Computational Geometry

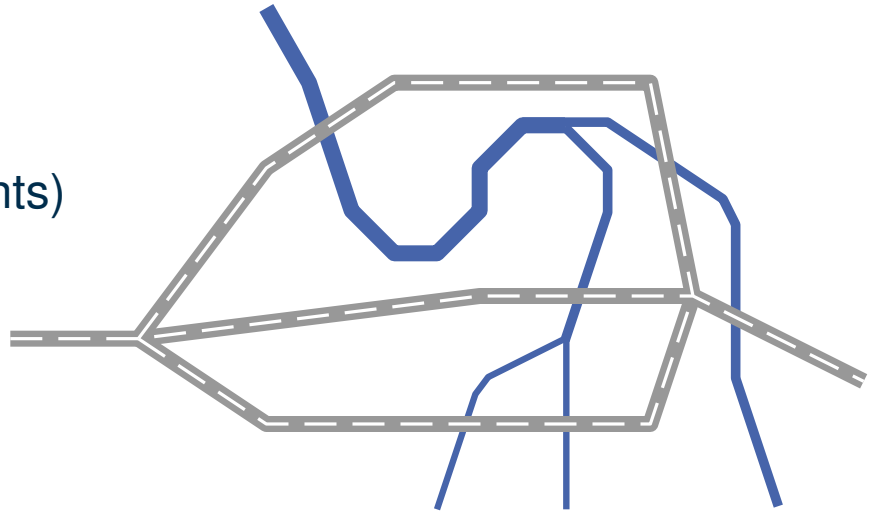
Line Segment Intersection

Thomas Bläsius

Line Segment Intersection: Motivation

Where Are Bridges?

- **given:** roads and rivers
  (each as sets of line segments)
- **goal:** find all bridges



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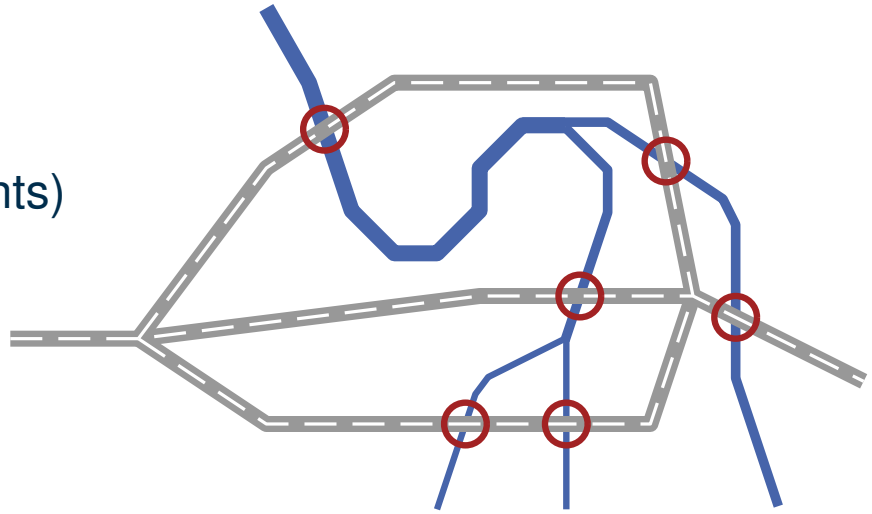

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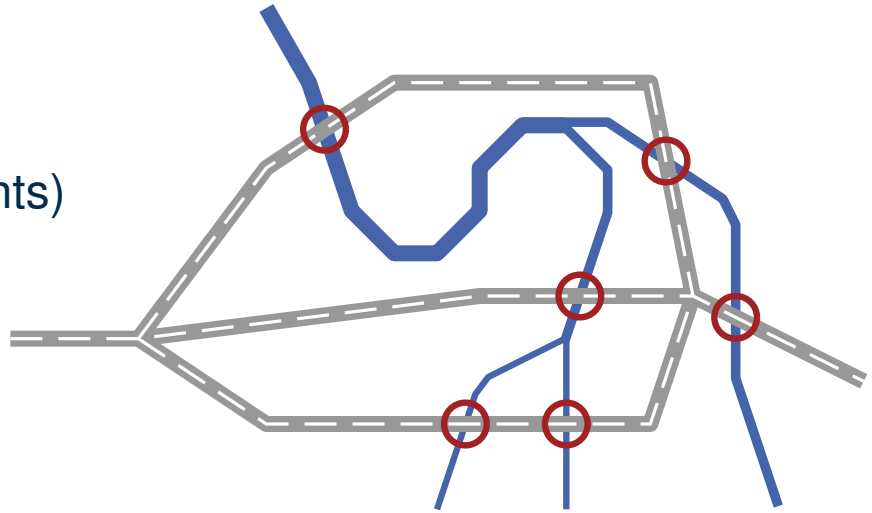
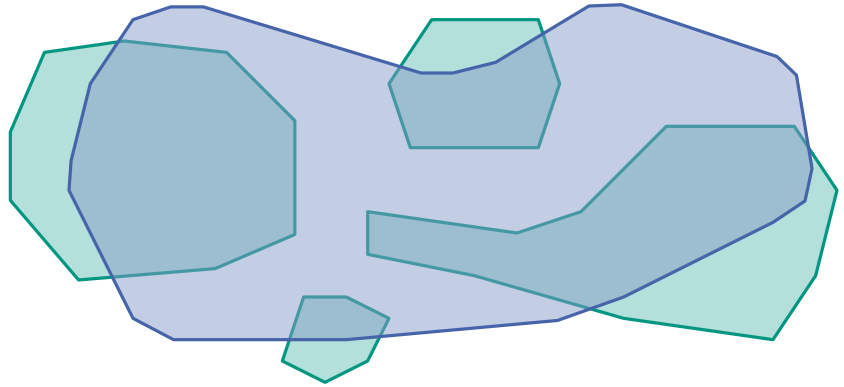


Forests With A Lot Of Rainfall

- **given:** forests and regions with more than 1500mm rainfall
  (each as polygons)
- **goal:** compute the intersection of both



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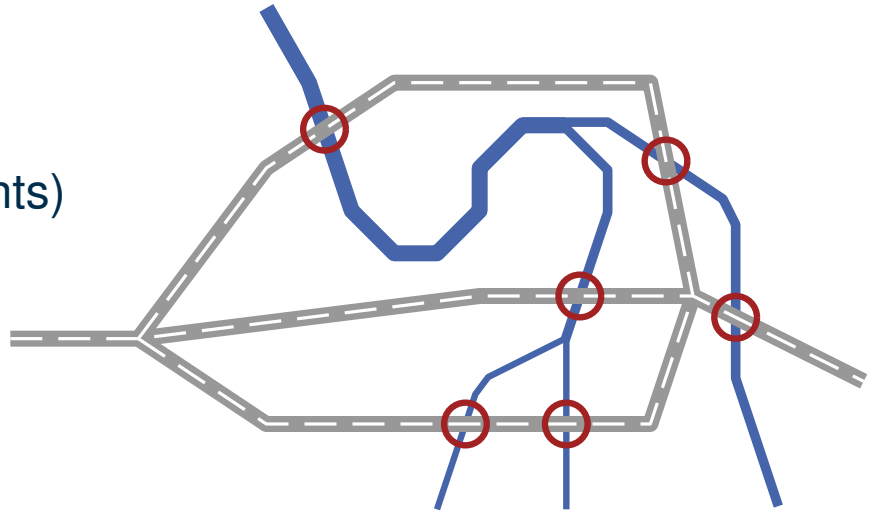

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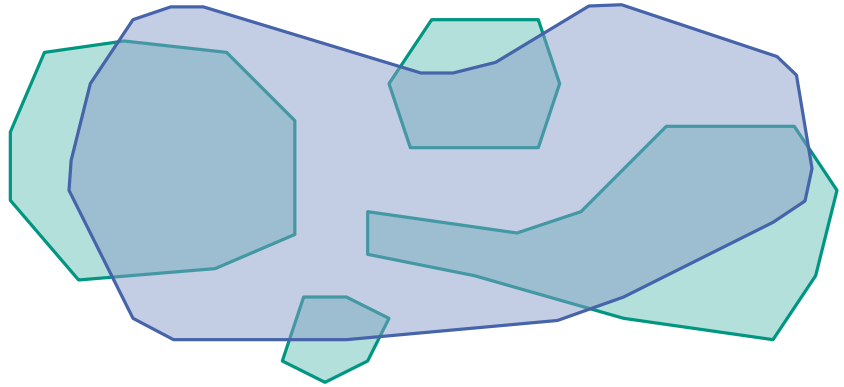

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Given n line segments, compute all pairwise intersections.

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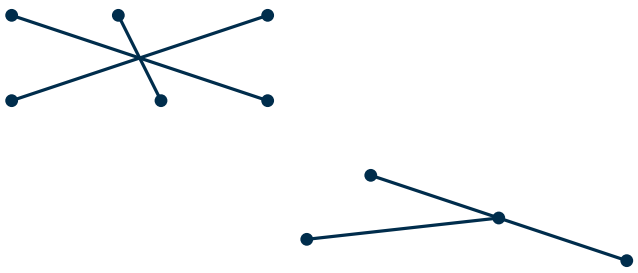

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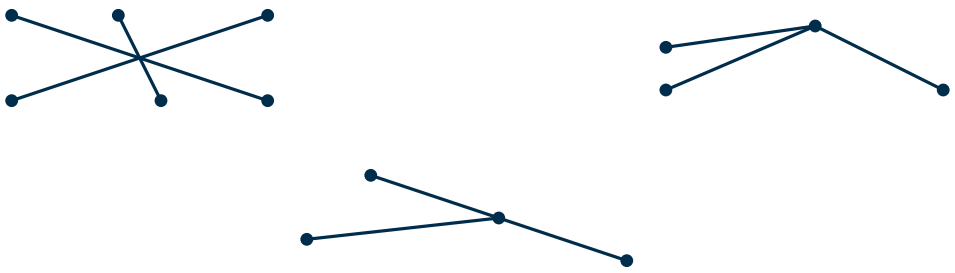

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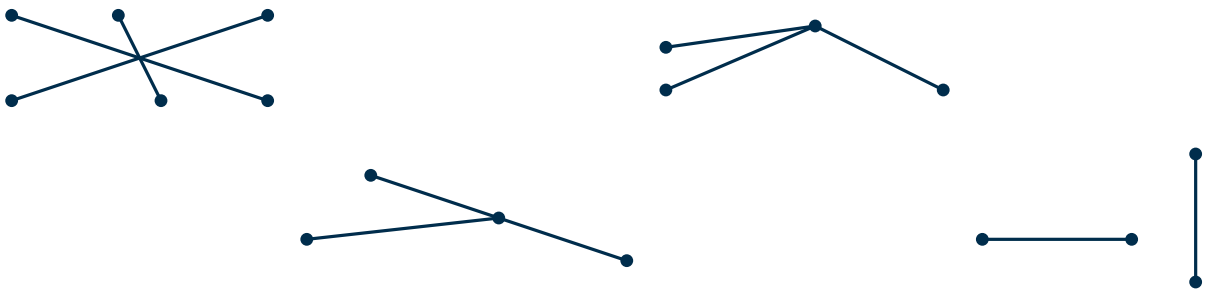

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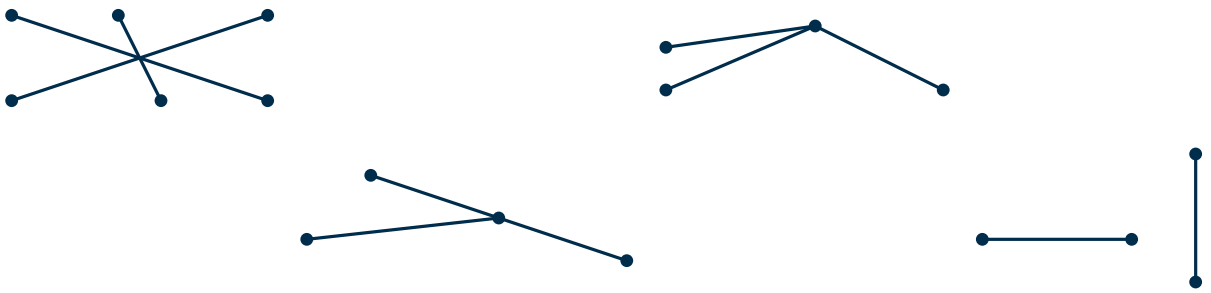

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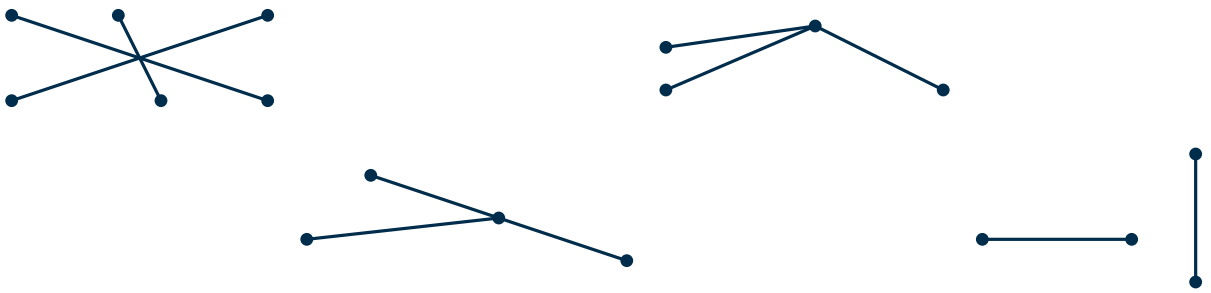

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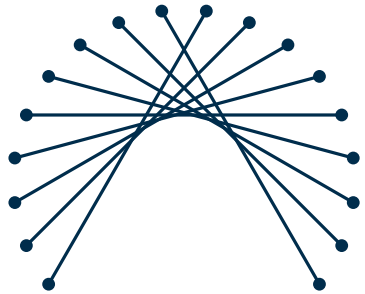

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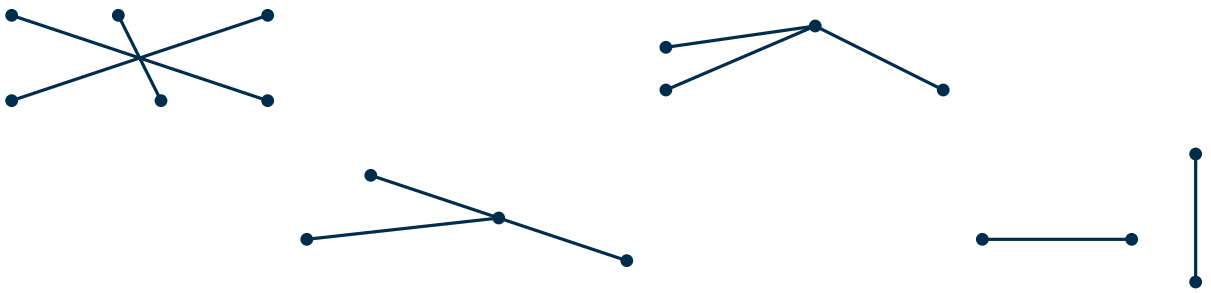

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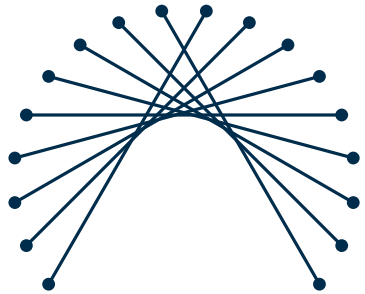

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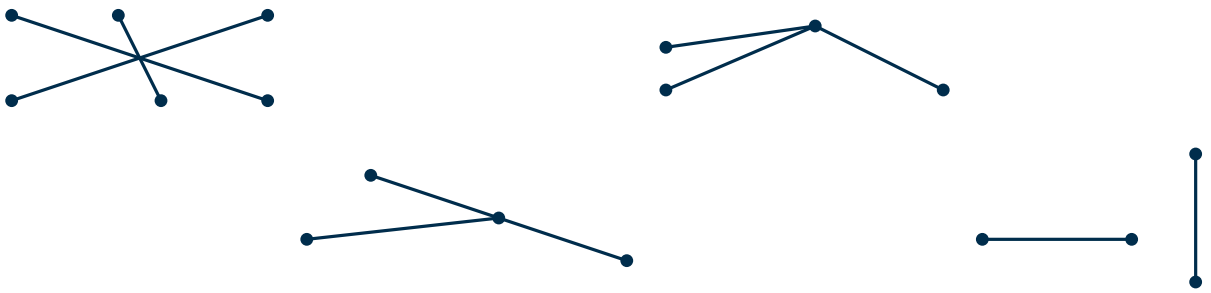

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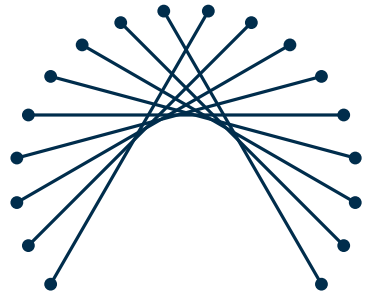

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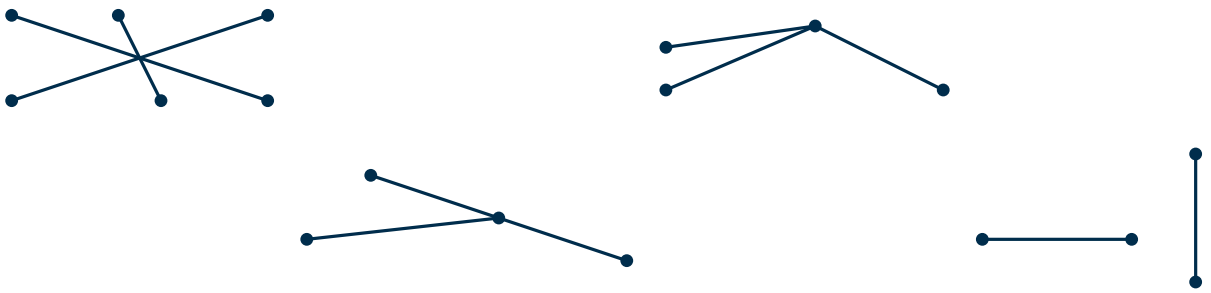

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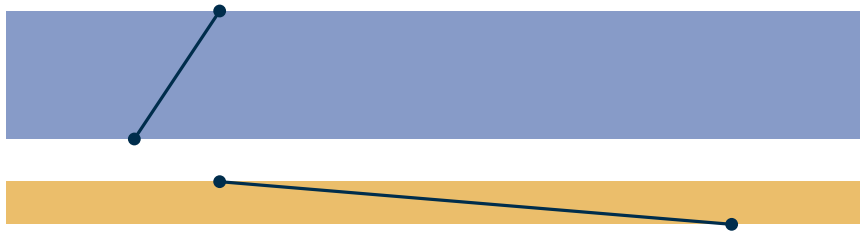
Potential Improvement

- few intersections $\Rightarrow$ better running time
- running time depends on output size
  $\rightarrow$ output sensitive algorithm

A Simple Sweep-Line Algorithm

Idea

- don’t compare segments if one lies fully above the other



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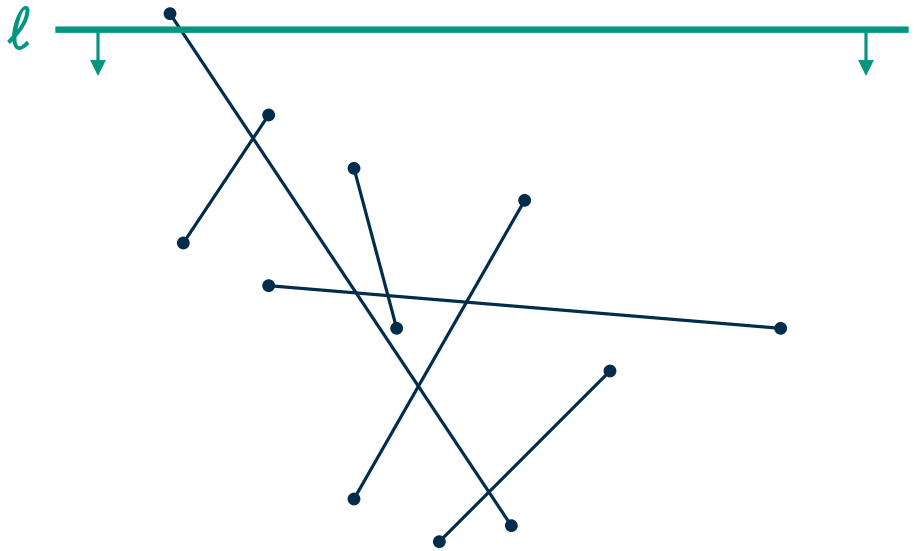

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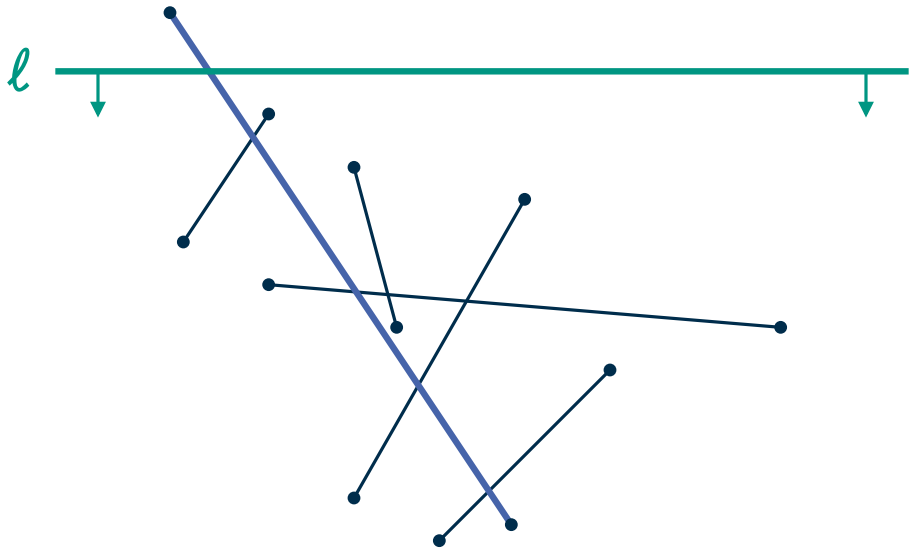

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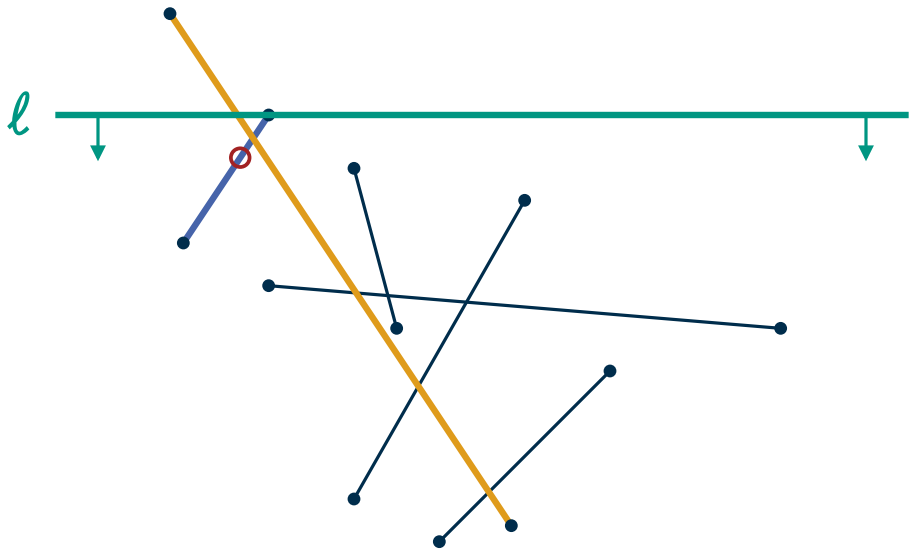

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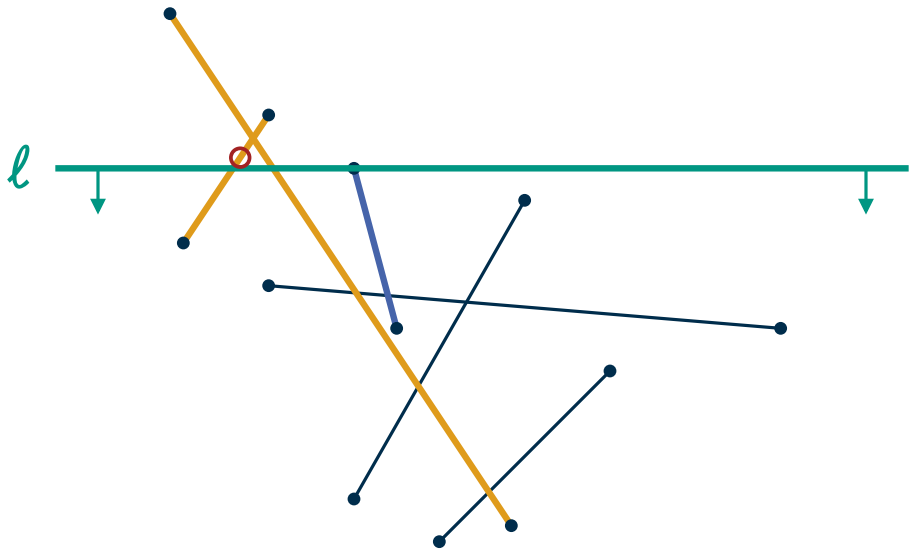

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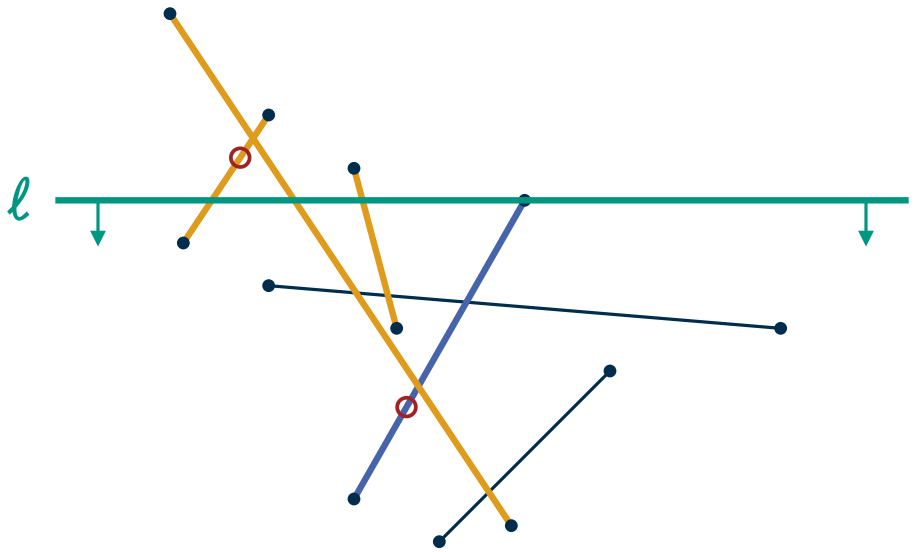

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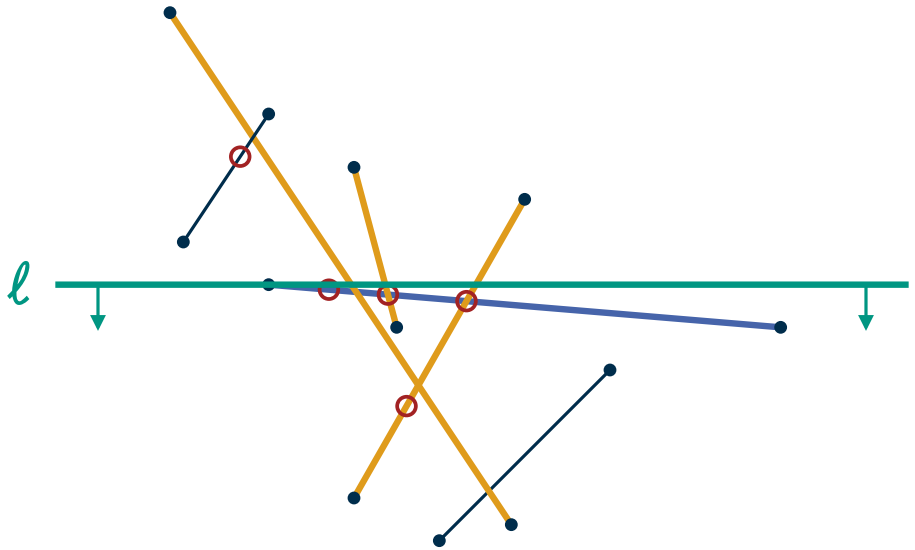

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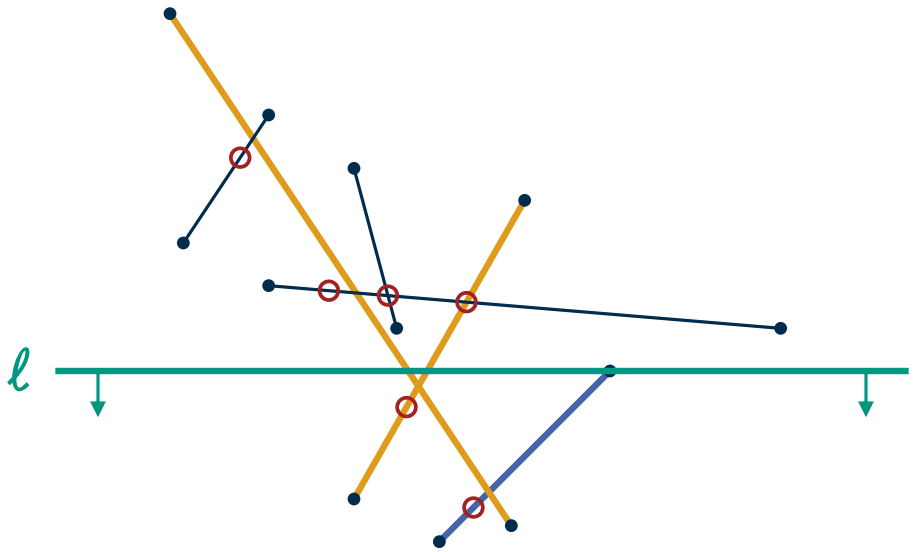

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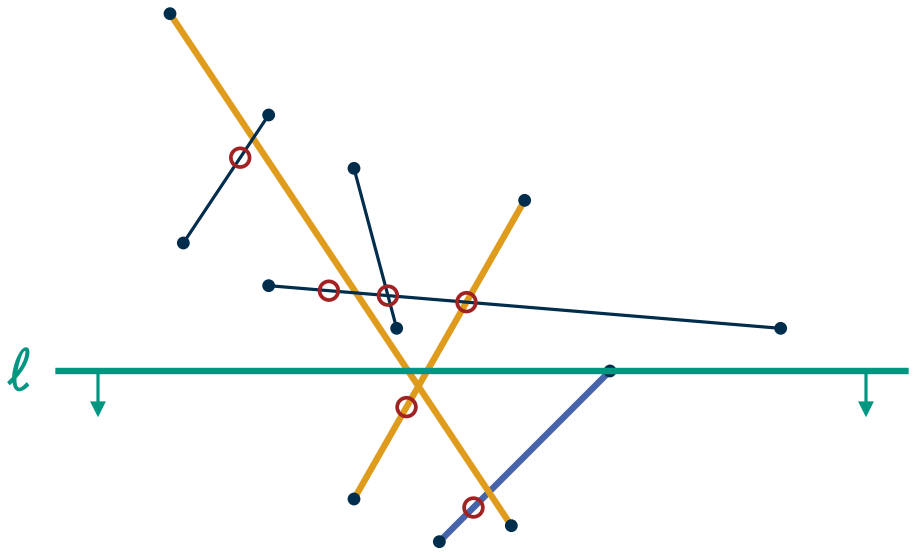

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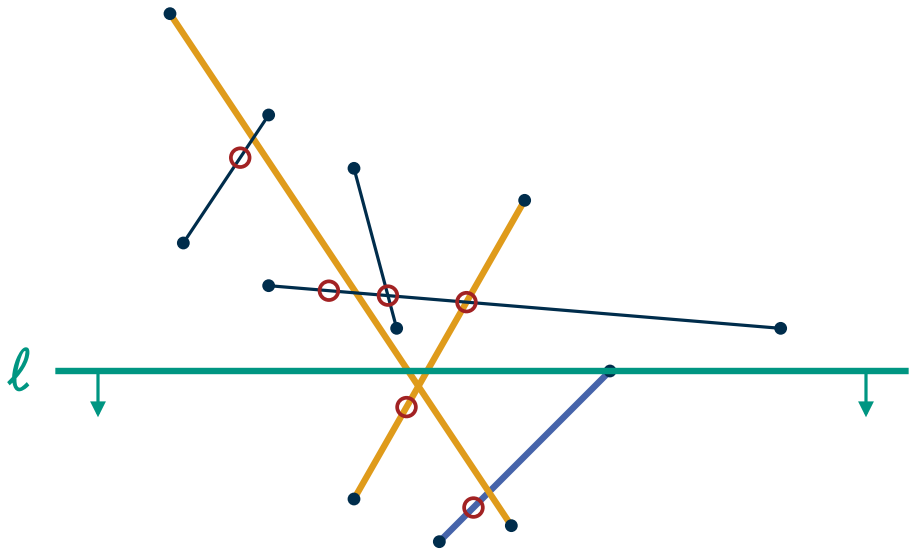

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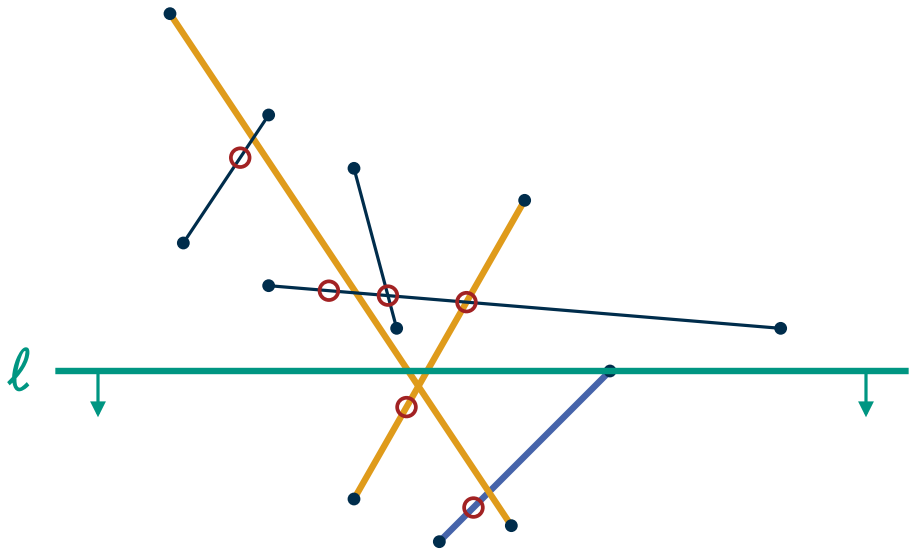

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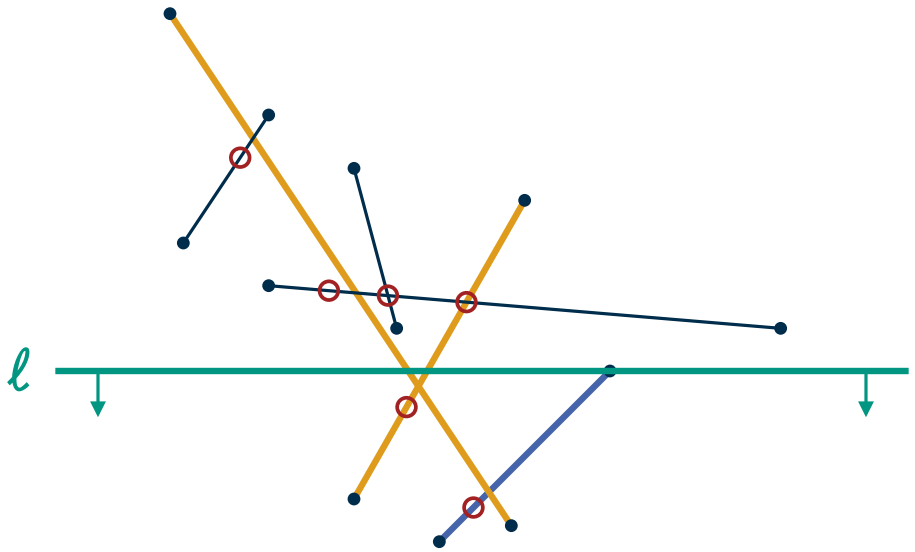

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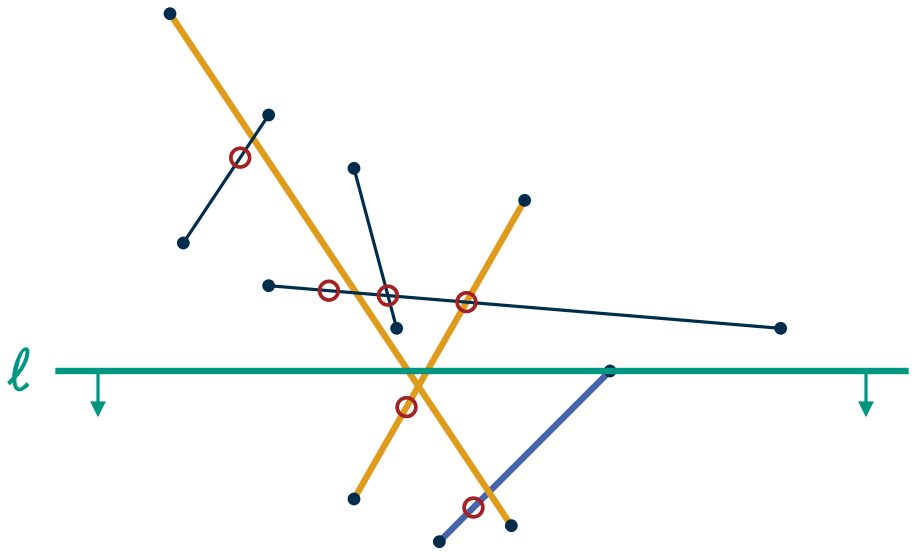

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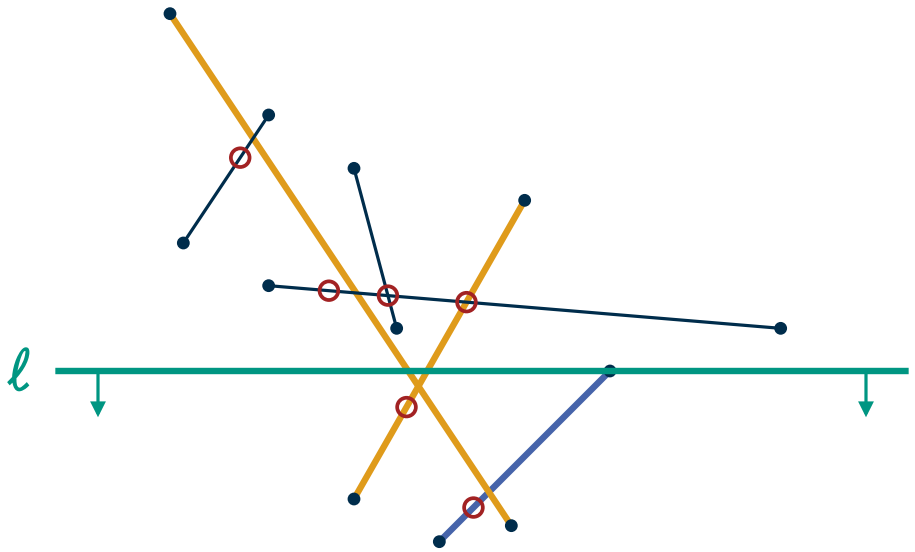

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  - endpoint of segment $s \rightarrow$ set $S_\ell = S_\ell - s$
  - start point of $s \rightarrow$ check if s intersects segments in S_ℓ and set $S_\ell = S_\ell + s$

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Solution: only compare adjacent segments (**adjacent:** next to each other on the current sweep line)

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Only Checking Adjacent Segments

What Changes?

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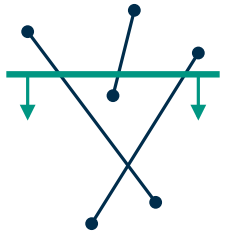

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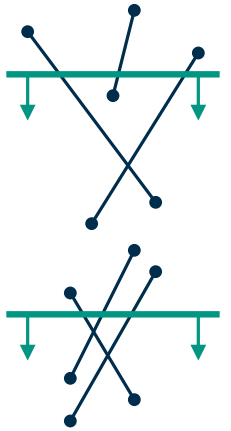

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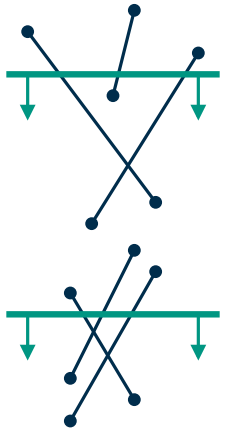

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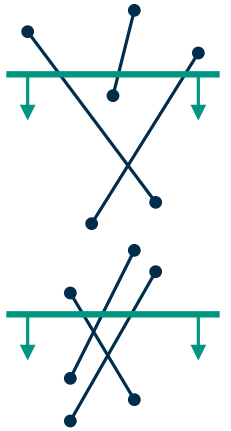

Which Data Structure?

Only Checking Adjacent Segments

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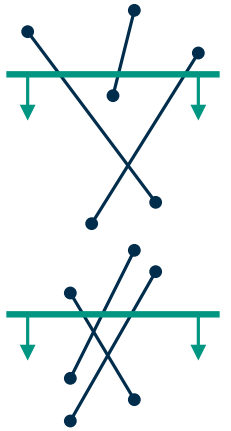

Which Data Structure?

- sweep-line status: insert, delete, find successor / predecessor
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Only Checking Adjacent Segments

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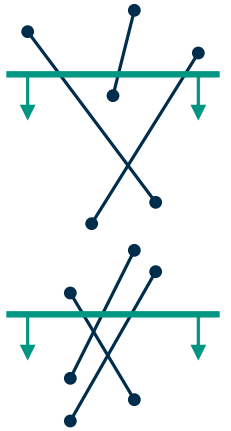

Which Data Structure?

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} search tree: $O(\log n)$ (e.g., (a, b)-tree, red-black tree)

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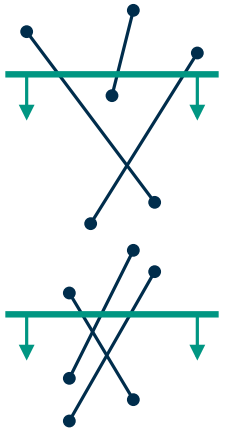

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Why not a heap for the event queue?

Only Checking Adjacent Segments

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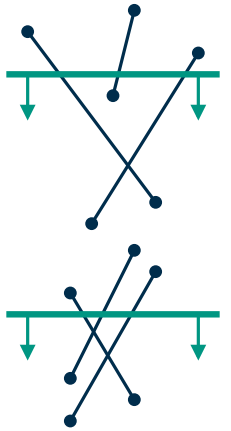

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Why do we need “search” for the event queue?

Only Checking Adjacent Segments

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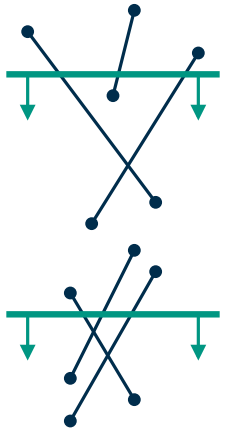

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one way to handle finding the same intersection multiple times

Improved Sweep-Line Algorithm

function FINDINTERSECTIONS(S)

Input: set of line segments S

Output: all intersections (with segment pairs)

```
Q = empty queue                 // event queue
for pq ∈ S
    Q = Q + p + q
T = search tree                 // status
while Q ≠ ∅
    p = min{Q} and Q = Q − p
    HANDLEEVENT(p)              // event handler
```

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function HANDLEEVENT(p)

```
if p is vertex of a segment
    HANDLEVERTEX(p)
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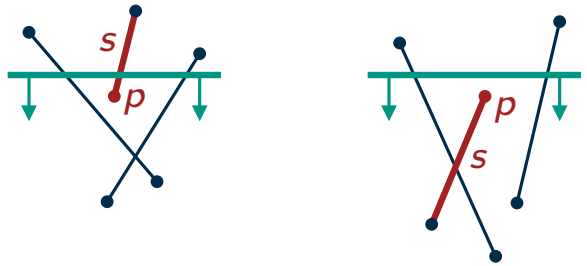

function HANDLEVERTEX(p)

```
s = segment with vertex p

if s ∈ T                           // segment ends

else                               // segment starts
```

Improved Sweep-Line Algorithm

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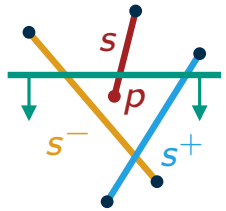

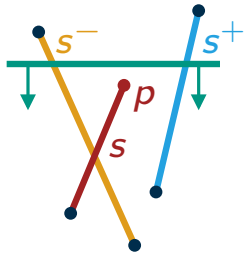

function HANDLEVERTEX(p)

```
s = segment with vertex p
s⁻ = predecessor of s in T
s⁺ = successor of s in T
if s ∈ T                           // segment ends


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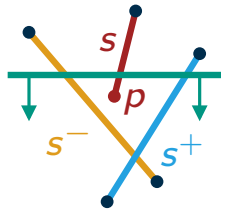

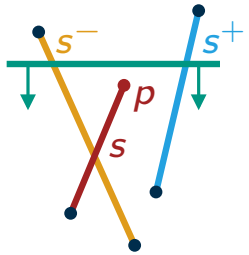

function HANDLEVERTEX(p)

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s = segment with vertex p
s⁻ = predecessor of s in T
s⁺ = successor of s in T
if s ∈ T                           // segment ends
    CHECKINTERSECTION(s⁻, s⁺, p)
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else                               // segment starts
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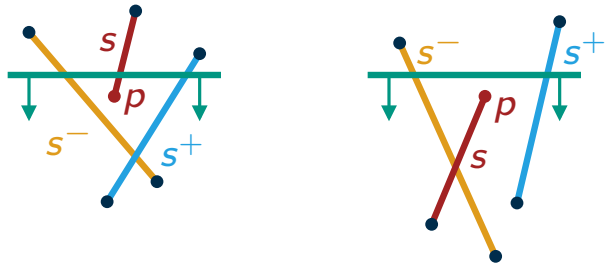

function HANDLEVERTEX(p)

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s = segment with vertex p
s⁻ = predecessor of s in T
s⁺ = successor of s in T
if s ∈ T                           // segment ends
    CHECKINTERSECTION(s⁻, s⁺, p)
    T = T − s
else                               // segment starts
    T = T + s
    CHECKINTERSECTION(s⁻, s, p)
    CHECKINTERSECTION(s⁺, s, p)
```

Improved Sweep-Line Algorithm

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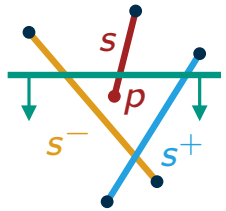

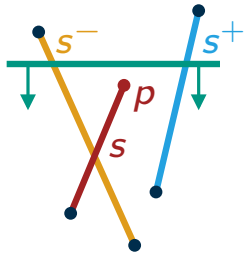

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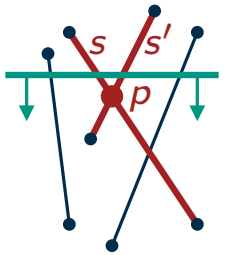

function HANDLEINTERSECTION(p)

```
s, s′ = segments intersecting in p
output (p, s, s′)                  (s < s′ in T)
```

Improved Sweep-Line Algorithm

function FINDINTERSECTIONS(S)
Input: set of line segments S
Output: all intersections (with segment pairs)

- $Q =$ empty queue // event queue
- **for** $pq \in S$
  - $Q = Q + p + q$
- $T =$ search tree // status
- **while** $Q \neq \emptyset$
  - $p = \min\{Q\}$ and $Q = Q - p$
  - HANDLEEVENT(p) // event handler

function HANDLEEVENT(p)

- **if** p is vertex of a segment
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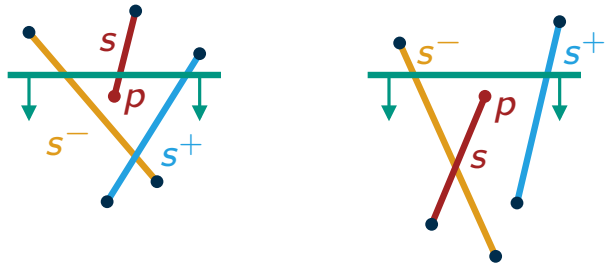

function HANDLEVERTEX(p)

- $s =$ segment with vertex p
- s^- = predecessor of s in T
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- **if** $s \in T$ // segment ends
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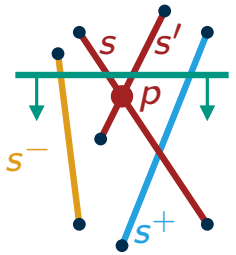

function HANDLEINTERSECTION(p)

- $s, s' =$ segments intersecting in p
- **output** (p, s, s') ($s < s'$ in T)
- swap s and s' in T ($s' < s$)
- s^- = predecessor of s' in T
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Improved Sweep-Line Algorithm

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  - HANDLEEVENT(p) // event handler

function HANDLEEVENT(p)

- **if** p is vertex of a segment
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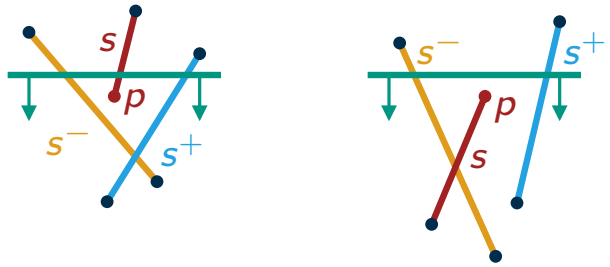

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- **else** // segment starts
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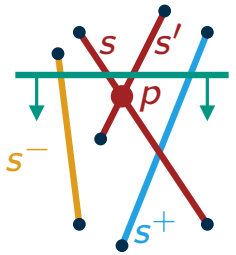

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function CHECKINTERSECTION(s^-, s^+, p)

- $q = s^- \cap s^+$
- **if** $q \neq \emptyset$ **and** $p_y < q_y$ **and** $q \notin Q$
  - $Q = Q + q$

Improved Sweep-Line Algorithm

function FINDINTERSECTIONS(S)
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- Q = empty queue // event queue
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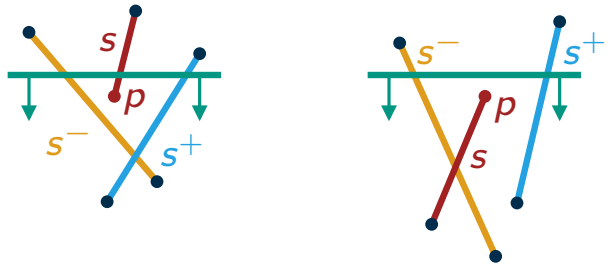

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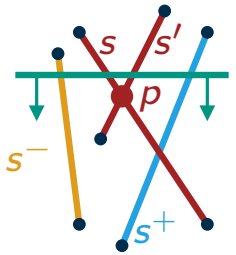
Running time?



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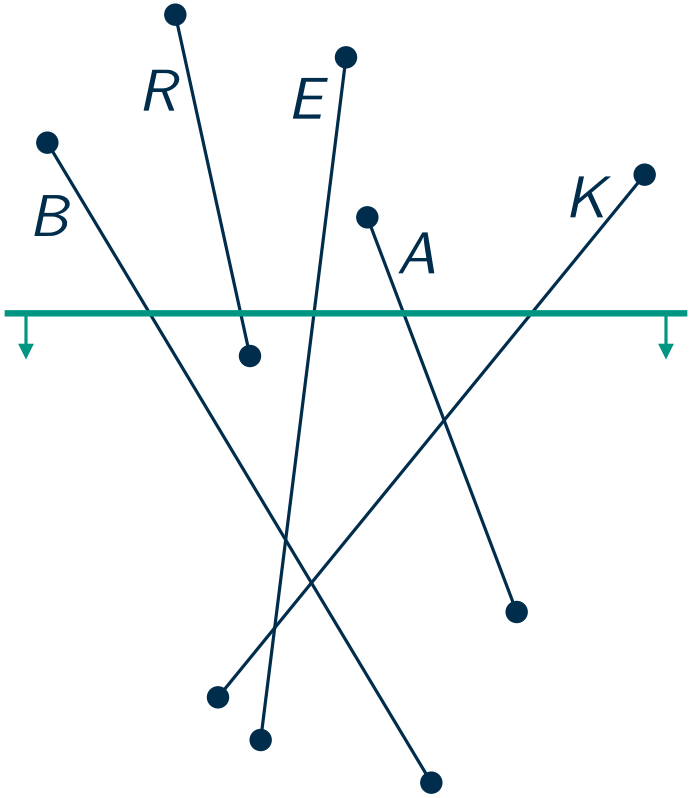
Improved Sweep-Line Algorithm

Theorem **assumption:** general position

The k intersections of n line segments can be computed in $O((n+k)\log n)$ time.

How Many Events Are In The Event Queue?



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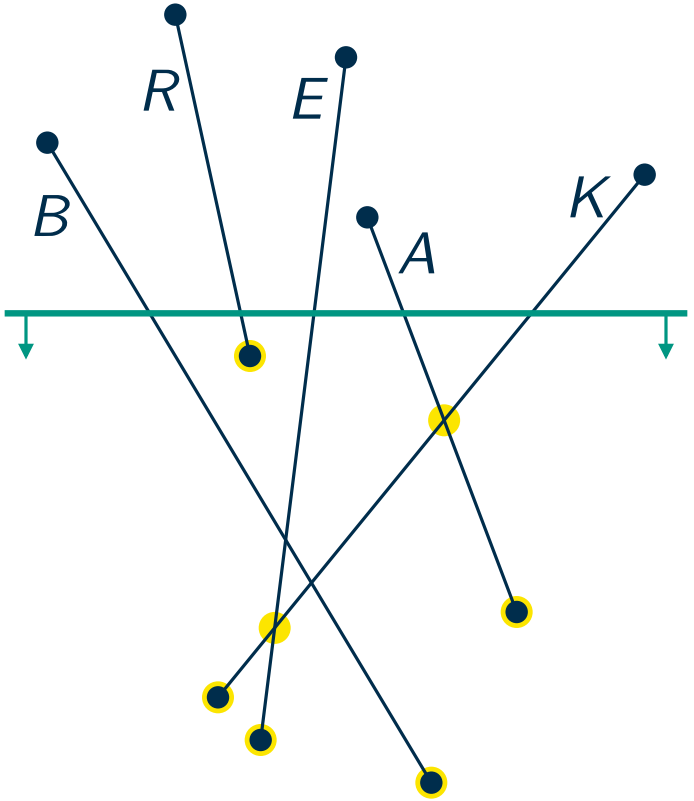


7

Do We Need General Position?

Theorem **assumption:** general position

The k intersections of n line segments can be computed in $O((n+k)\log n)$ time.

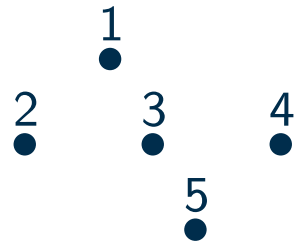
Do We Need General Position?

Theorem **assumption:** general position

The k intersections of n line segments can be computed in $O((n+k)\log n)$ time.

Problem: Event Points With Equal y-Coordinate

- use lexicographical order with respect to (y, x)
- equivalent to slight clockwise rotation



Do We Need General Position?

Theorem **assumption:** general position

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Problem: Multiple Events At The Same Point

- three things can happen at an event point:
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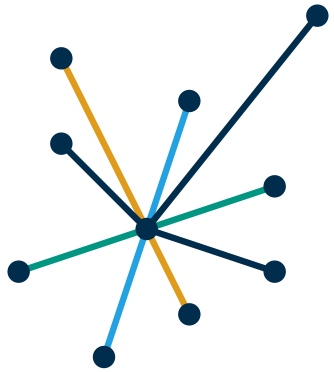

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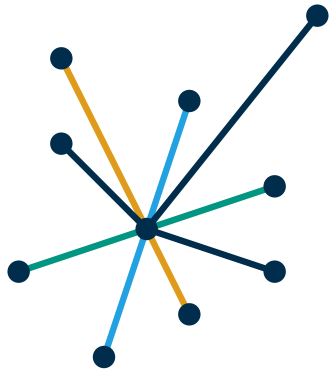

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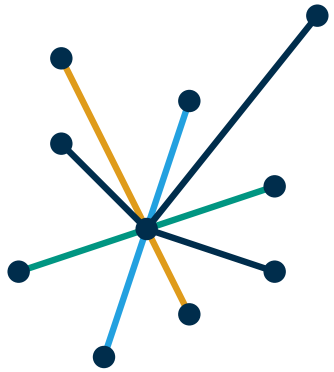

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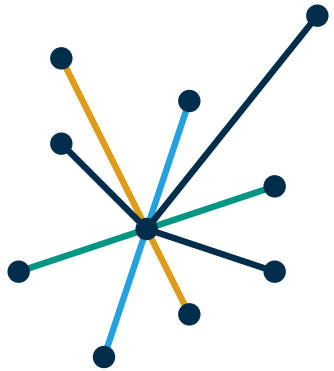

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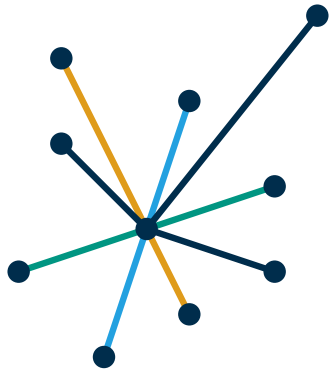

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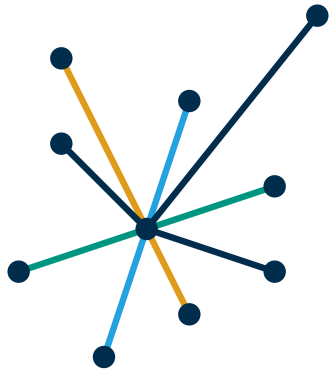

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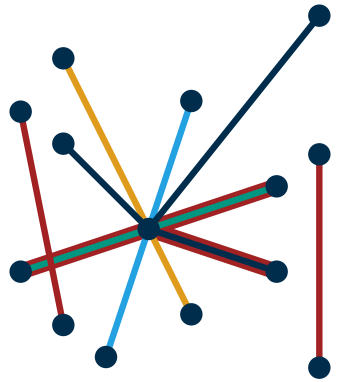

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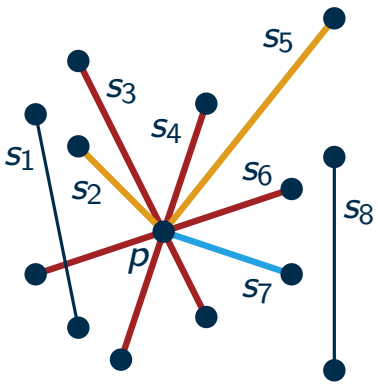

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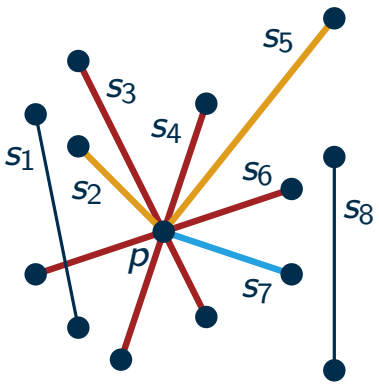

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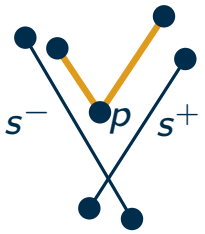

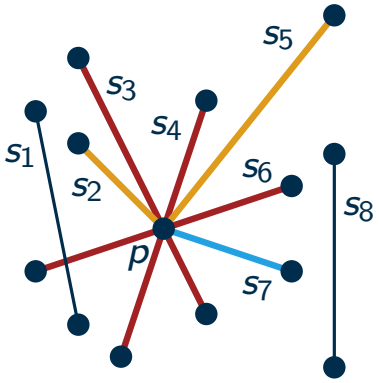

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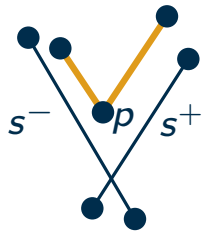

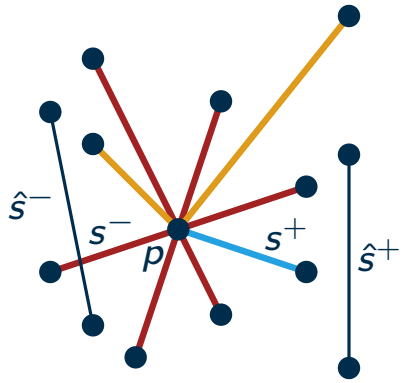

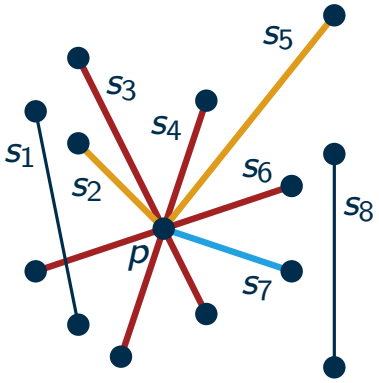

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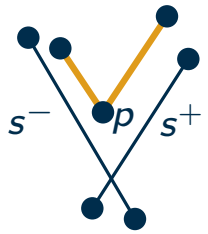

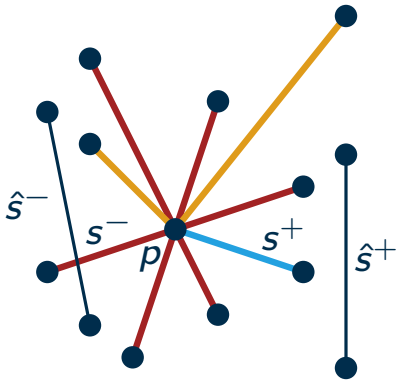

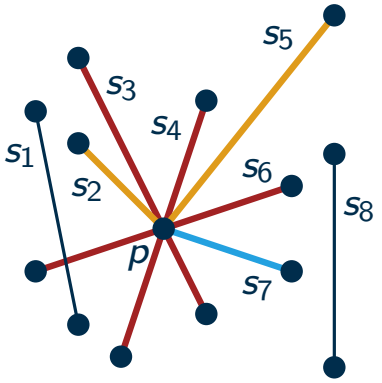

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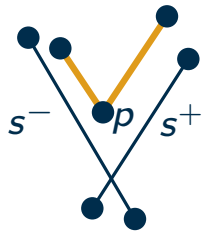

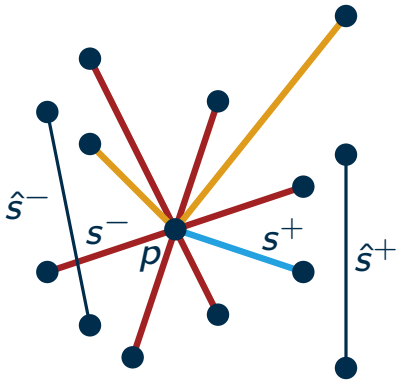

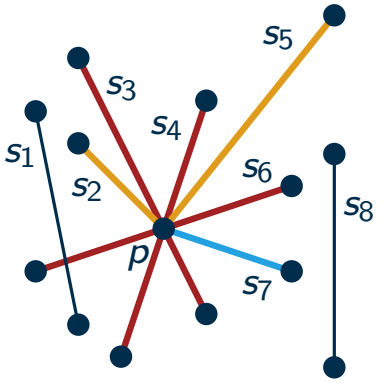

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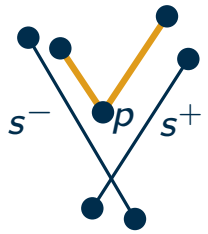

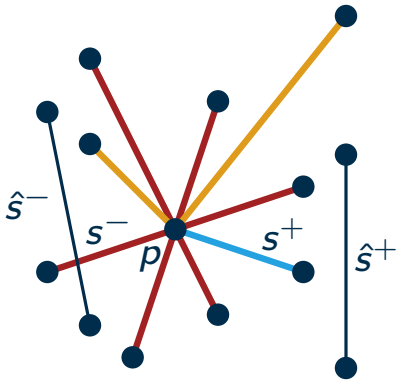

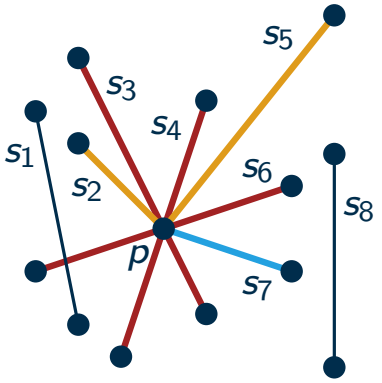

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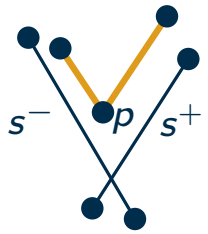

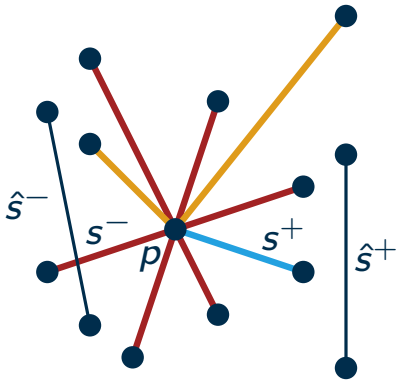

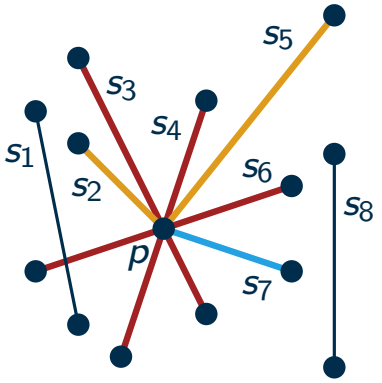

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What is left to show?

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Does $m \in O(n+k)$ hold?

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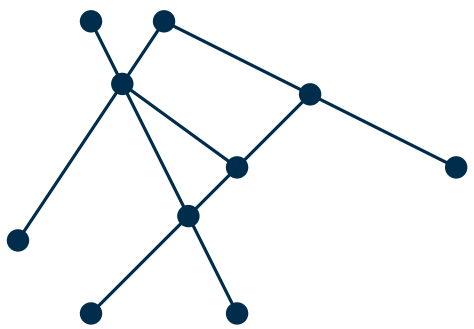

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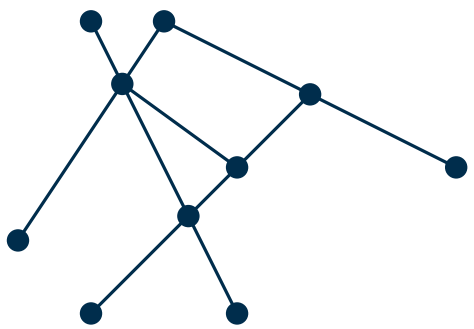

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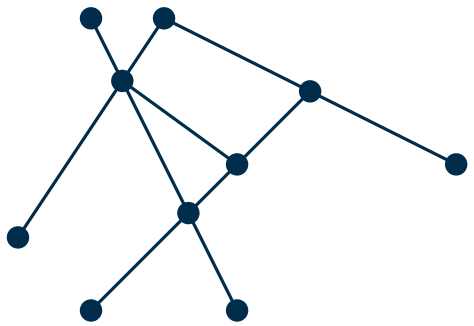
- for planar graphs: $|E| \leq 3|V| - 6$
  $\Rightarrow m \in O(n+k)$



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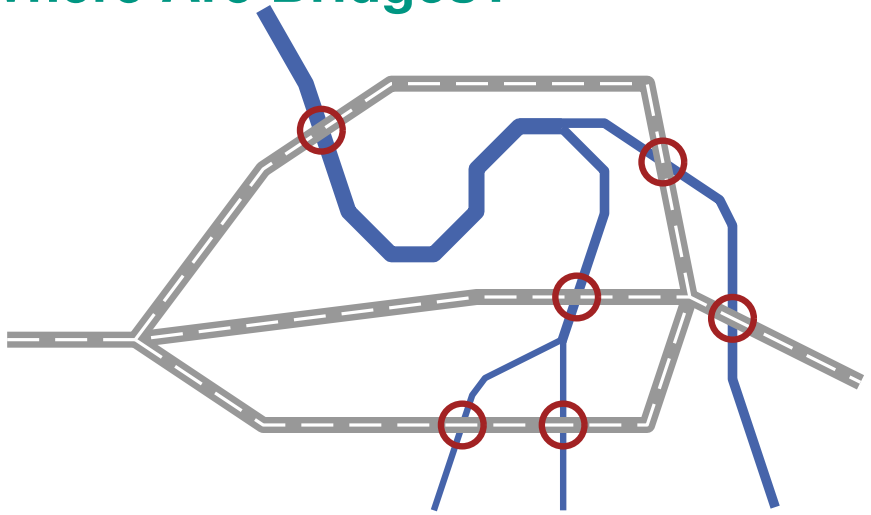
- option: only keep intersections in the queue corresponding to adjacent segments $\rightarrow O(n)$

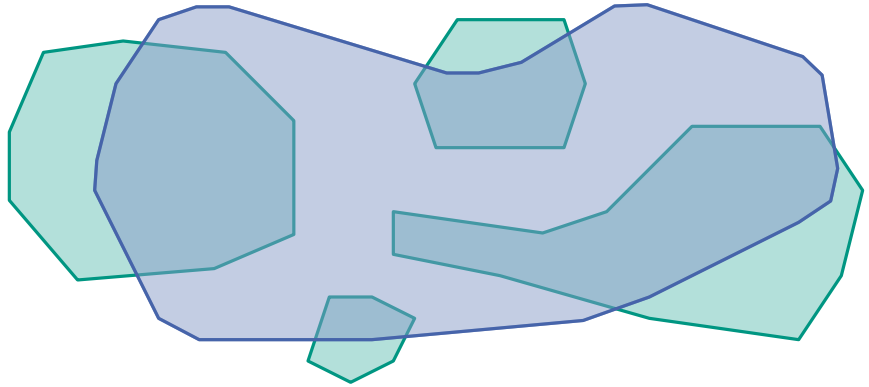
Back To Our Motivation

Did We Reach Our Goal?

Where Are Bridges?



Forests With A Lot Of Rainfall



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- we can find bridges

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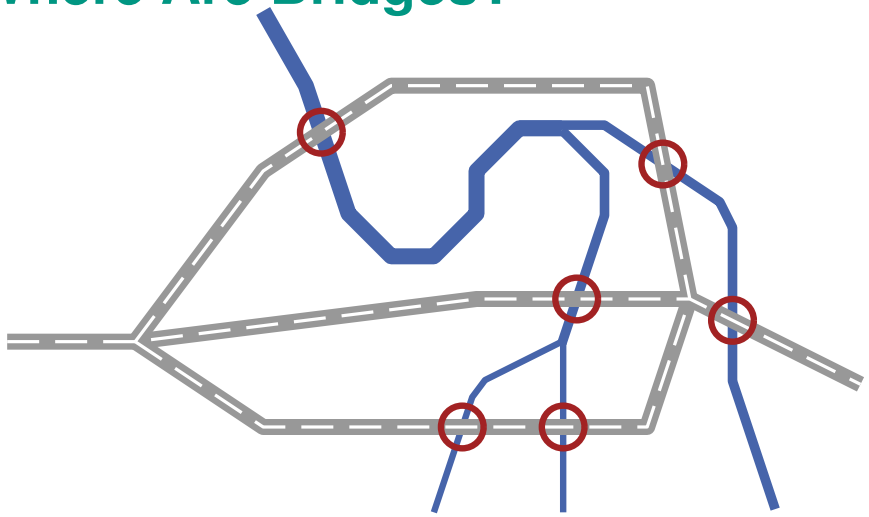

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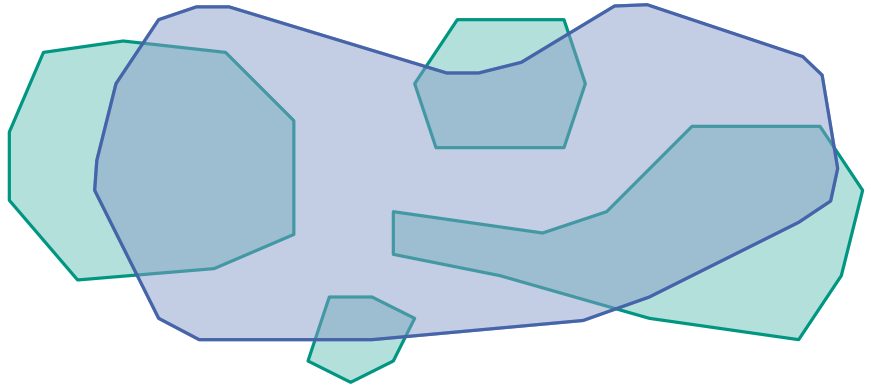

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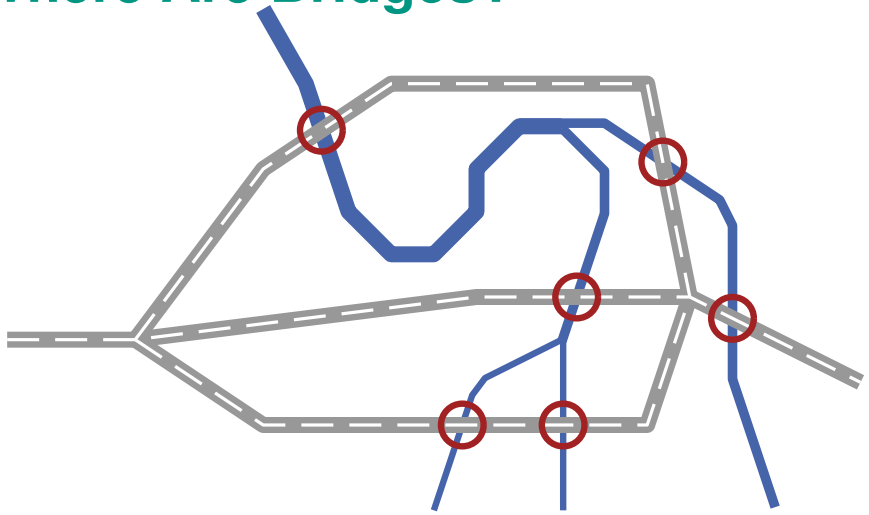

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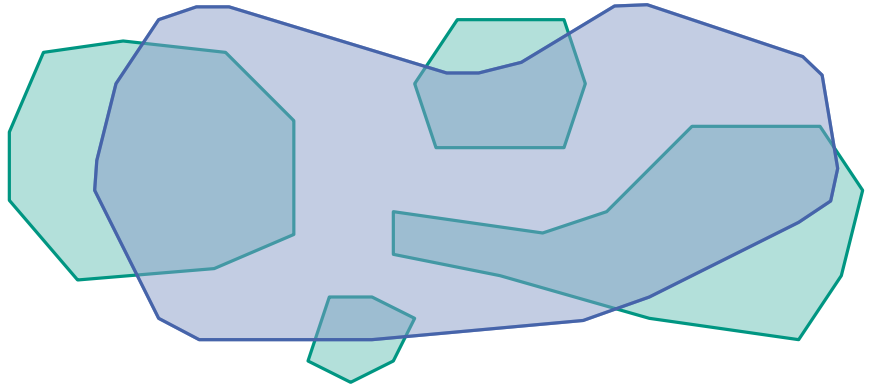

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In The Following

- data structure that helps with computing the intersection
- actually using it: exercise sheet

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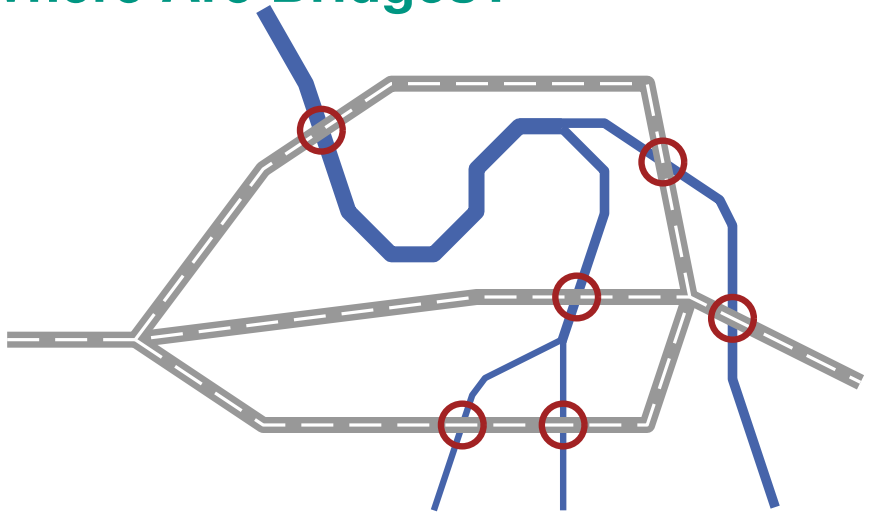

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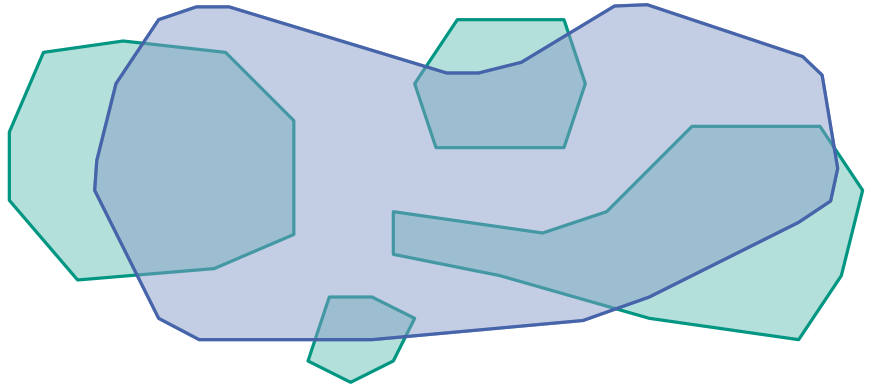
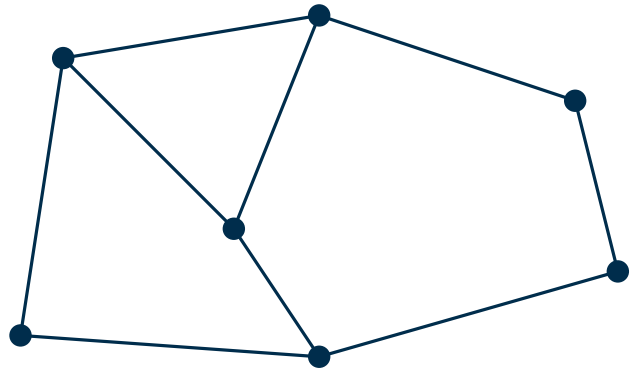


Doubly-Connected Edge List

Geometric Graphs



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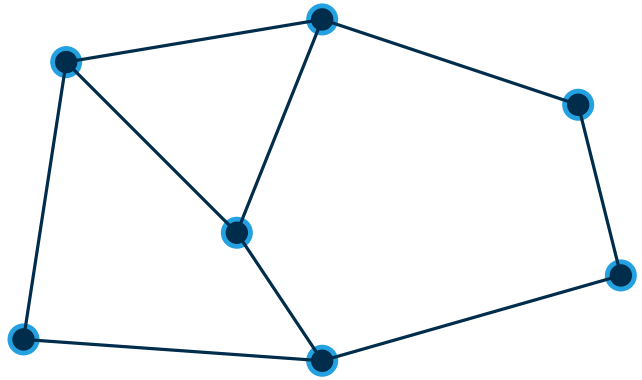
- vertices with coordinates



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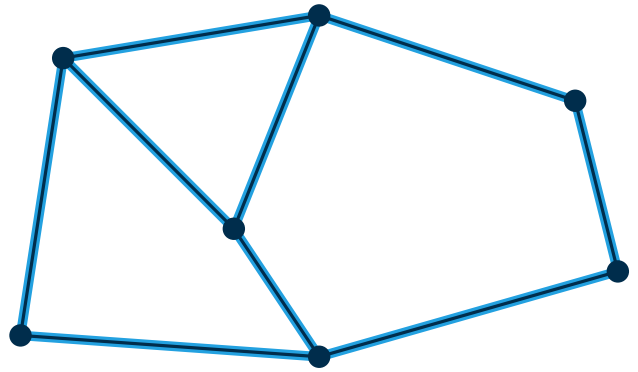
- edges



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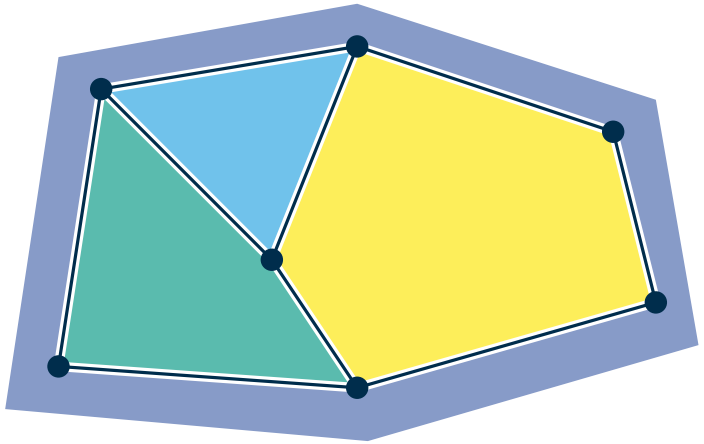
Geometric Graphs

- vertices with coordinates
- edges
- faces



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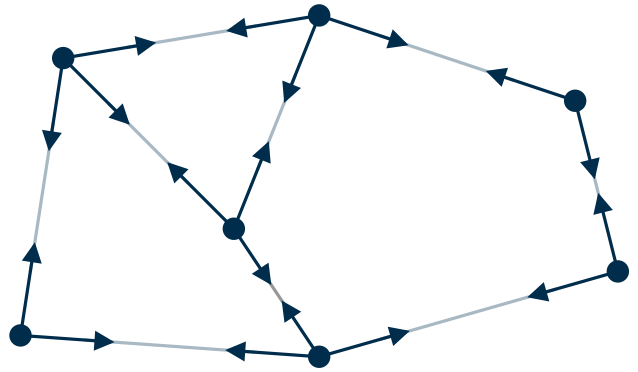

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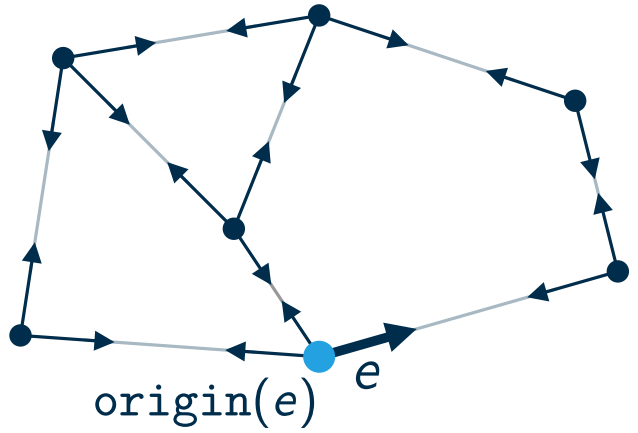

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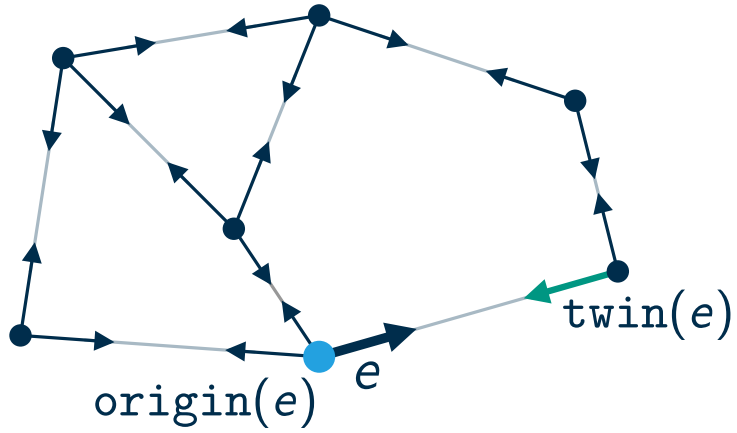

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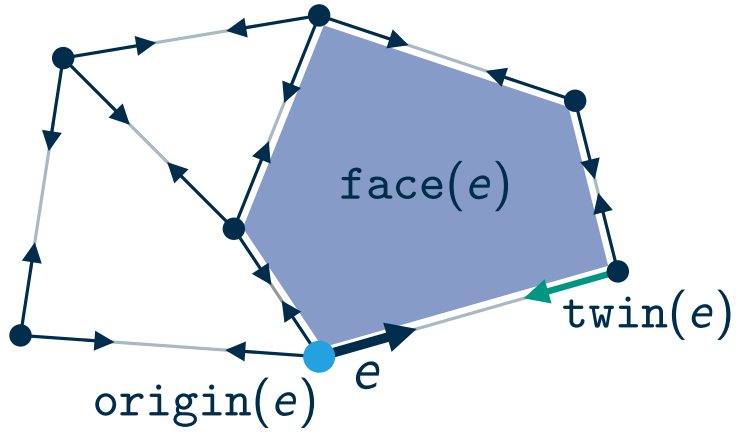

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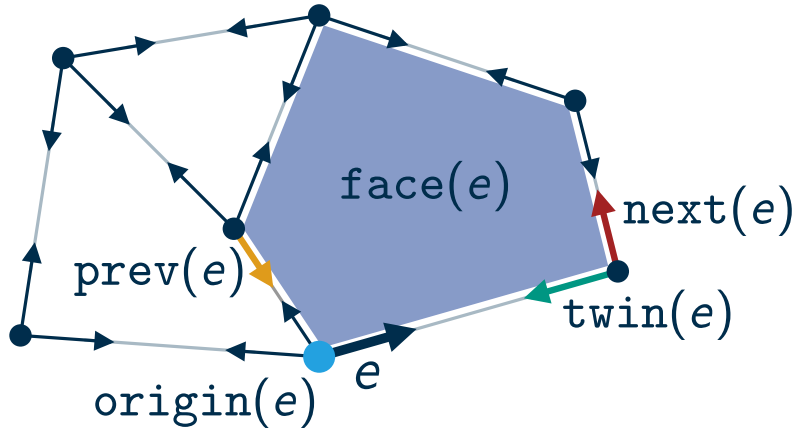

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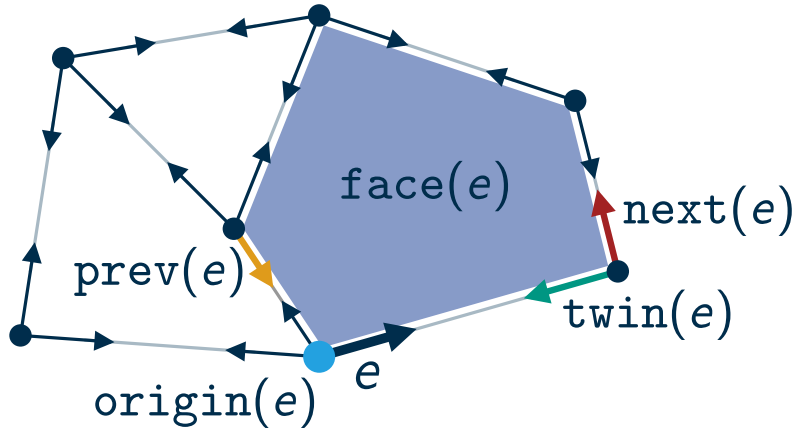

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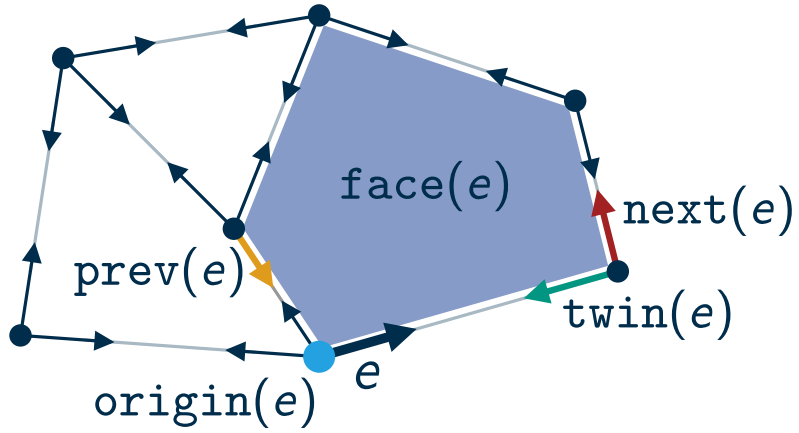

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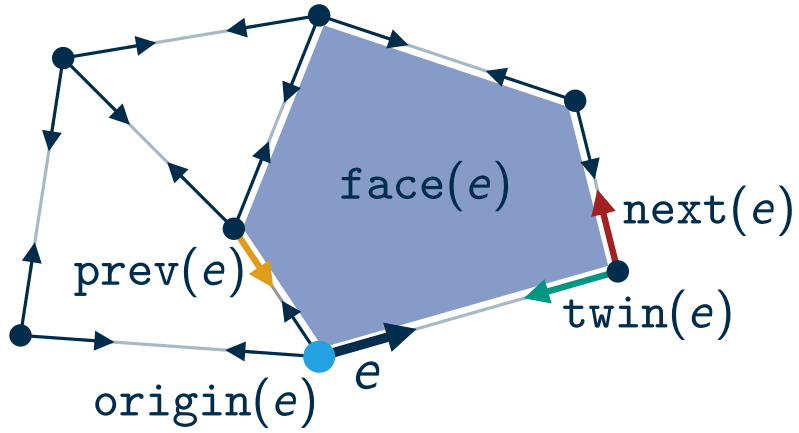

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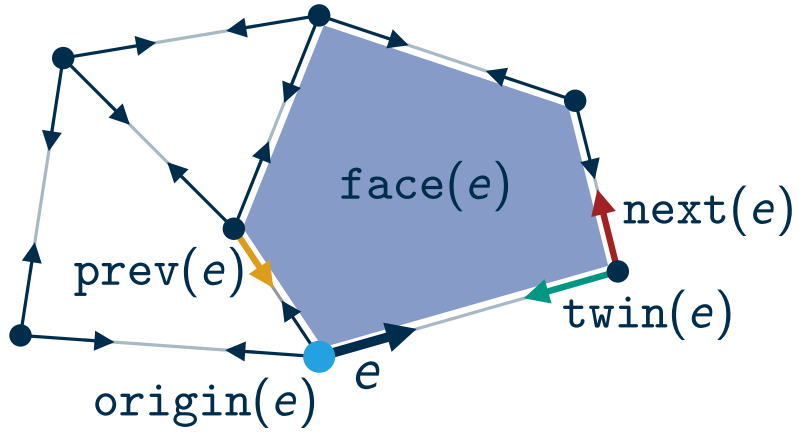

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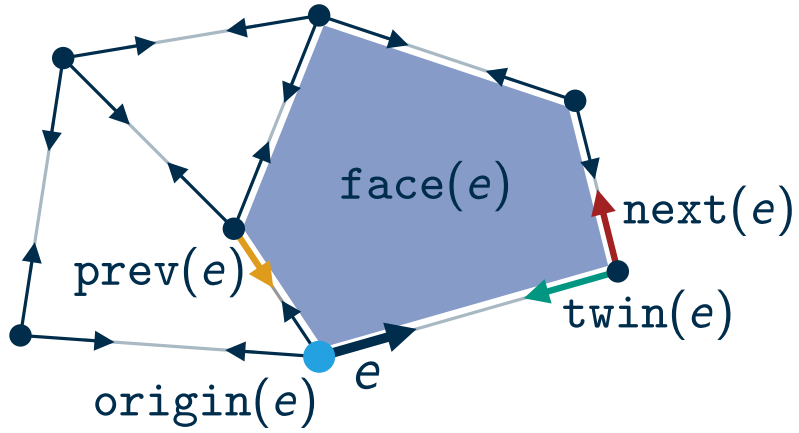

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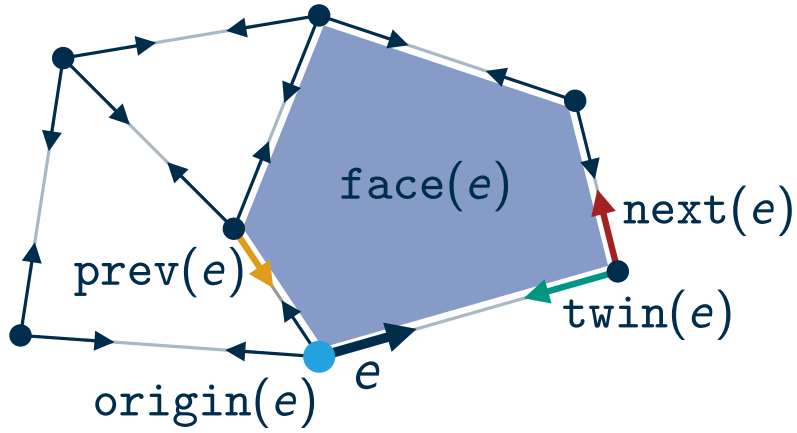

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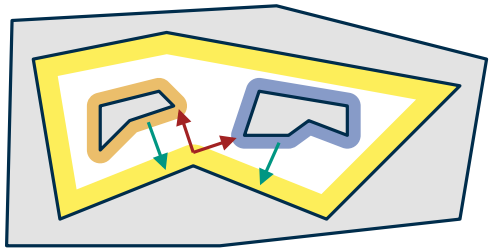

adaptation for multiple connected components

Wrap-Up

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- extensions to the sweep-line approach
  - the sweep line might move differently (e.g., rotate)
  - the sweep line does not need to be a line