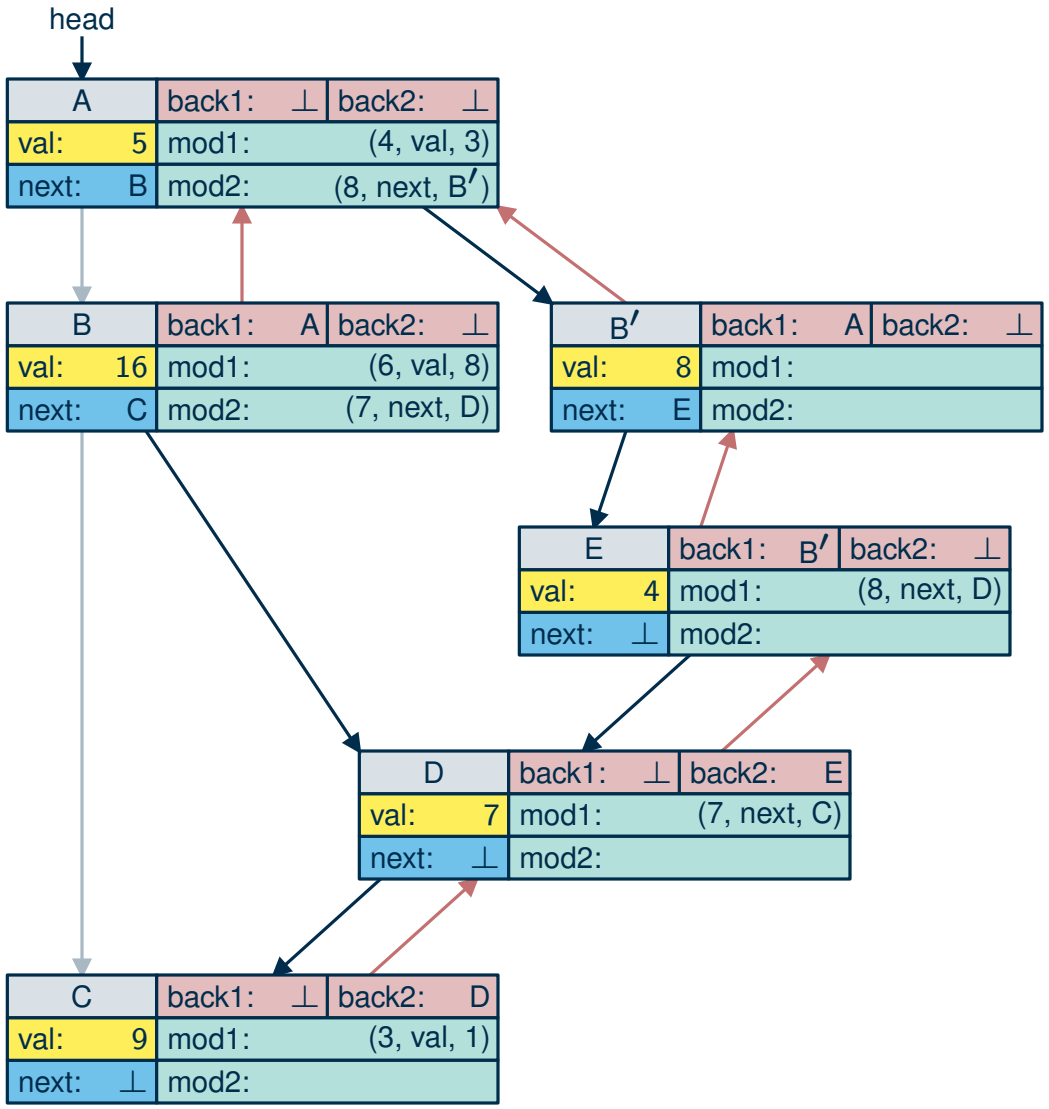

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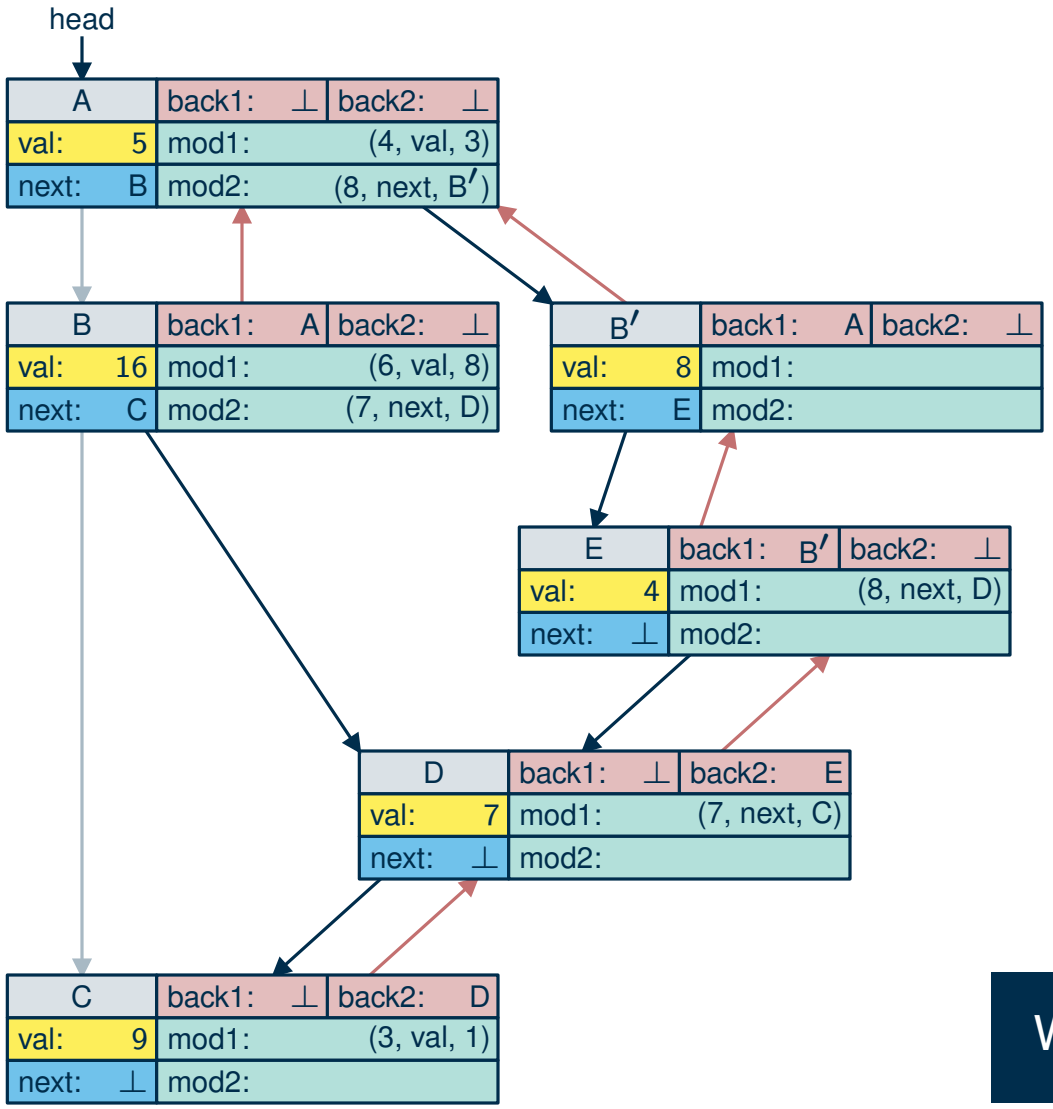

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What would have happened, if A.mod2 was already occupied?

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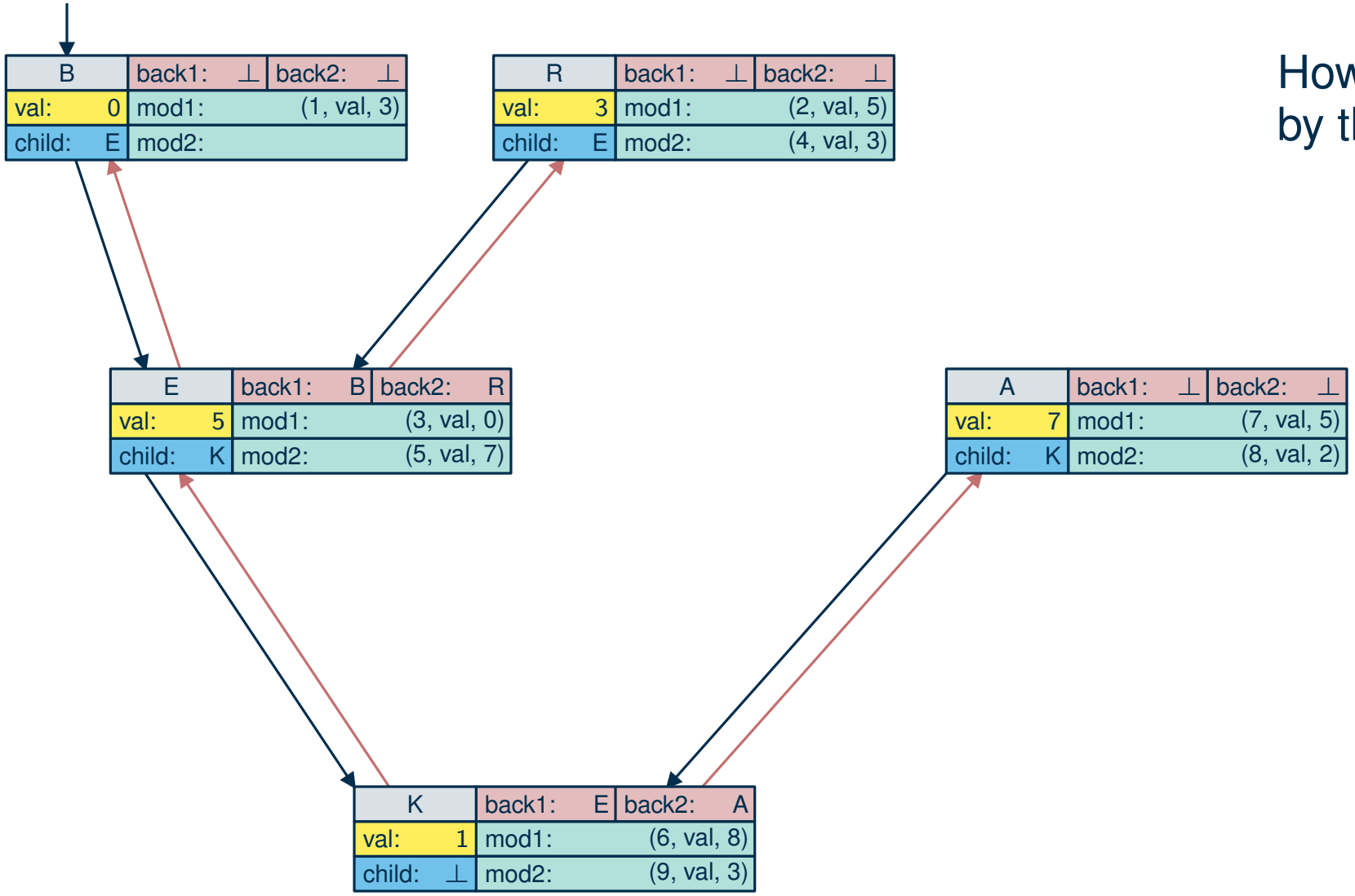

How many new nodes are created by the operation K.val $= 2$?

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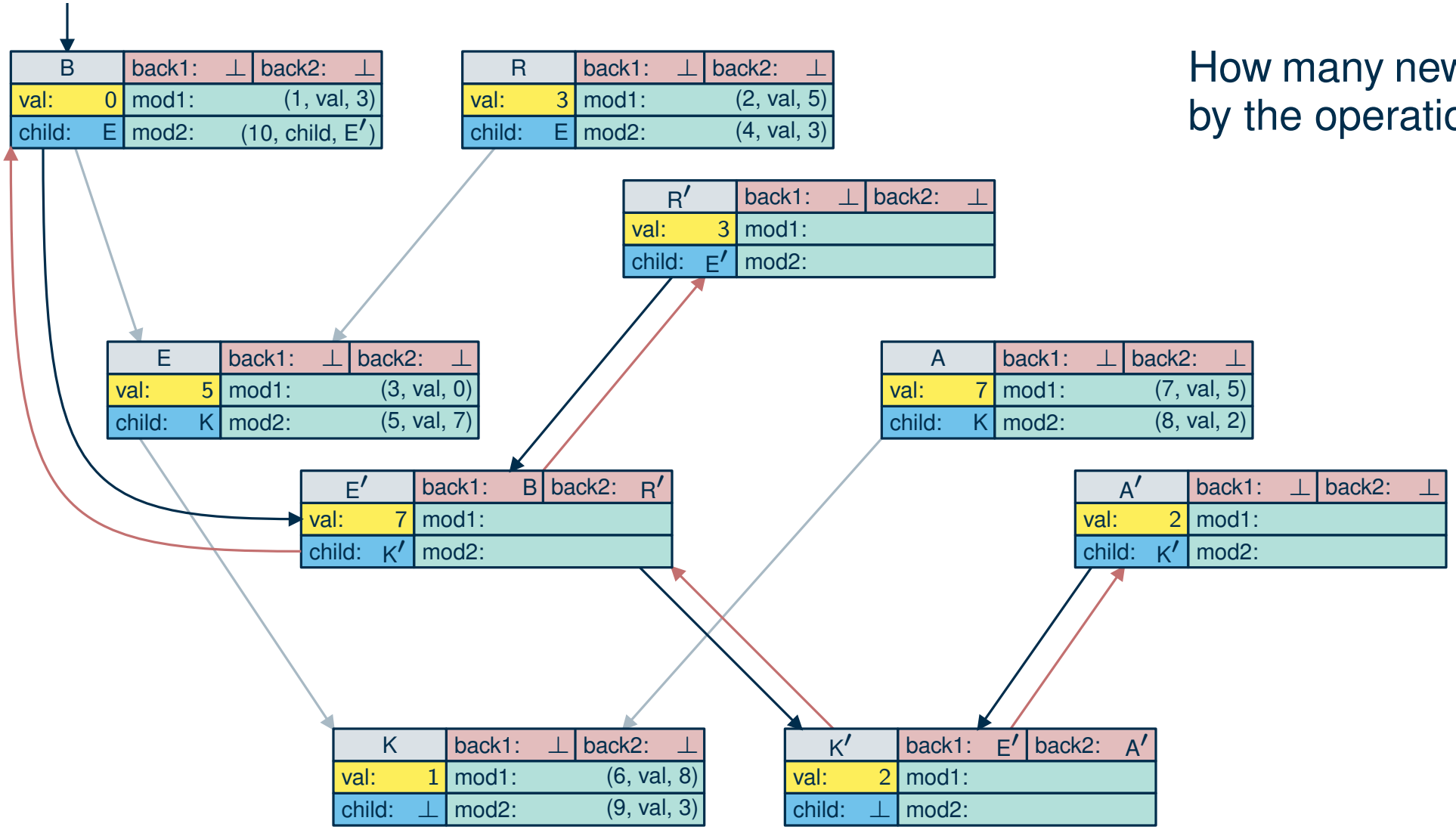

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(Almost) Everything Is Constant

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- case 2: copy node of size $O(1)$, apply $O(1)$ modifications, update $O(1)$ back pointers

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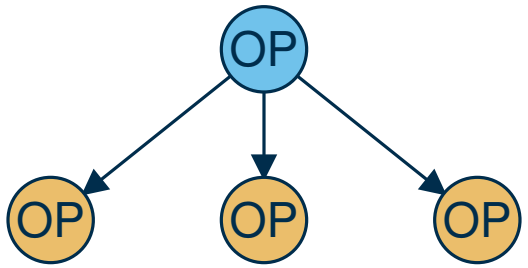

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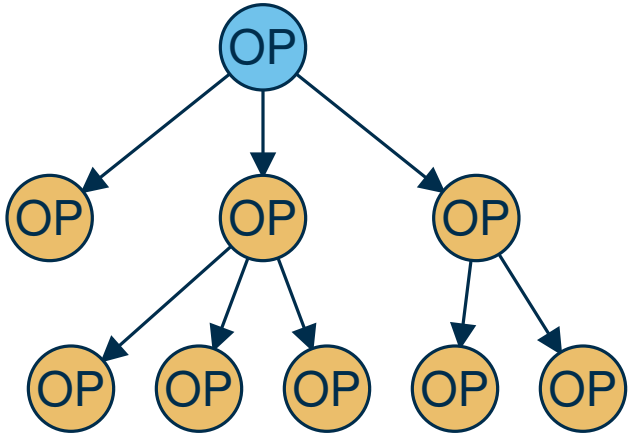

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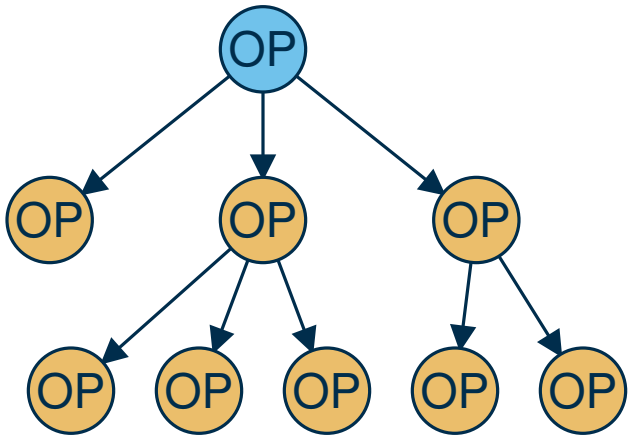

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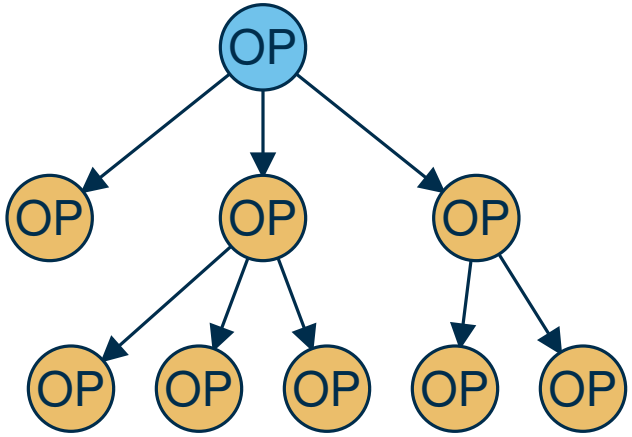

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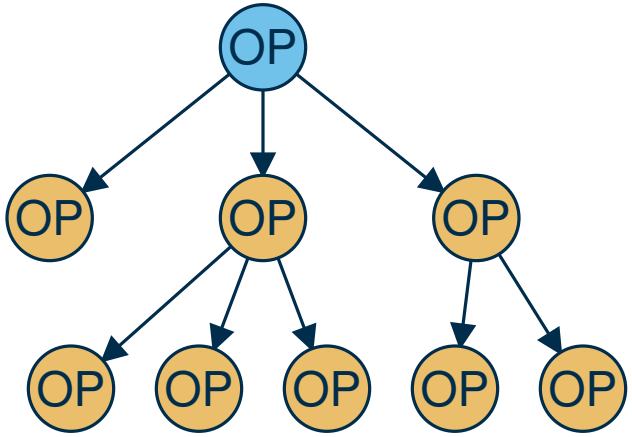

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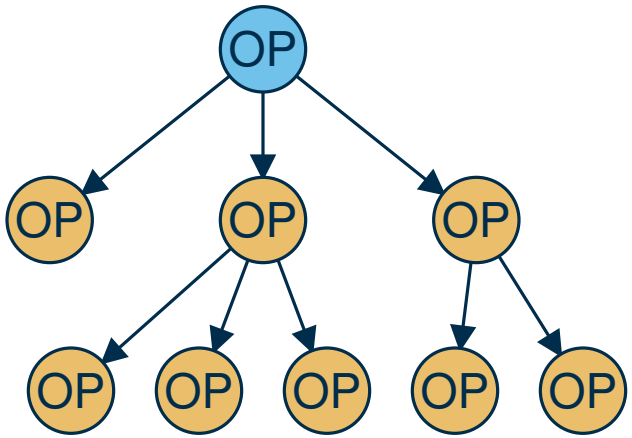

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- amortized analysis: let case-1 operations pay for case-2 operations

Amortized Analysis

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- case-1 operations pay into account
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Amortized Analysis

Some Variables
n_ℓ: #leaves in the tree
n_i: #inner nodes
k: #mod fields
d: in-degree in the DS

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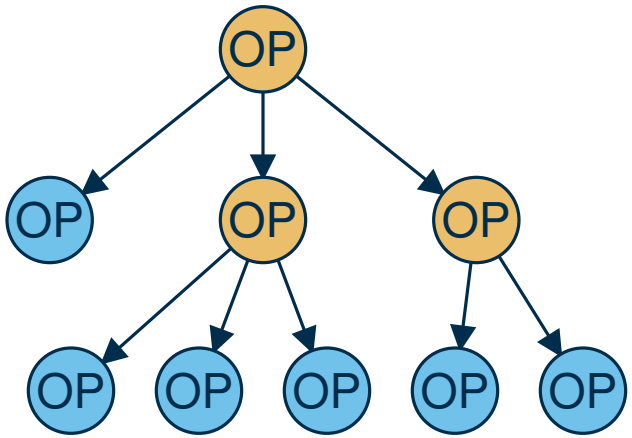

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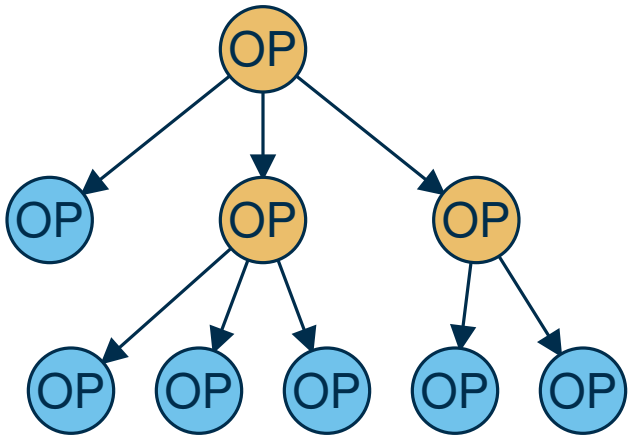

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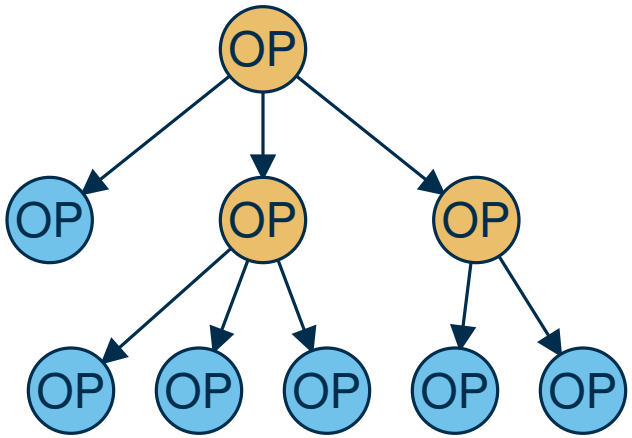

Amortized Analysis

Some Variables
n_ℓ: #leaves in the tree
n_i: #inner nodes
k: #mod fields
d: in-degree in the DS

Accounting/Potential Method

- case-1 operations pay into account $\rightarrow$ deposit 1
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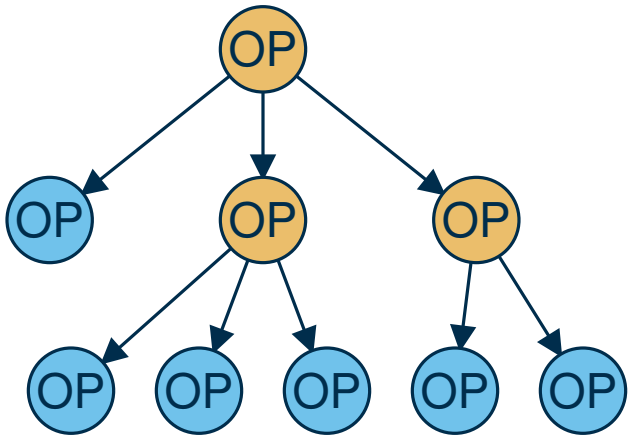

How Much Can A Case-2 Operation Withdraw?

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Bounding The Amortized Cost Of A Case-2 Operation

$$\underbrace{n_\ell + n_i}_{\text{actual cost}} + \underbrace{n_\ell - kn_i}_{\text{account balance change}}$$

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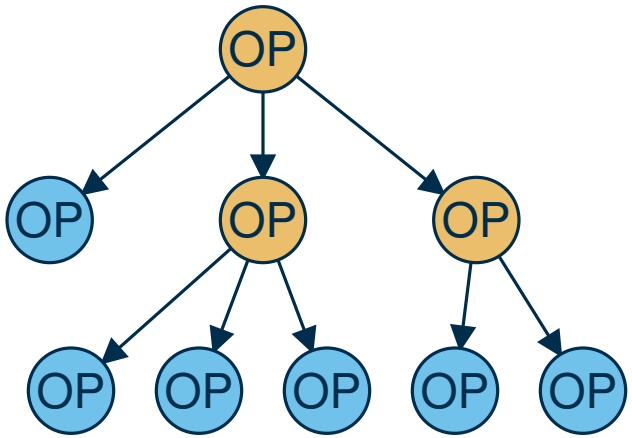

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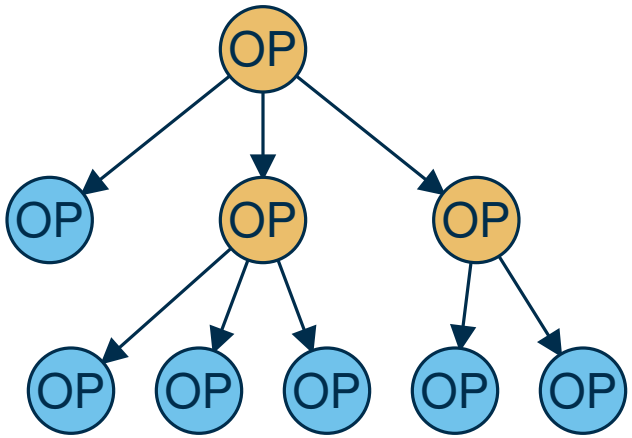

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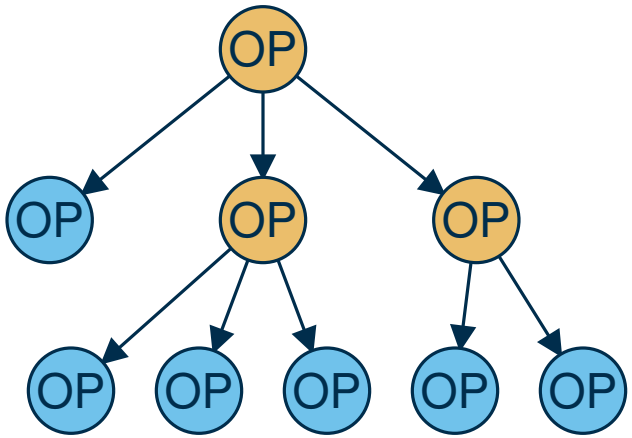

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as $n_\ell \leq dn_i$

using $k = 2d + 1$ mod fields

Wrap-Up: Persistence

Cost Of An Atomic Operation

- amortized constant time overhead
- amortized constant additional memory

Cost Of An Atomic Query

- constant time overhead

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Theorem **(partial persistence for everyone)**
Every data structure in the pointer machine model with constant in-degree can be made partially persistent, such that every atomic operation has amortized constant time and space overhead. Every query has constant overhead.

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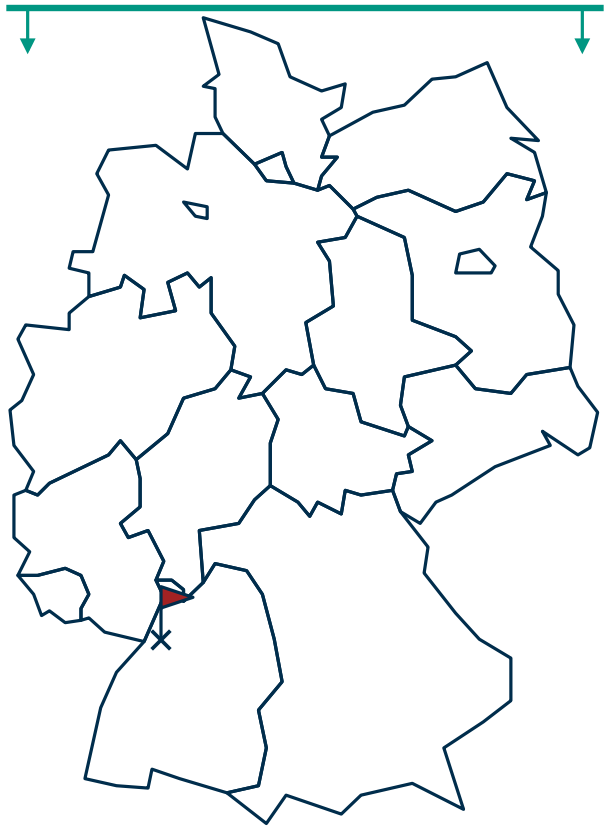
Where does our approach fail for full persistence?

Where am I?

Problem: Point Location
In which face of a geometric graph does a given point p lie?



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- graph G is fixed
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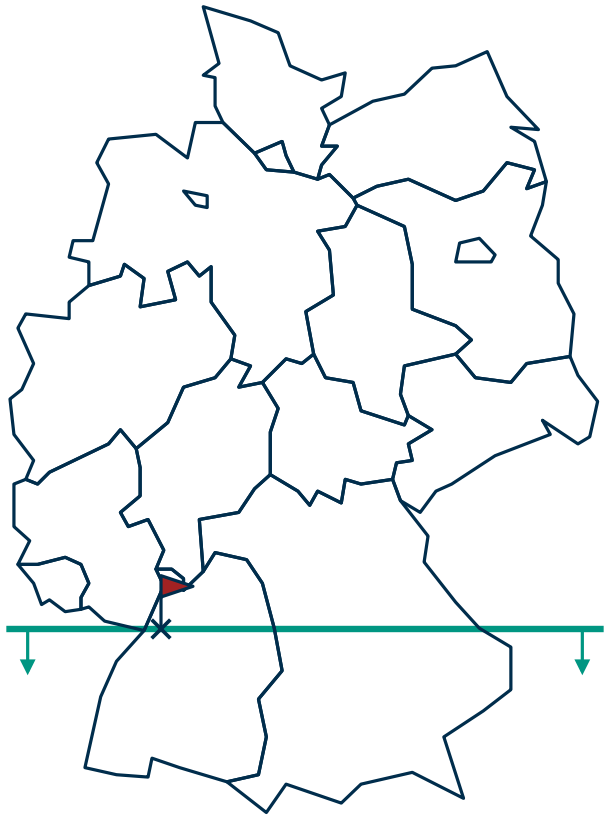

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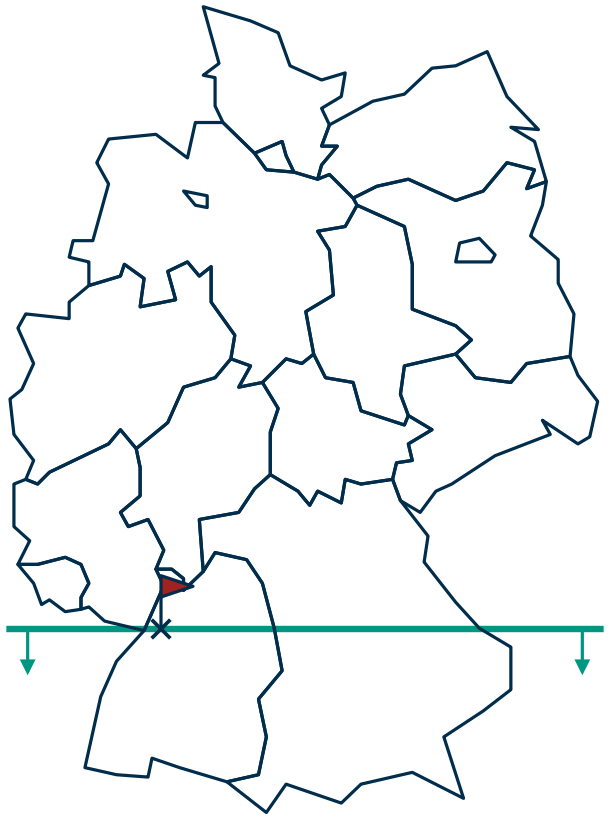

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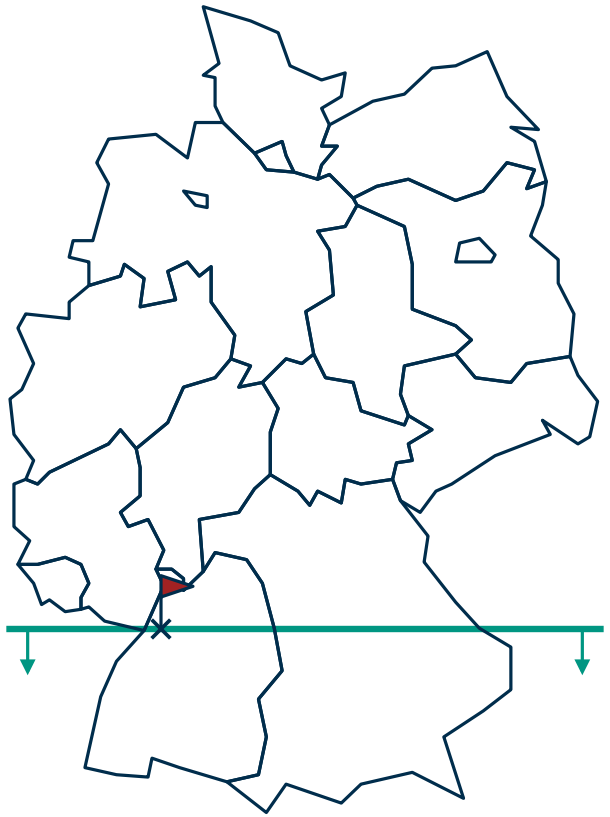

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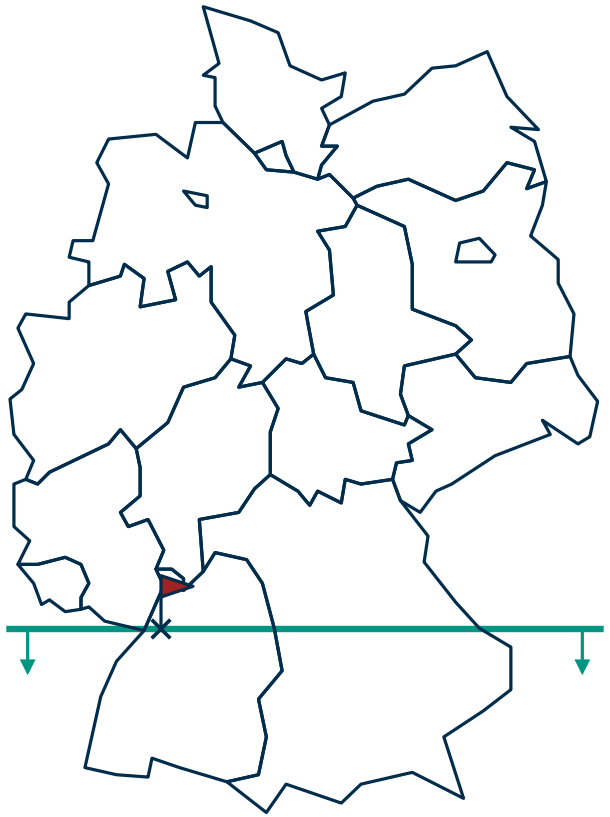

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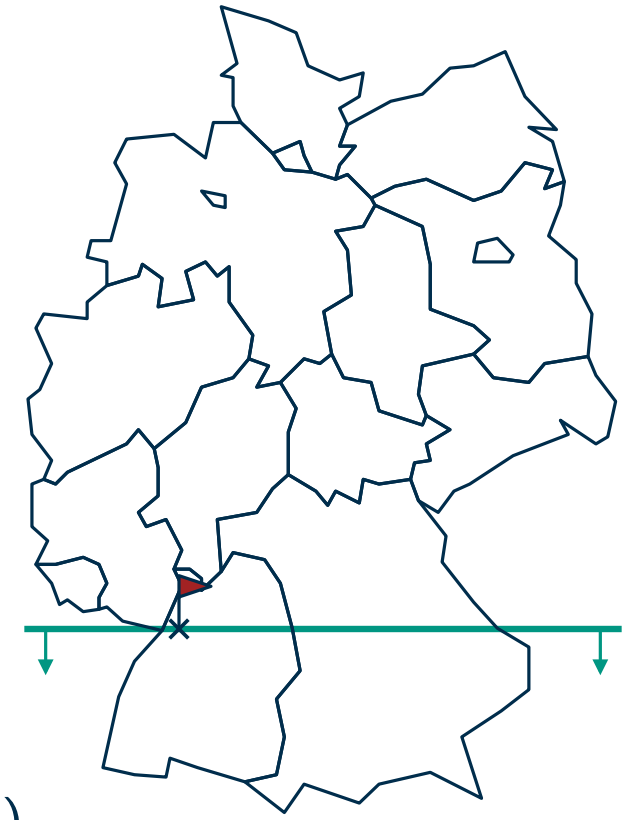

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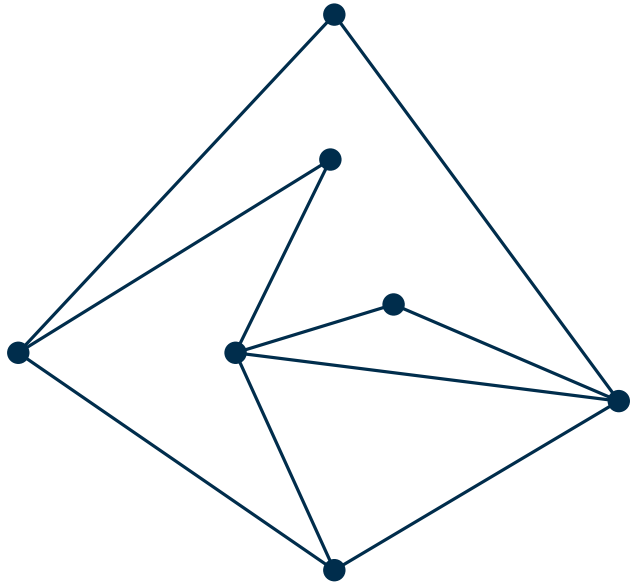
Query: $O(\log n)$
Precomputation: $O(n \log n)$
Memory: $O(n)$

Is There A Solution Withouth Time Travel?



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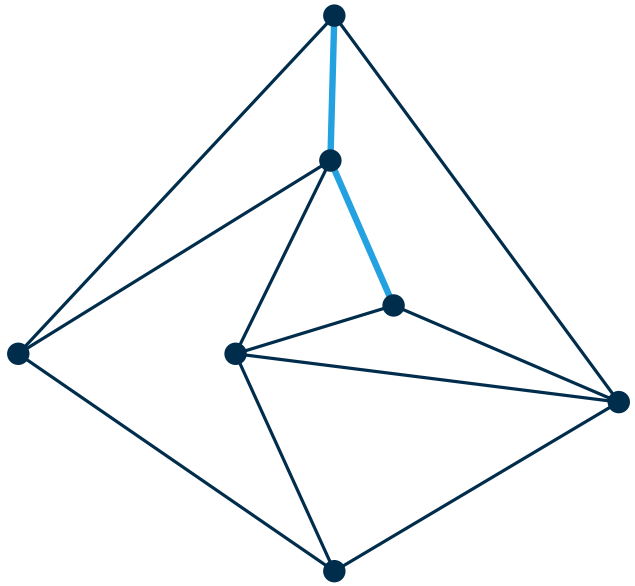
Bringing Order Into Chaos

- make faces y-monotone



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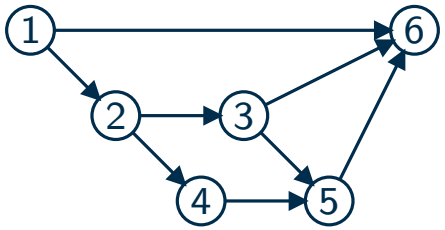
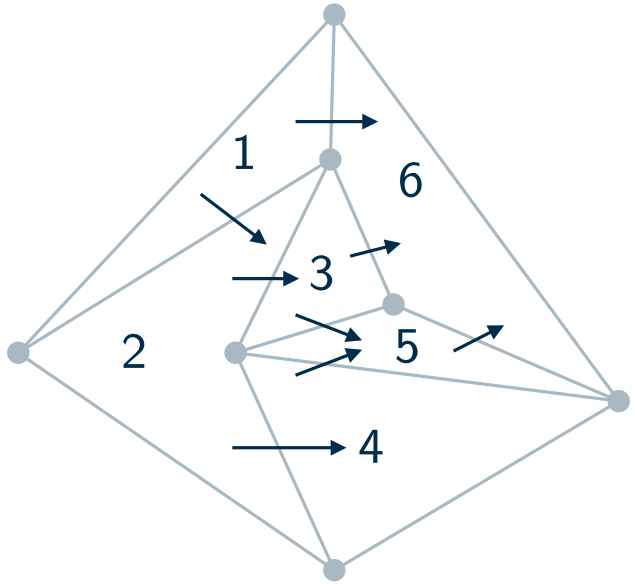
- make faces y-monotone
- order faces from left to right





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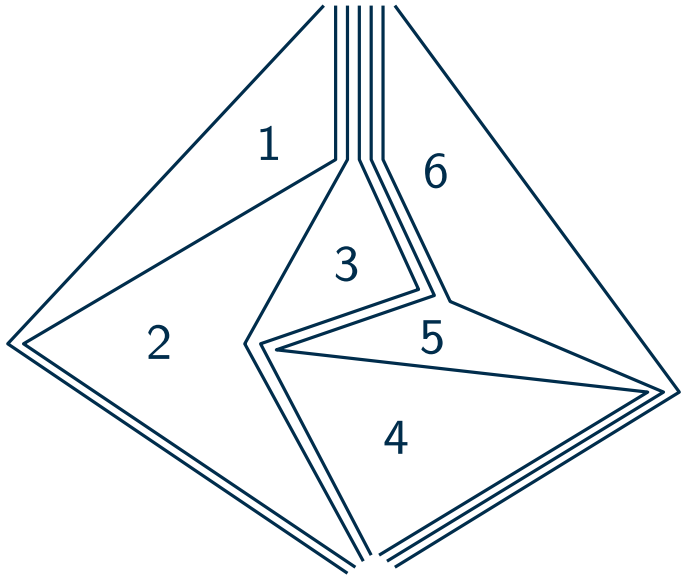

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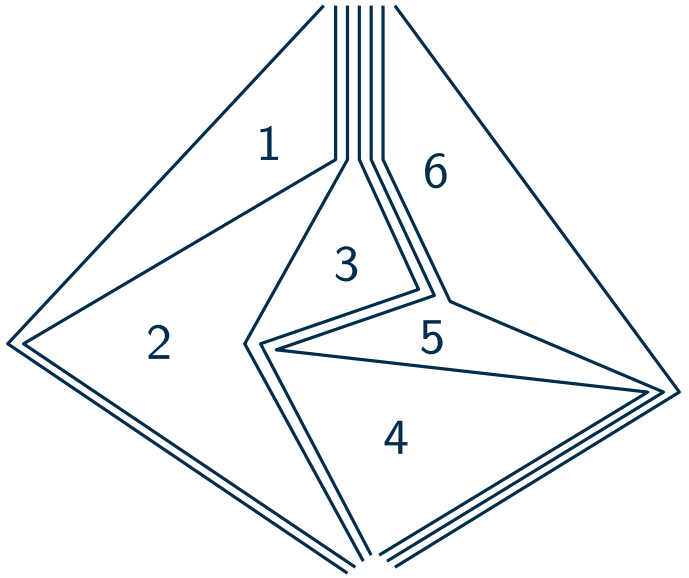

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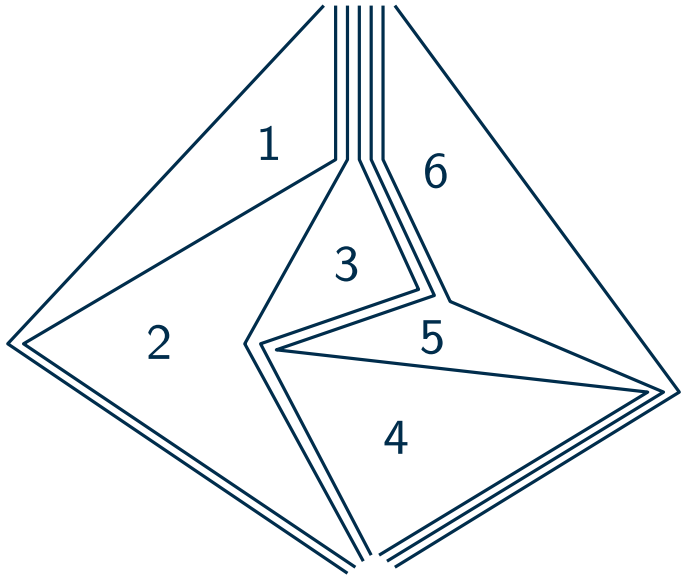

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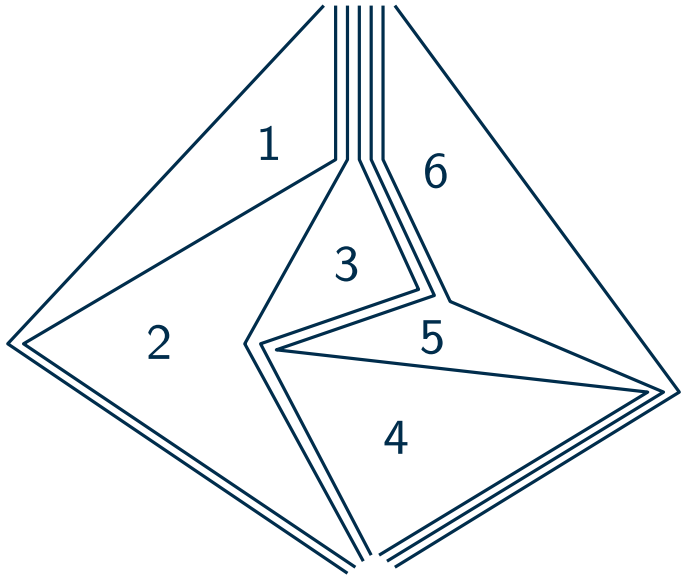

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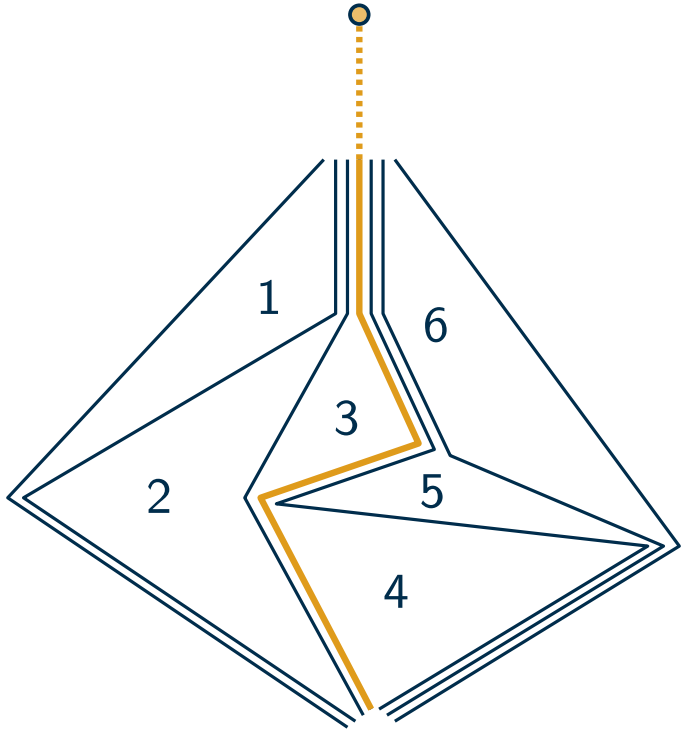

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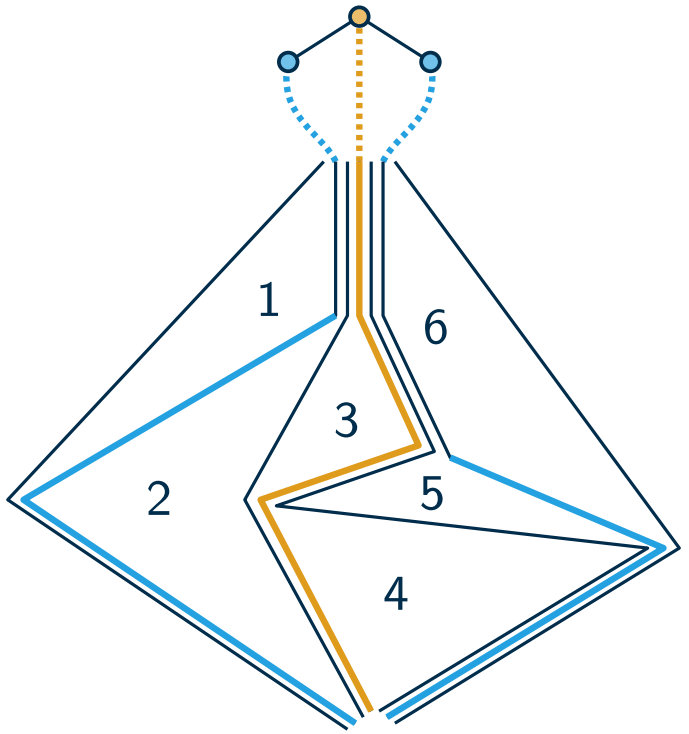

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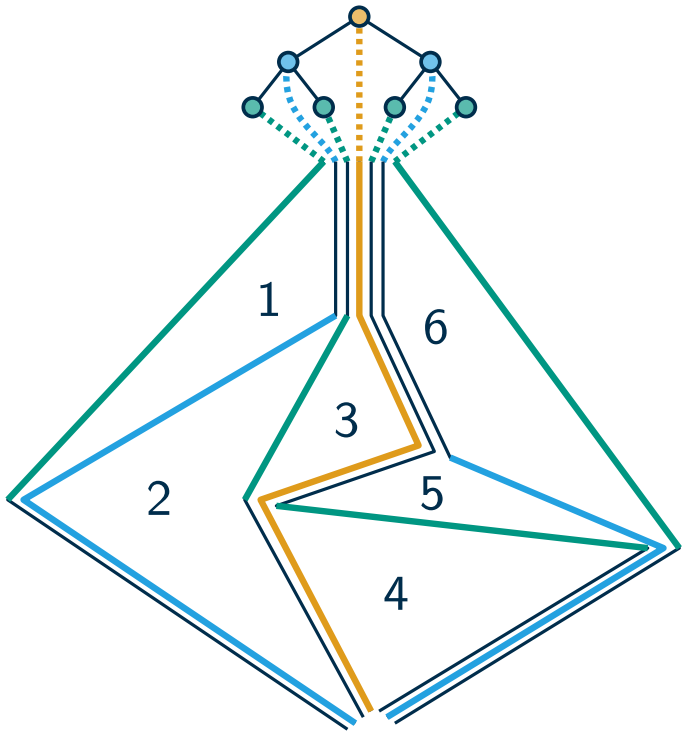

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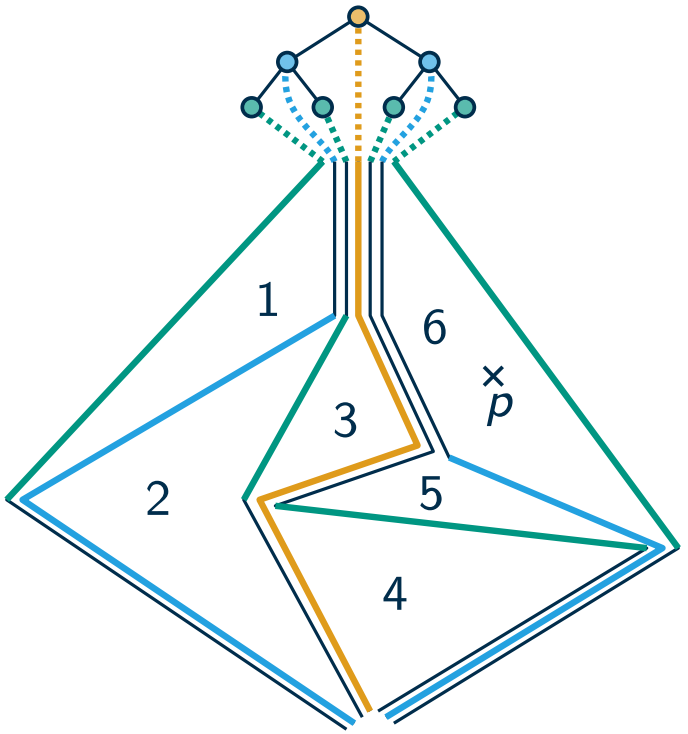
- decision at each tree node

How can we decide whether to walk left or right down the tree?



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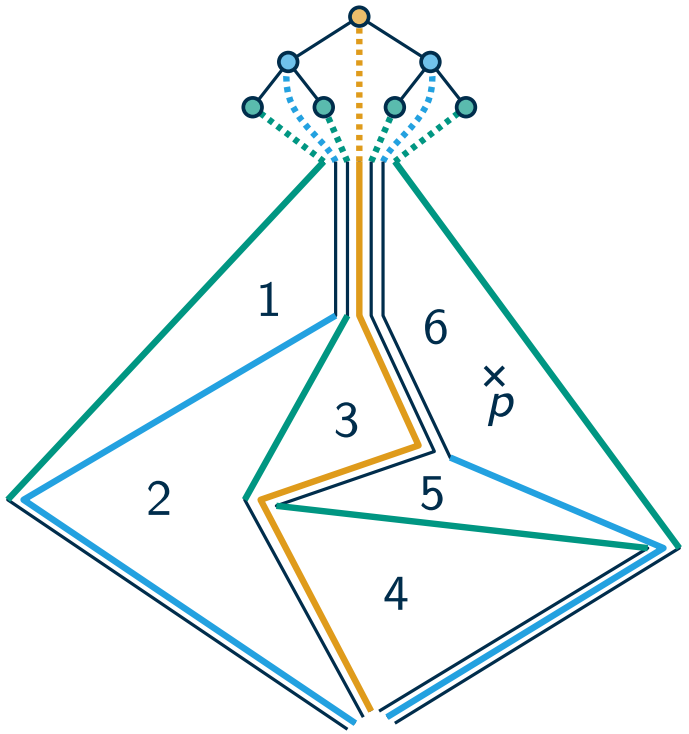

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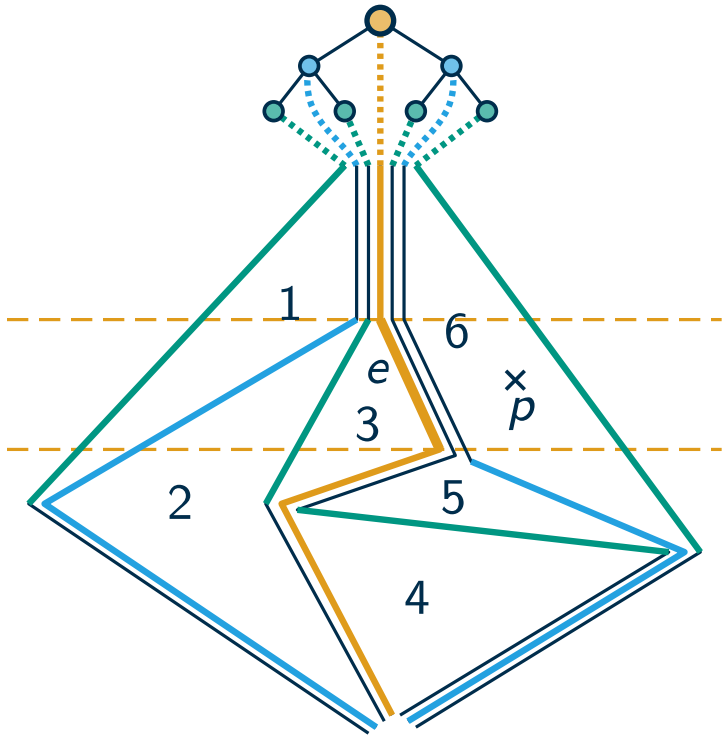

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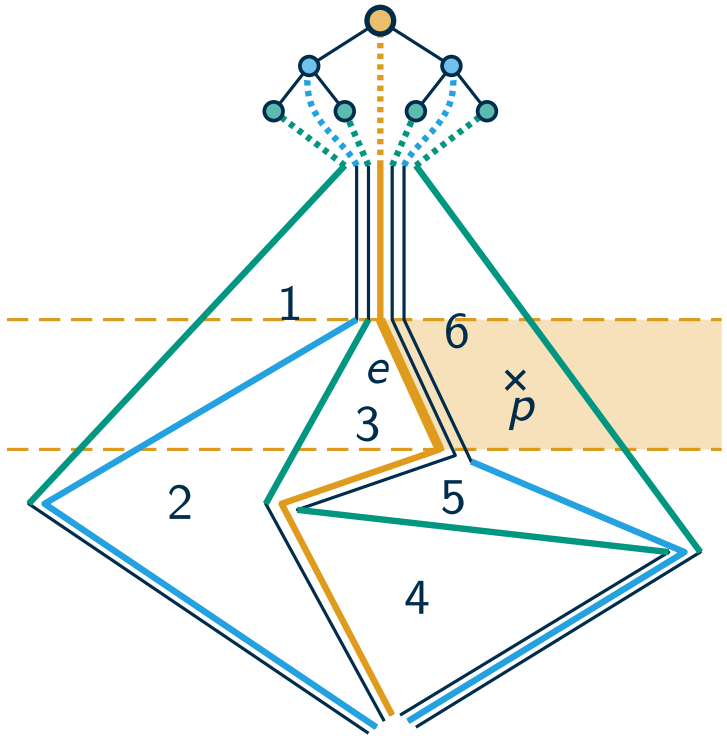

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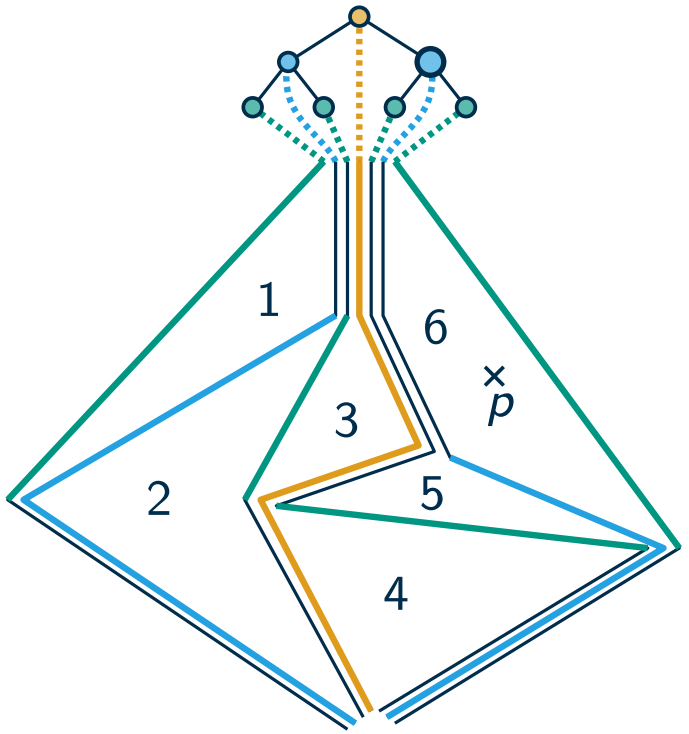

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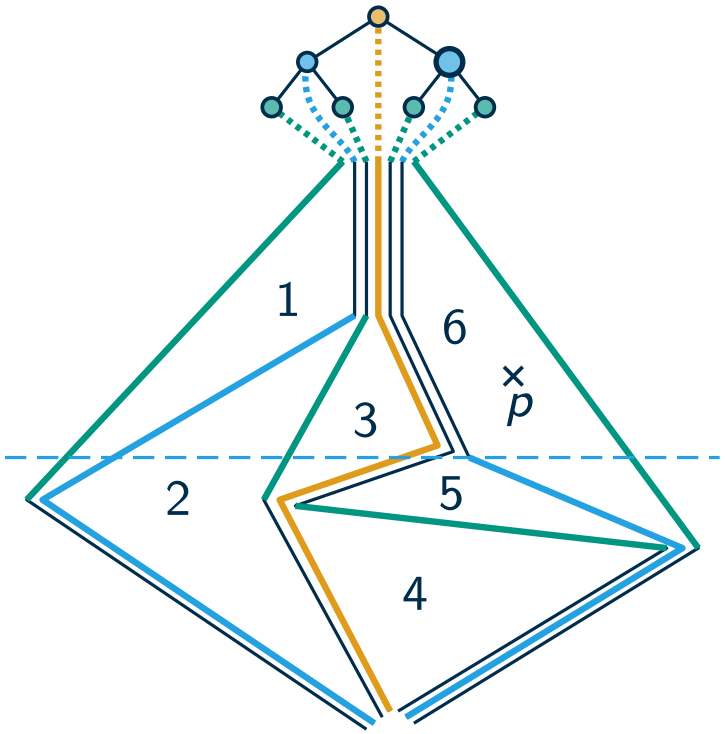

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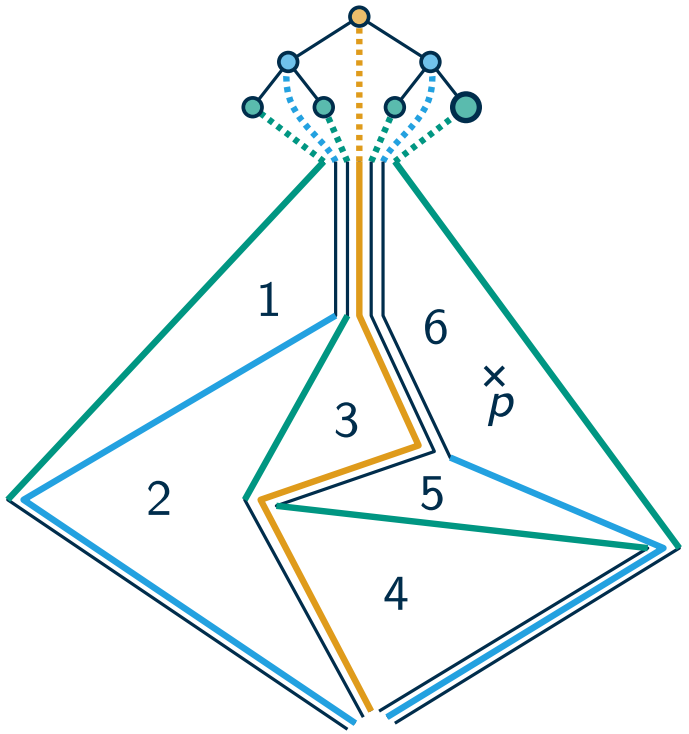

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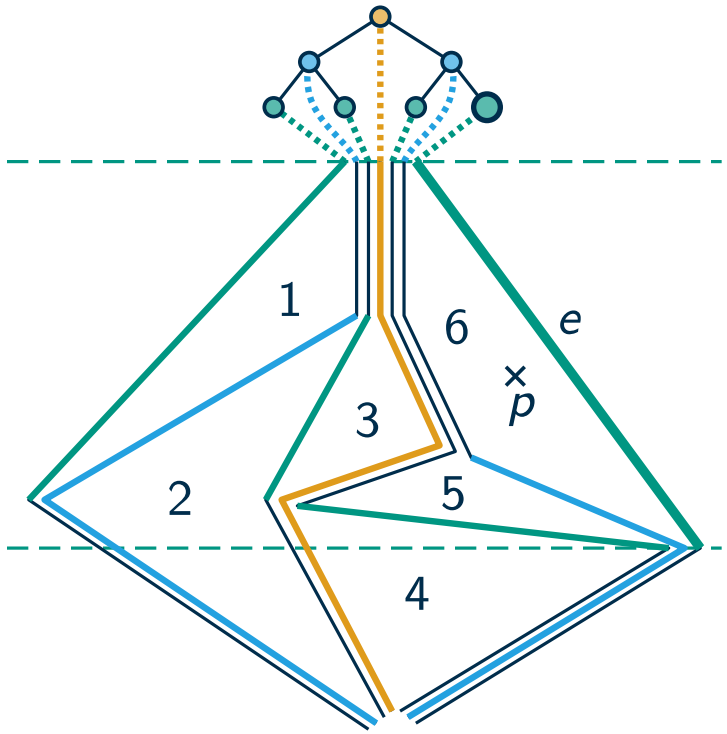

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Bringing Order Into Chaos

- make faces y-monotone
- order faces from left to right
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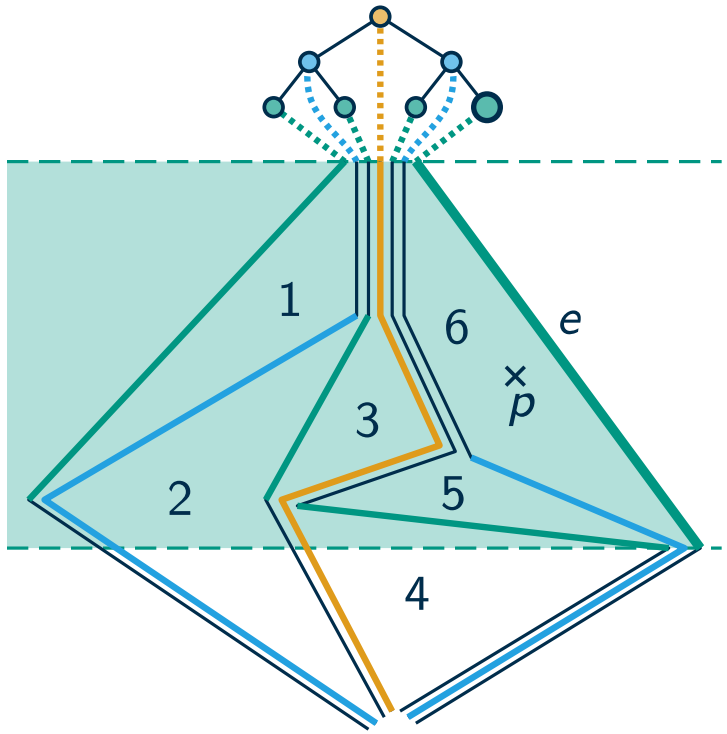

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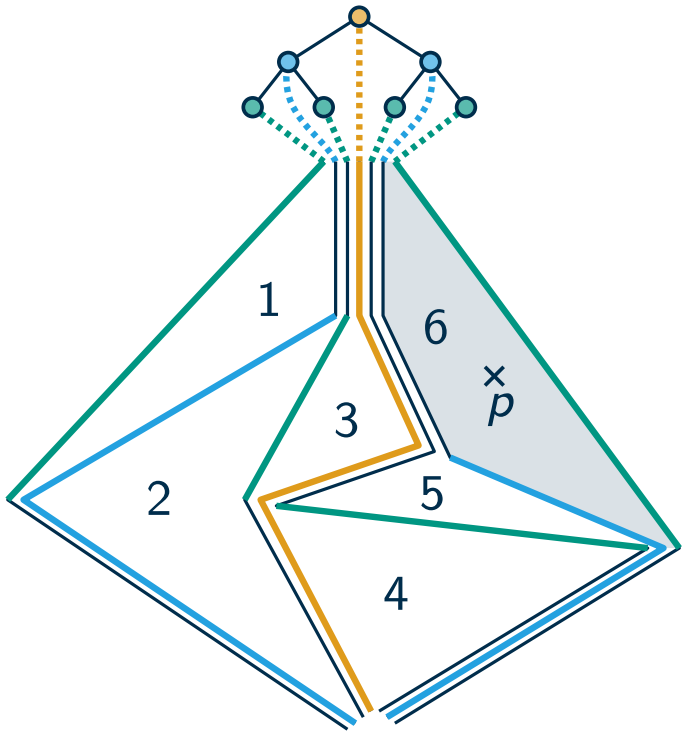

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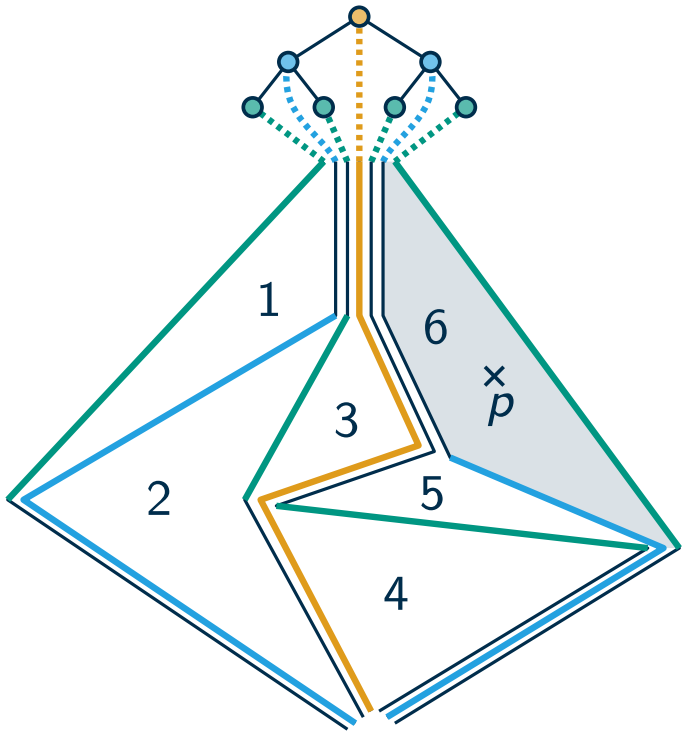

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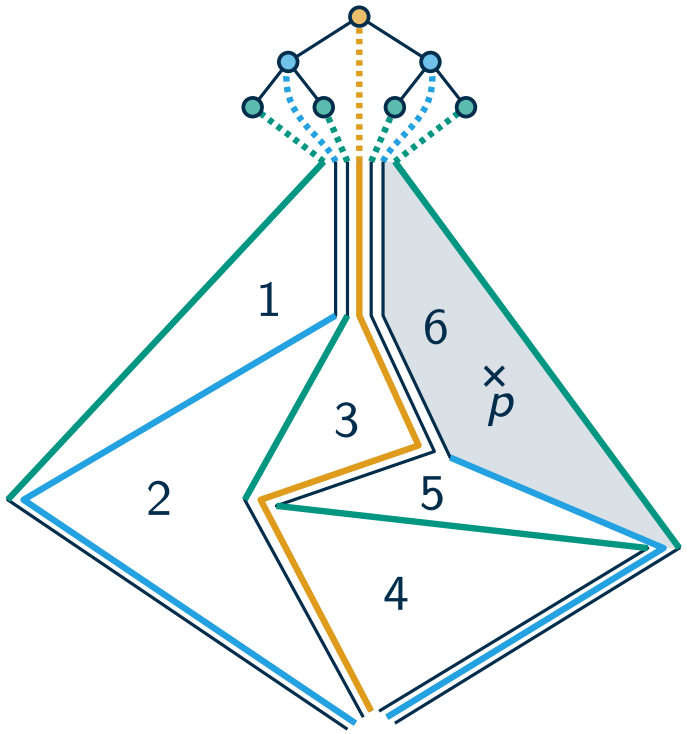

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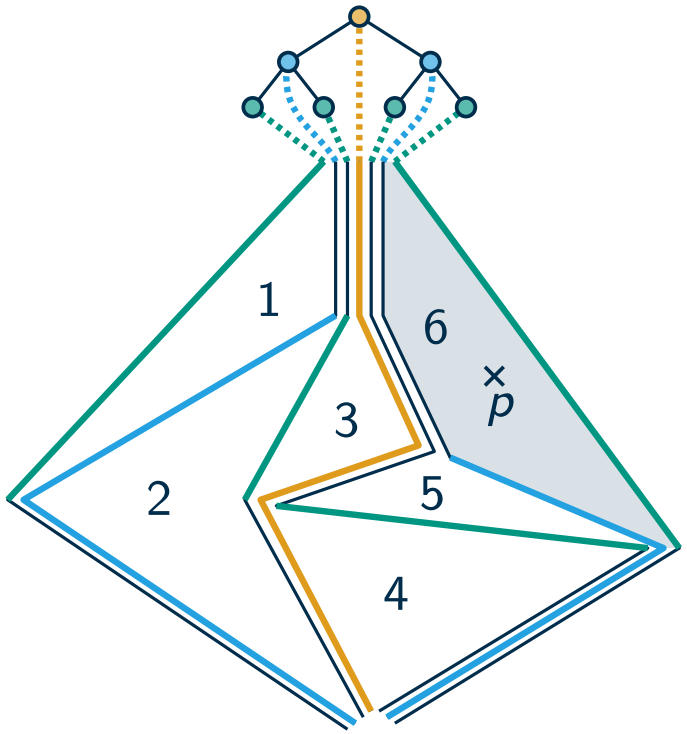
- **precomputation** $O(n \log n)$ and **memory** $O(n)$



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- dynamic variants
- retroactive data structures: allow operations in the past affecting the present state