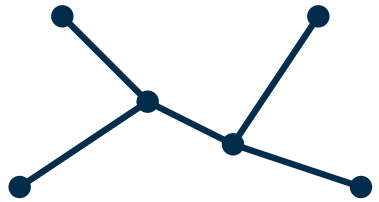
Computational Geometry

EMST & Clustering

Thomas Bläsius

Euclidean MST

Problem: EMST
Let $P \subset \mathbb{R}^2$ be a set of points. Compute a tree $T = (P, E)$ with vertices P such that its total edge length $\sum_{pq \in E} \|p - q\|$ is minimized.



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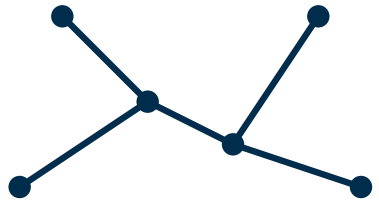

How fast can you compute the EMST?
(assuming a real-RAM)

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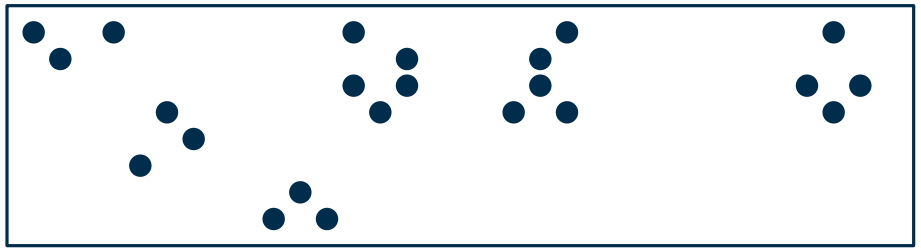
(assuming integer coordinates and a word-RAM)

Is EMST $\in$ P?

Clustering

Goal

- given a set of points, find a reasonable clustering



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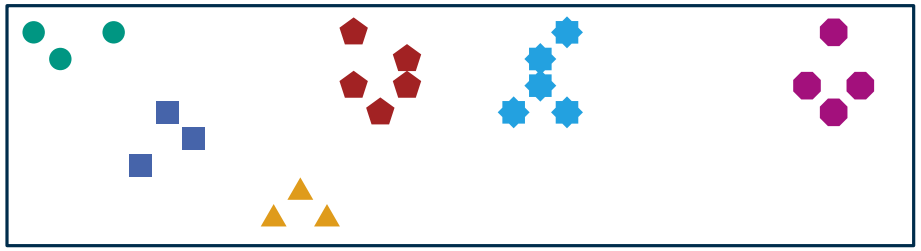

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- requirements for the algorithm:
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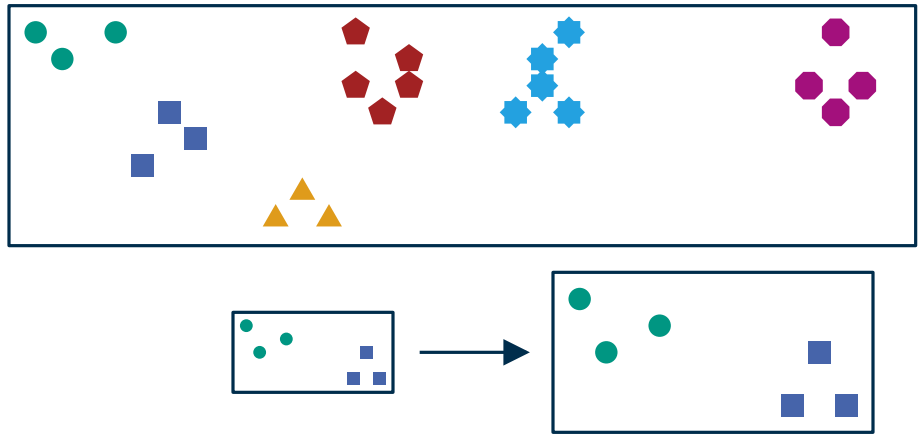

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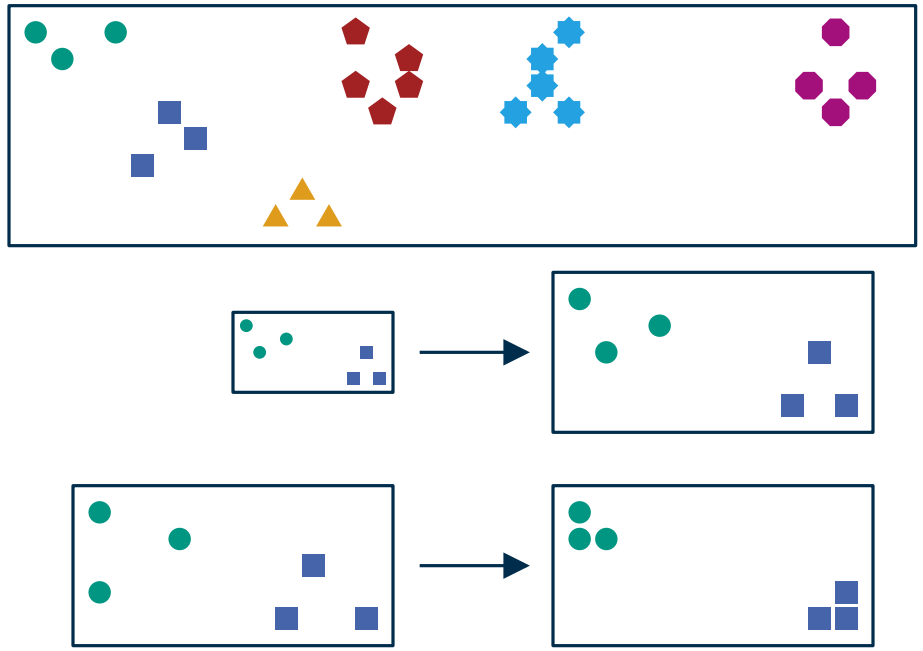

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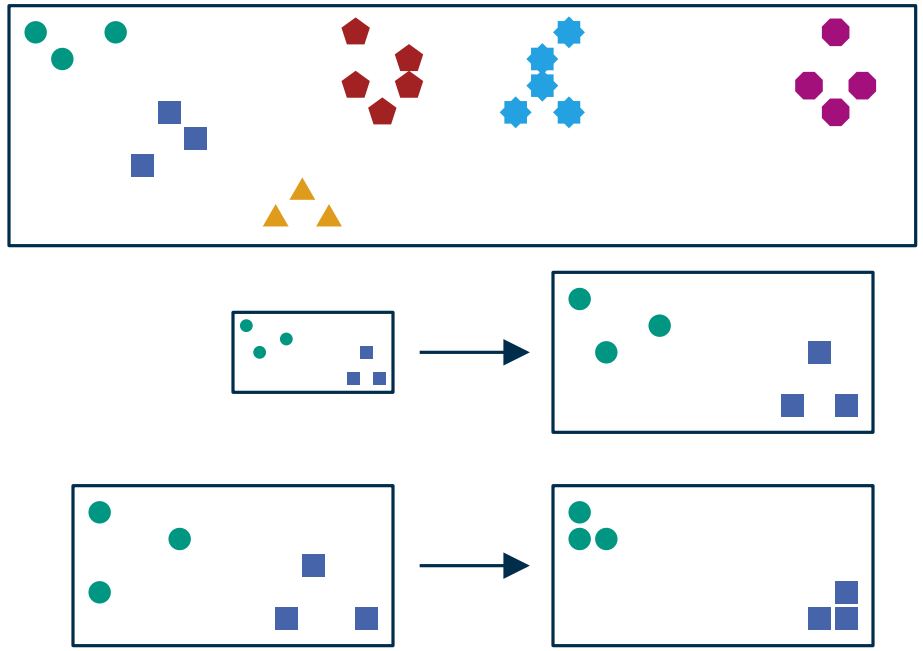

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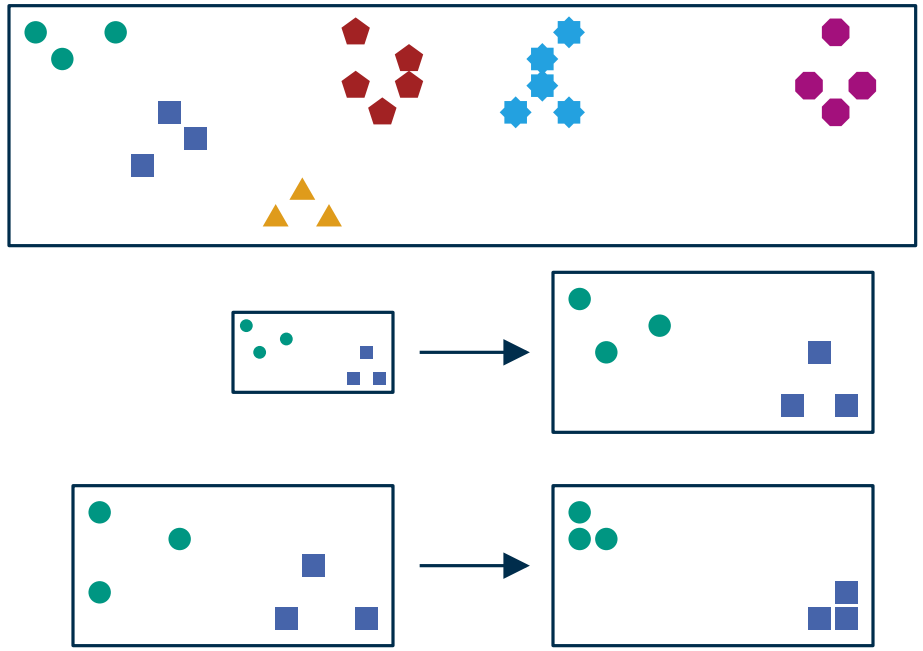

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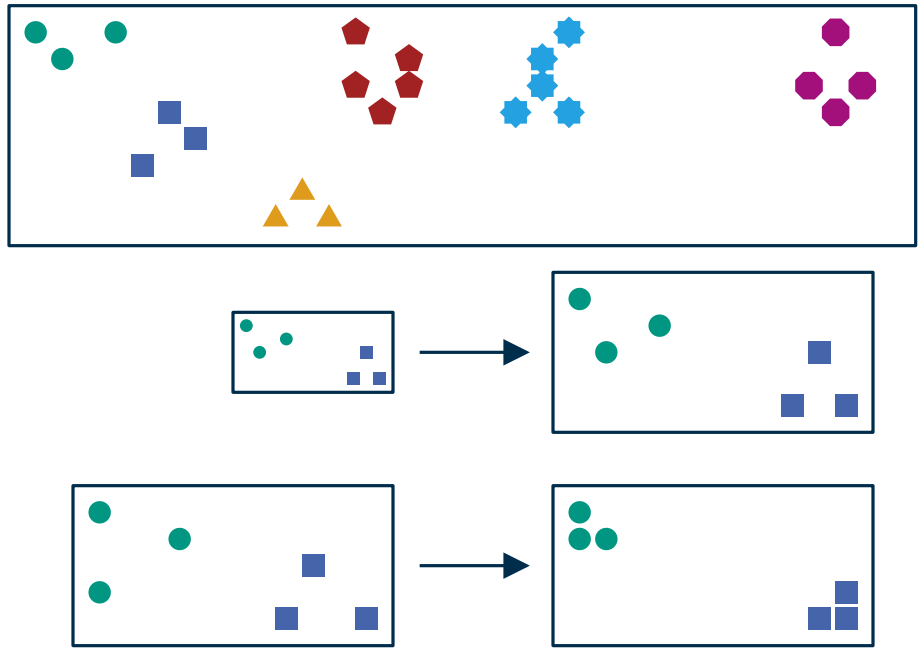

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Which of the three requirements would you drop or relax?