Karlsruhe Institute of Technology

Hasso Plattner Institut
IT Systems Engineering | Universität Potsdam

Competitive Programming

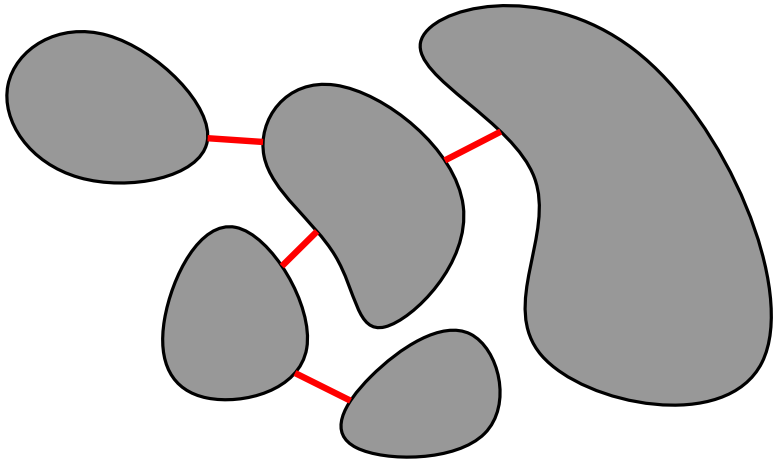
Winter Term 23/24

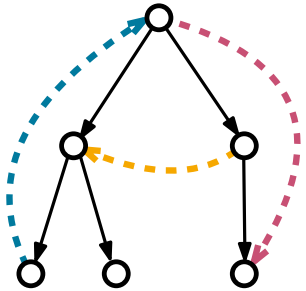
DFS-Tree and Applications

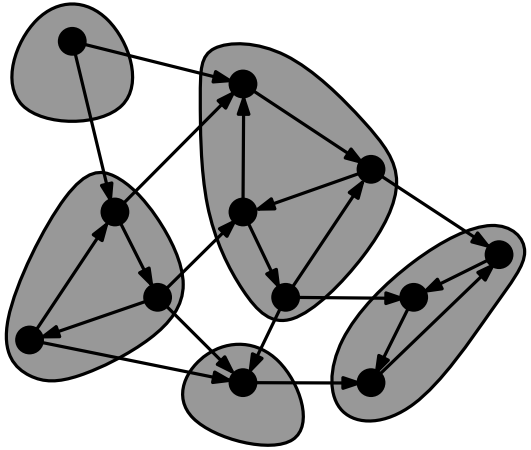
Kirill Simonov
Algorithm Engineering Group
HPI

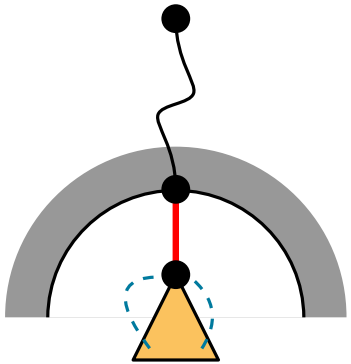
Michael Zündorf
Scalable Algorithms Group
KIT

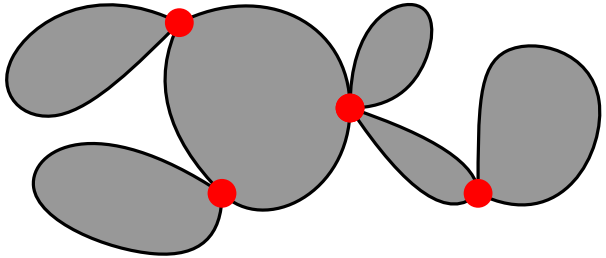
Overview

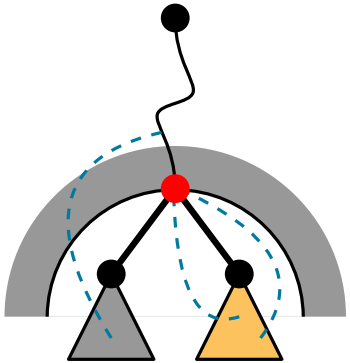












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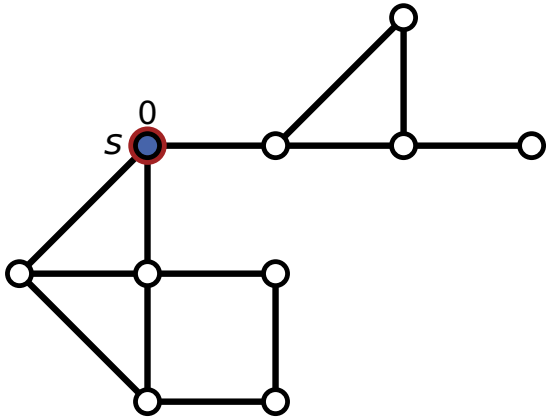

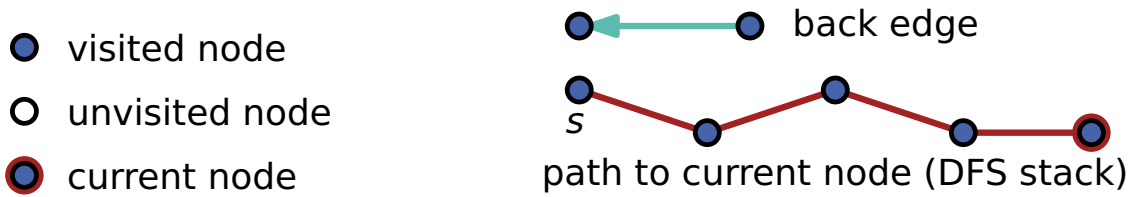

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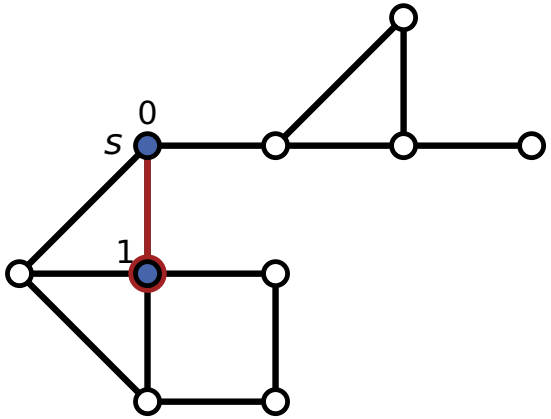

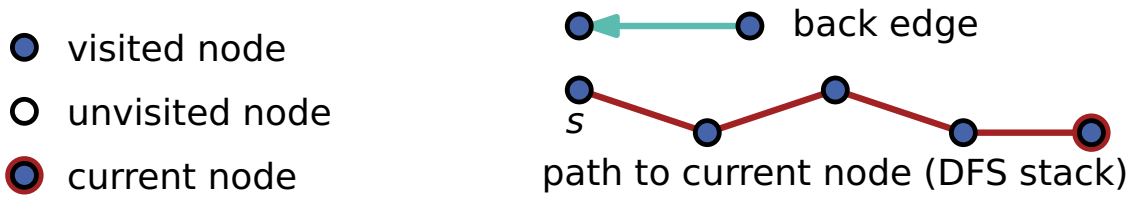

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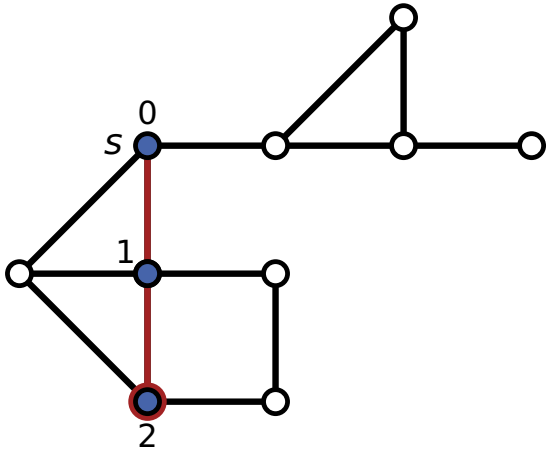

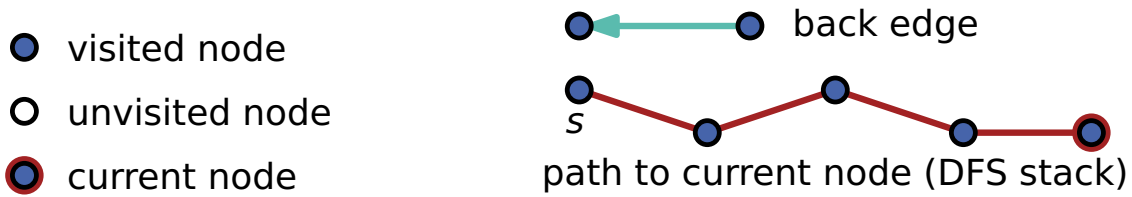

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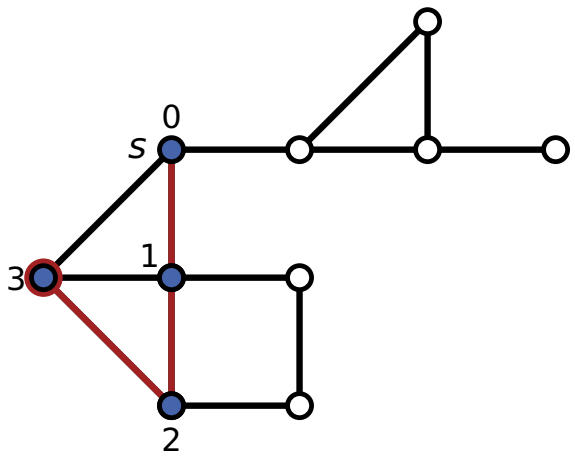

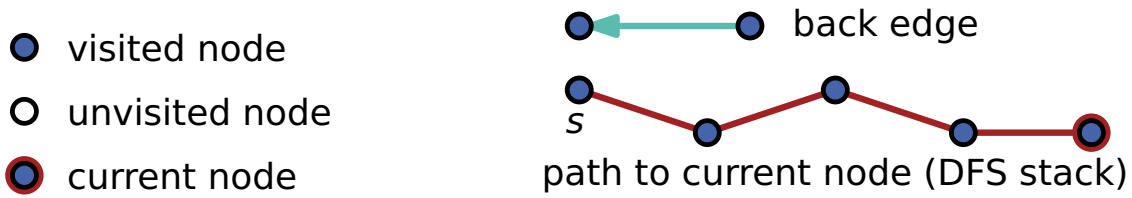

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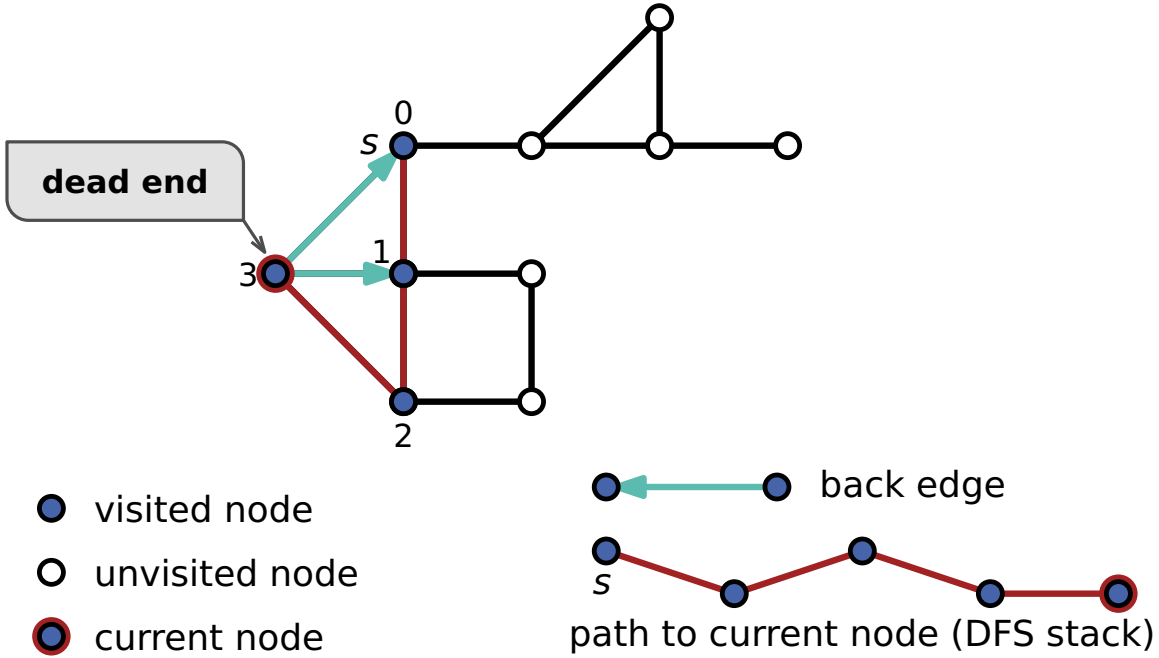

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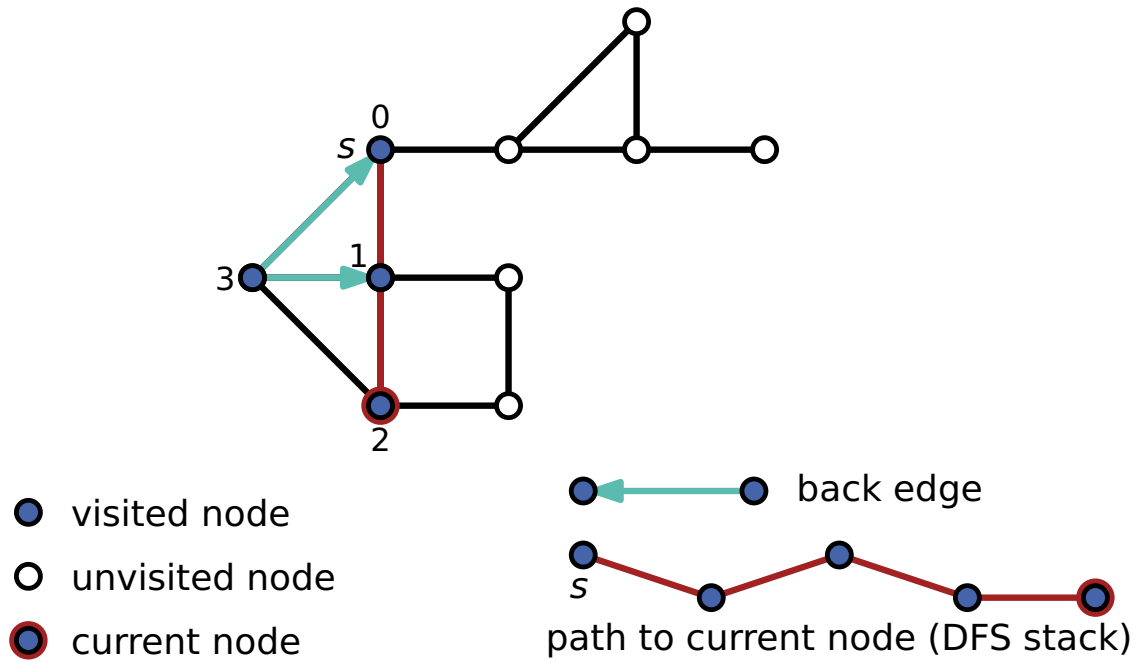

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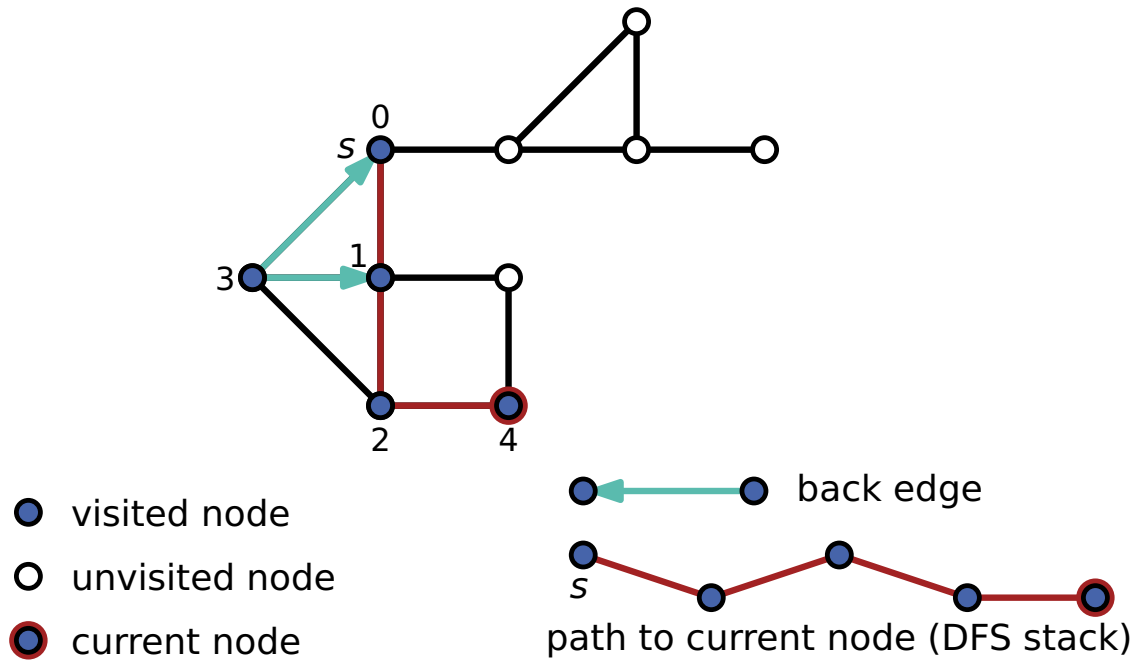

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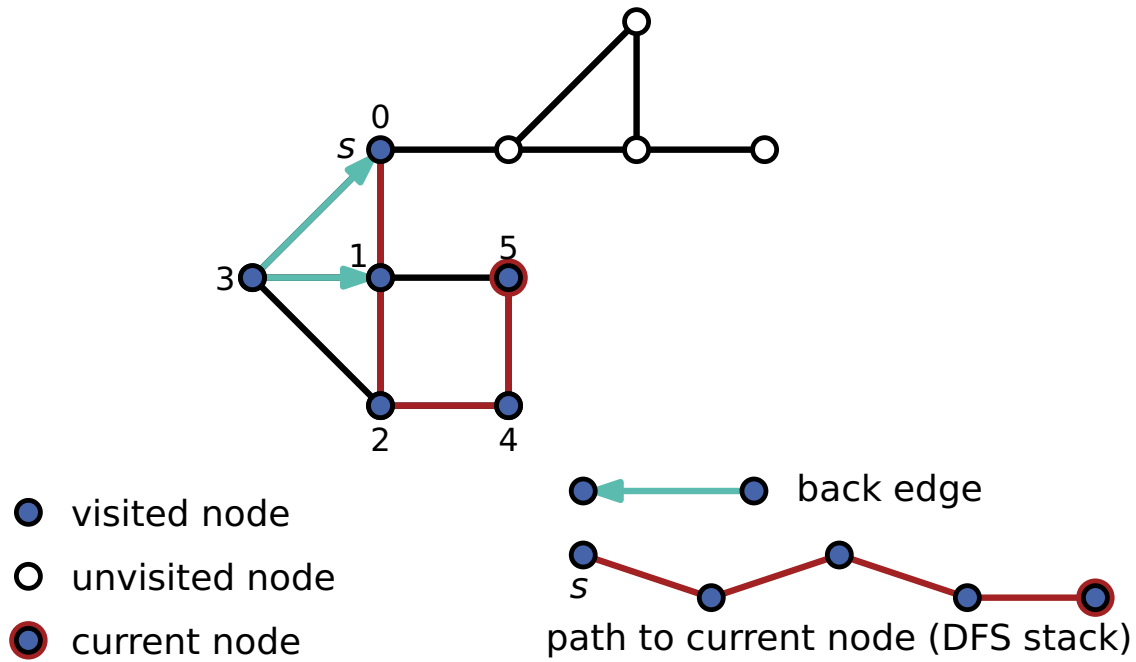

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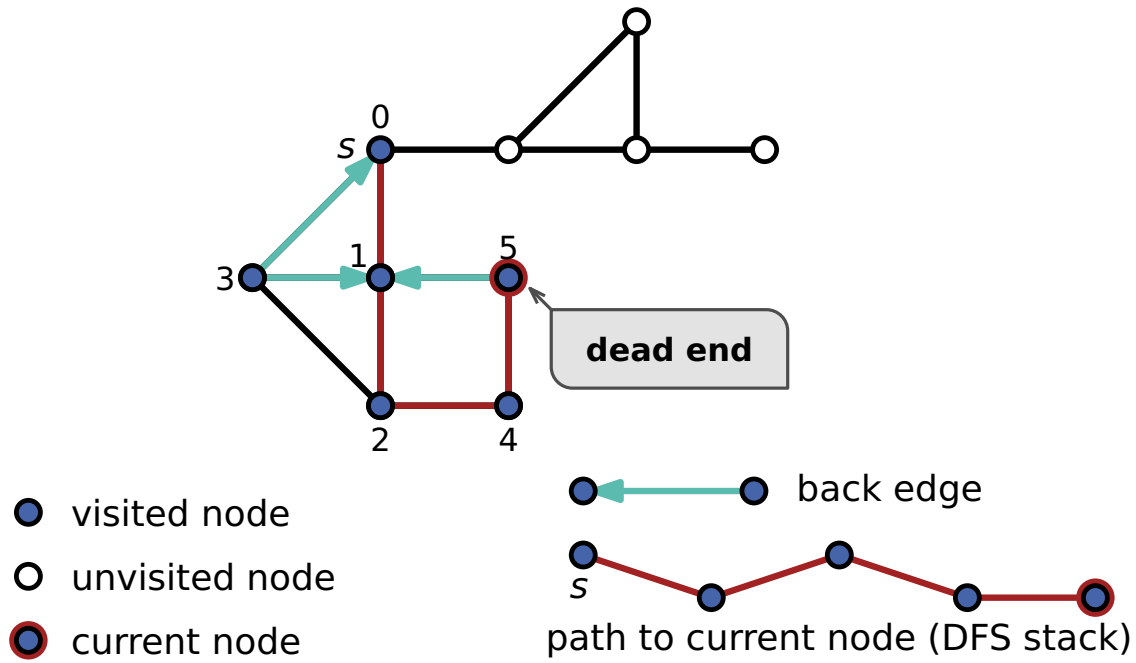

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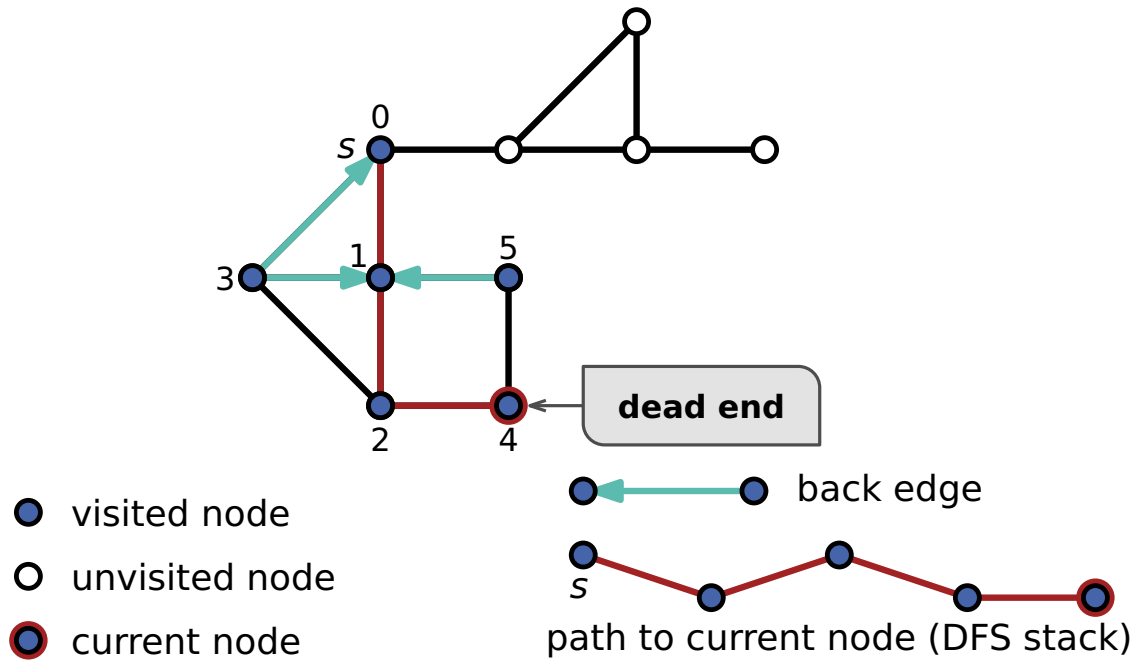

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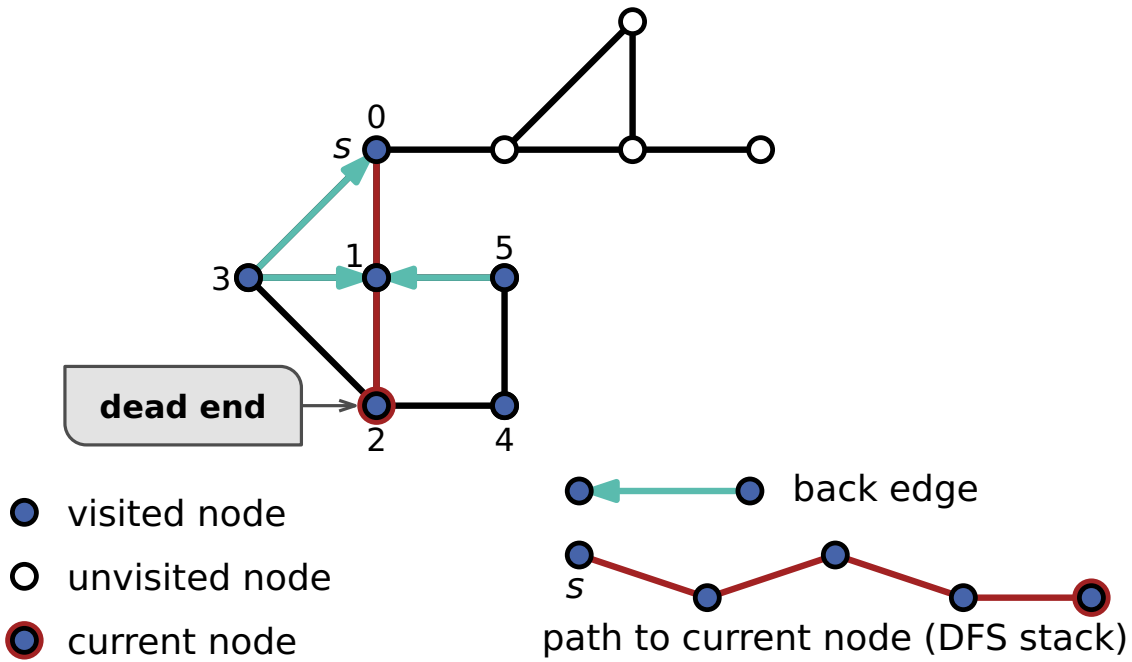

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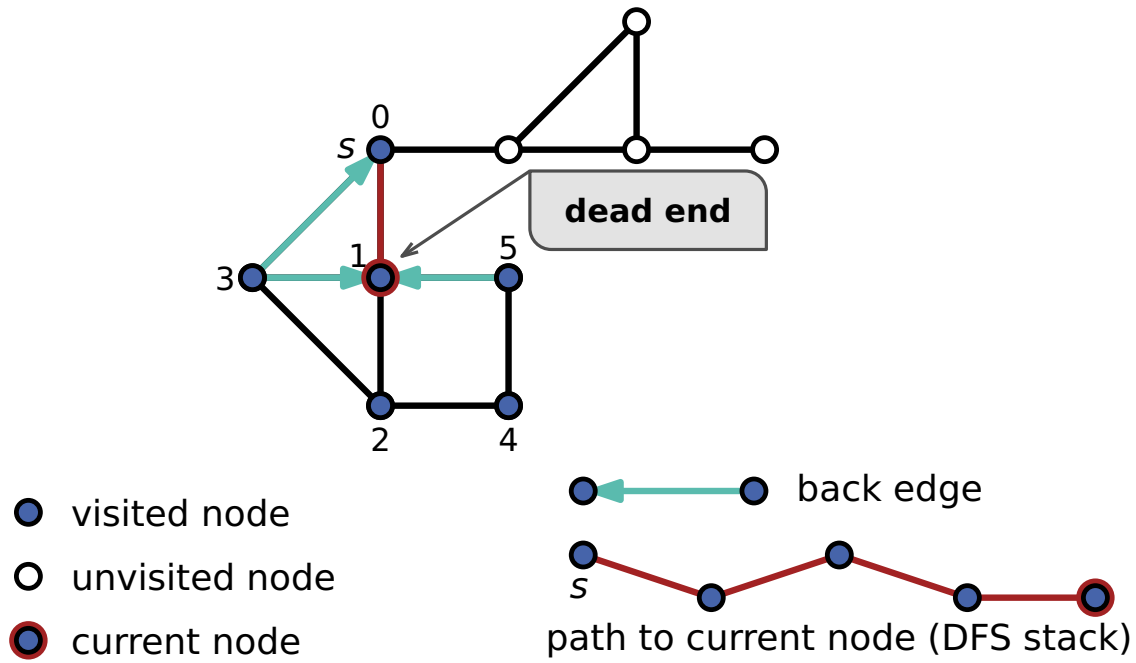

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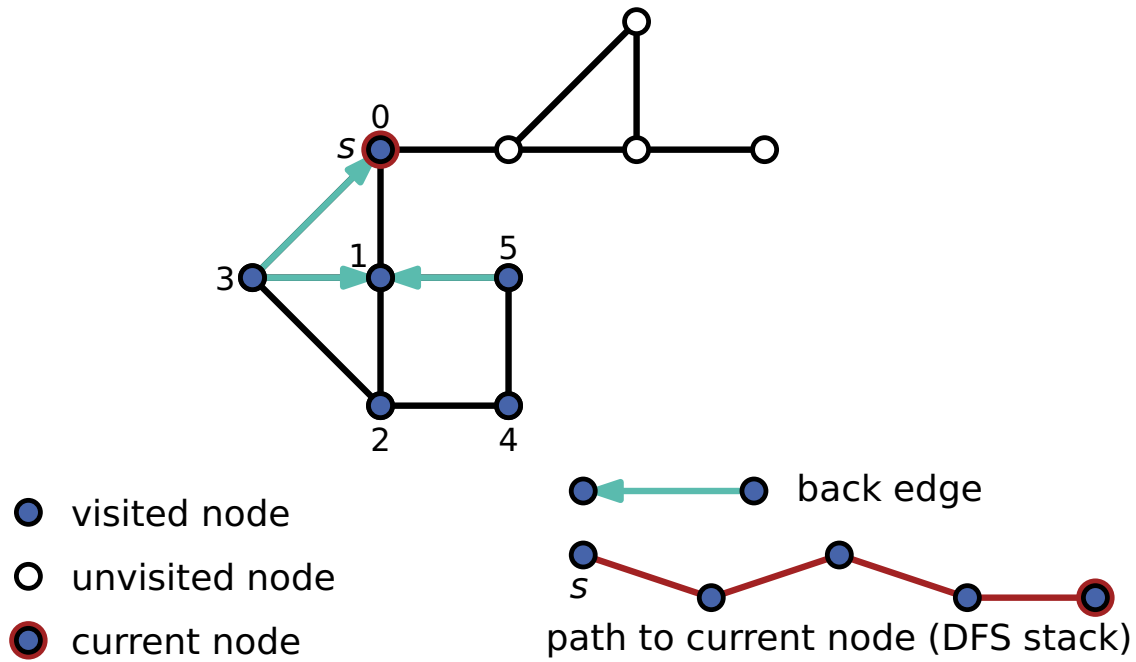

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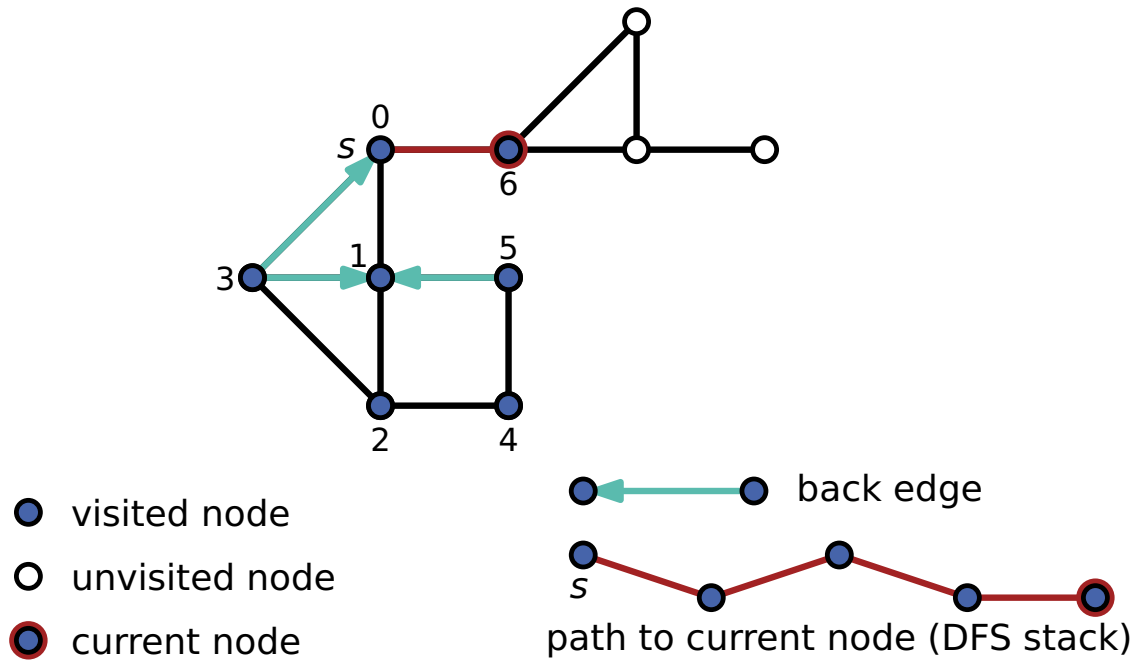

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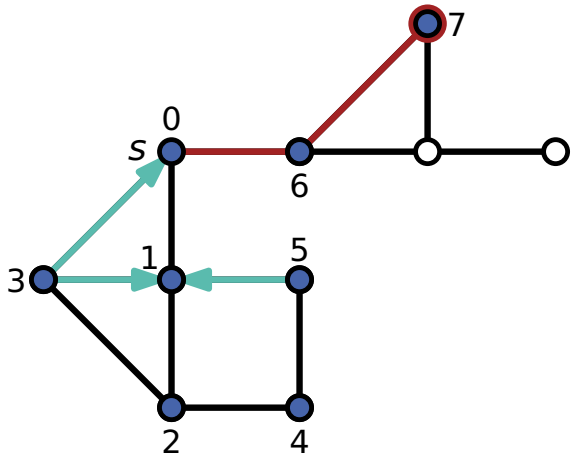

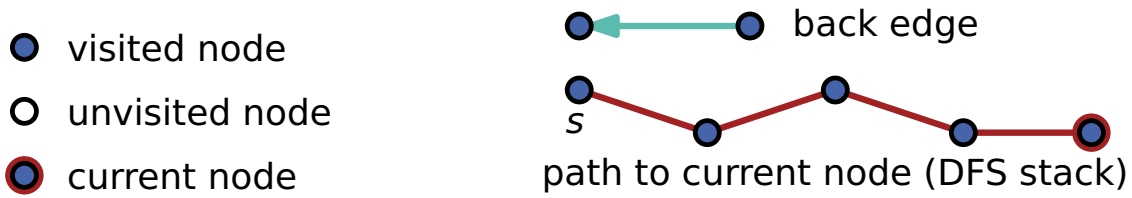

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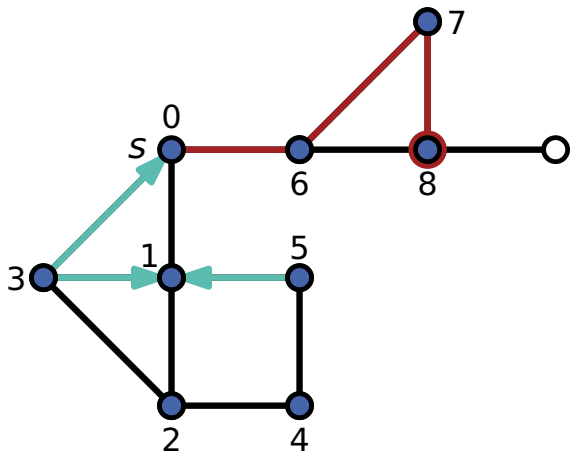

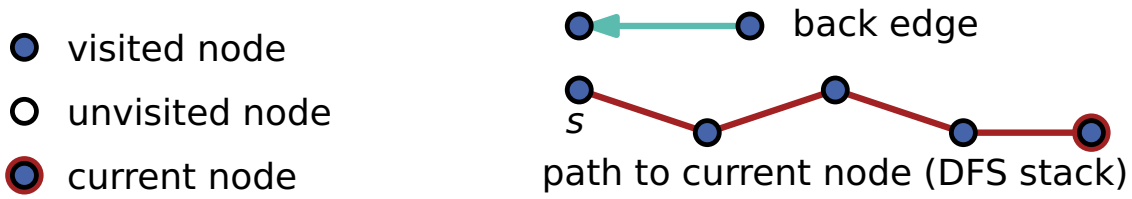

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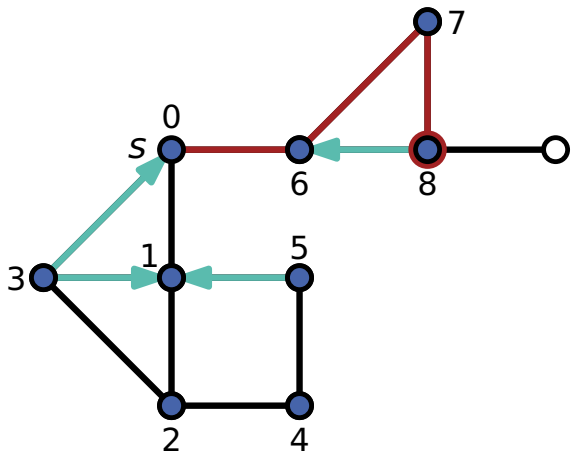

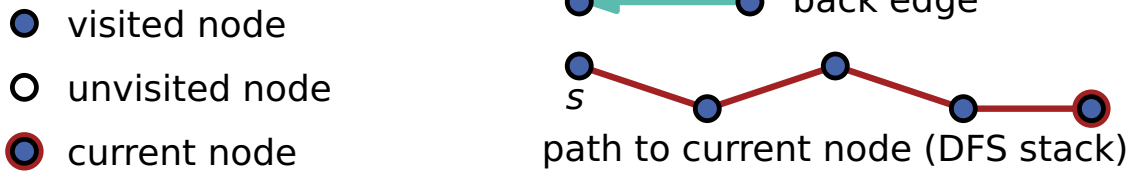

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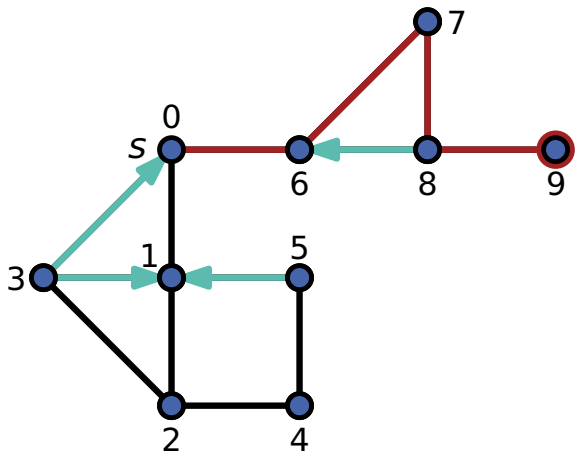

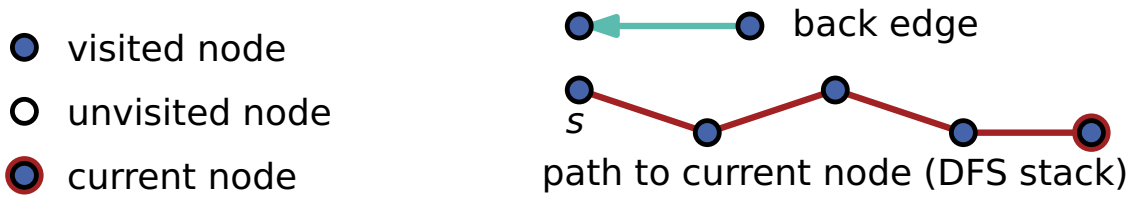

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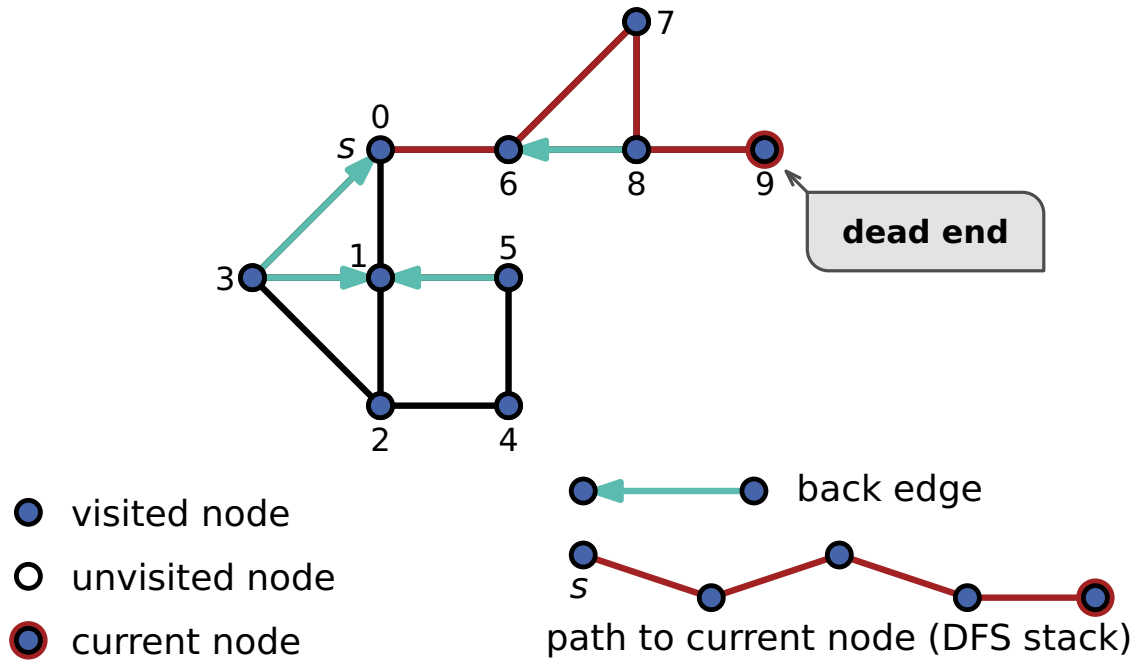

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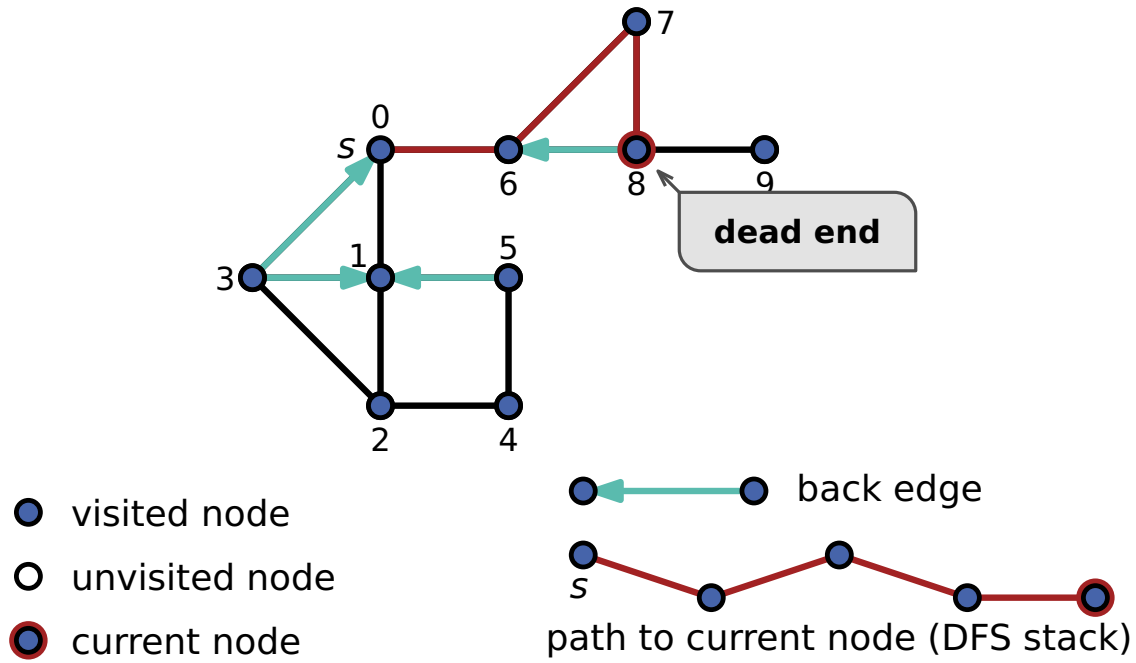

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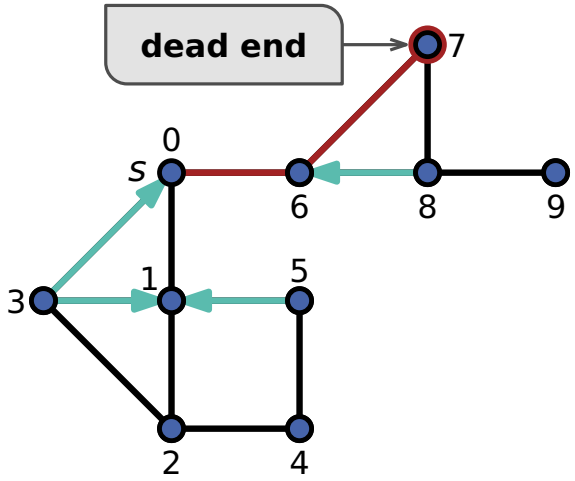

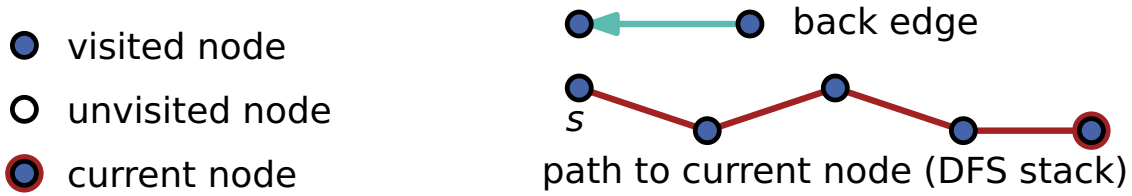

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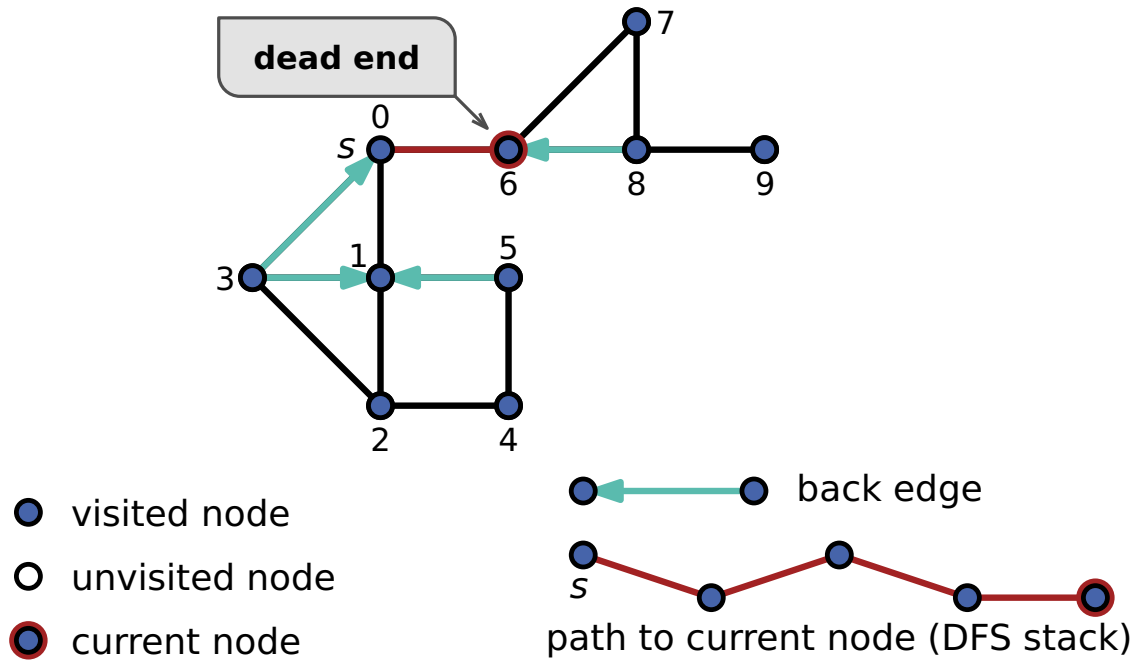

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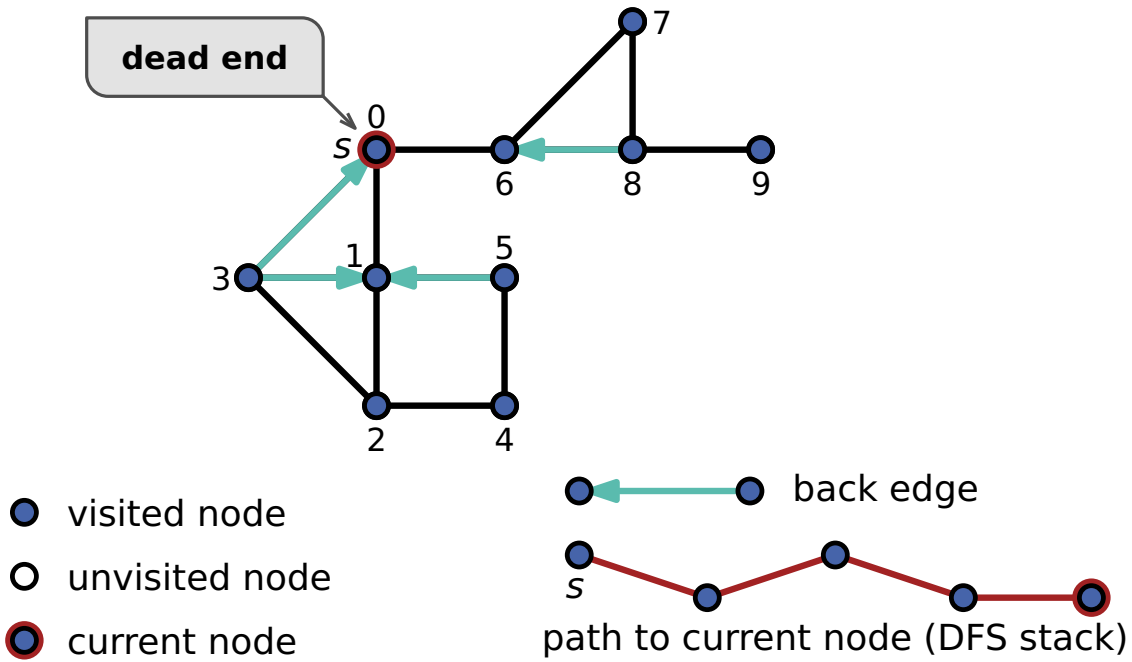

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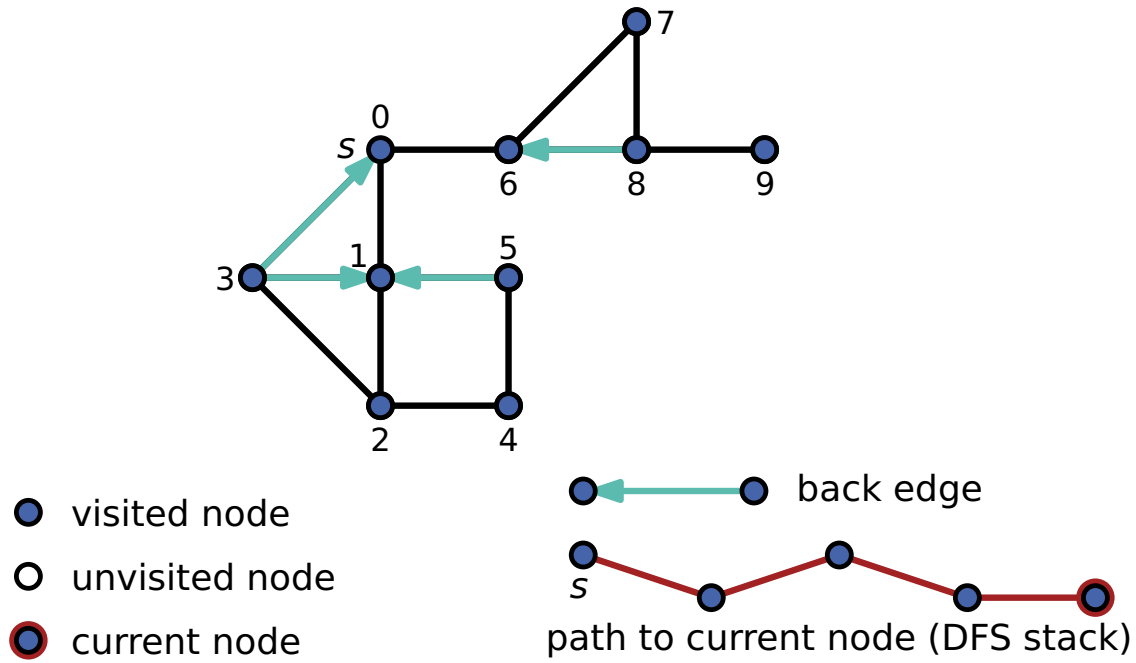

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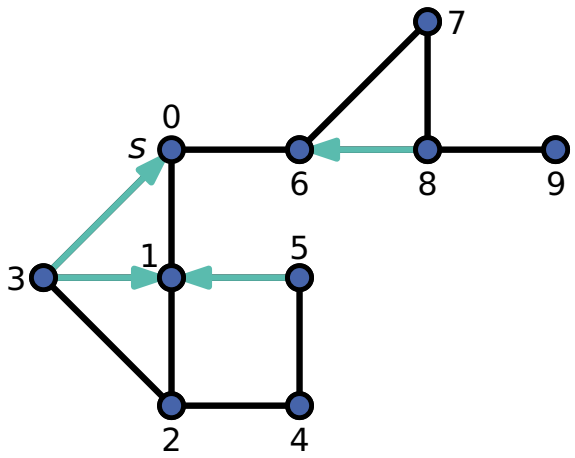

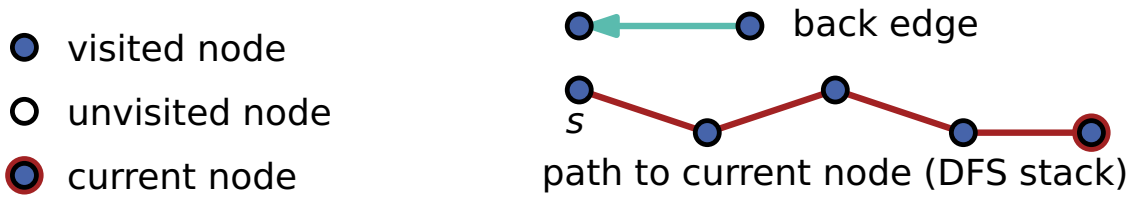

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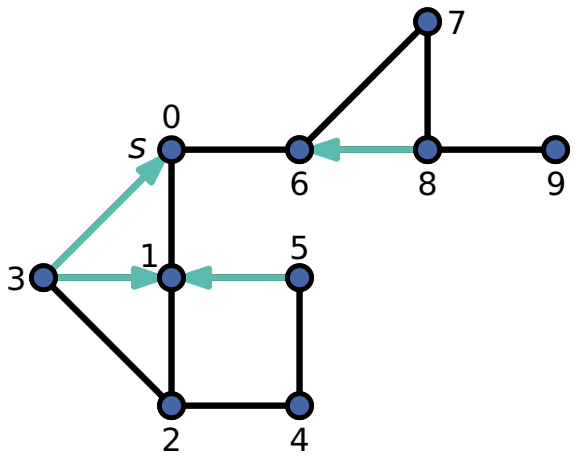

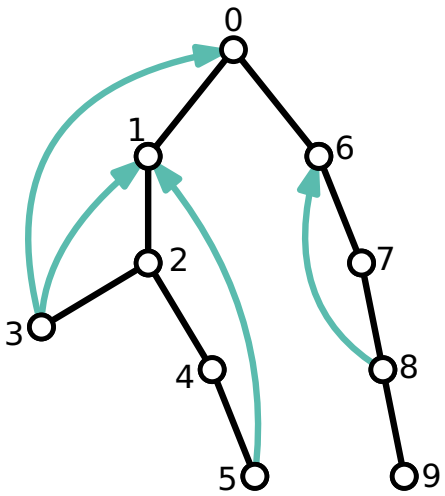

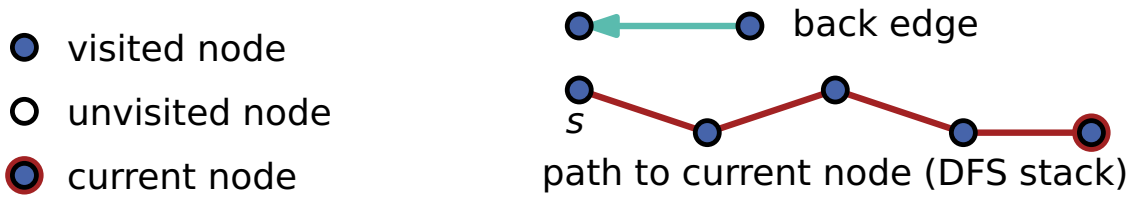

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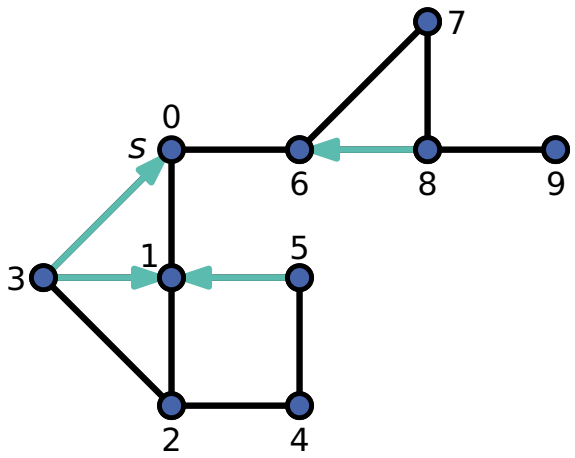

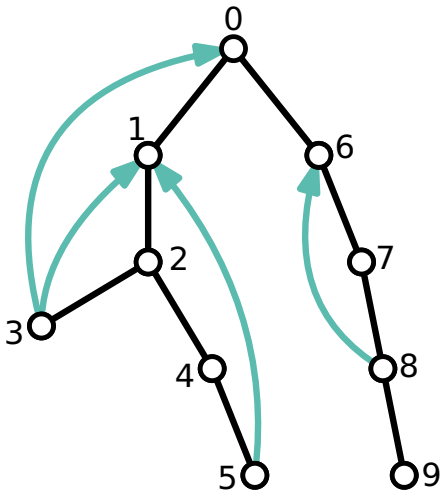

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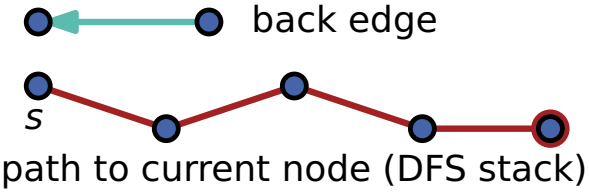
unvisited node

current node



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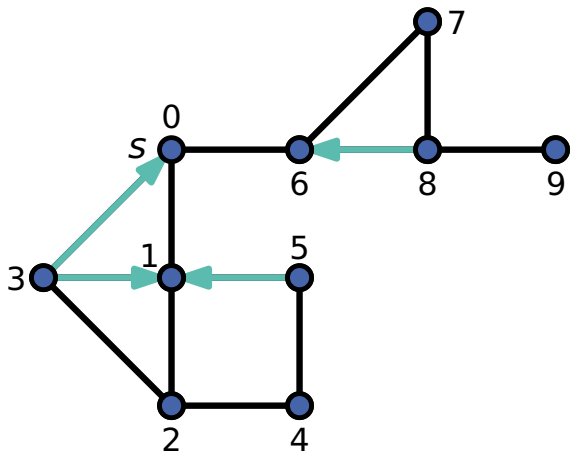

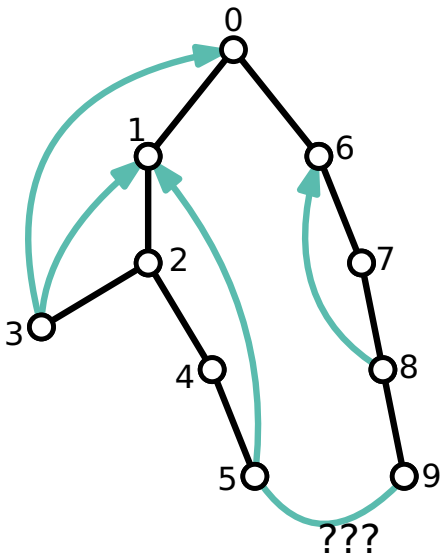

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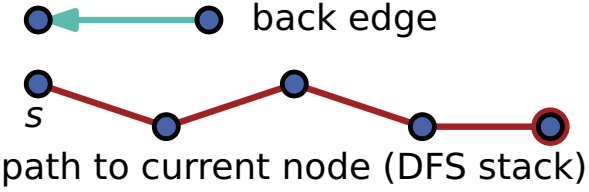

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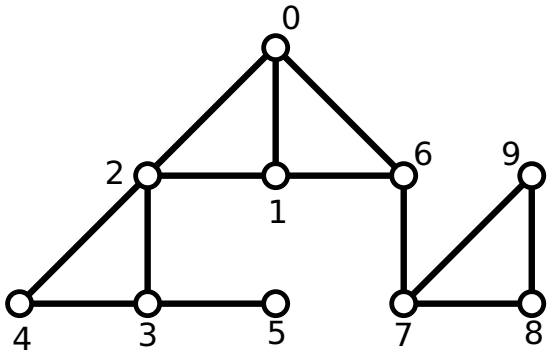

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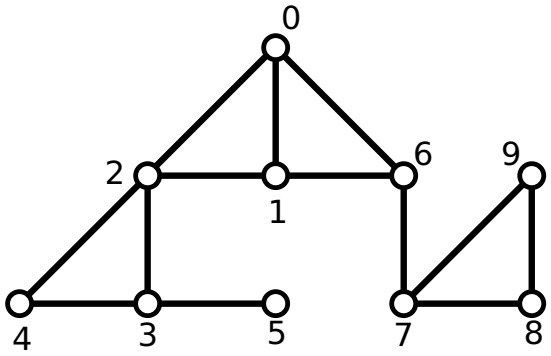

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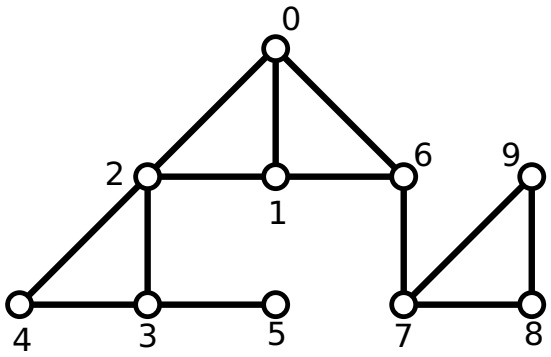

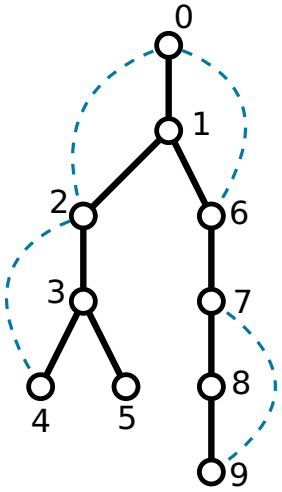

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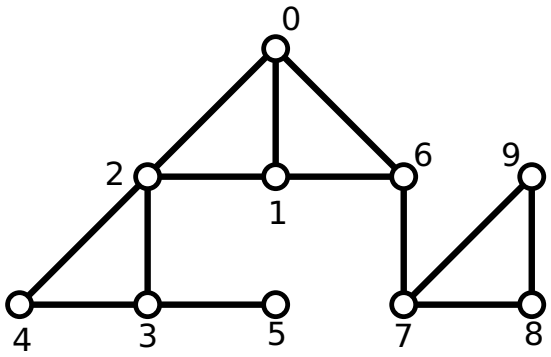

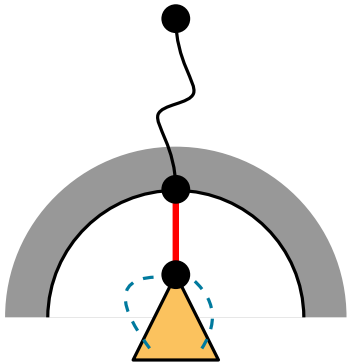

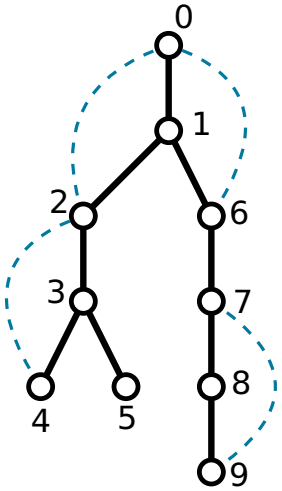

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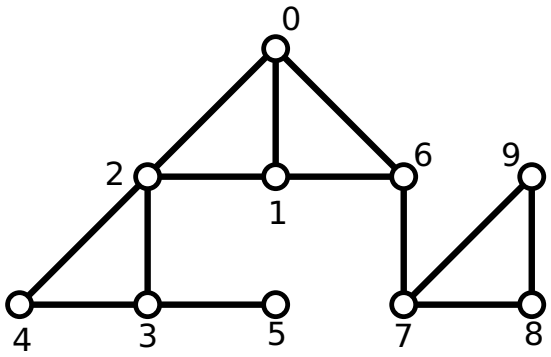

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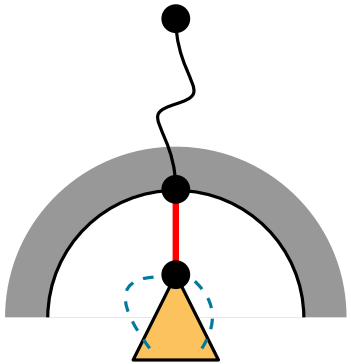

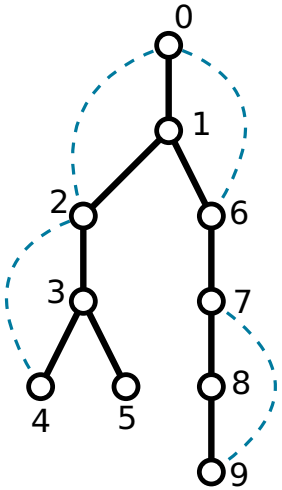

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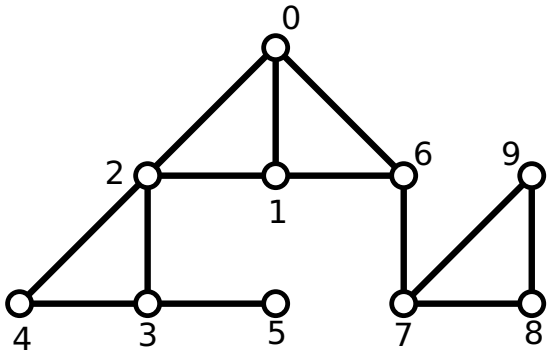

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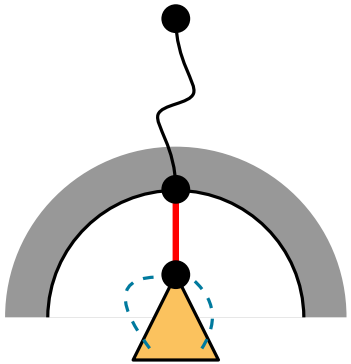

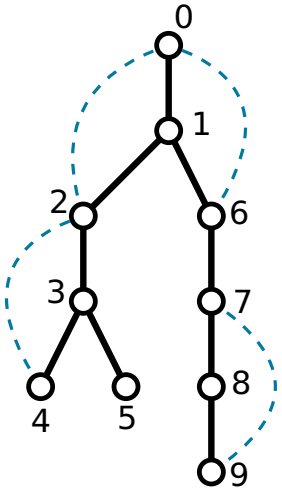

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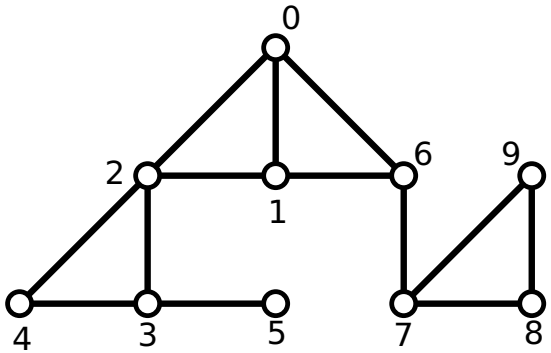

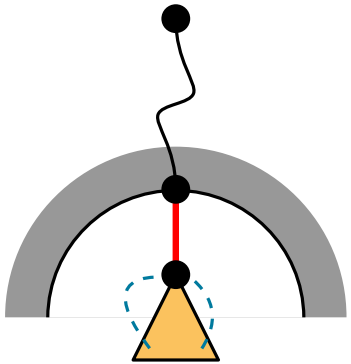

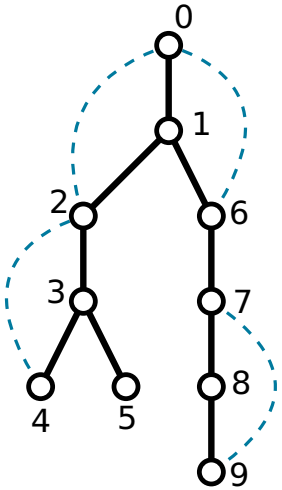

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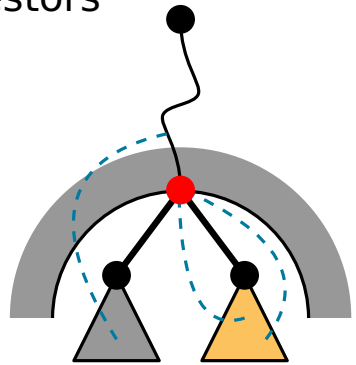

Computing Bridges

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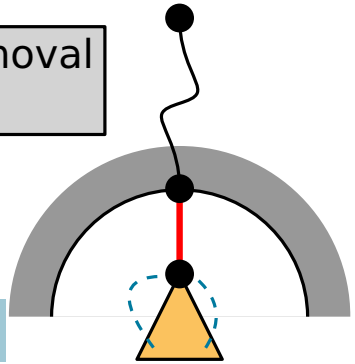

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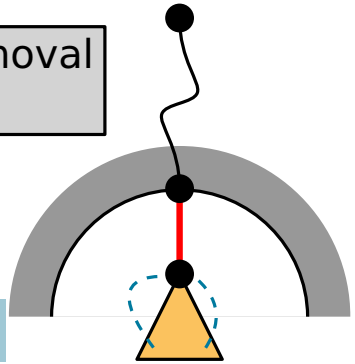

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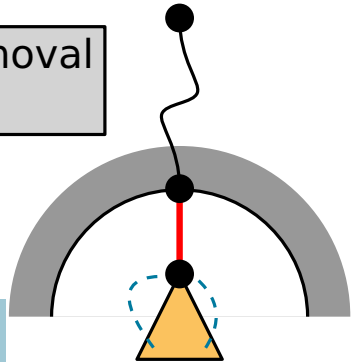

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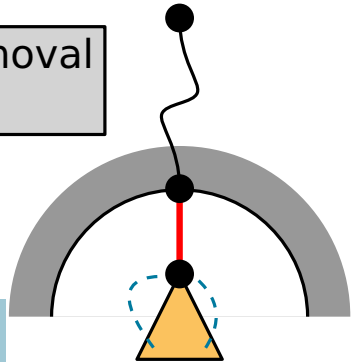

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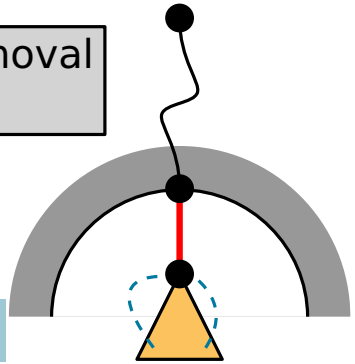

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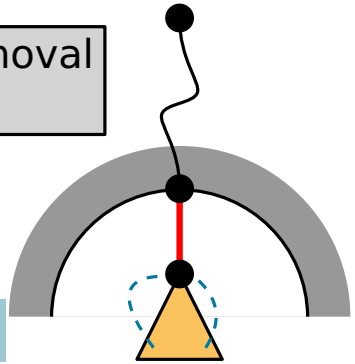

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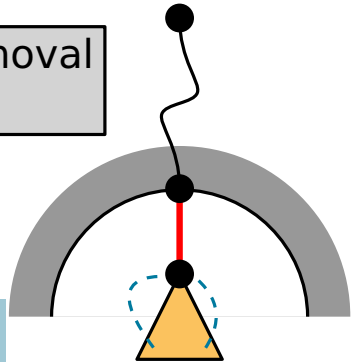

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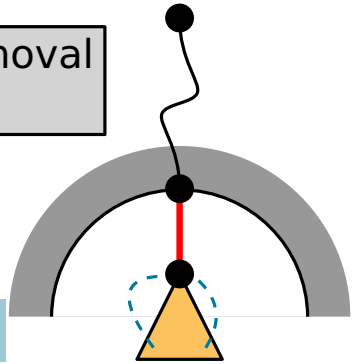

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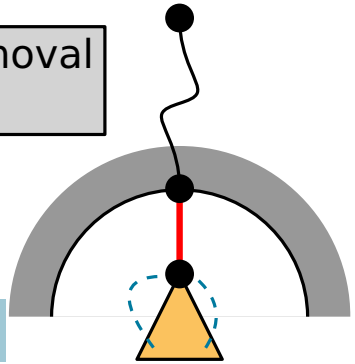

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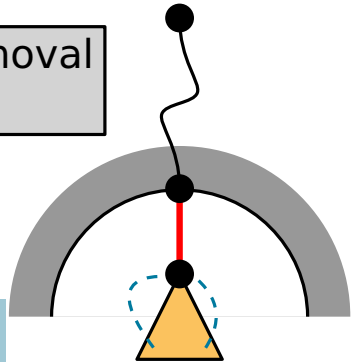

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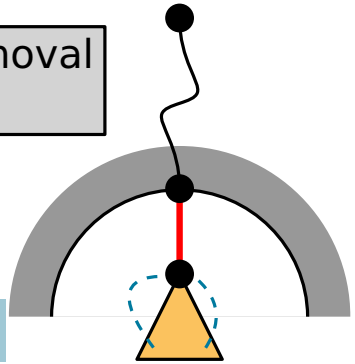

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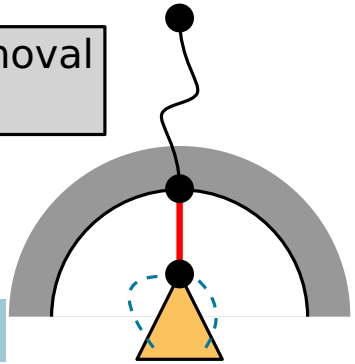

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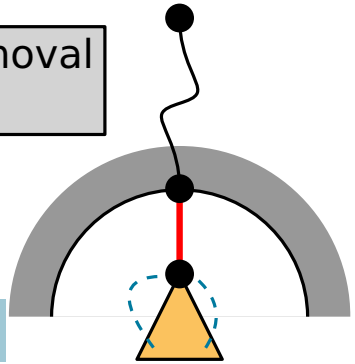

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- **Algorithm Idea:** compute this number of each subtree in tree-DP
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- most other tutorials find the highest back-edge of each subtree, which also works for multi-graphs

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cp-algorithms.com/graph/bridge-searching.html
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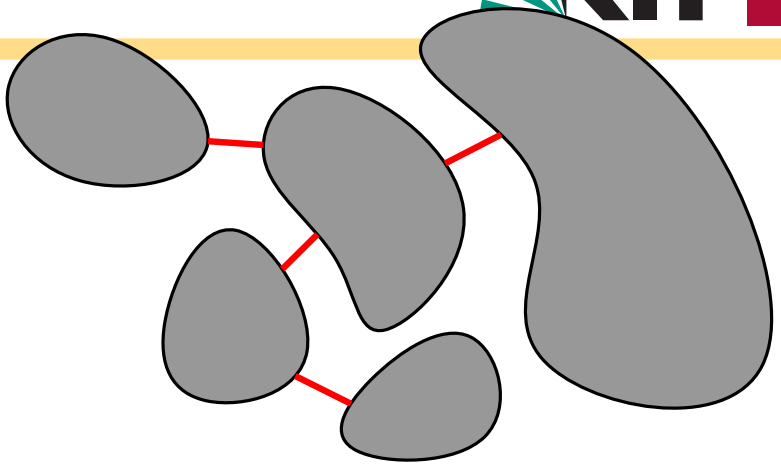
2-Edge-Connected Components

- bridges partition the nodes of a graph into 2-edge-connected components
- those components form a tree :)



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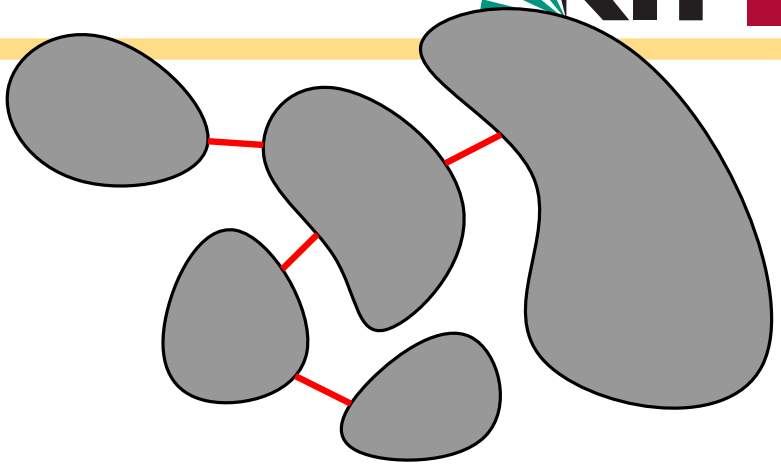

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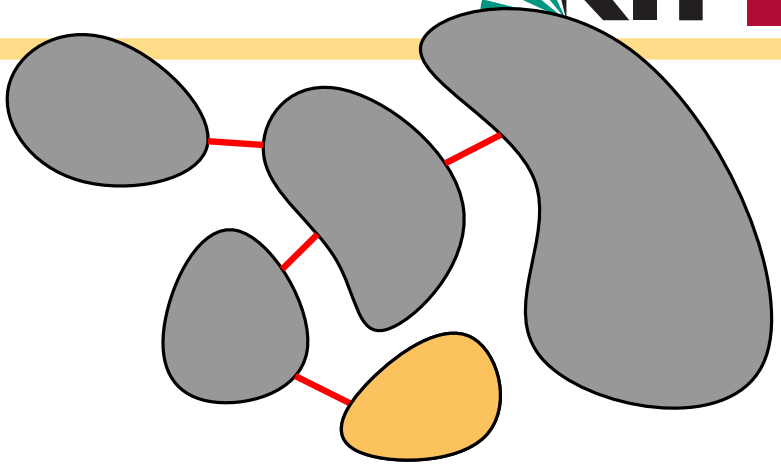

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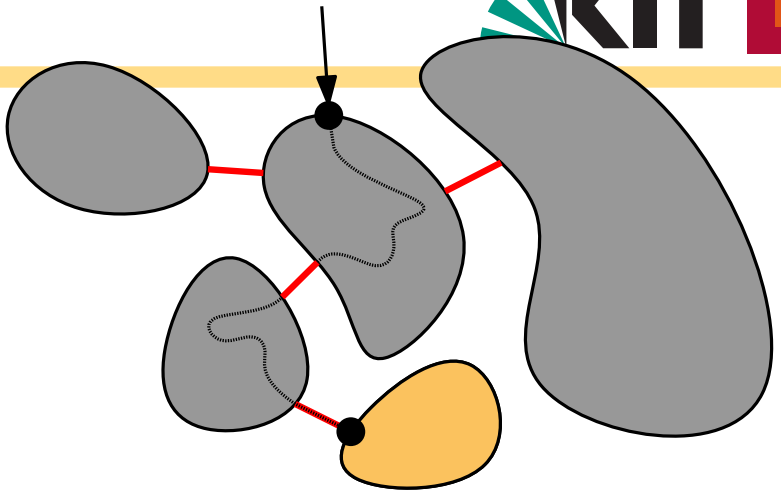

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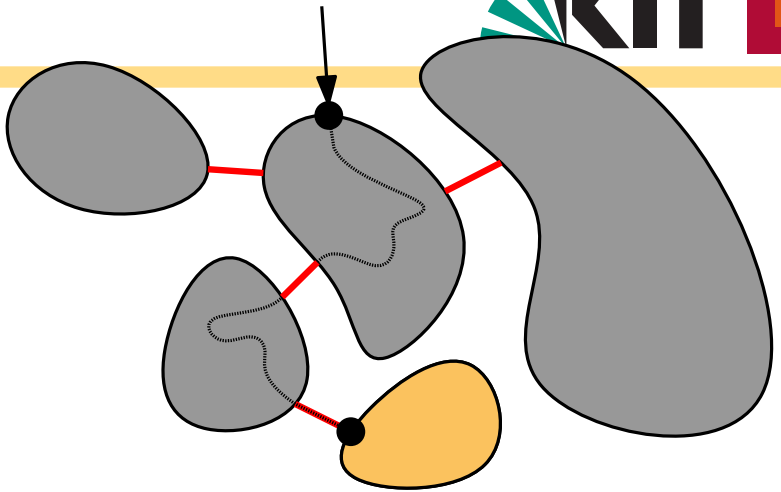

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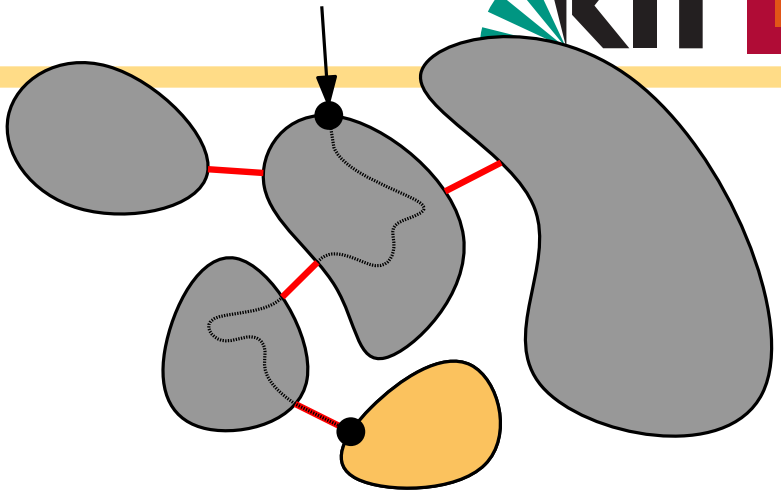

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- allows to identify the nodes visited between entering and exiting the current component

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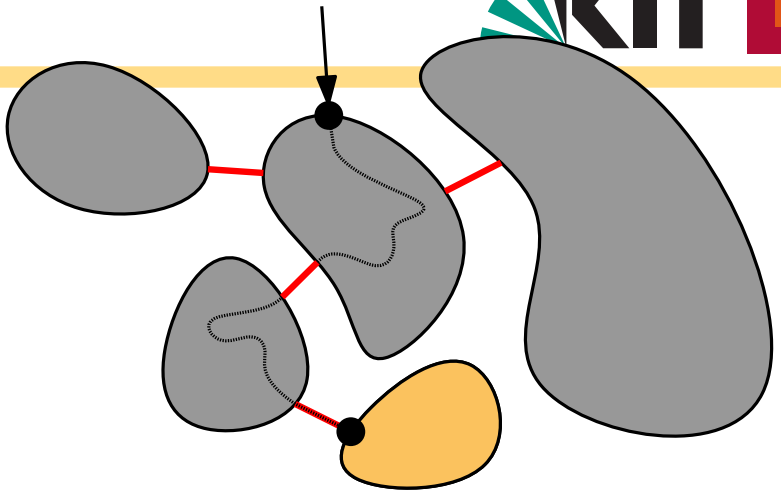

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- can be computed by removing all bridges and then computing connected components
- instead, look at a leaf in component tree
- at some point DFS enters the component via the bridge, then visits all its nodes, and then backtracks back over the bridge
- use a stack to track visited nodes
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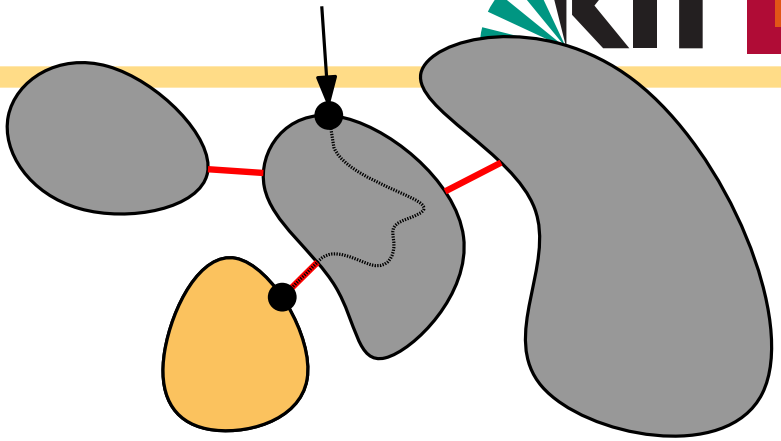
- **eat away** components from the outside

2-Edge-Connected Components



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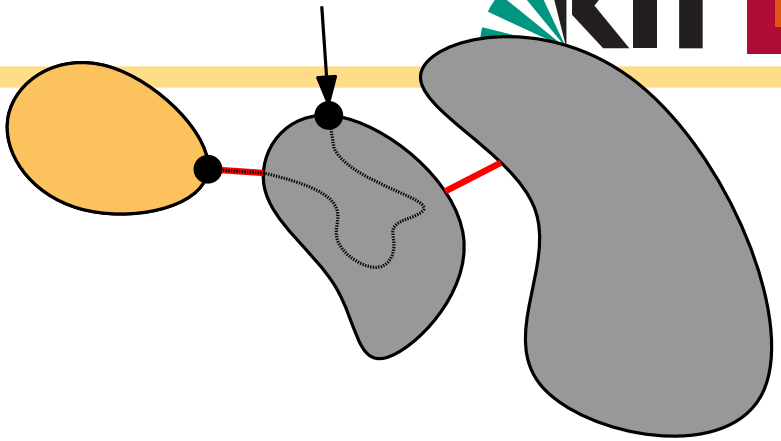

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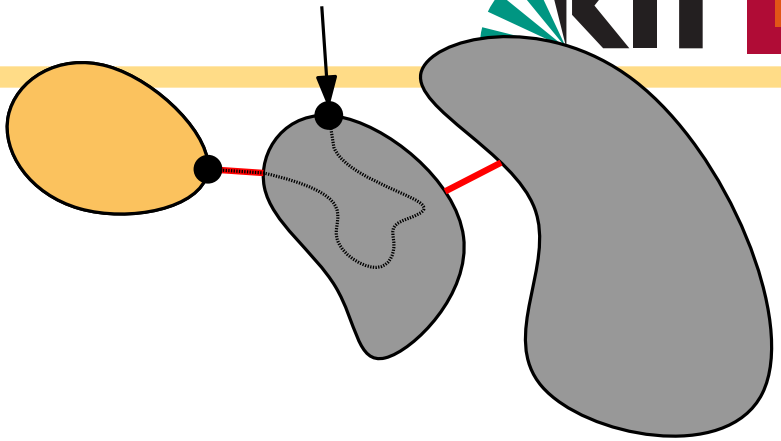

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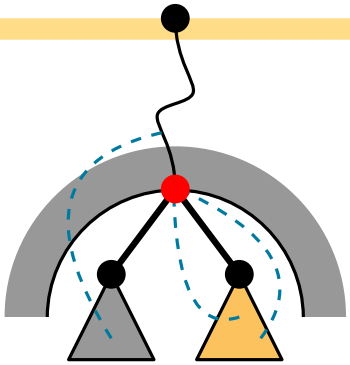
```
vector<int> st;
vector<vector<int>> comps;
int dfs(int v, int d) {
    int st_pos = size(st);
    st.push_back(v);
    //TODO: cool DFS stuff from last slide
    if(d==0 || up==1) { // exiting comp
        comps.emplace_back(
            begin(st)+st_pos, end(st));
        st.resize(st_pos);
    }
    return up;
}
```

Cut-Vertices

Cut-Vertex: vertex whose removal disconnects the graph.

- a cut-vertex has at least one DFS-subtree that has no back-edges going to his ancestors



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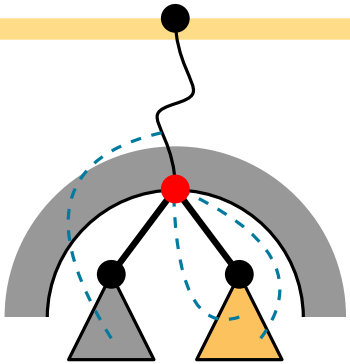

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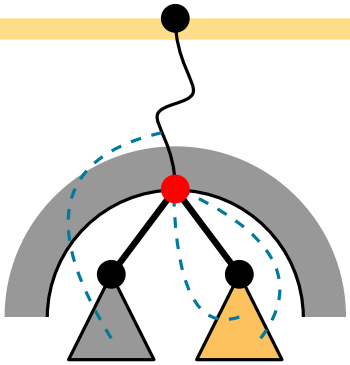

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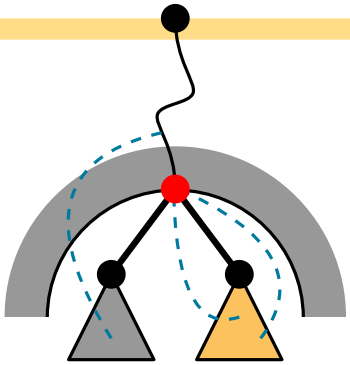

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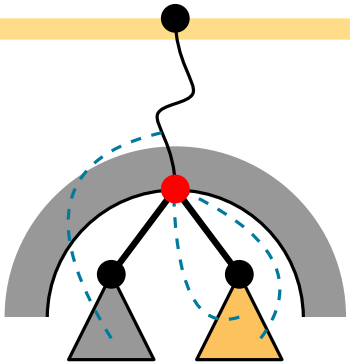

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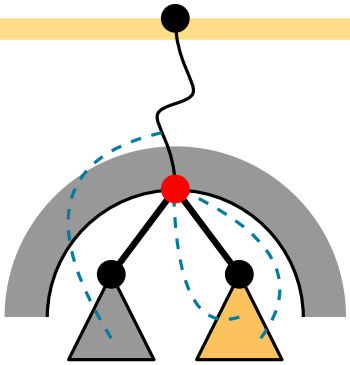

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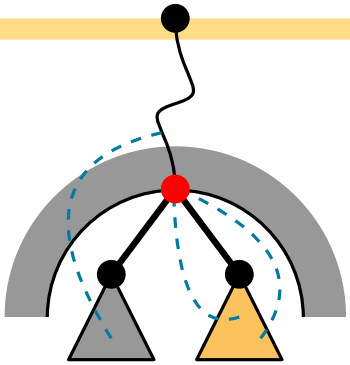

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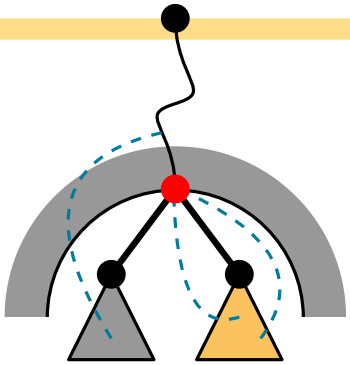

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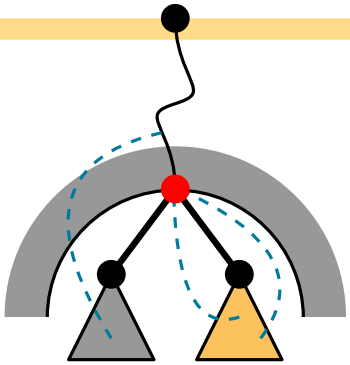

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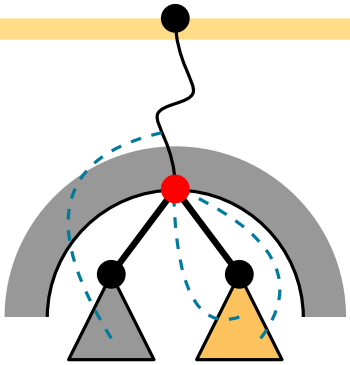

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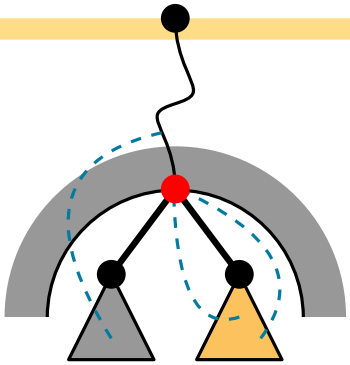

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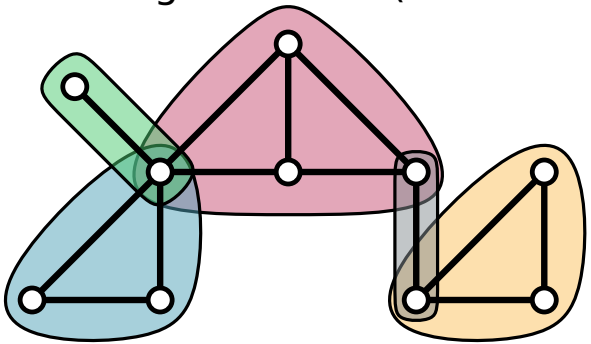
- partition graph into biconnected subgraphs
- a graph is biconnected if it cannot be disconnected by removing one node (i.e. has no cut-vertex)



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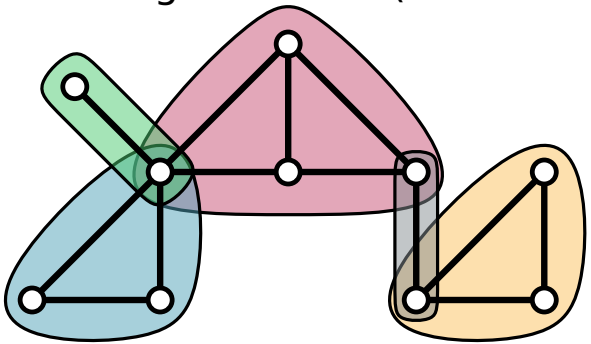

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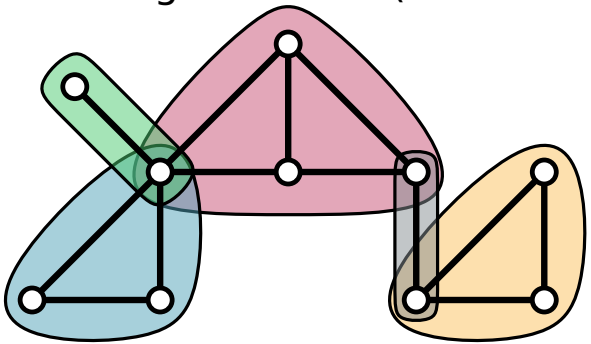

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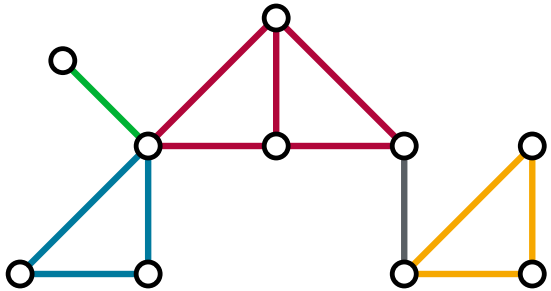

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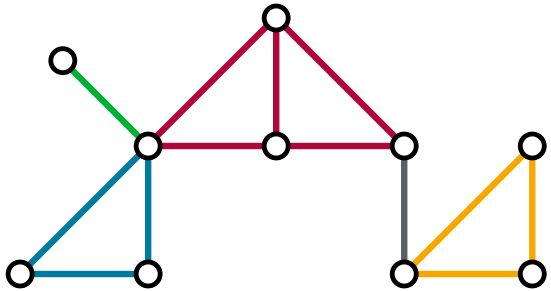

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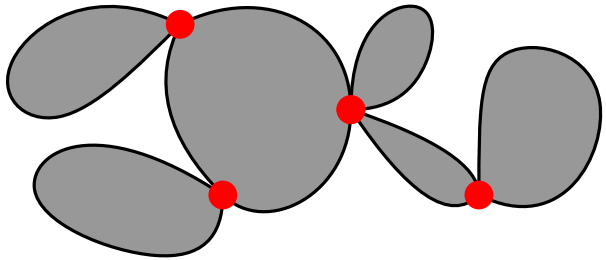

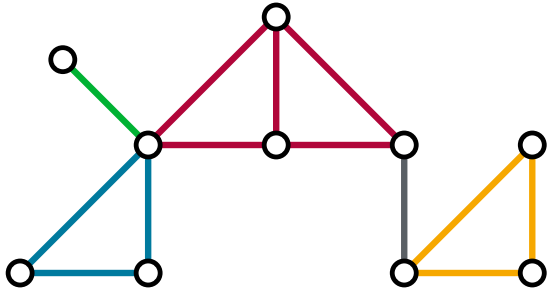

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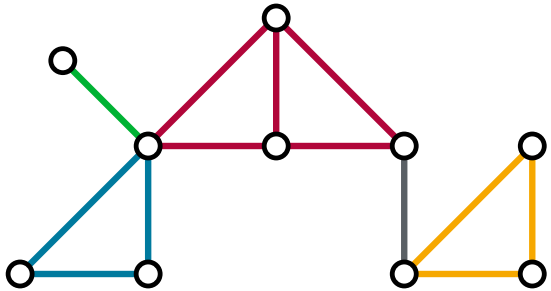

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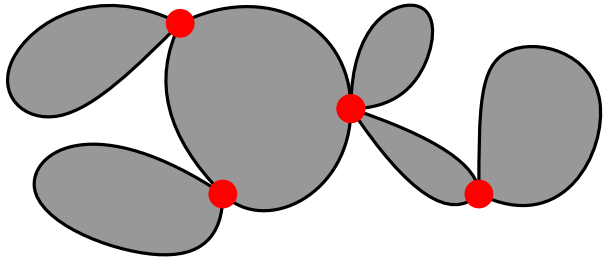

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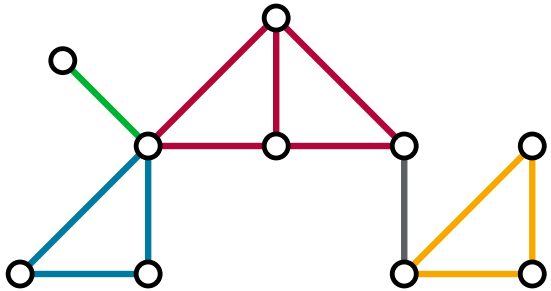

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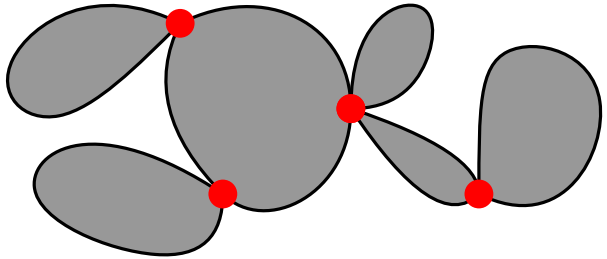

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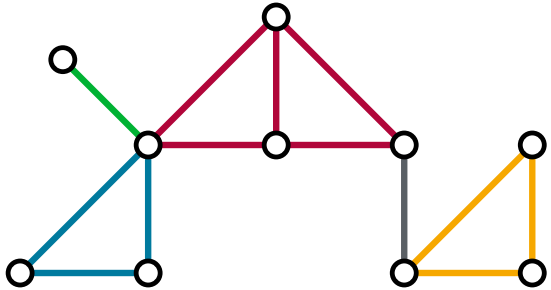

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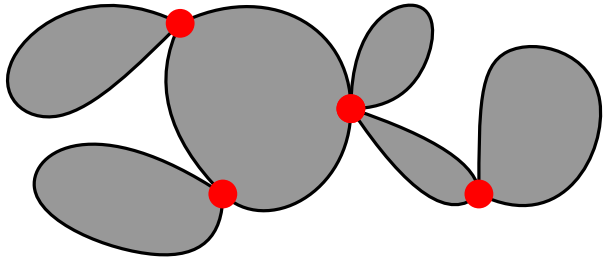

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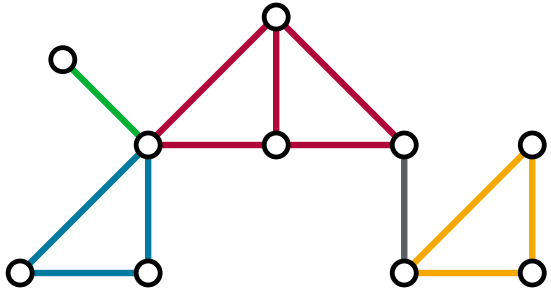

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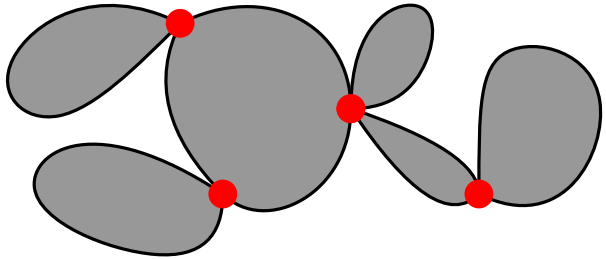

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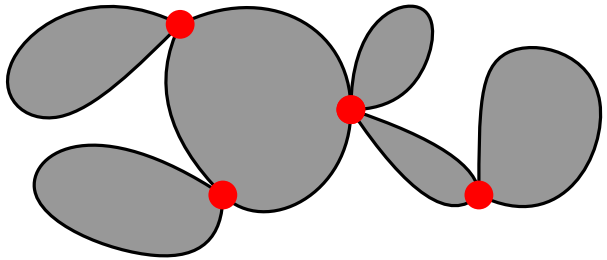

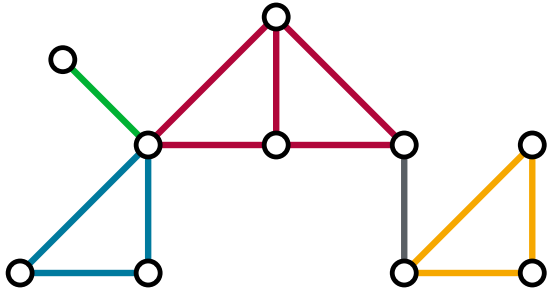

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- small trick: you can skip the root-edge-case from last slide
- because when backtracking out of a subtree with no higher back-edges, then you always exit a component

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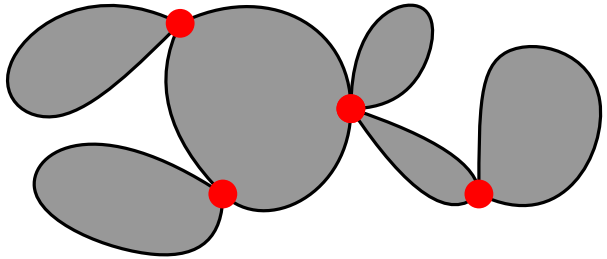
    int low = disc[v] = time++;
    for(int v2 : adj[v]) {

        if(disc[v2]==-1) { // tree edge
            int low2 = dfs(v2);
            low = min(low, low2);
            if(low2>=disc[v]) { // exiting comp



            }
        }


        low = min(low, disc[v2]);
    }
    return low;
}
```



- as for 2-edge-connected components, use a stack but push edges instead of vertices
- be careful not to push edges from both sides
- small trick: you can skip the root-edge-case from last slide
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2-Vertex-Connected Components

```
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int dfs(int v) {
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            }
        }
        if(disc[v2]<disc[v]) // from below
            st.push_back({v,v2});
        low = min(low, disc[v2]);
    }
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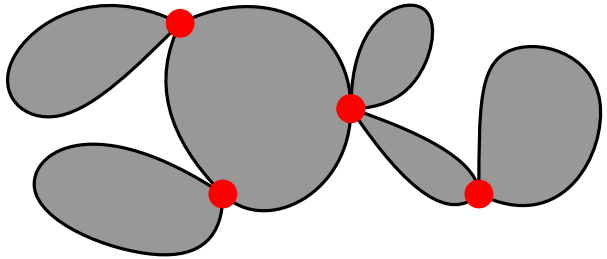
```



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```
// returns earliest discover time
// reachable via 1 edge from my subtree
int dfs(int v) {
    int low = disc[v] = time++;
    for(int v2 : adj[v]) {
        int st_pos = size(st);
        if(disc[v2]==-1) { // tree edge
            int low2 = dfs(v2);
            low = min(low, low2);
            if(low2>=disc[v]) { // exiting comp
                comps.emplace_back(
                    begin(st)+st_pos, end(st));
                st.resize(st_pos);
            }
        }
        if(disc[v2]<disc[v]) // from below
            st.push_back({v,v2});
        low = min(low, disc[v2]);
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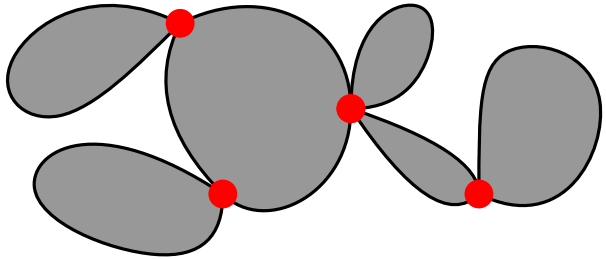

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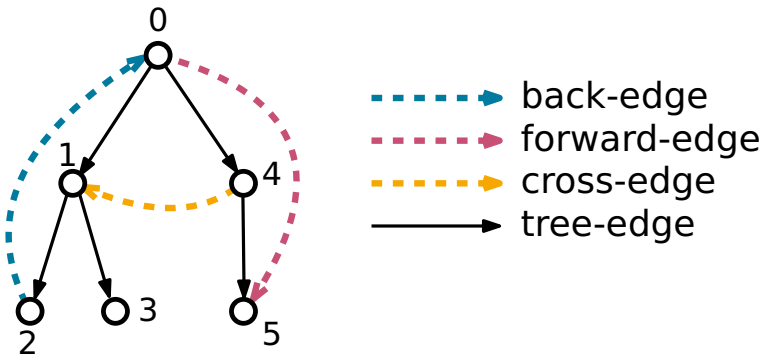

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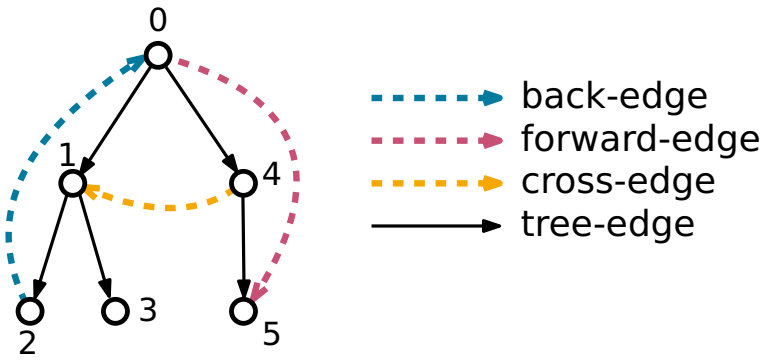

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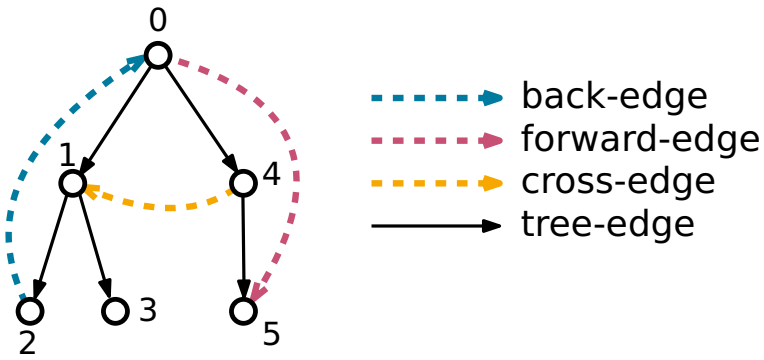

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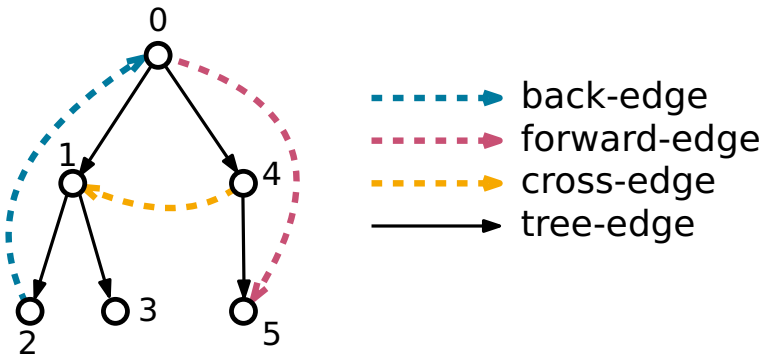

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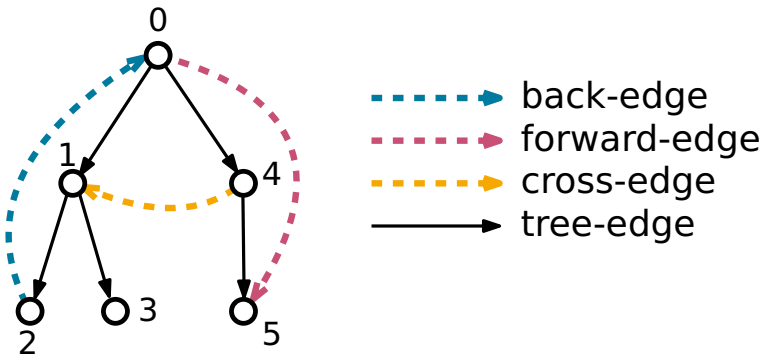

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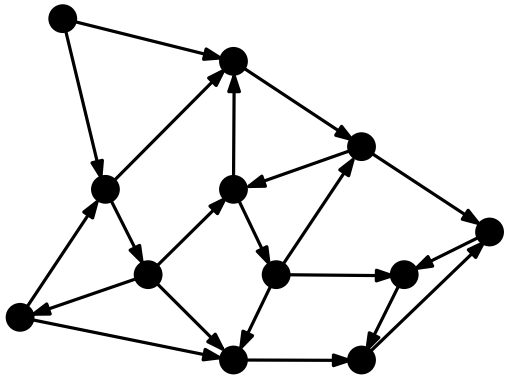
- multiple DFS needed to explore the full graph unless it's strongly connected

Strongly Connected Components (SCCs)

- a graph is strongly connected, if we can reach each node from every other node



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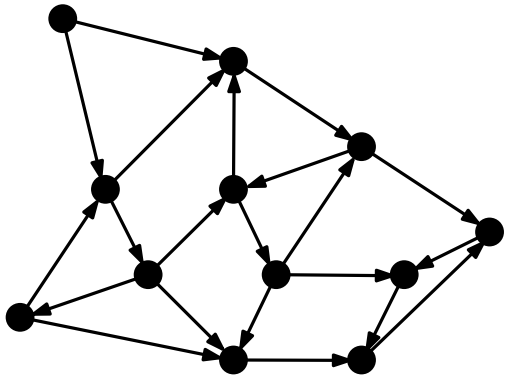

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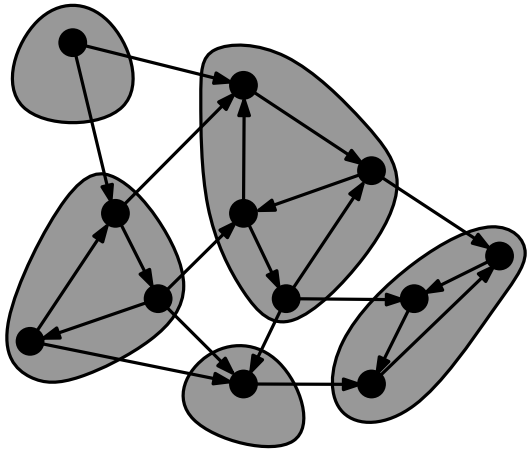

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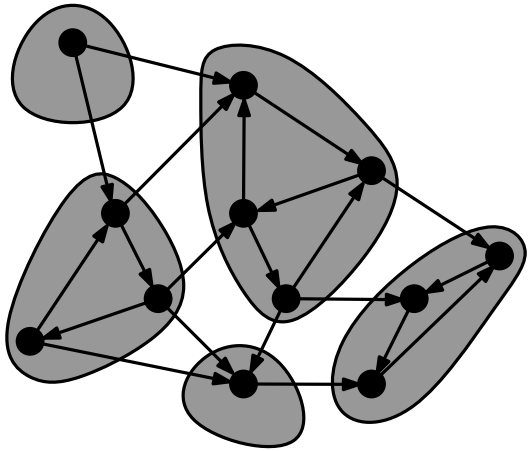

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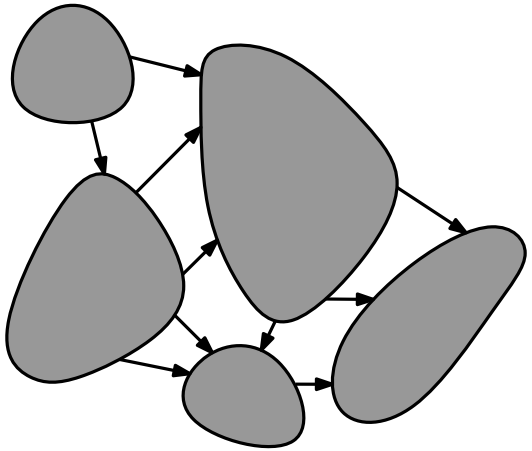

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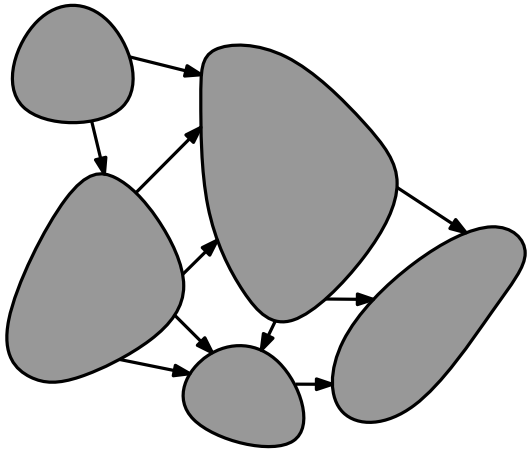

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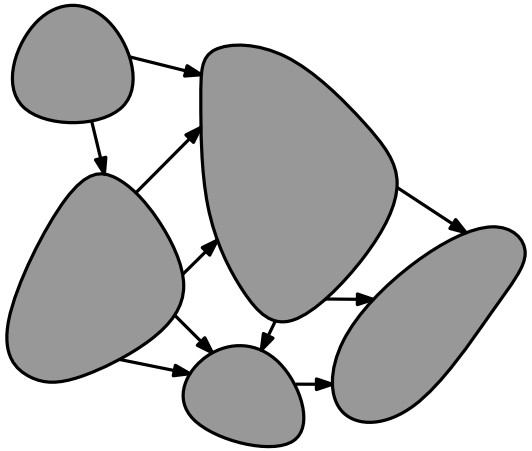

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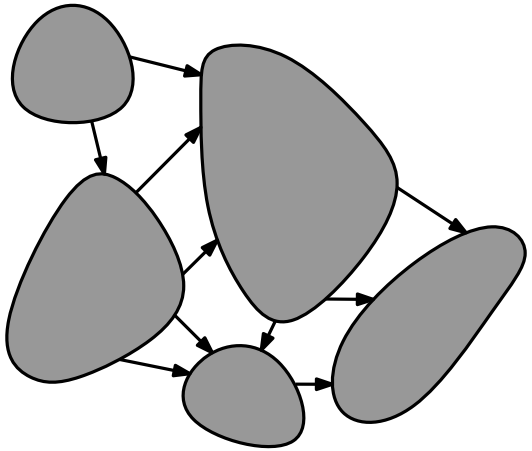

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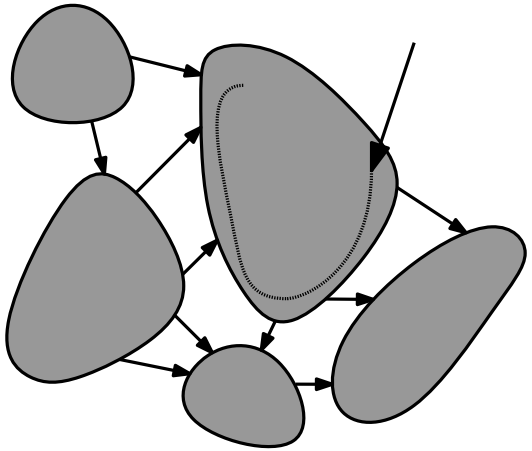

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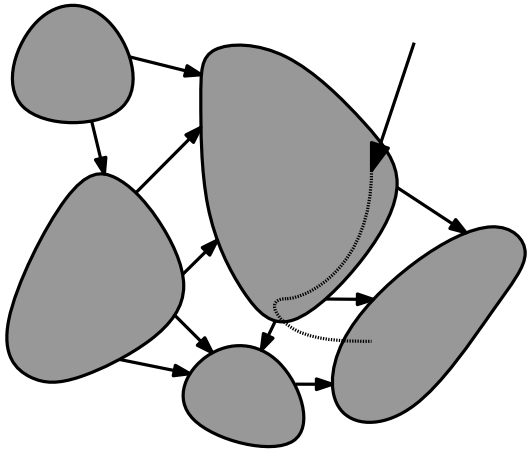

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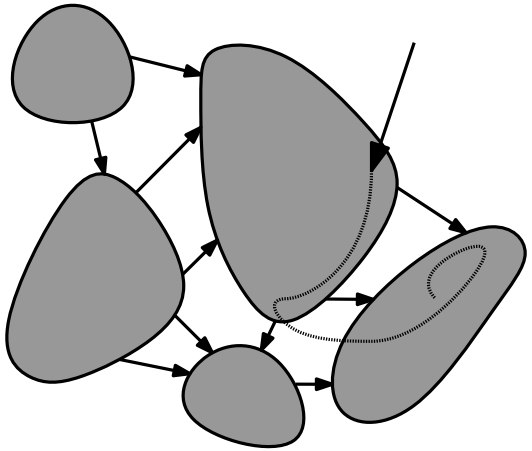

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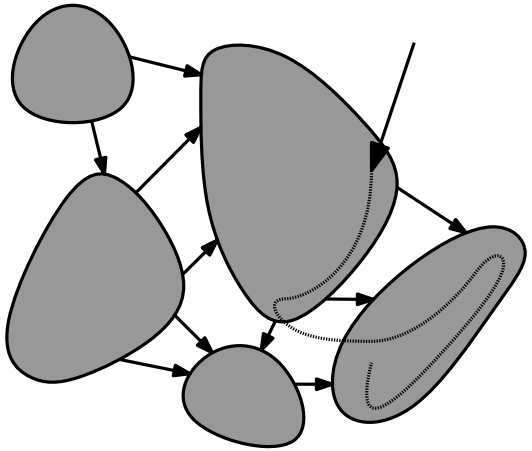

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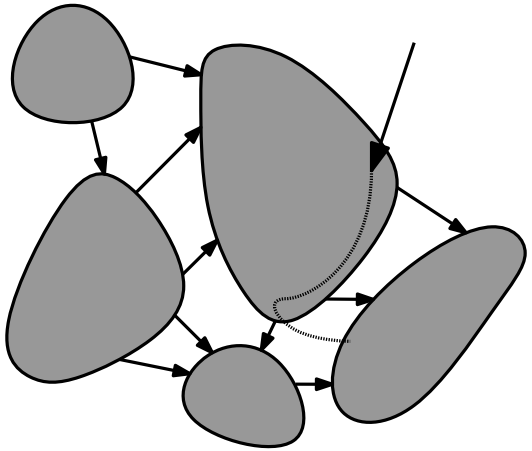

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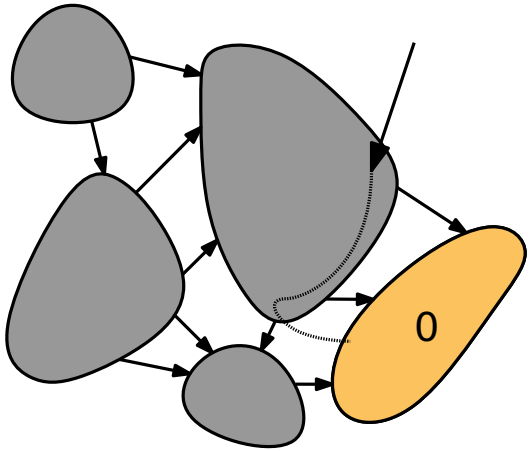

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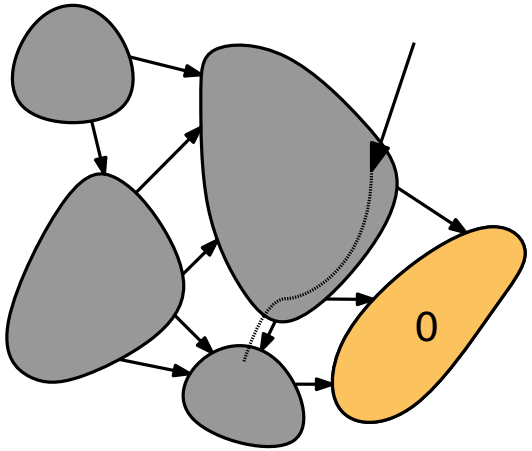

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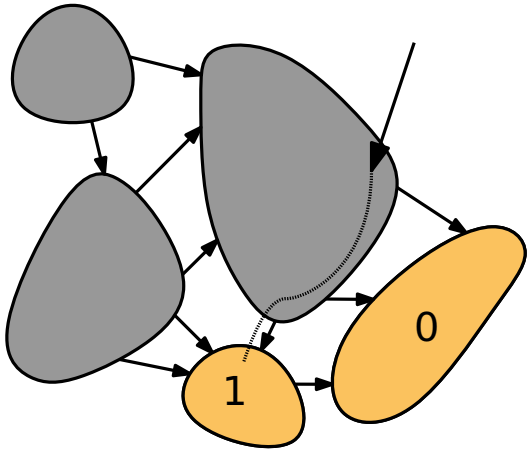

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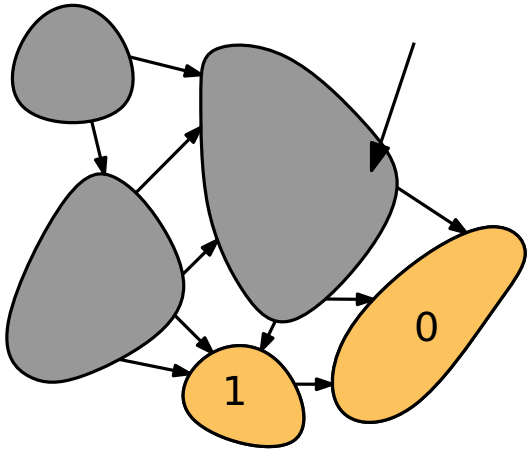

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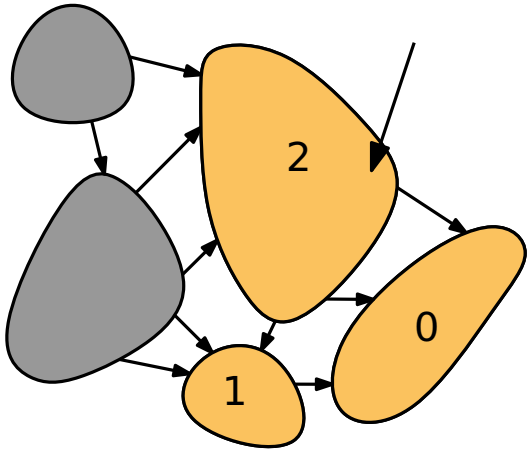

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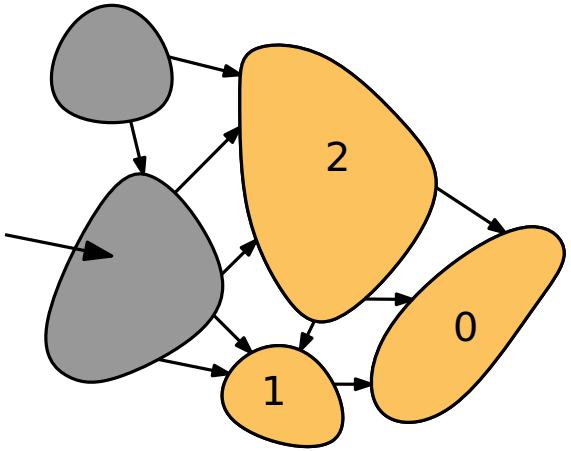

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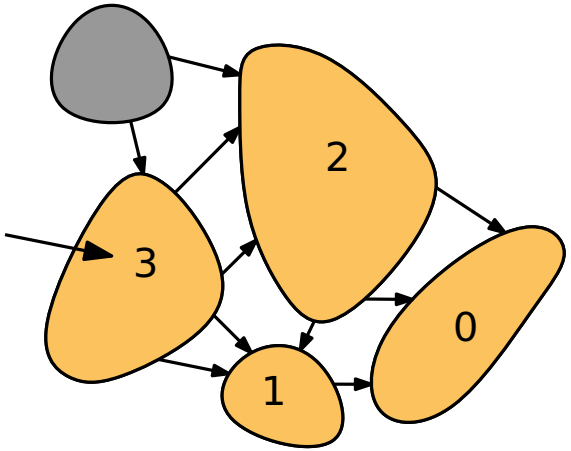

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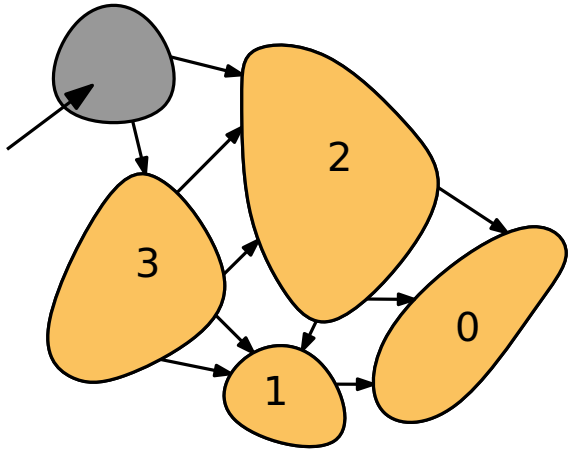

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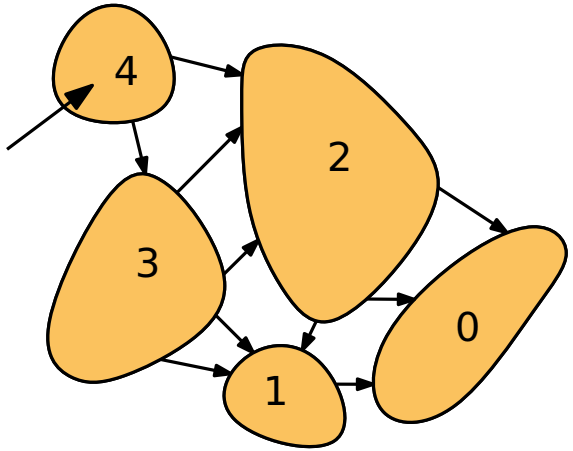

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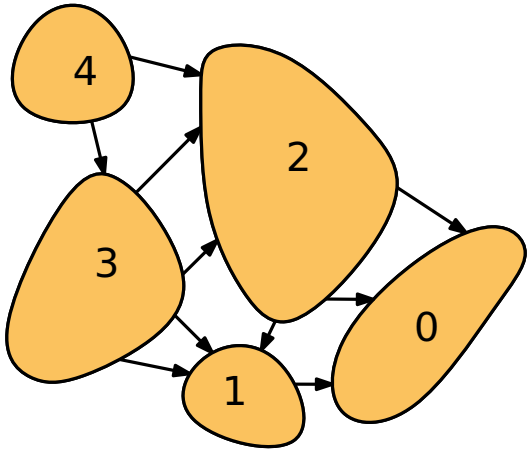

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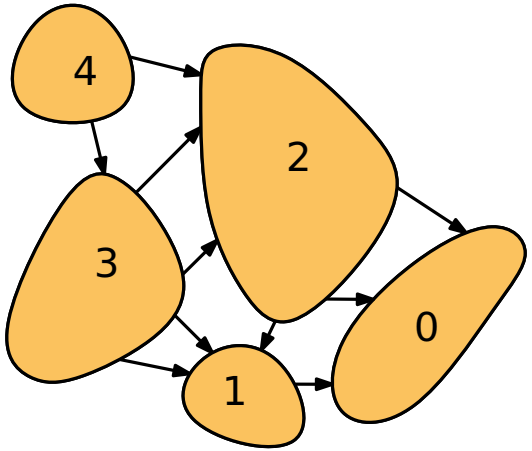

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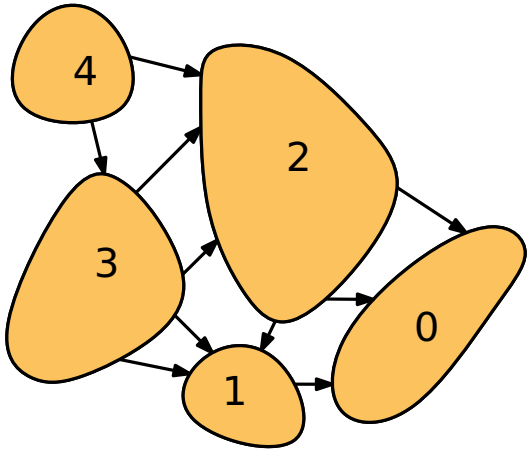

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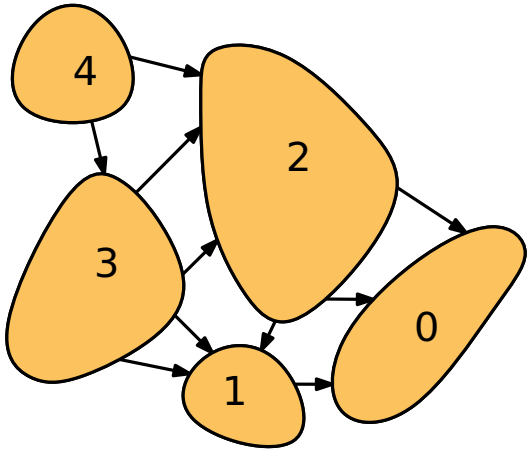

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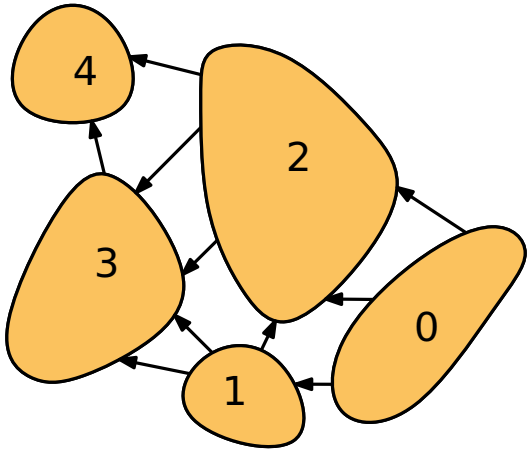

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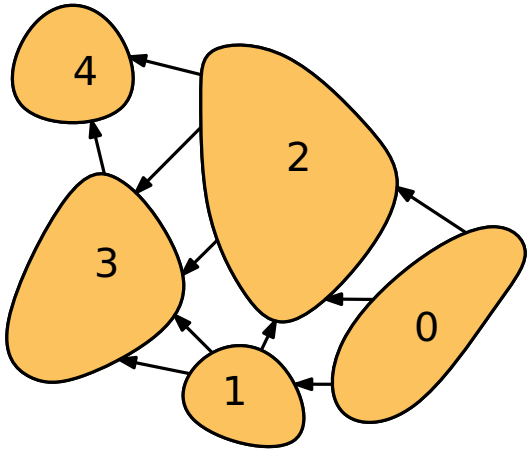

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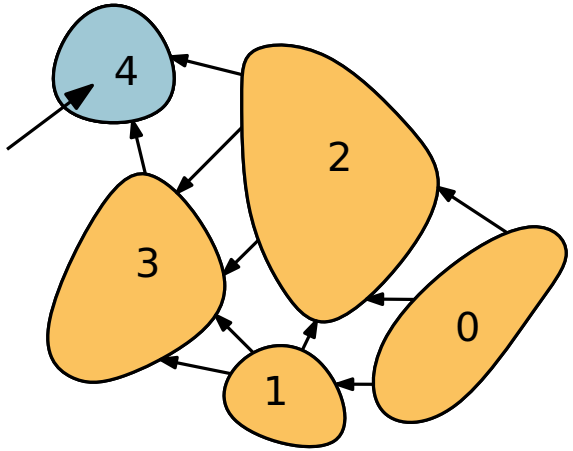

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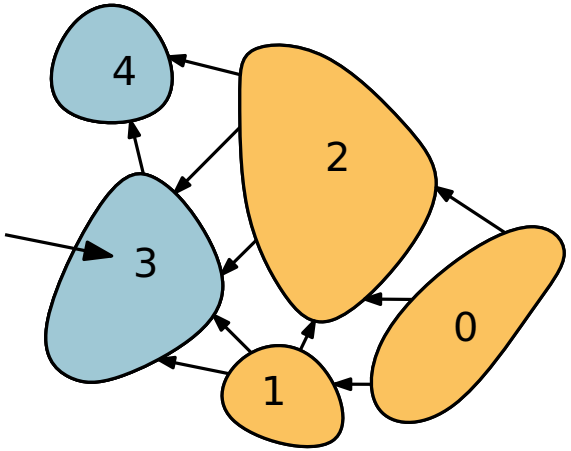

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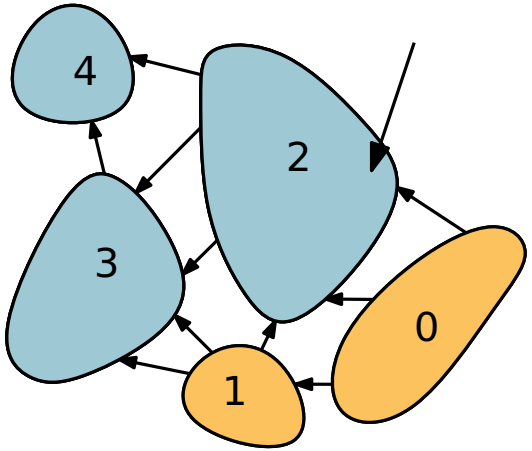

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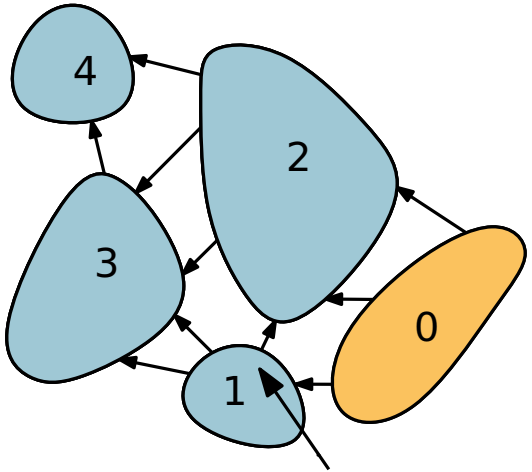

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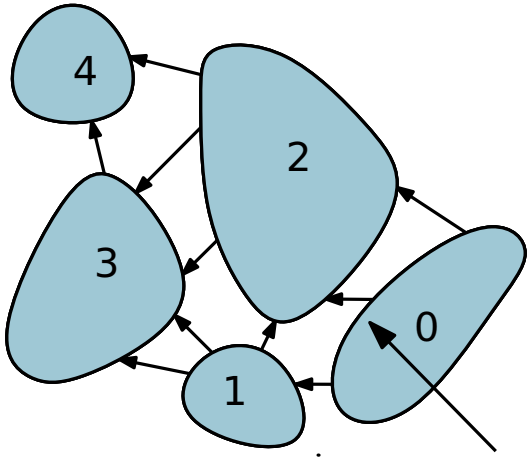

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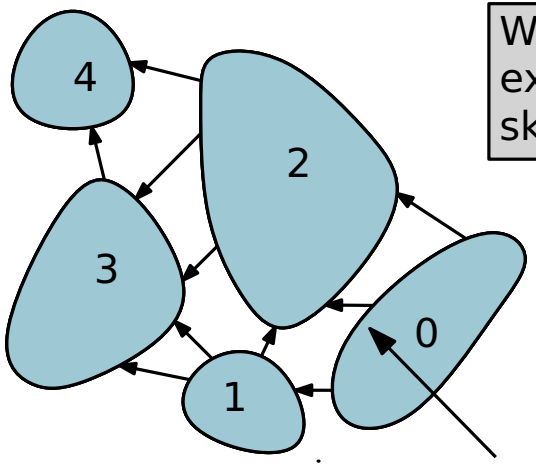

Why not lowest exit time and skip transpose?

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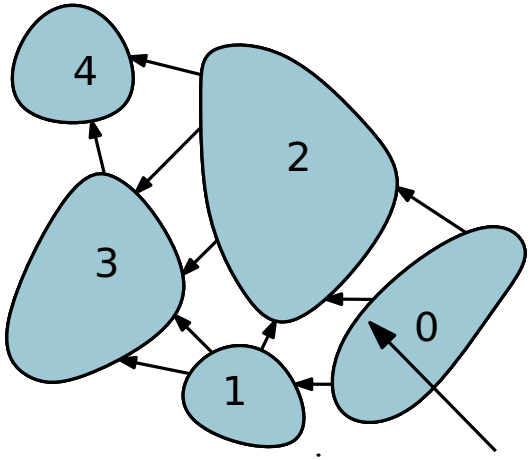
- **Algo 2**: use stack trick again to eat away components in reverse topological order during first traversal → Tarjan's Algorithm (next slide)



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Tarjan’s SCC Algorithm

- **Algo**: use stack trick to eat away components in reverse topological order

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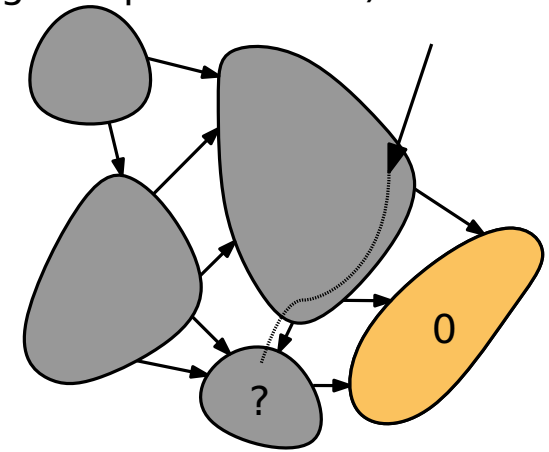

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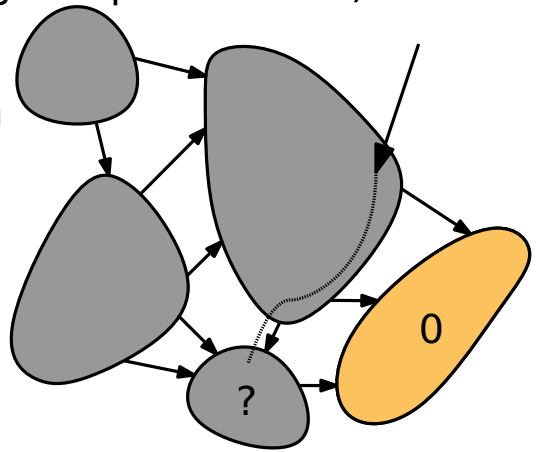

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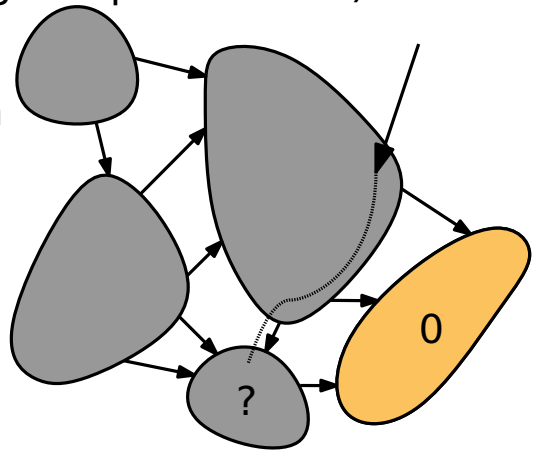
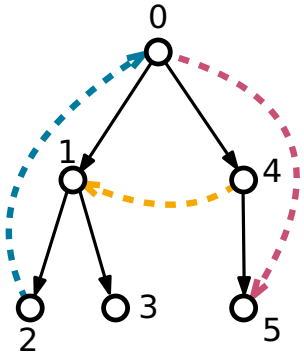


- back-edges point to an ancestor
- will be to node in same SCC only



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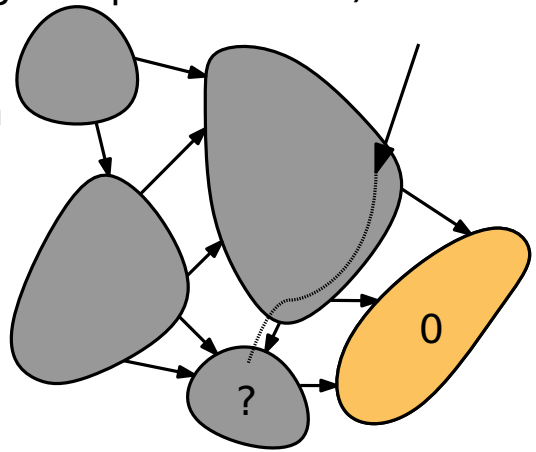

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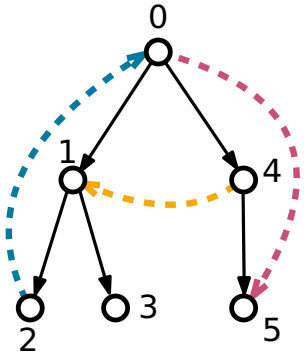

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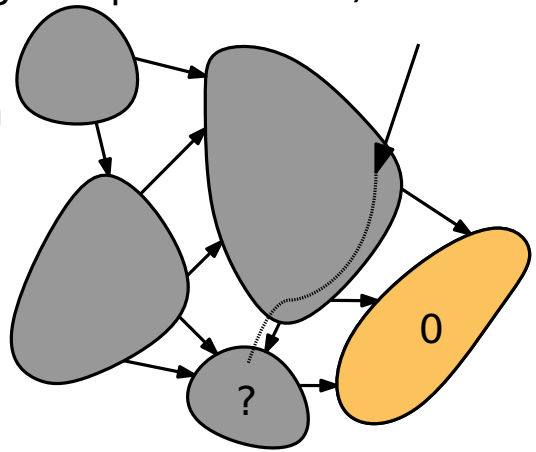

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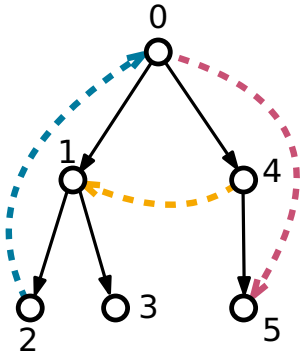

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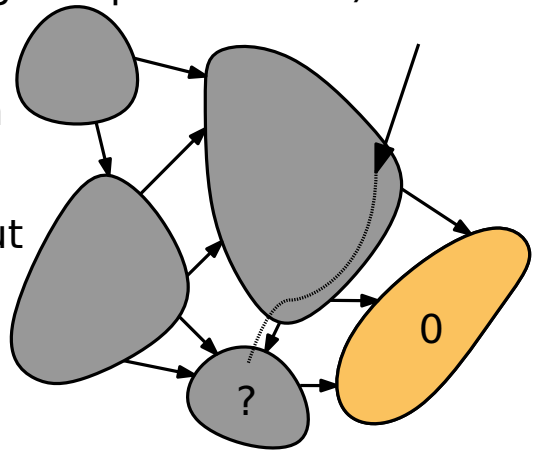

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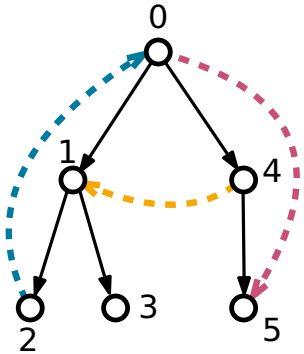

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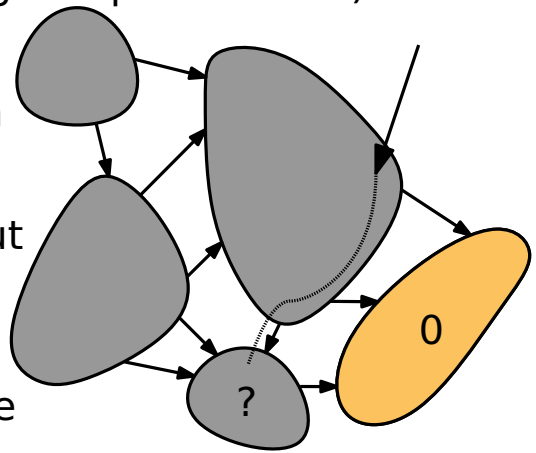

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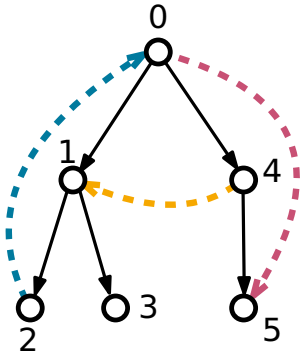

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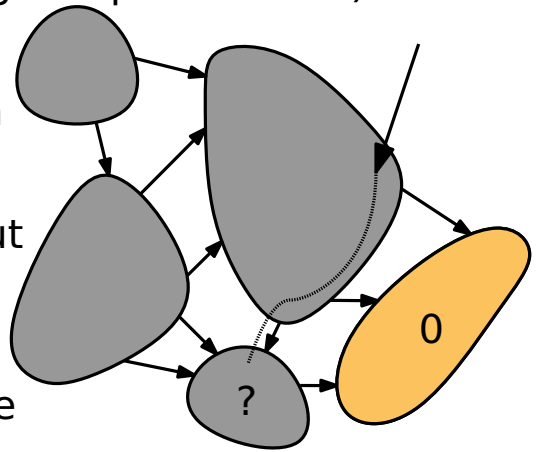

how to ignore completed SCC?

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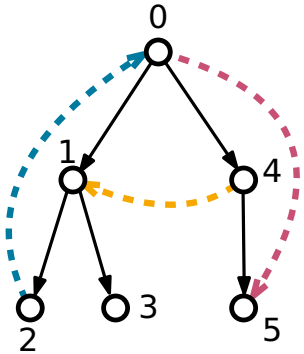

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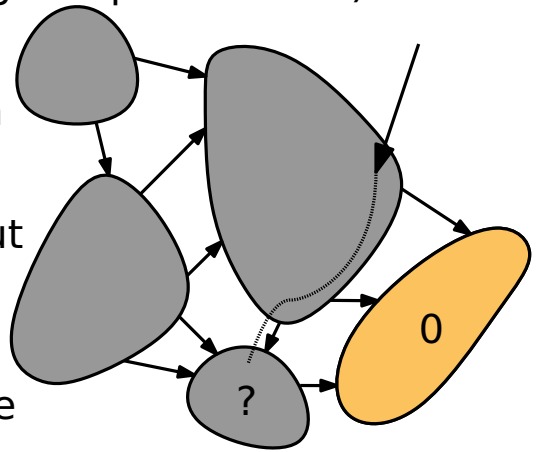

how to ignore completed SCC?

```
// returns lowest disc. time of nodes in SCC
// reachable via 1 edge from my subtree
int dfs(v) {
    int st_pos = size(st);
    st.push_back(v);
    in_stack[v] = true;
    int low = disc[v] = time++;
    for(int v2 : adj[v]) {
        if(disc[v2]==-1) // tree edge
            low = min(low, dfs(v2));
        if(in_stack[v2]) // not yet assigned
            low = min(low, disc[v2]);
    }
    if(low==disc[v]) { // exiting SCC
        // SCC is in st[st_pos..]
        // TODO: remove SCC from stack
    }
    return low;
}
```

Hint: `in_stack[v2]` → `v2` reaches `v`