Karlsruhe Institute of Technology

Hasso Plattner Institut
IT Systems Engineering | Universität Potsdam

Competitive Programming

Winter Term 23/24

Treaps: the only data structure you'll ever need

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HPI

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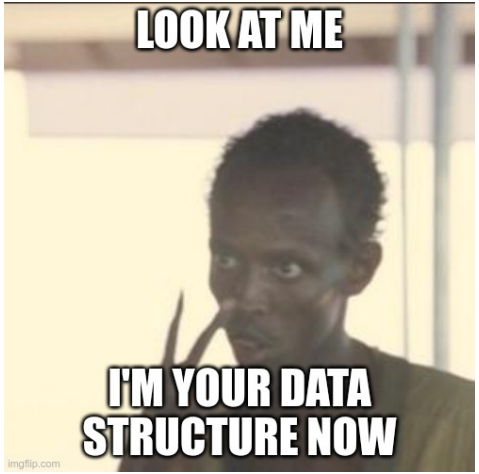
Scalable Algorithms Group
KIT

Overview for Today

- Treaps
- ...
- profit



Impressions

”D. A. Heger (2004) presented a performance comparison of binary search trees. Treap was found to have the best average performance”

— Wikipedia

”Treaps are hella slow, but easy to implement and customize.”

— some russian dude
Gleb Evstropov
ICPC 2013, 14, 15 Gold Medalist

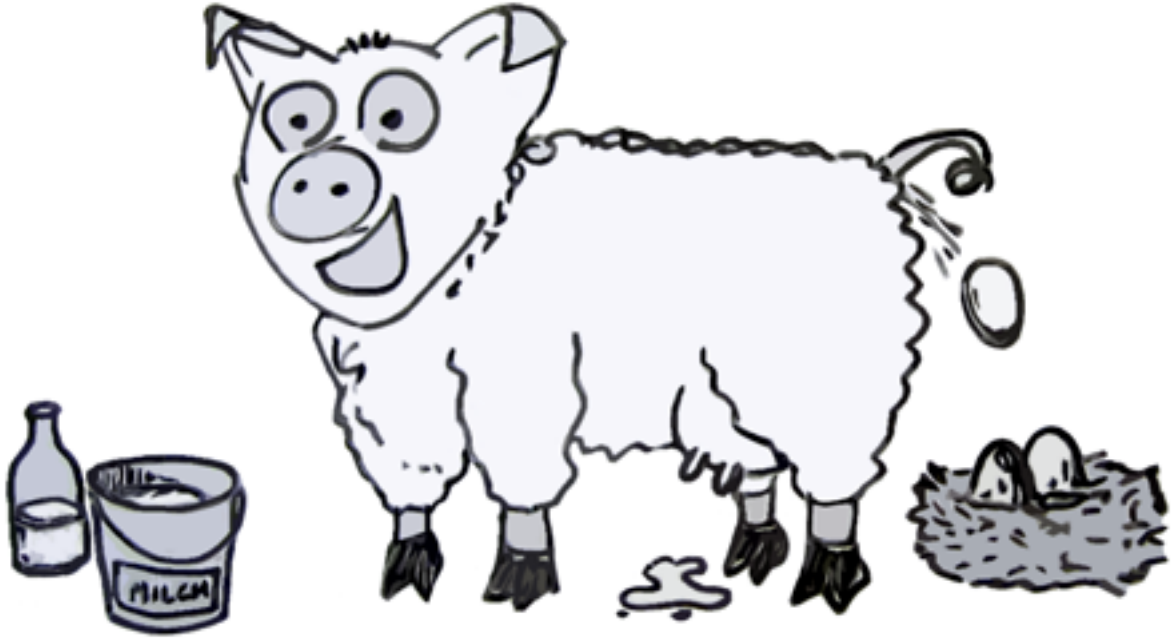
”Would recommend.”

— 9/10 Dentists

”First implementation was broken, but worked fine after refund.”

— Me

Jack of all Trades



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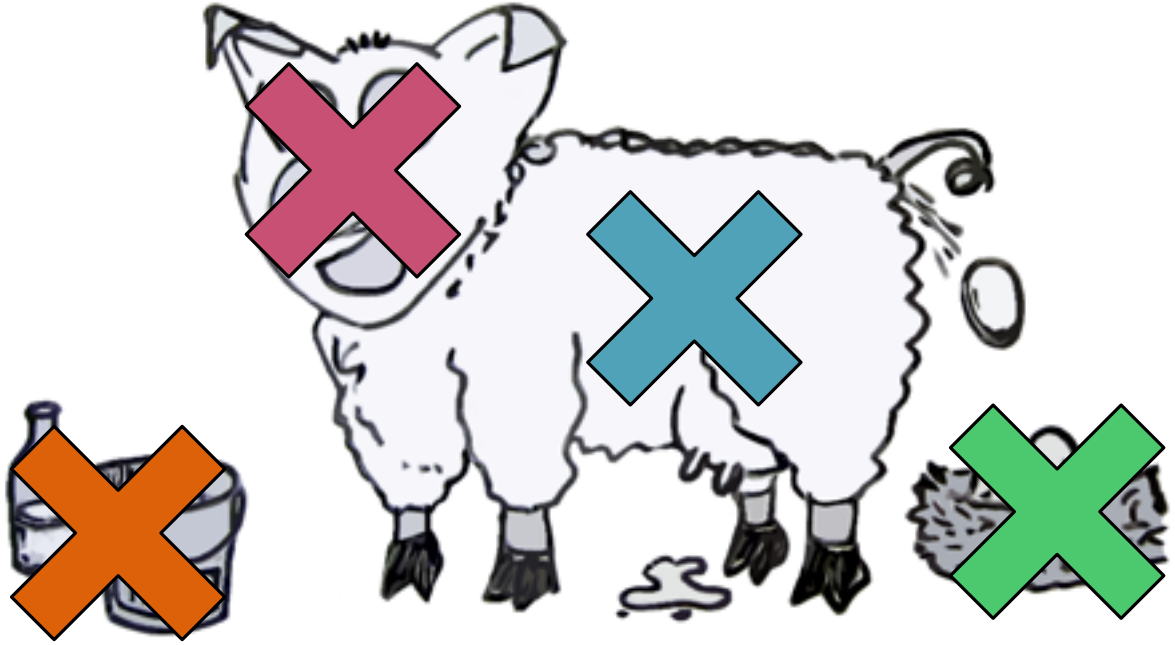
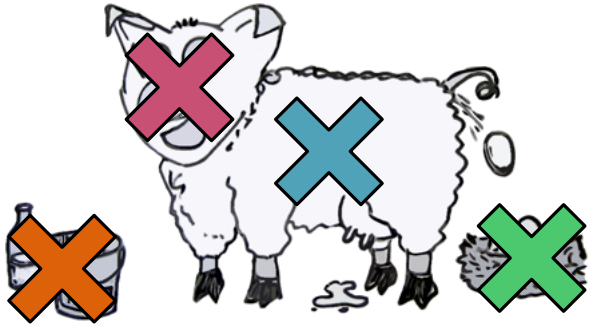


What a Treap can do

Self balancing BSTs
(like a set)

- find value
- insert value
- delete value



binary search tree

Treap = BS**T** + Heap

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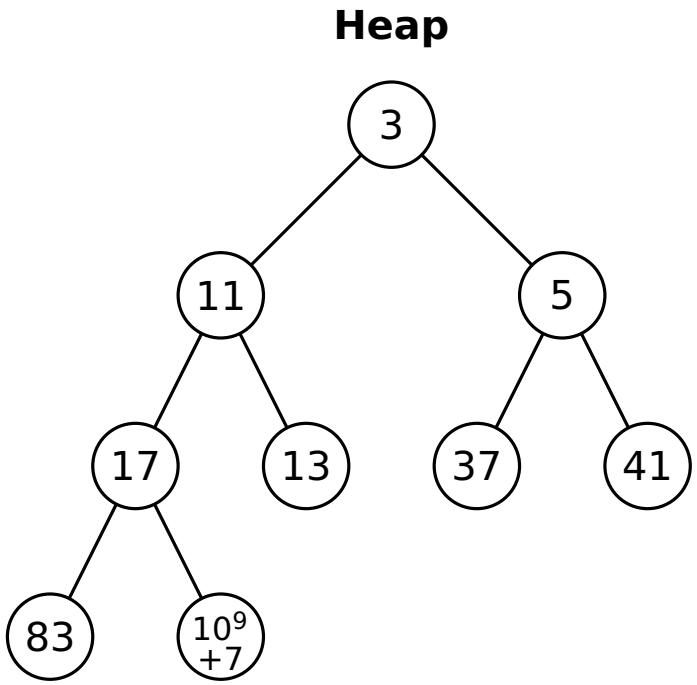
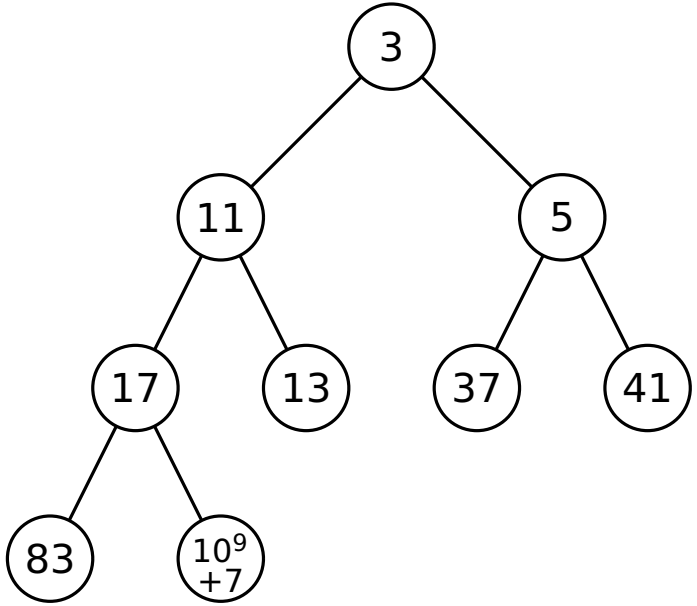


parent smaller than children

Treap = BS**T** + Heap

Heap



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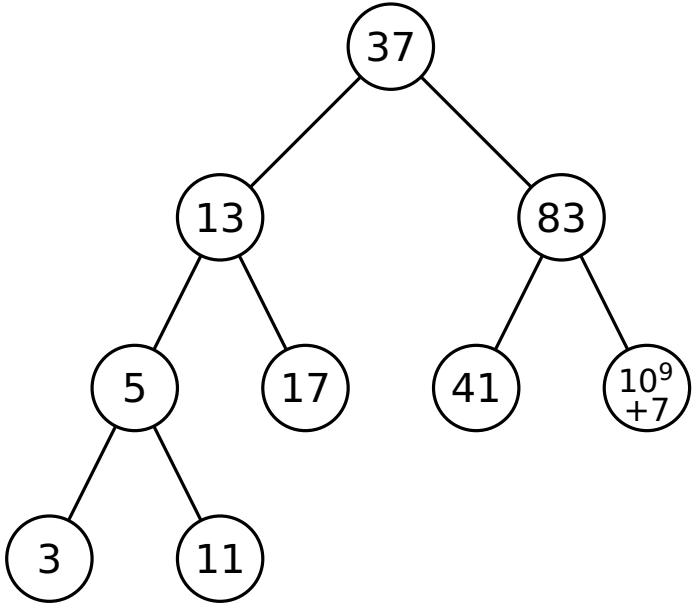
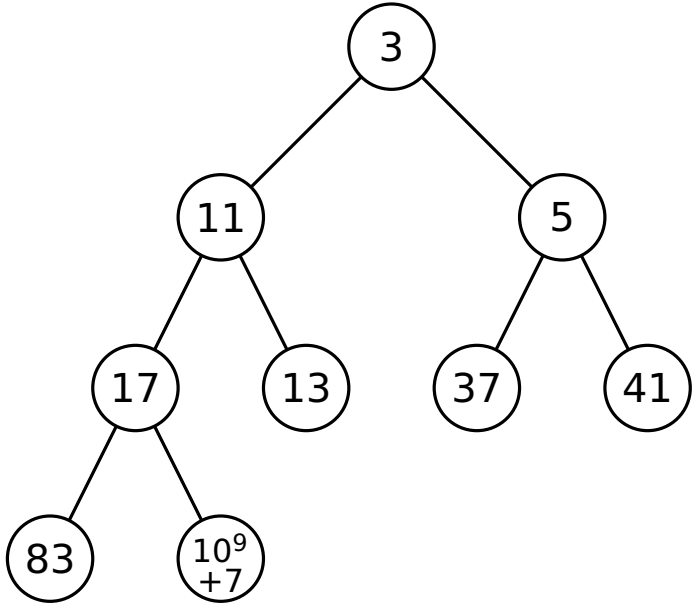
BST



left child $<$ parent $<$ right child

Treap = BS**T** + Heap

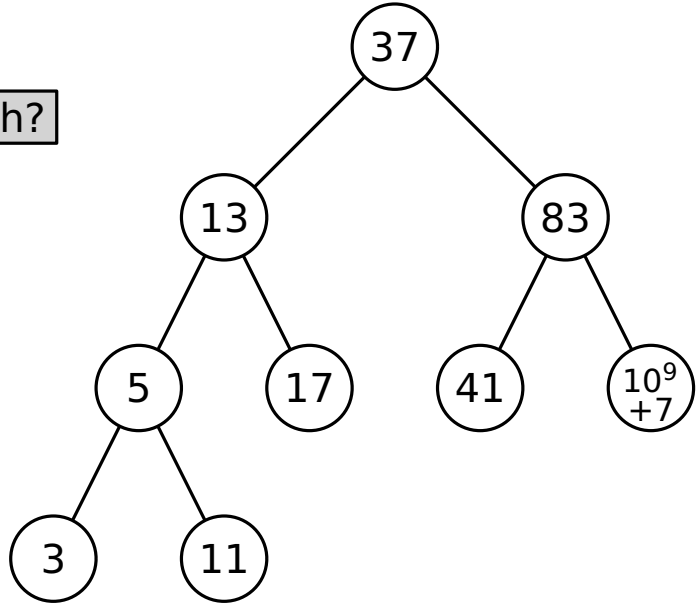
Heap



parent smaller than children

Can we do both?

BST



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Treap = BS**T** + Heap

- yes, we can do both, but ...

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- yes, we can do both, but ...
- parent must be smaller than children (heap)
- parent must be larger than left child (BST)

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  $\rightarrow$ there cannot be a left child

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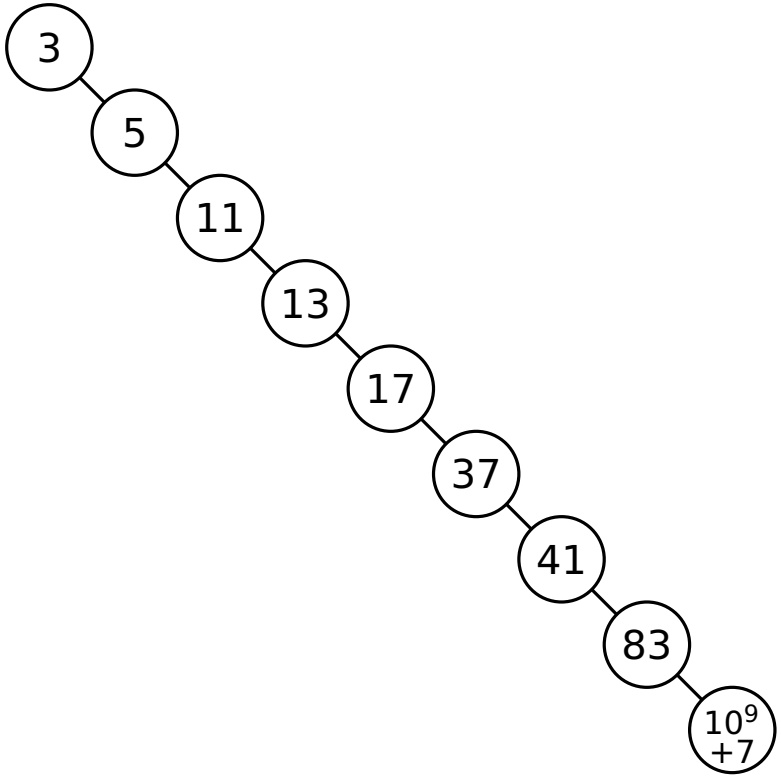

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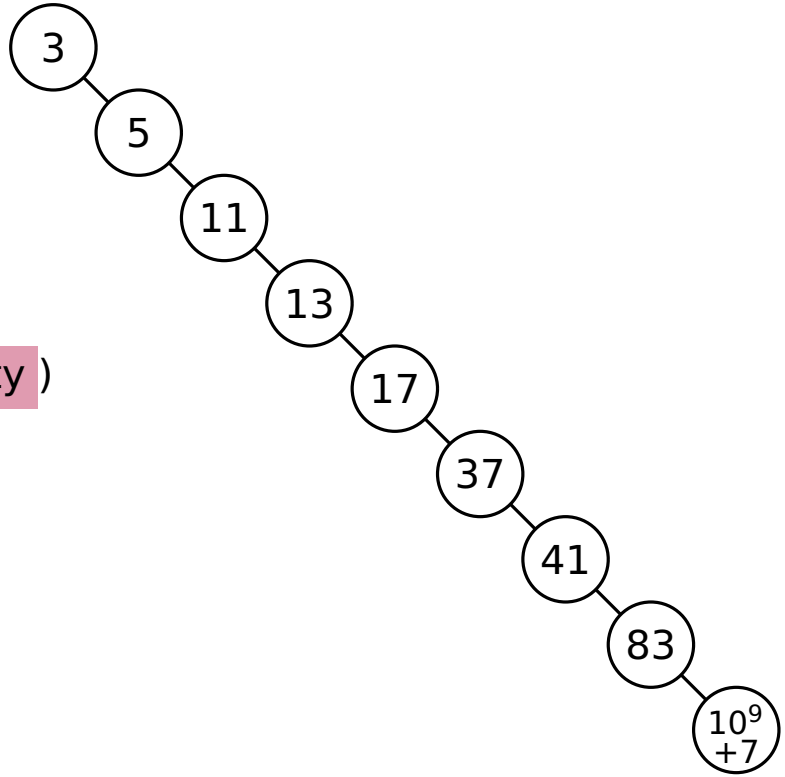
- let's give nodes a random second value (Priority)
- heap by priority; BST by key



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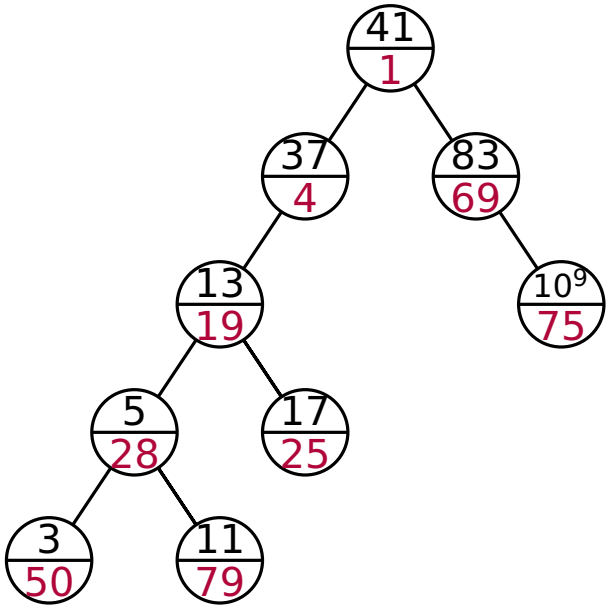

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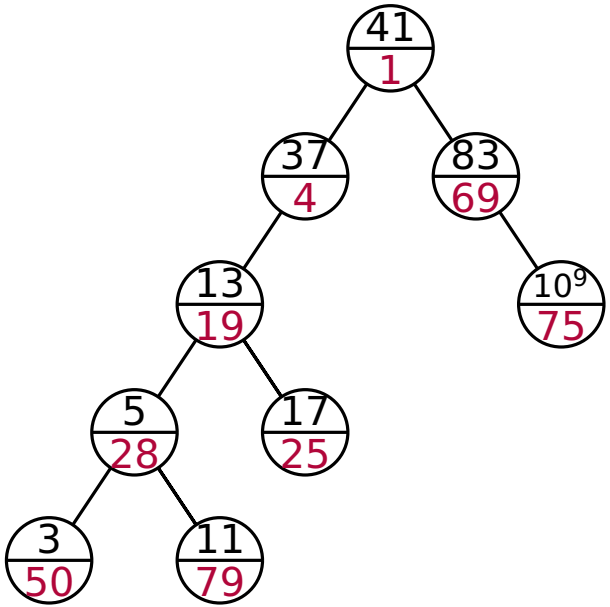
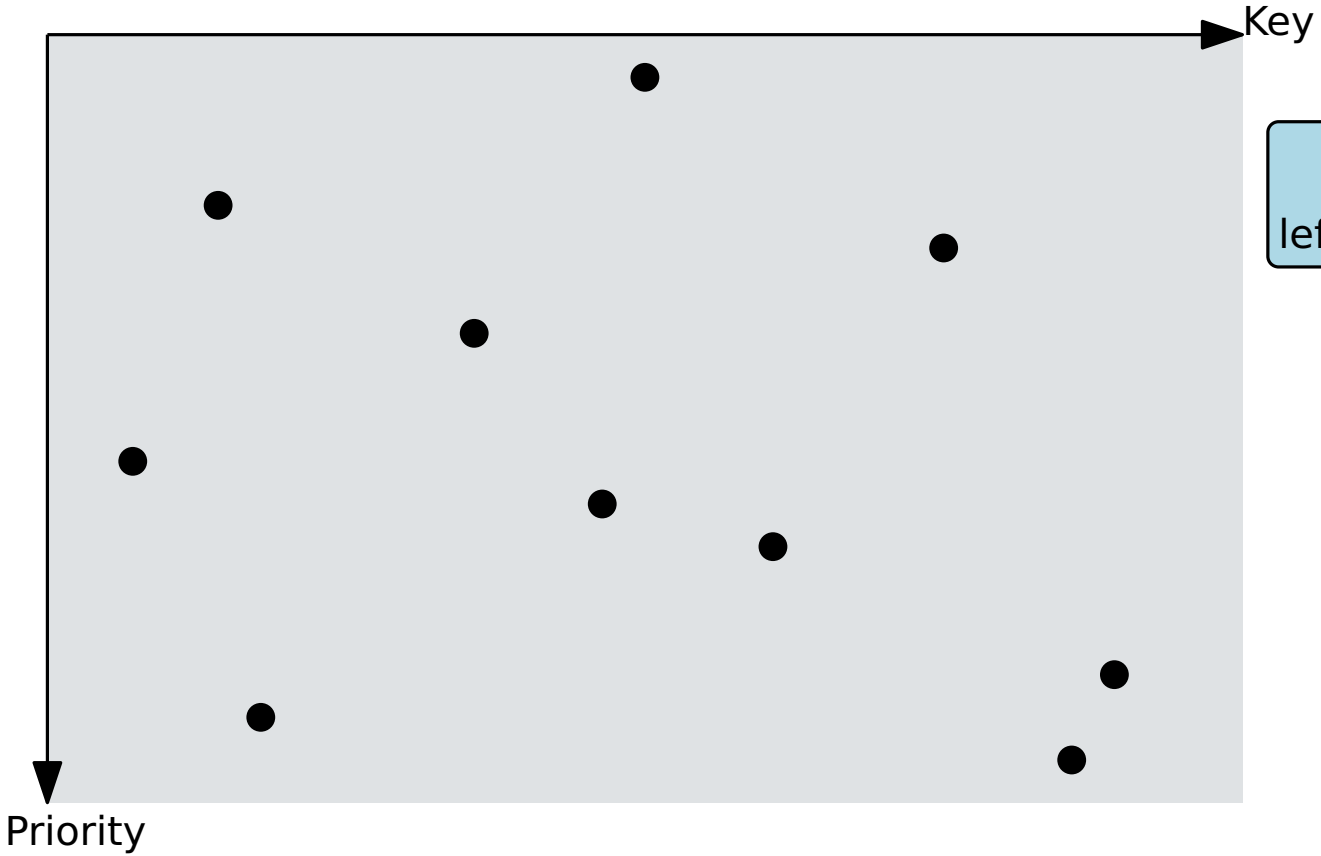
can be shown that:

- expected depth of a node is $\log n$
- expected depth of a treap is $\log n$
- treap has $\log n$ depth w.h.p.

  probability that a treap is deeper than $10 \log n$ is smaller than $1/n$



Unique?



BST by key:

left child $<$ parent $<$ right child

Heap by Prio:

parent $<$ children

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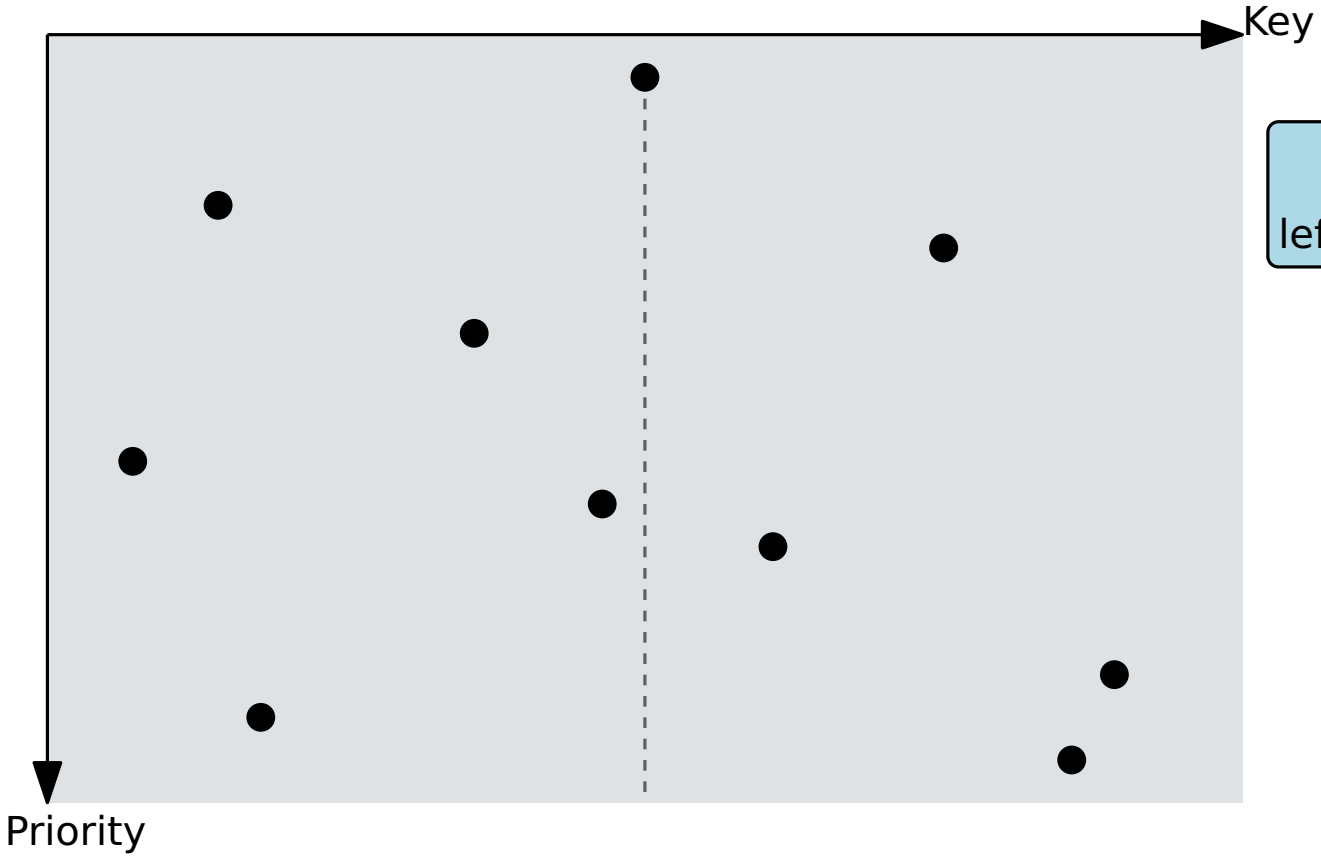

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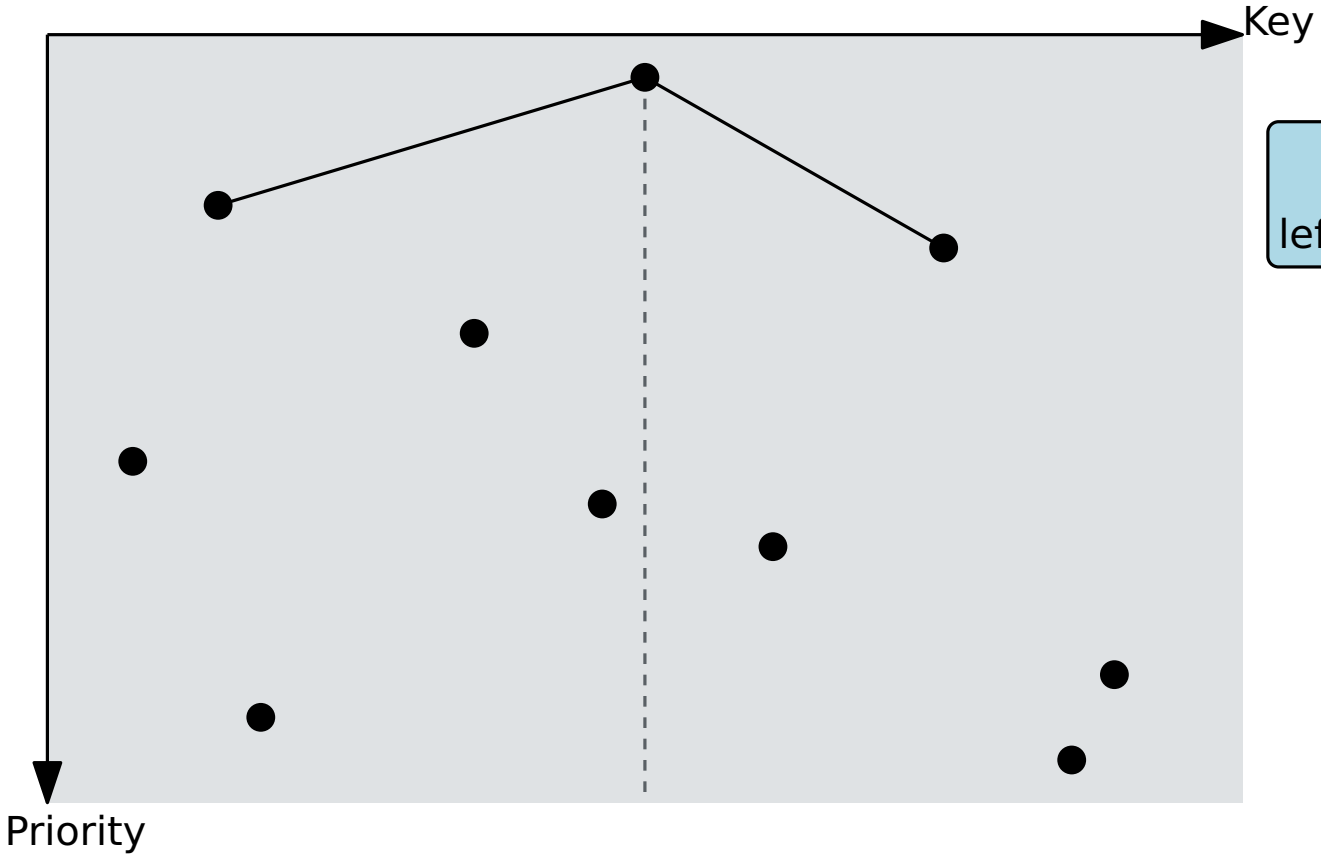

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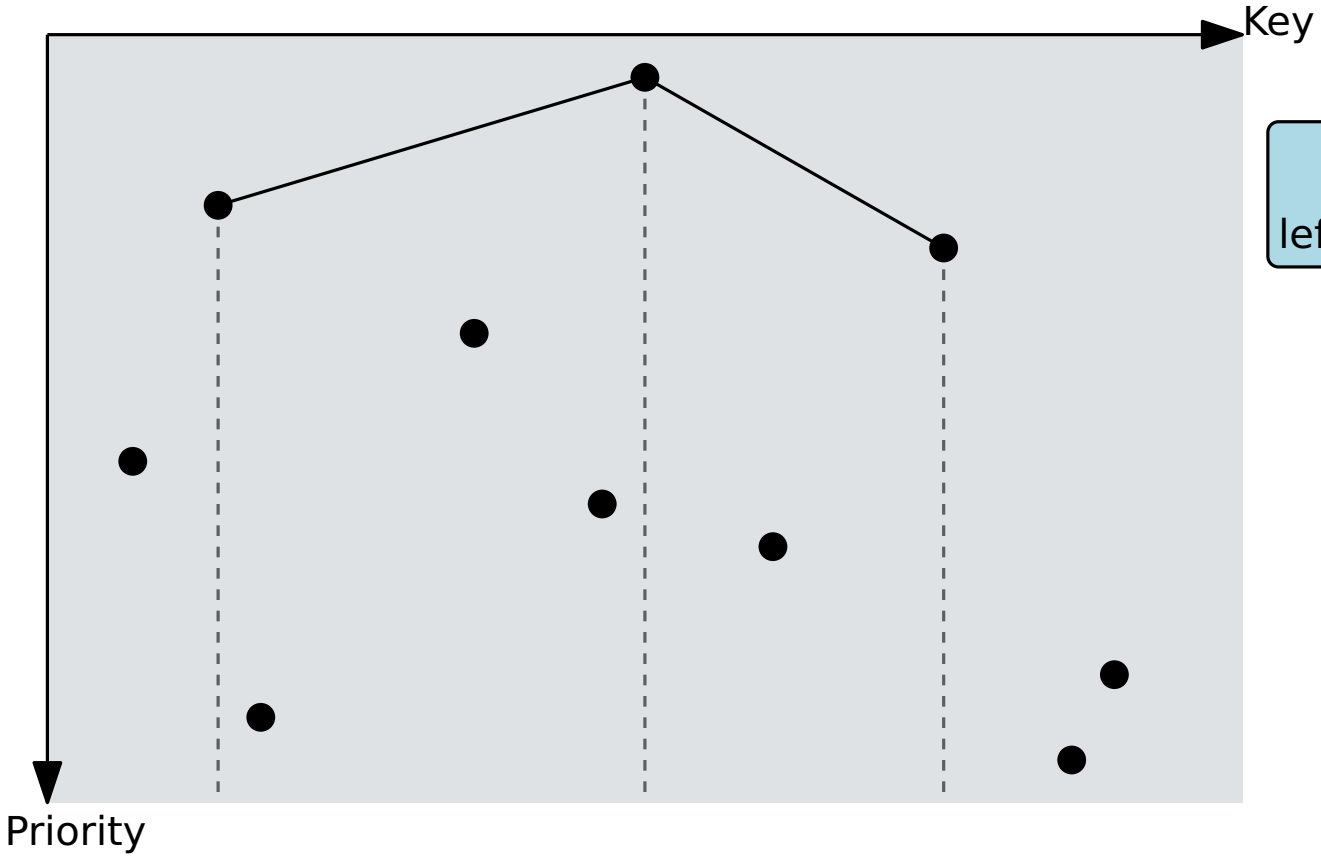

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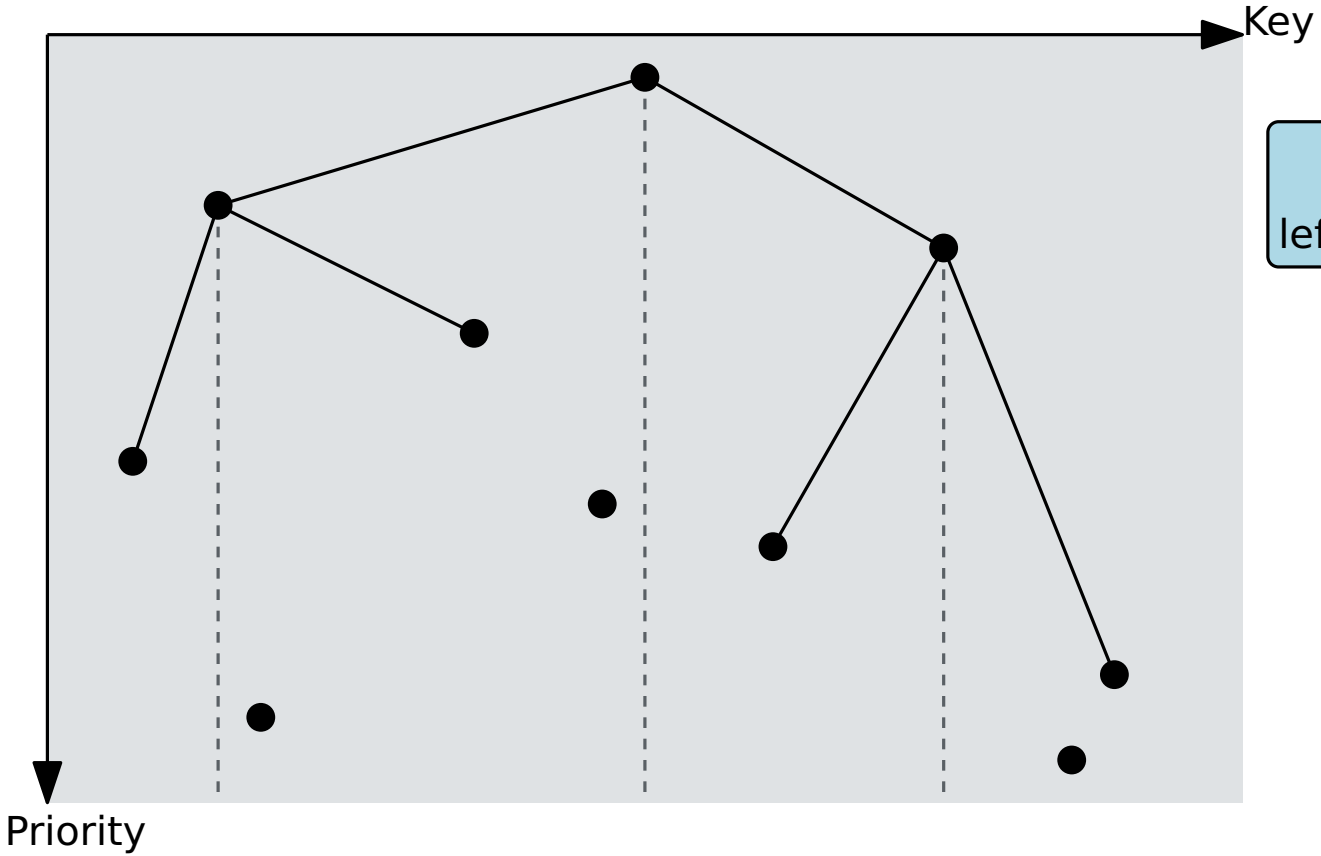

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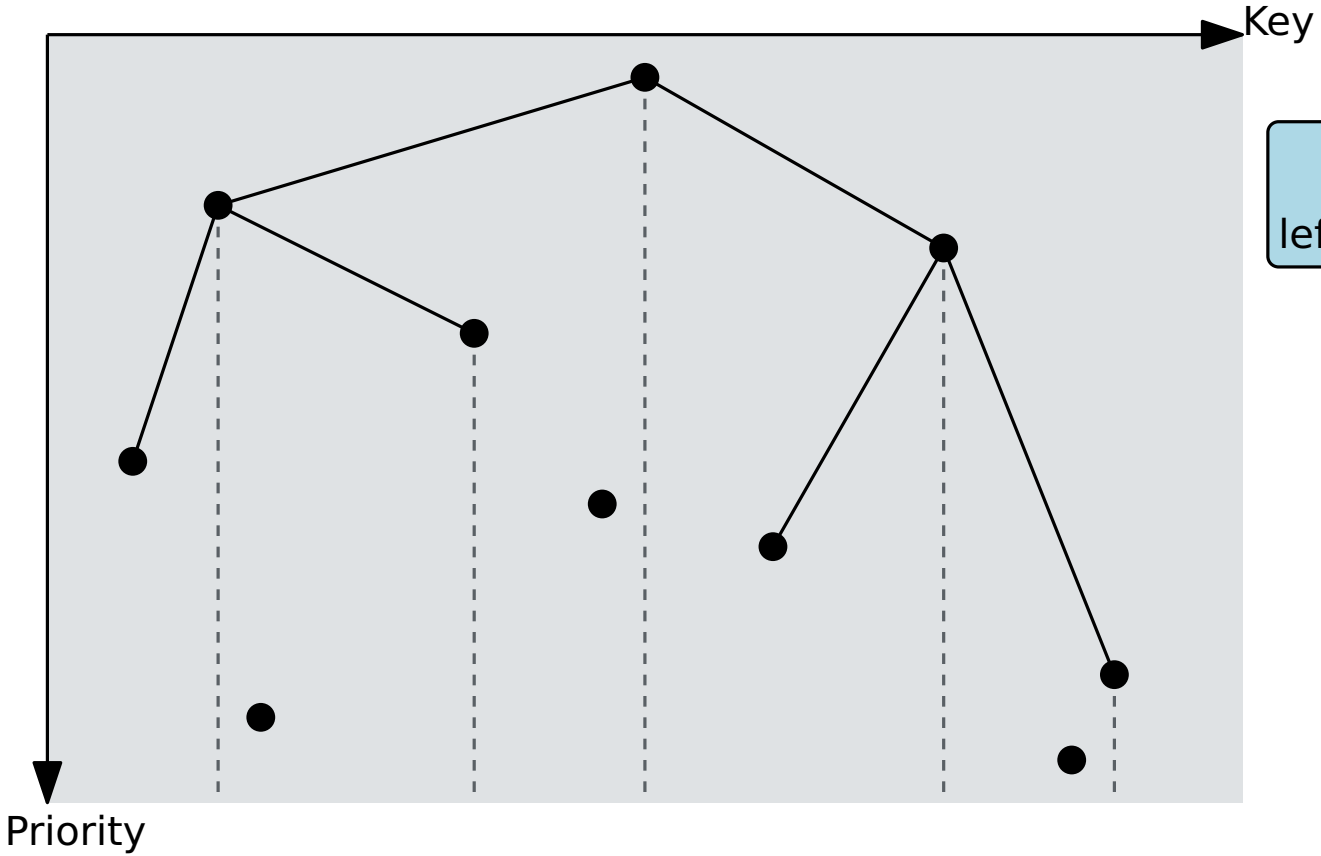

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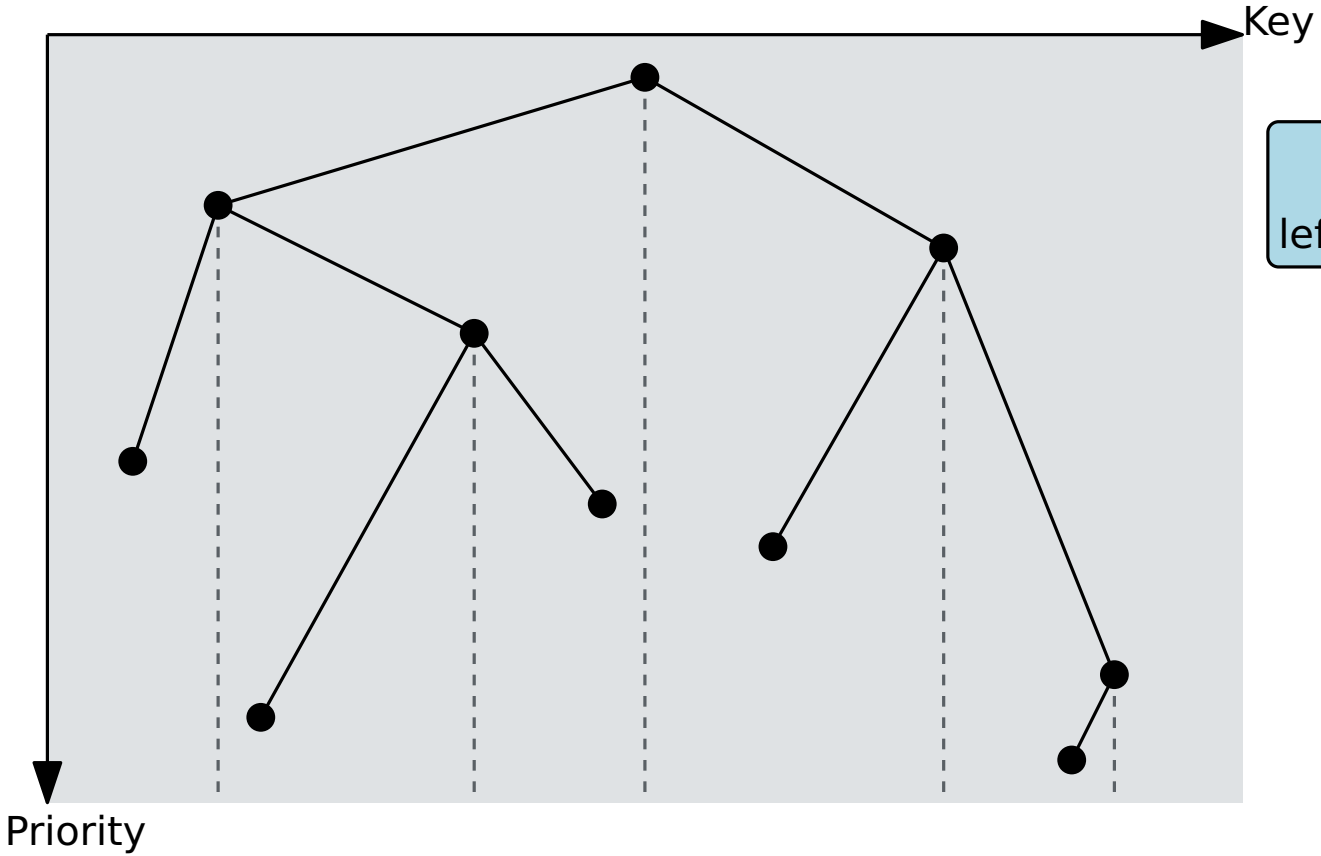

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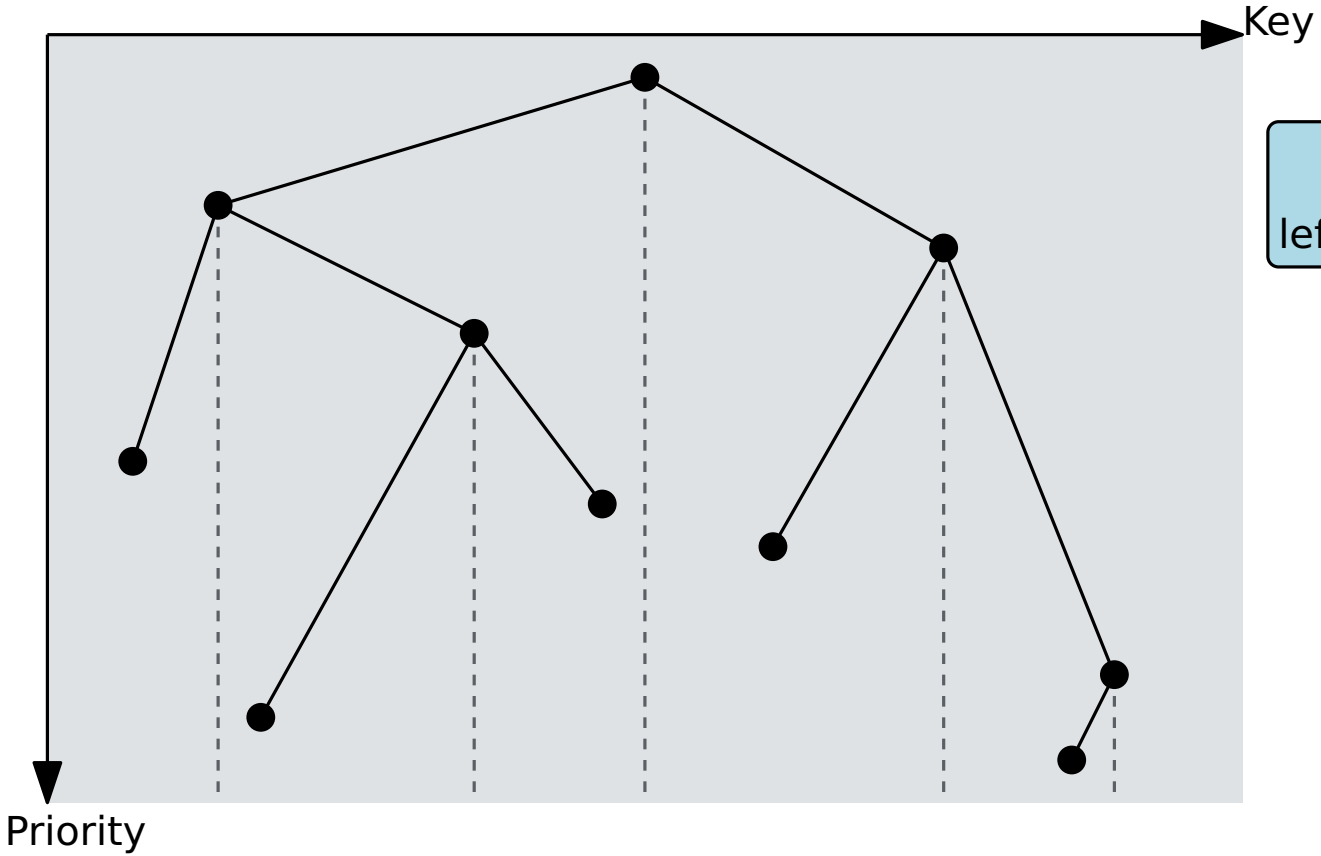

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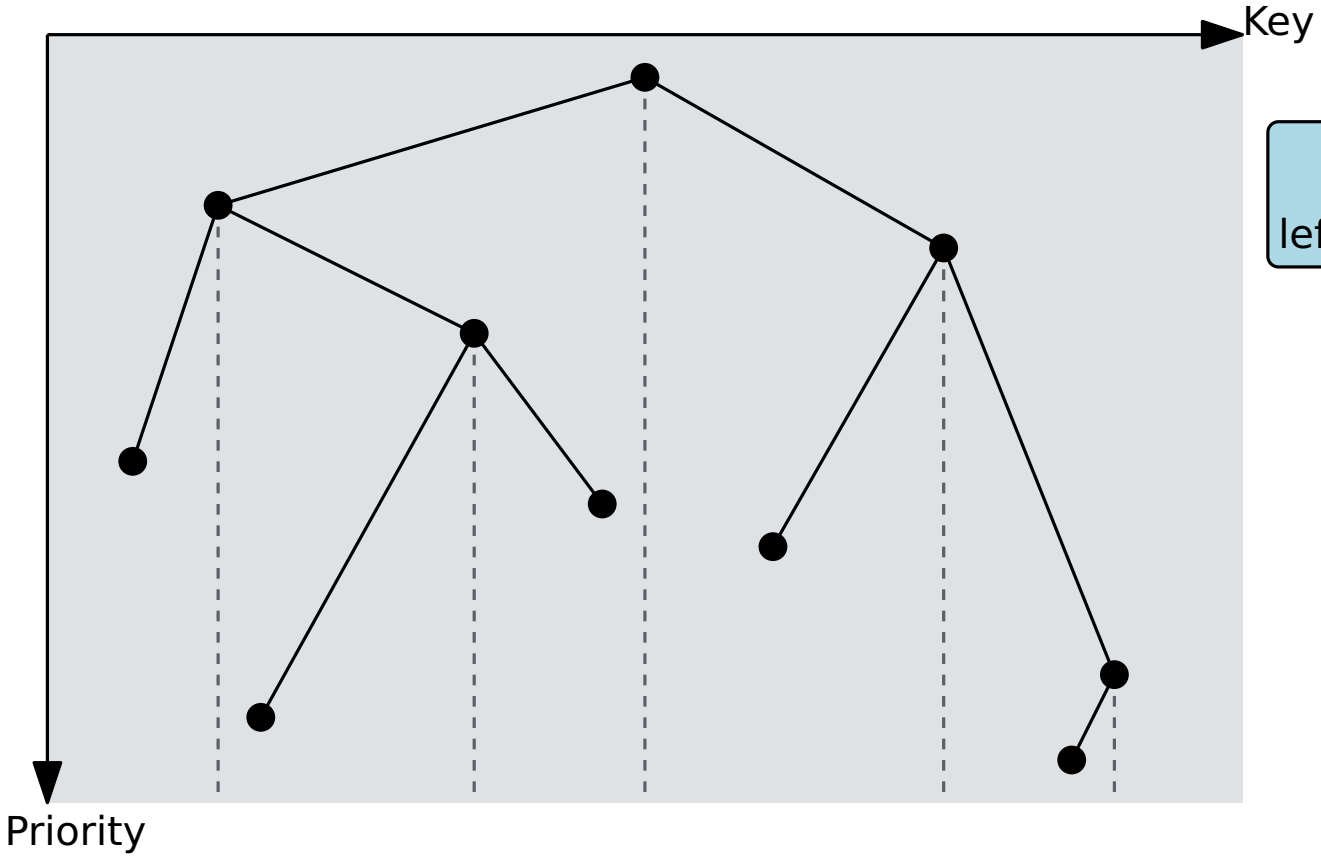

BST by key:

left child $<$ parent $<$ right child

Heap by Prio:

parent $<$ children

- for now assume unique values
- keys/prios uniquely determine a treap

Implementation

we break down everything to two operations

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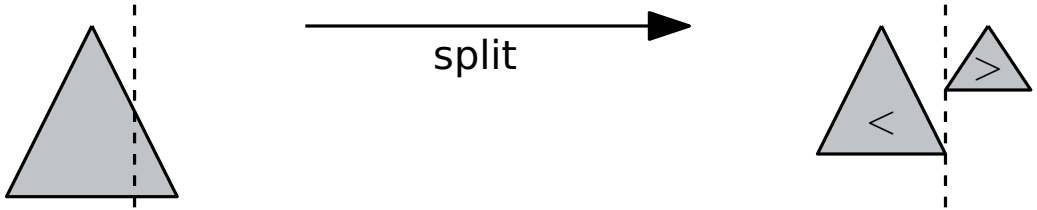

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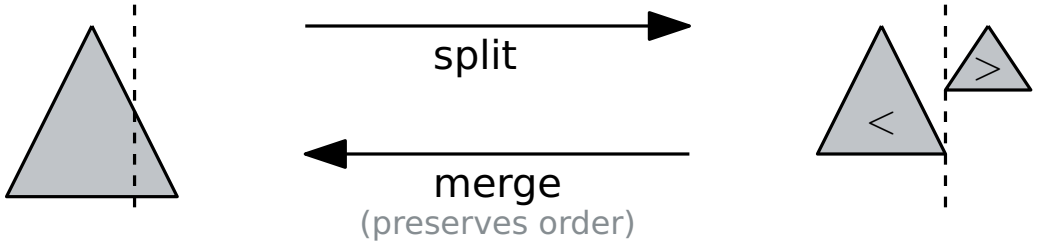

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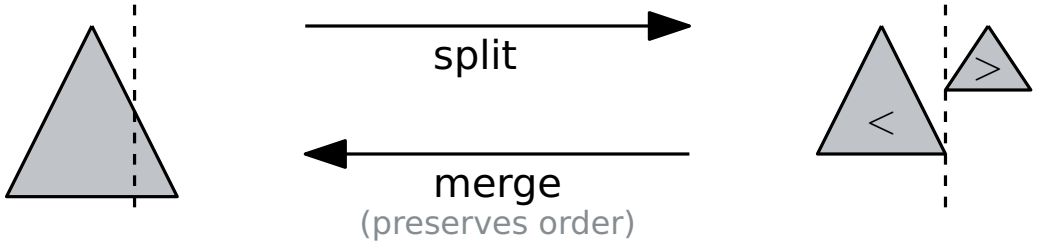

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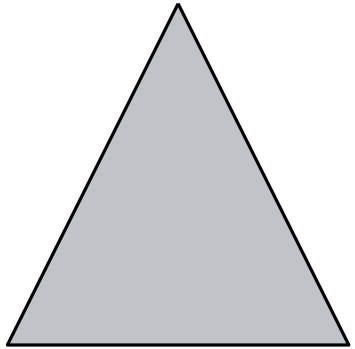

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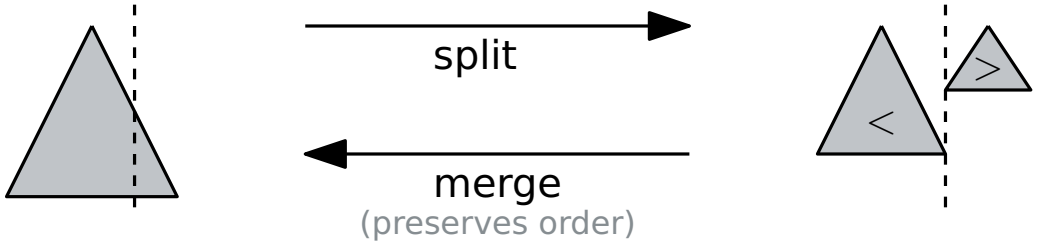

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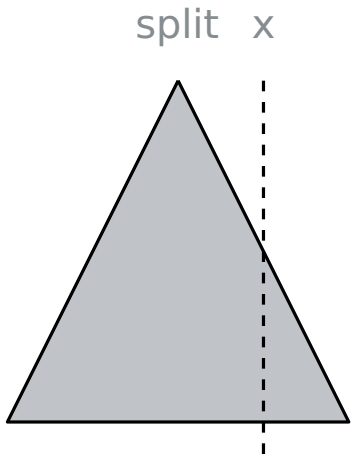

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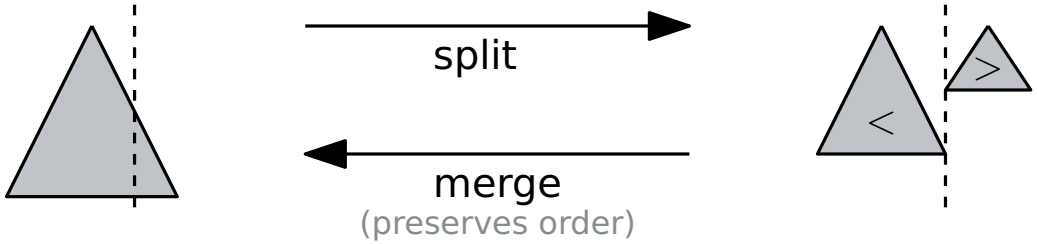

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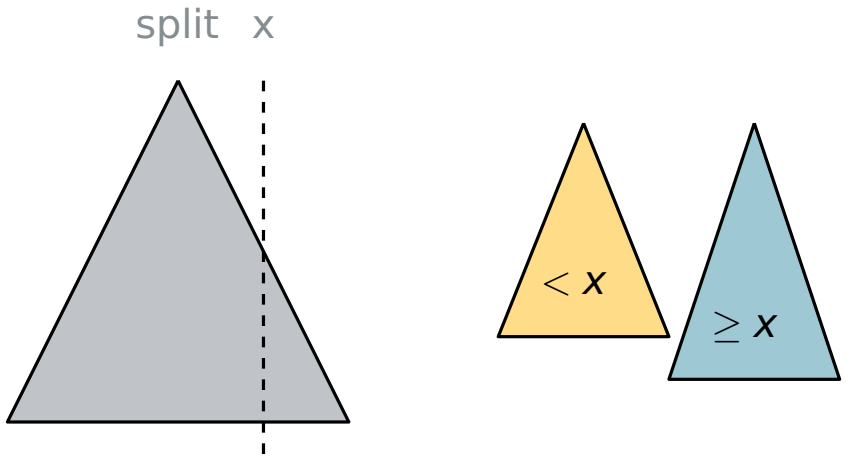

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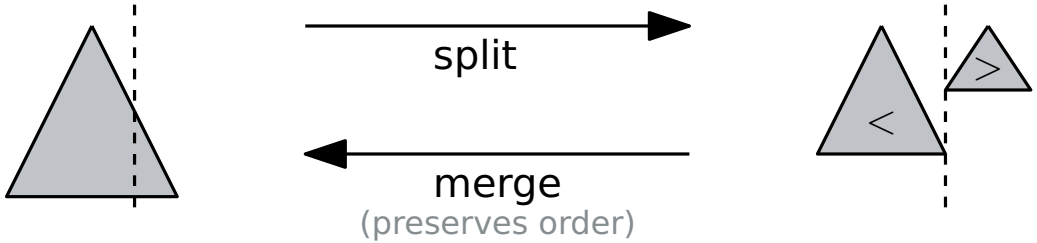

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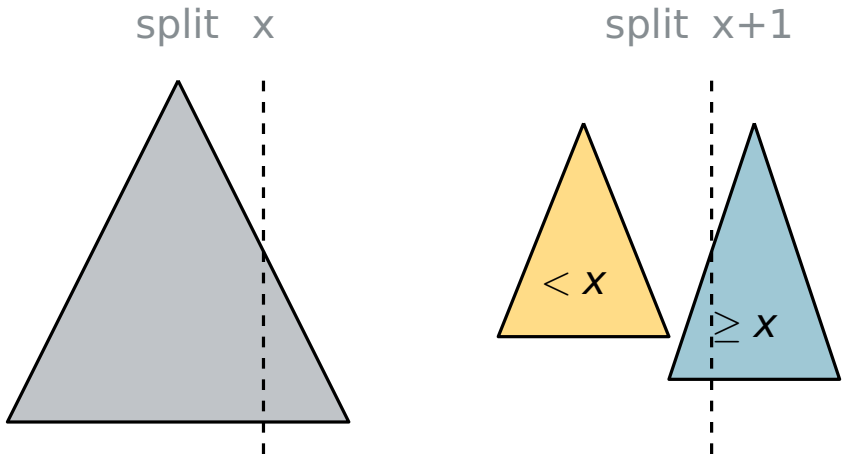

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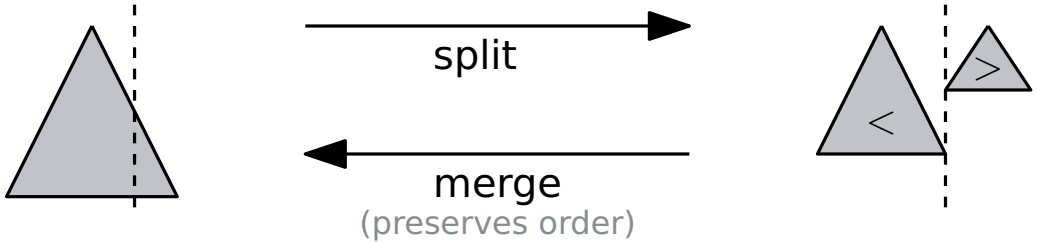

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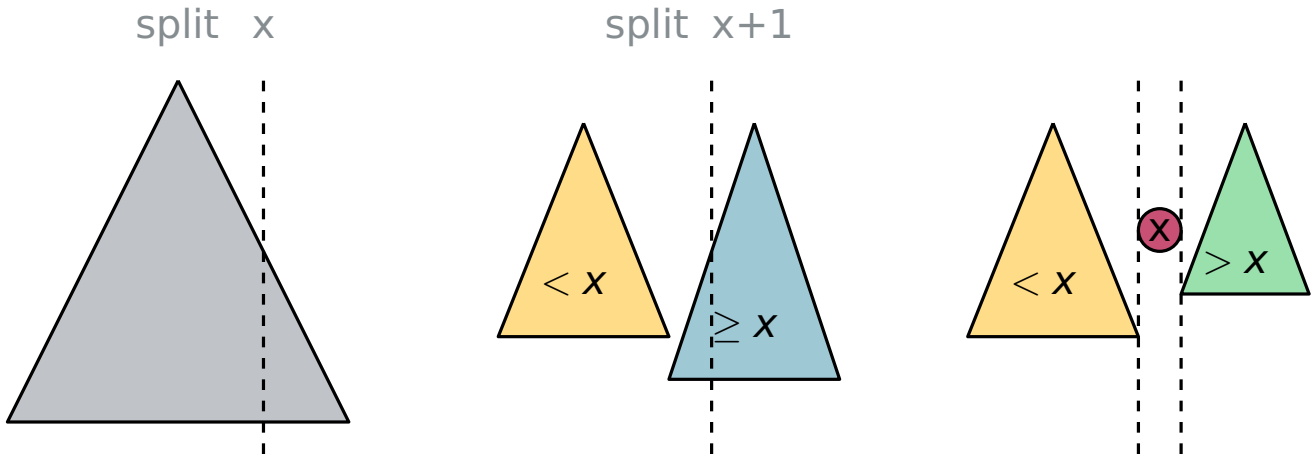

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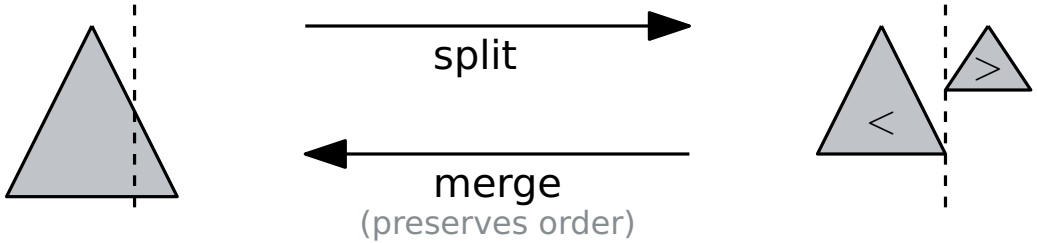

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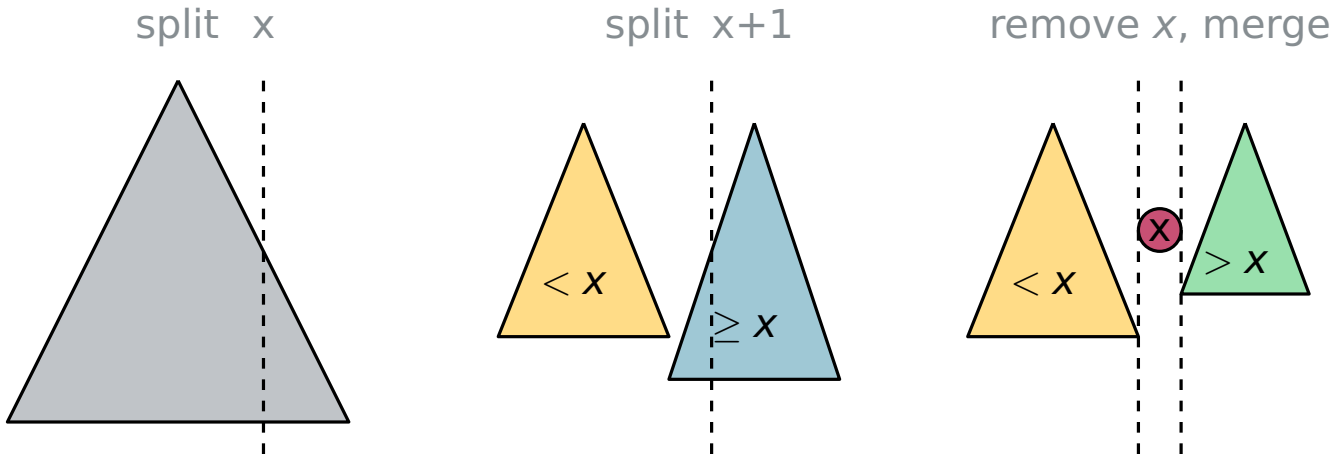

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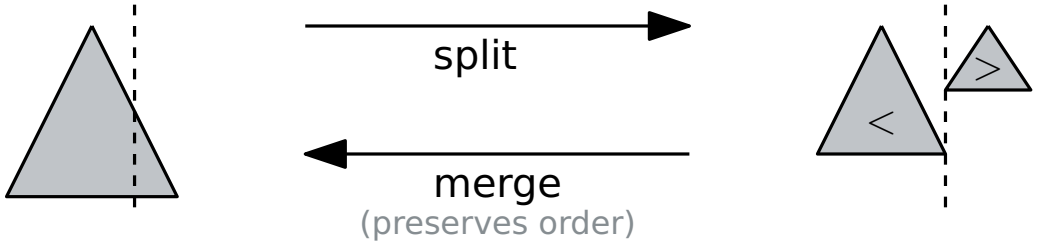

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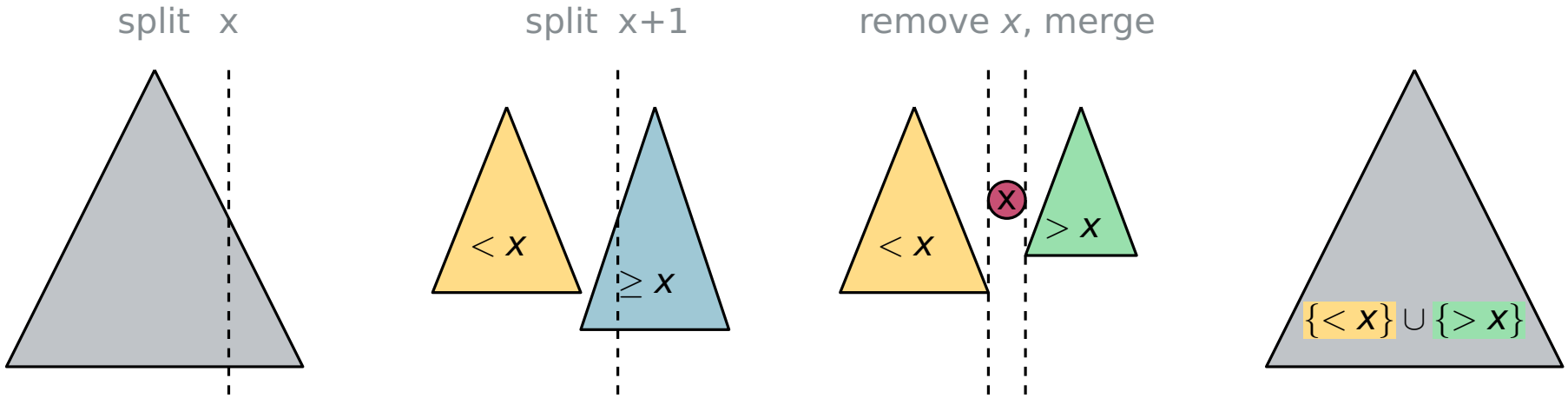
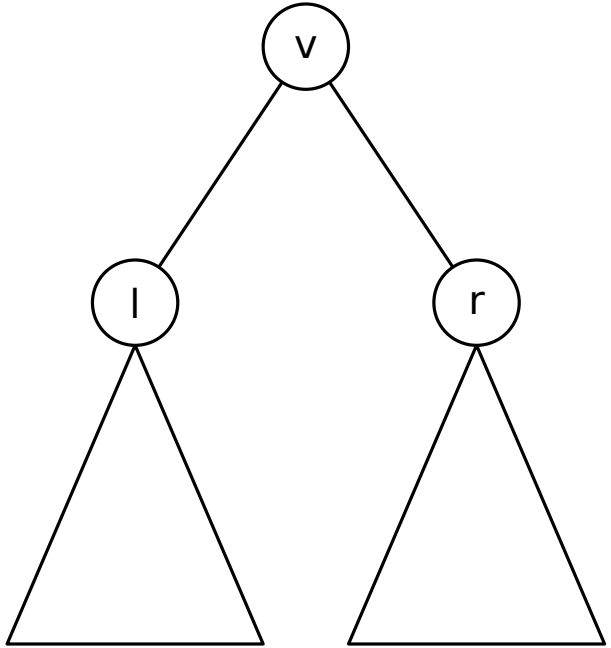


Split

returns two treaps containing all nodes $<$ and $\geq$ the given key respectively



split(Node v, Key x)

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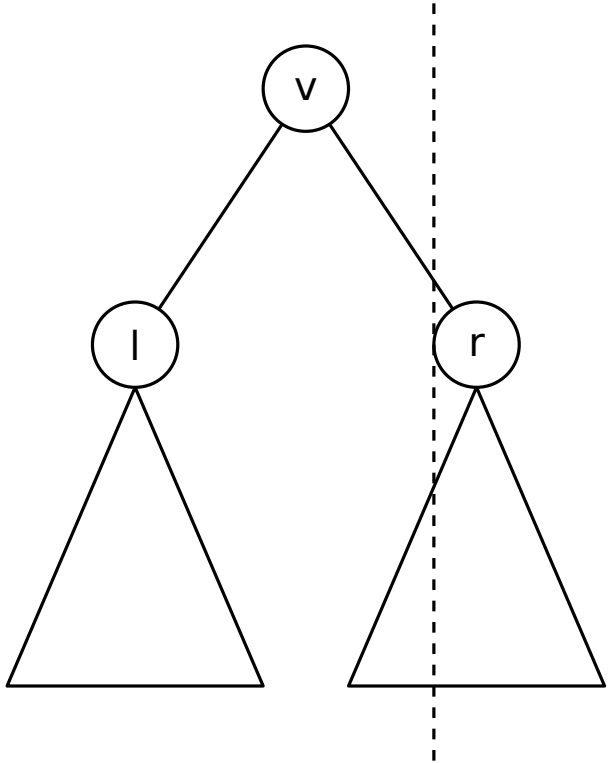


```
split(Node v, Key x)
    if v.key < x:
        ...
    else
        ...
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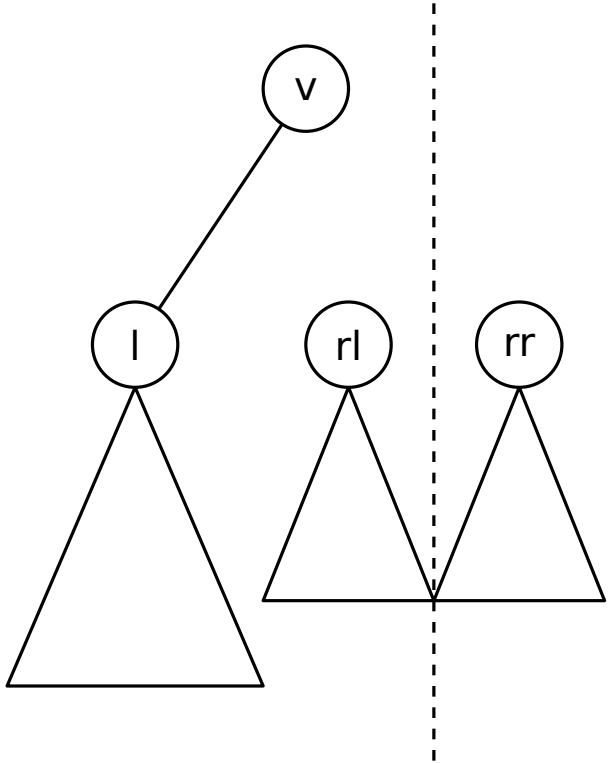


```
split(Node v, Key x)

    if v.key < x:
        rl, rr = split(v.right, x)



    else
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```

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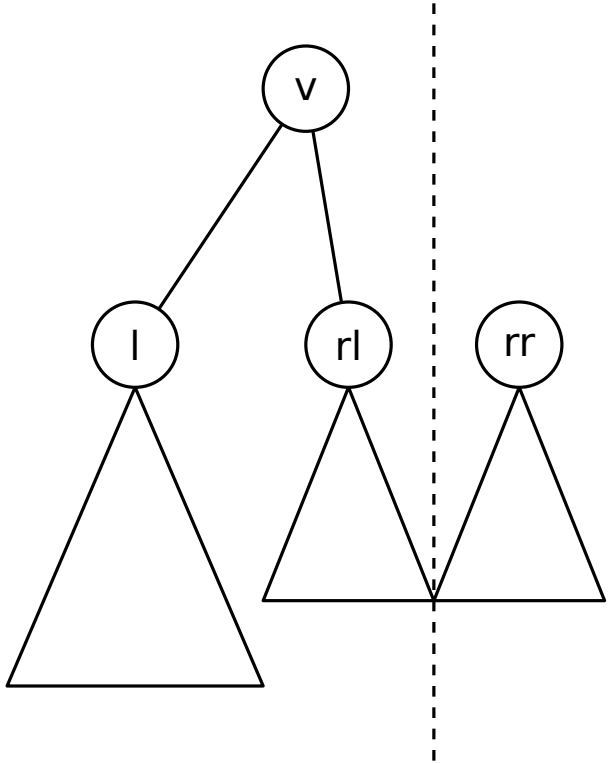


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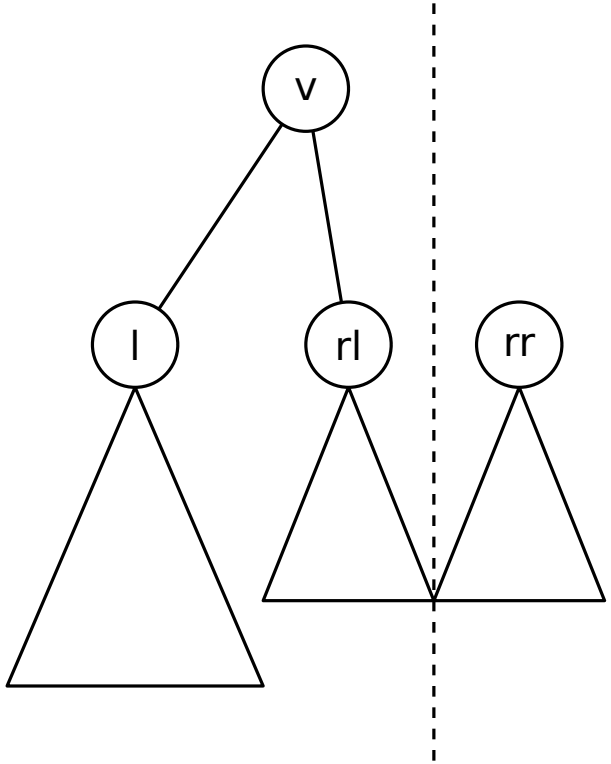


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        return {v, rr}
    else
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```
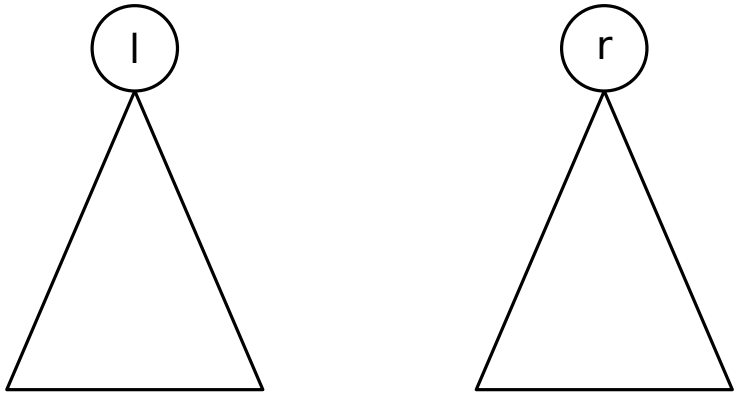

Merge

merge two treaps into one preserving horizontal ordering (left, right)



merge(Node l, Node r)

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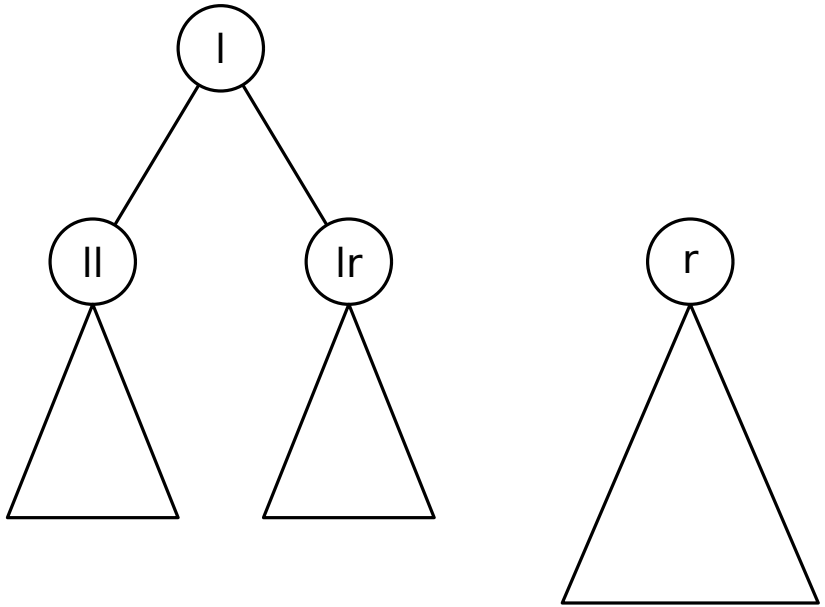


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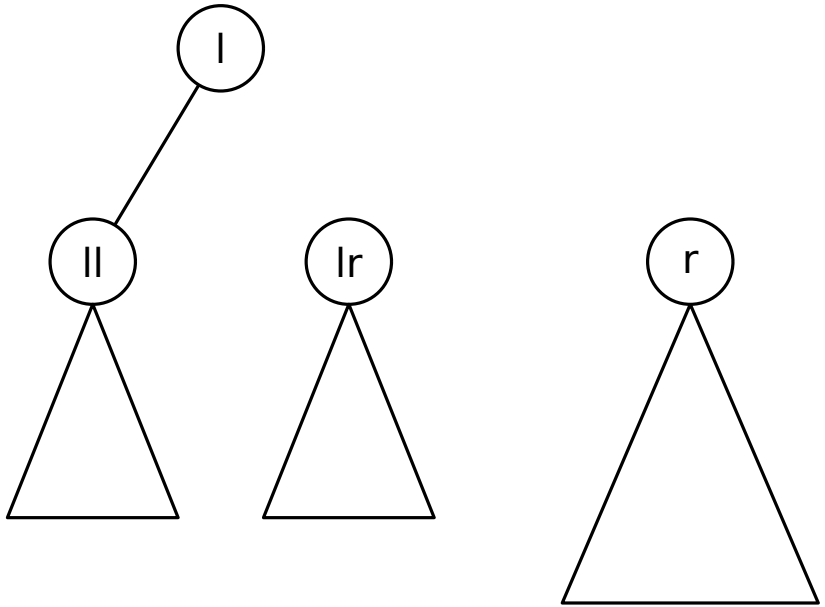


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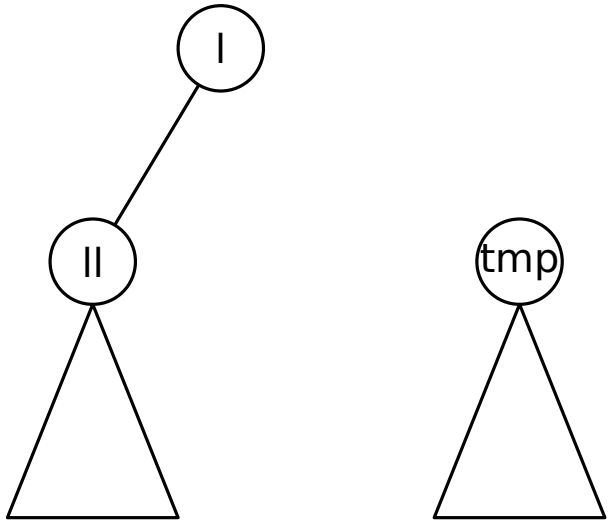


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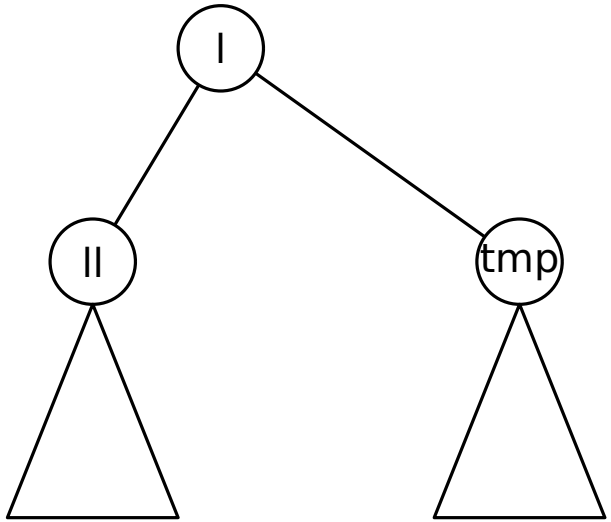


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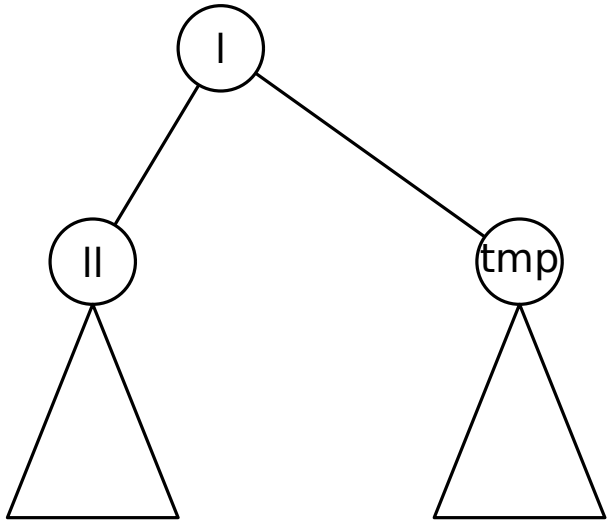
Merge

merge two treaps into one preserving horizontal ordering (left, right)

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What a Treap can do

Self balancing BSTs
(like a set)

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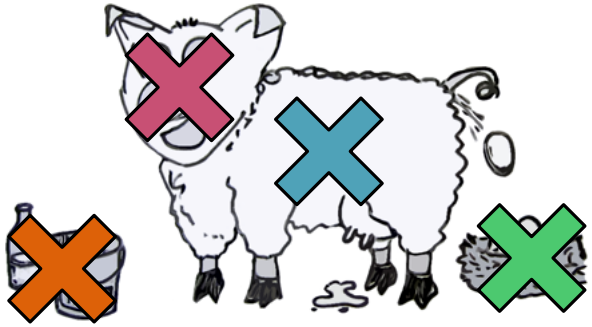

binary search tree

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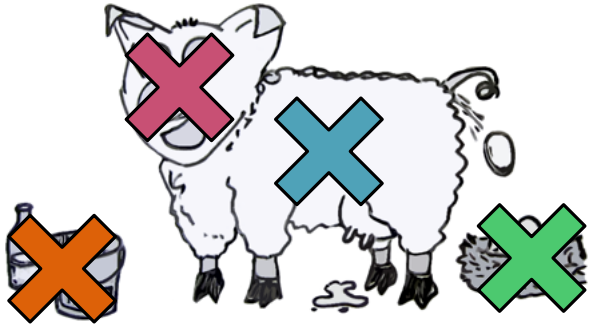

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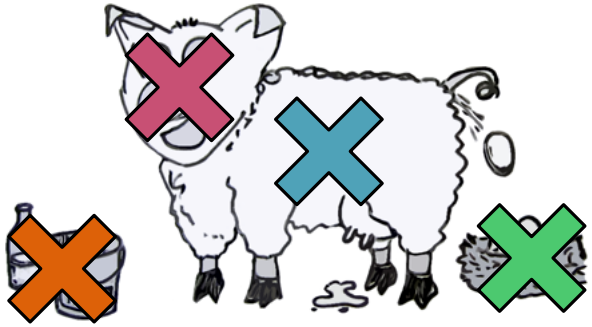

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we now want to answer queries about our data

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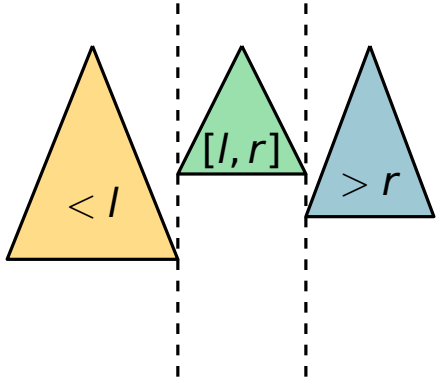

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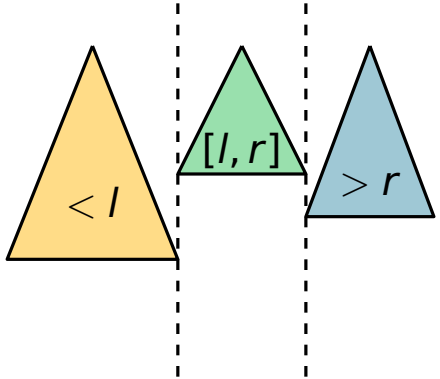

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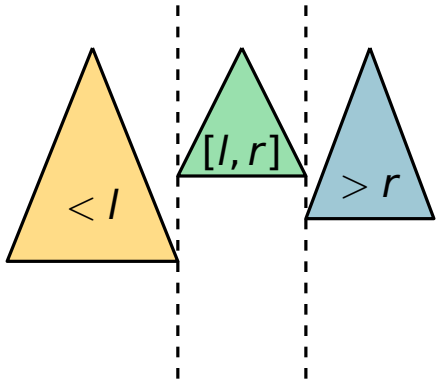

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        tmp = merge(l.right, r)
        l.right = tmp
        update(l)
        return l
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            ...
```

5 min break

- an important operation on sets is called **order of key**
- given a key k, it returns the number of elements in the set that are smaller than k

How would you support **order of key** in a treap?

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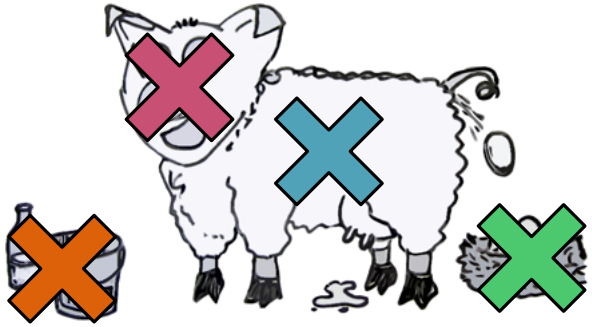

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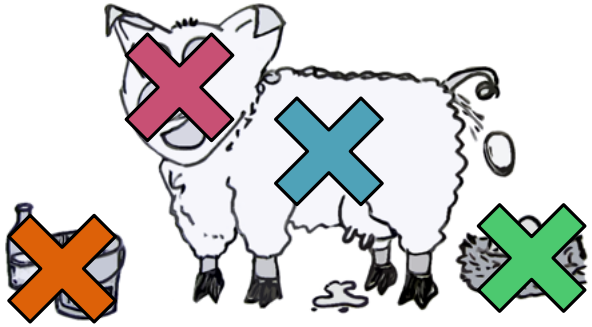

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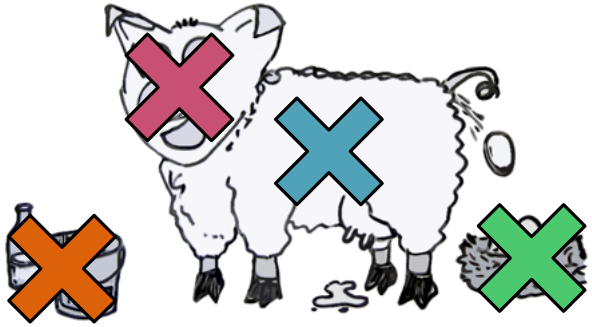

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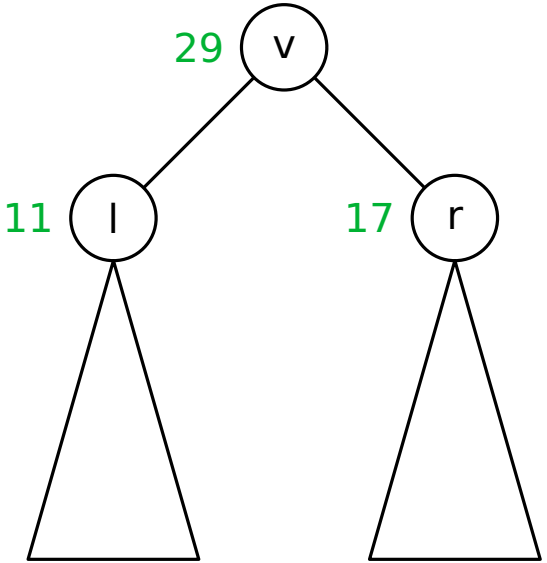

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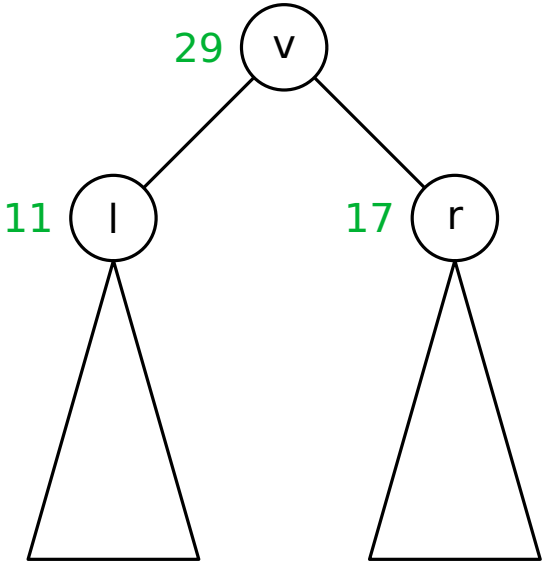

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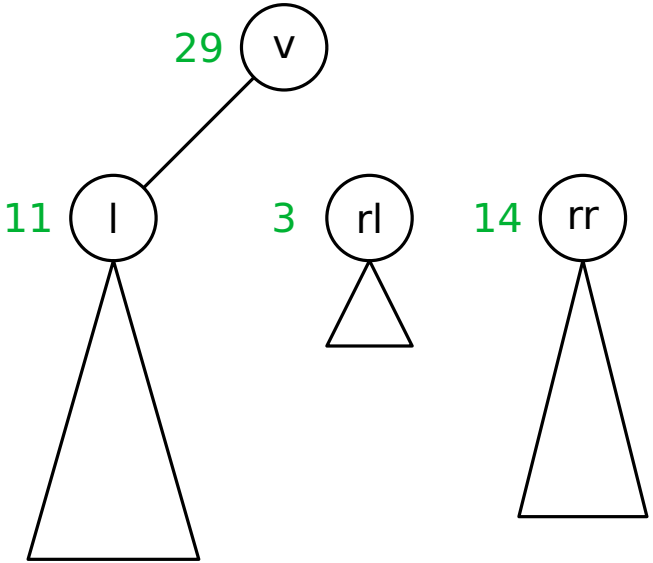

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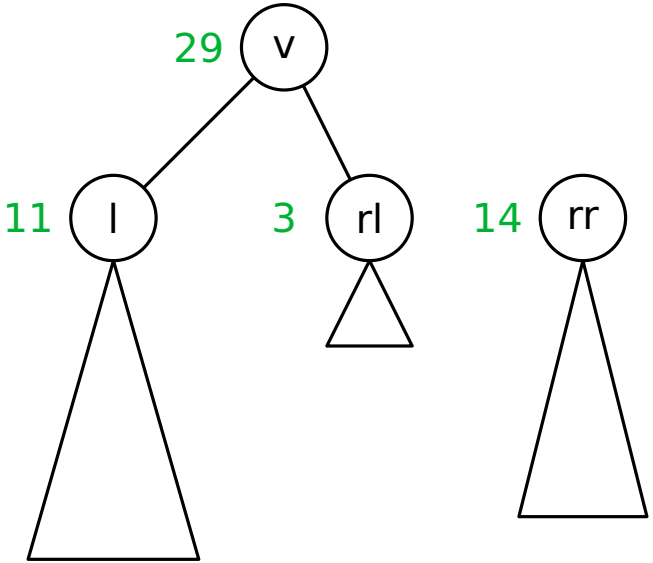

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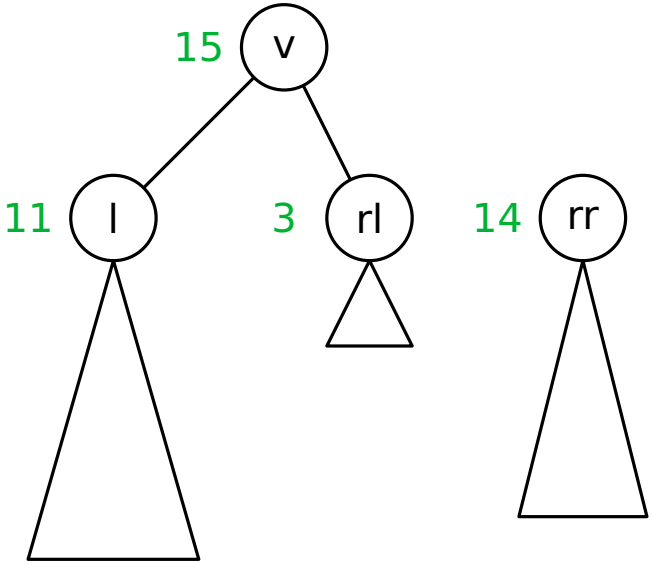

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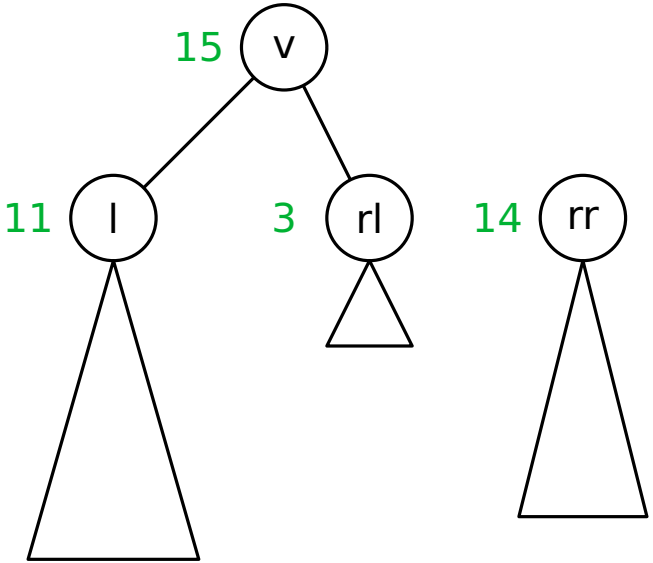

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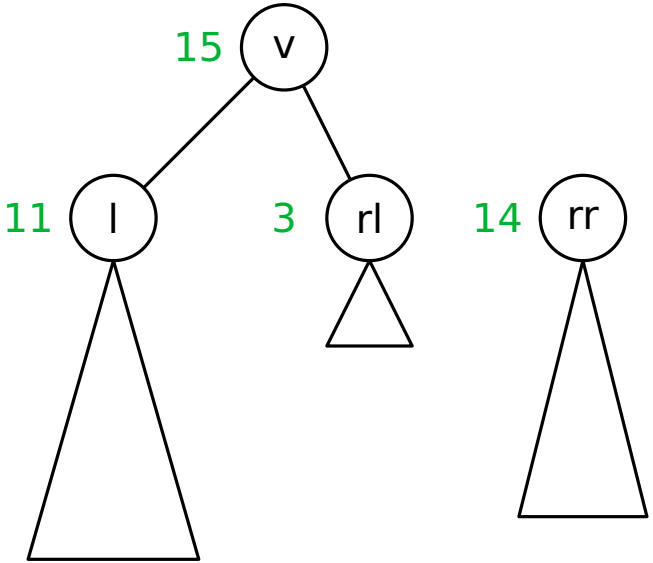

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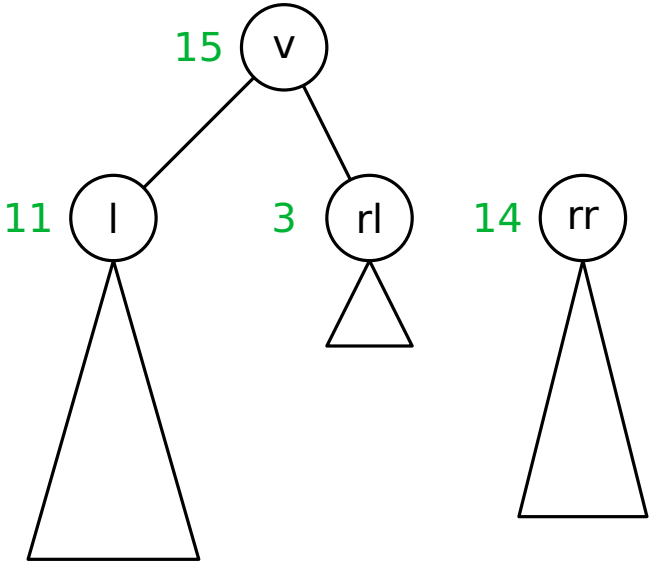

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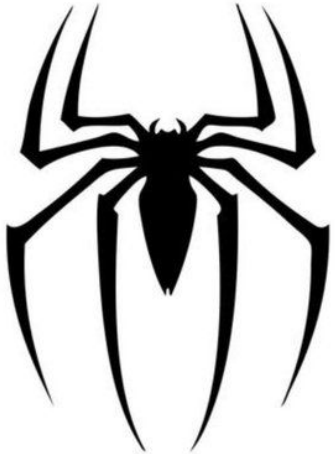

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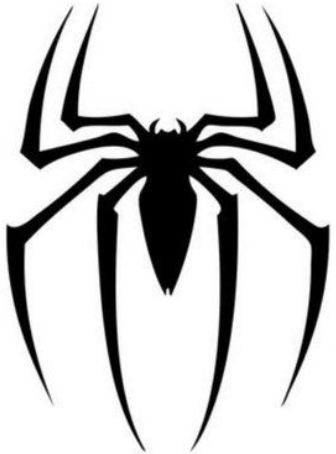

with great power comes great responsibility

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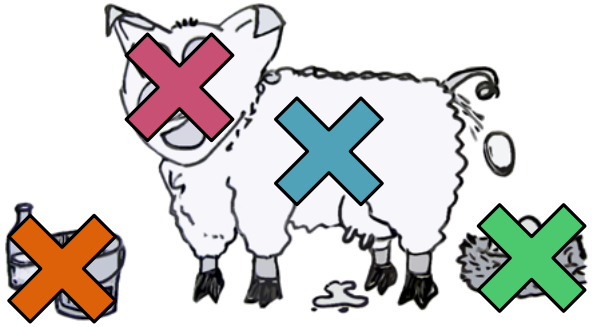

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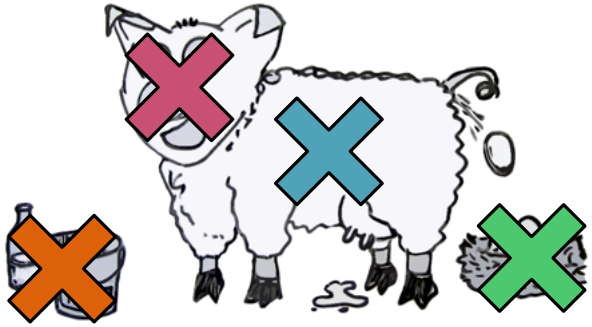

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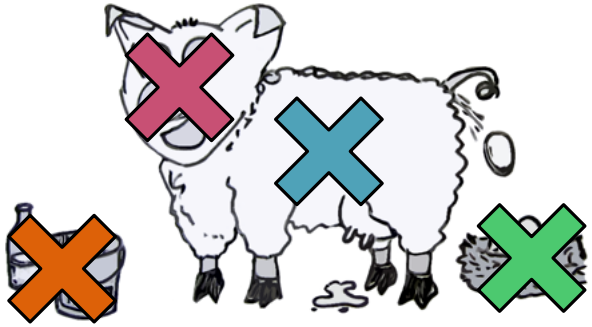

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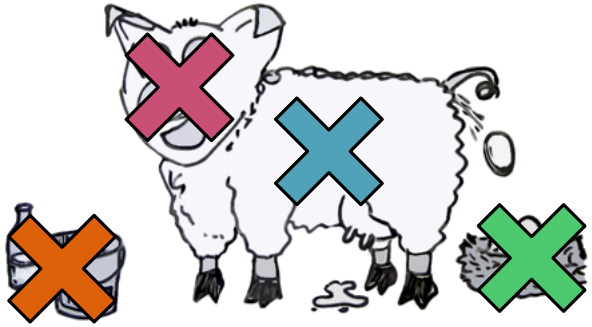

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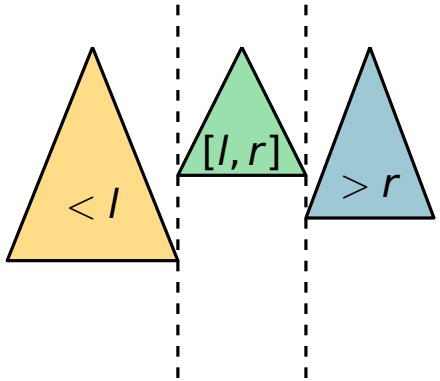

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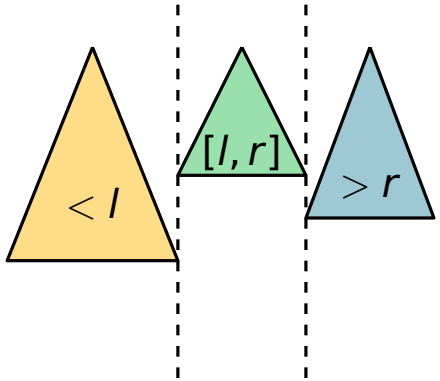

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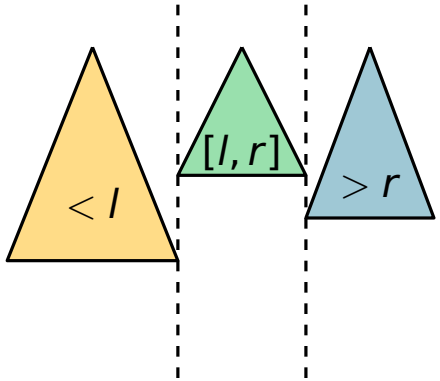

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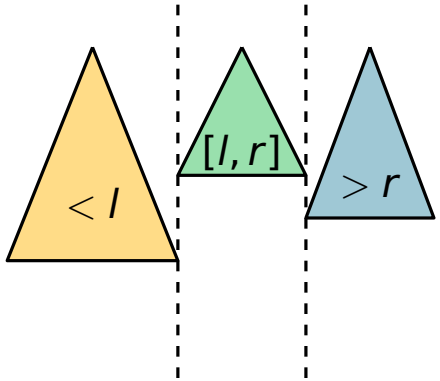
- cool application: reverse order of range

  reverse order $\equiv$ for all nodes swap left and right child




```
push(Node v)

    if v.reversed:
        swap(v.left, v.right)
        v.reversed = False
        if v.left:    v.left.reversed    ^= 1
        if v.right:   v.right.reversed   ^= 1
```

Treaps + Lazy Propagation

- lazy propagation again is a one-line change
- don't forget to add it to **split-size**

```
split(Node v, Key x)
    if v.key < x:
        push(v)
        rl, rr = split(v.right, x)
        v.right = rl
        update(v)
        return {v, rr}
    else
            ...
```

```
merge(Node l, Node r)
    if l.prio < r.prio:
        push(l)
        tmp = merge(l.right, r)
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if you read from children here (e.g. split-size), push must be earlier!!!

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What a Treap can do

Self balancing BSTs
(like a set)

- split by value
- find value
- insert value
- delete value
- find all values in interval
- delete interval of values
- query interval for aggregates
- order of key

Implicit Treaps
(like a really cool array)

- split by size
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- insert after index
- delete at index
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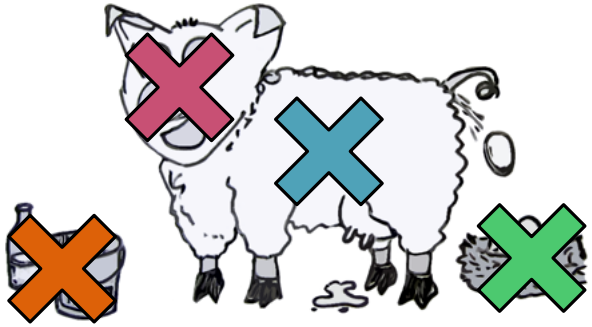
- move range



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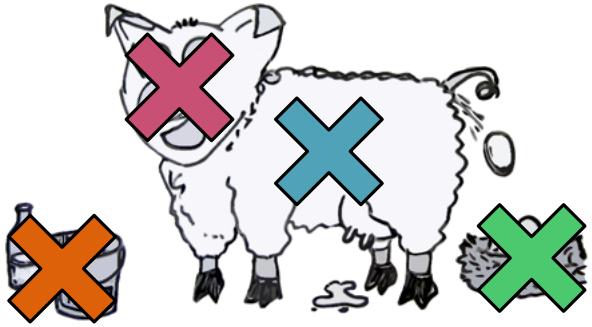

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    pros: Node stores two 32-bit ints instead of two 64-bit pointers
    cons: ugly

It’s dangerous to go alone! Take this.

```
void debug(Treap* t) {
        if(!t) return;
        push(t);
        debug(t->left);
        cout << t->key << ' ';
        debug(t->right);
 }
```