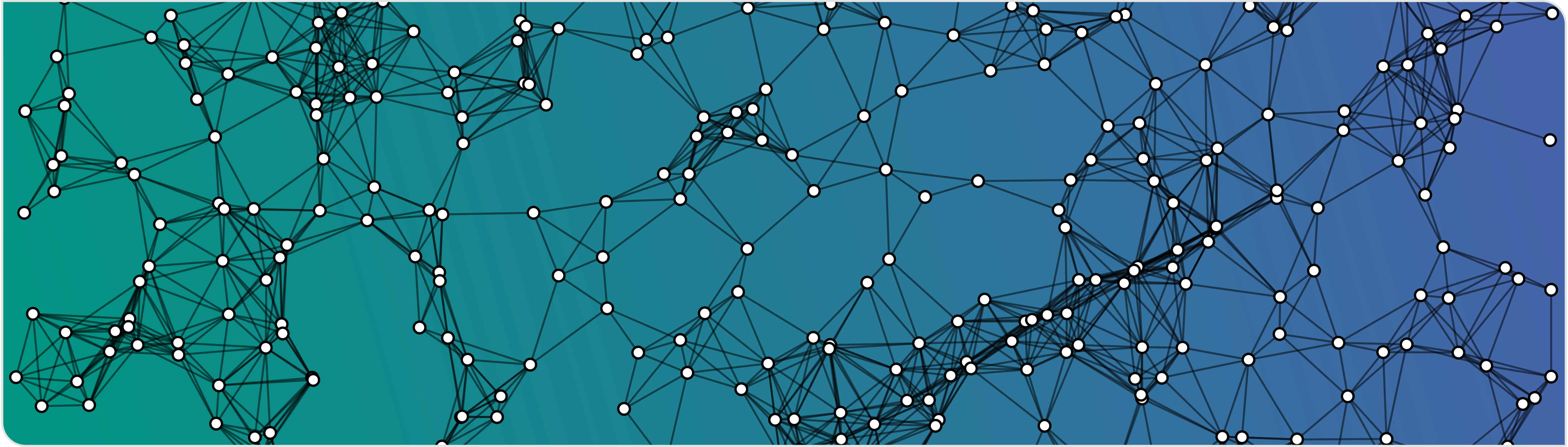
Karlsruher Institut für Technologie

A Simple Algorithm for Graph Reconstruction

presented by Jannik Westenfelder



Introduction

Problem

- Vertices (V) are known
- Edges (E) are unknown

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- Vertices (V) are known
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Goal

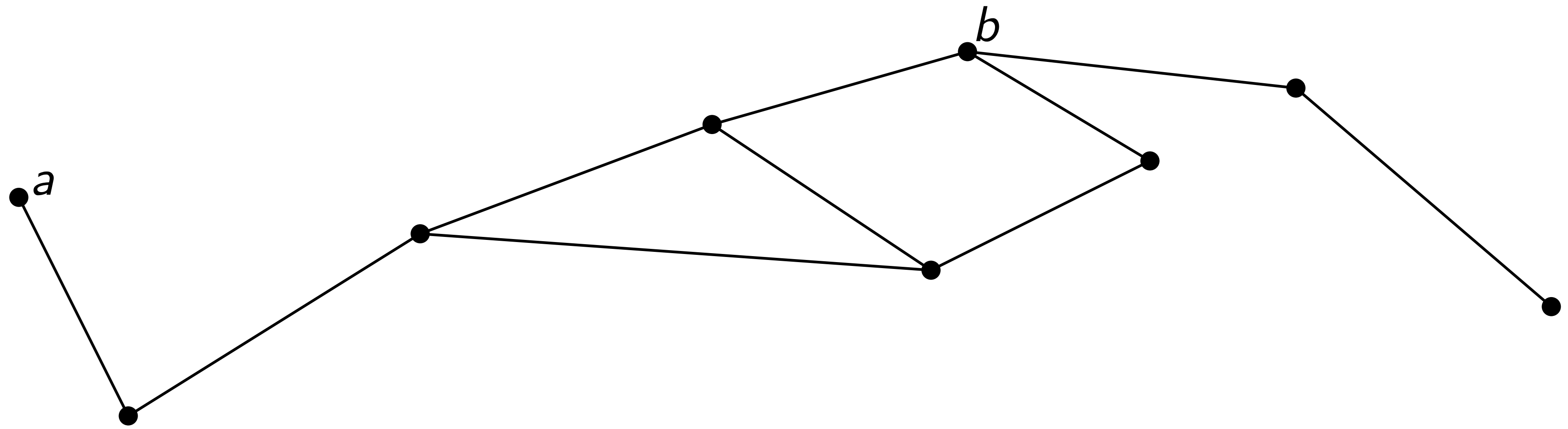
- Oracles give distance between vertices
- minimize Oracle queries
- Δ-regular graphs

Algorithm: Simple (V,s)

$S \leftarrow$ sample of s vertices selected uniformly and independently at random from V
for $u \in S$ and $v \in V$ **do**
    Query(u, v)
end for
$\hat{E} \leftarrow$ set of vertex pairs $\{a, b\} \subseteq V$ such that, for all $u \in S$, $|\delta(u, a) - \delta(u, b)| \leq 1$
for $\{a, b\} \in \hat{E}$ **do**
    Query(a, b)
end for
return set of vertex pairs $\{a, b\} \in \hat{E}$ such that $\delta(a, b) = 1$

Algorithm: Simple

$S = \{a, b\}$



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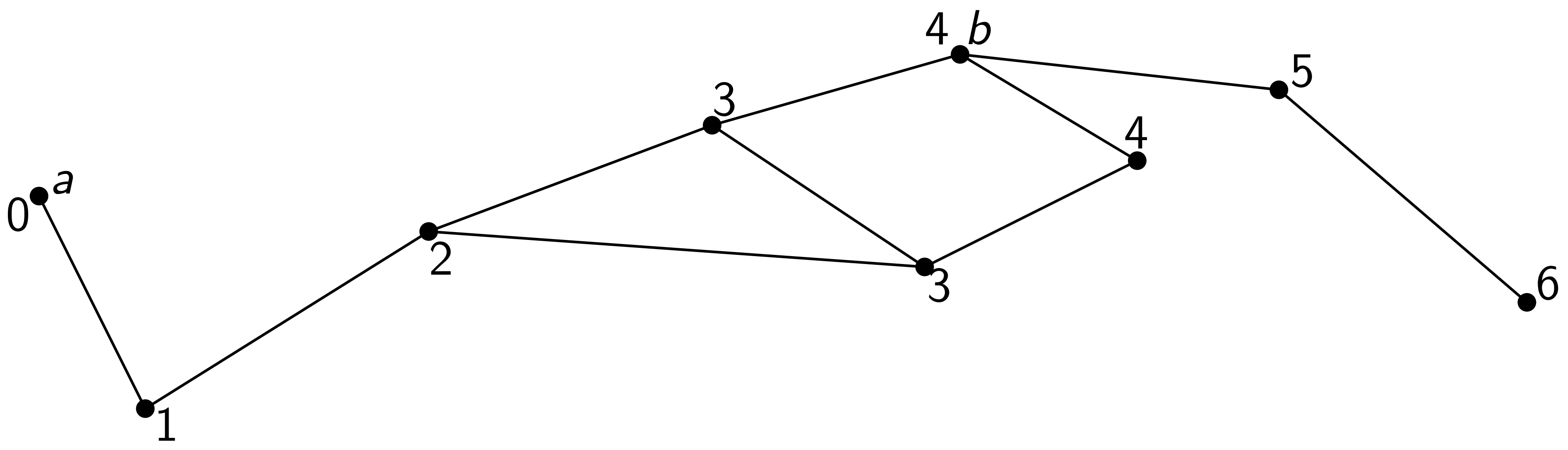
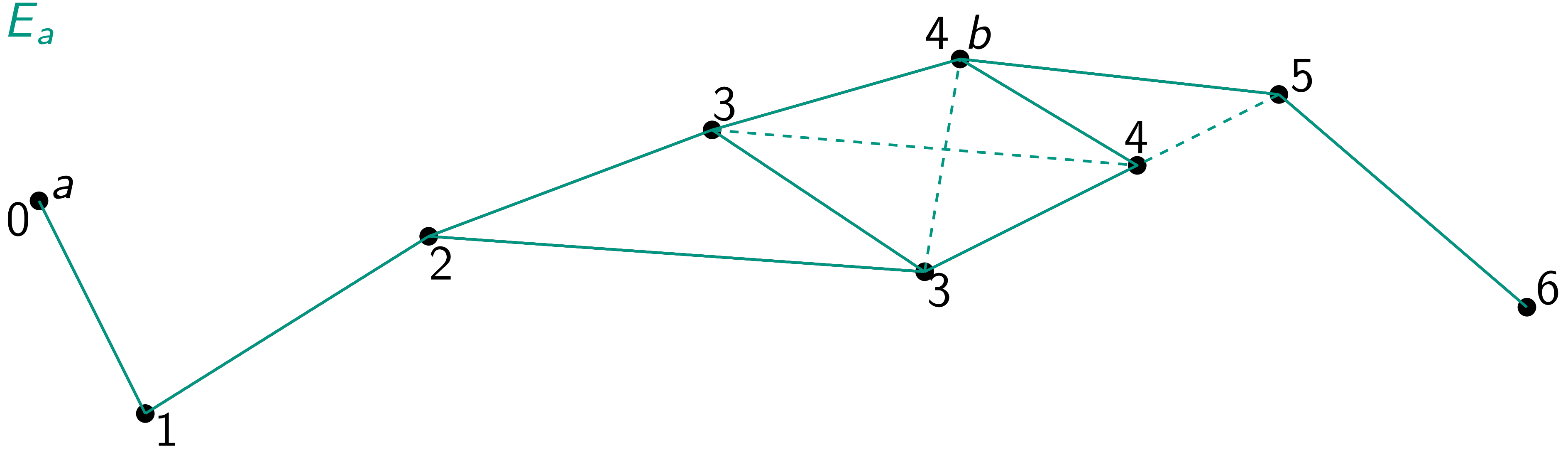


Algorithm: Simple

$S = \{a, b\}$

$\hat{E} = \hat{E}_a$



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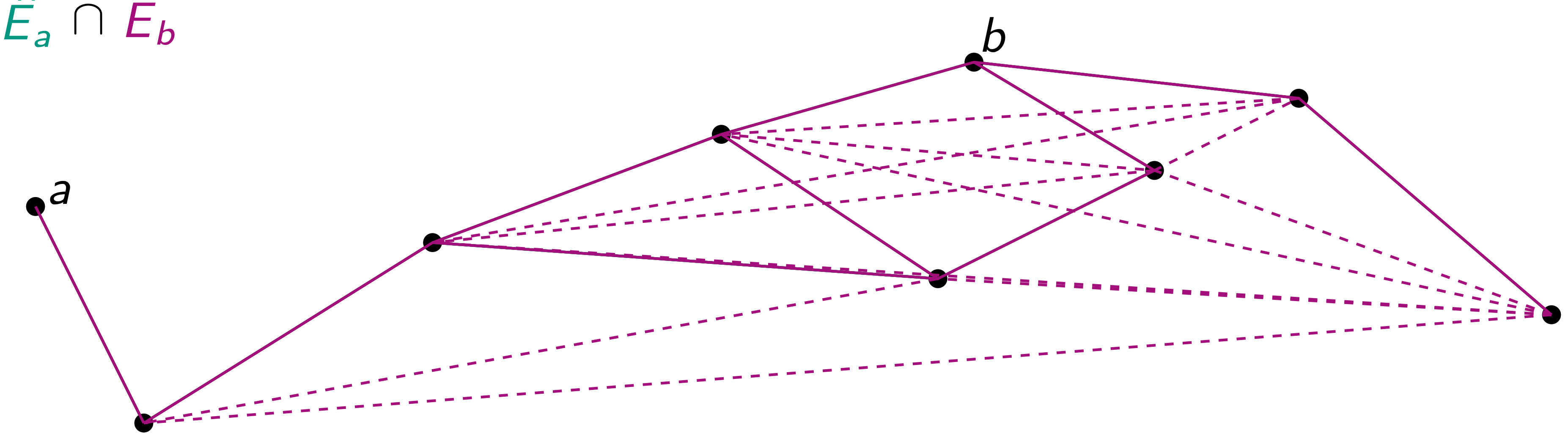

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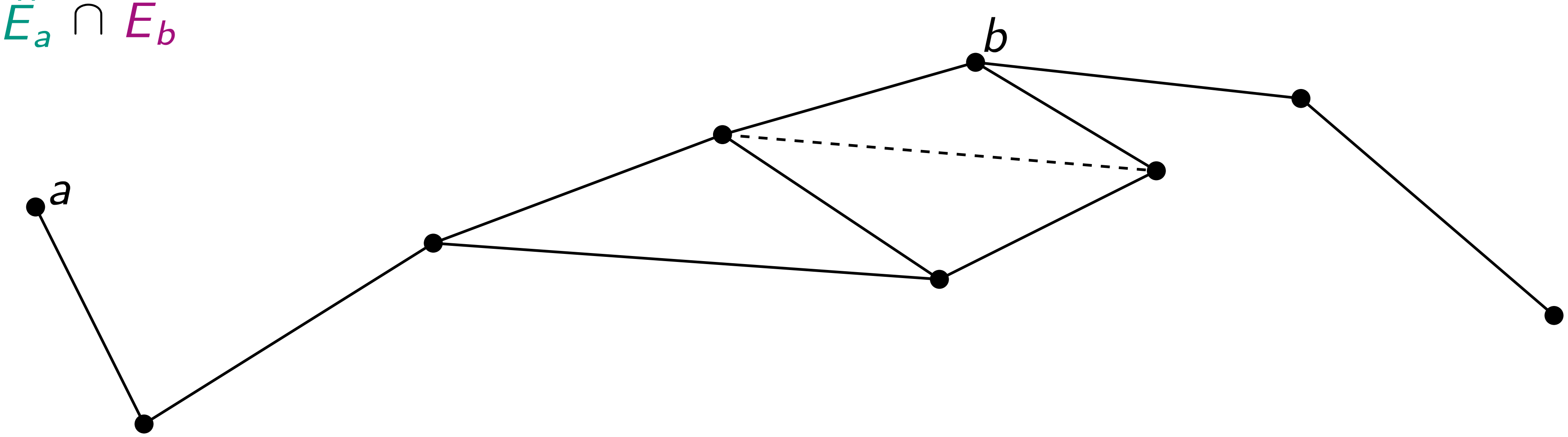

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$\}\ n \cdot s$ (lines 2–4)

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Lines 2–4: $n \cdot s$

Lines 6–8: $|\hat{E}|$

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$$\Rightarrow n \cdot s + |\hat{E}| \qquad |\hat{E}| = |E| + |\hat{E} \setminus E| \qquad \Rightarrow n \cdot s + |E| + |\hat{E} \setminus E|$$

Analysis of Simple

Definition (Distinguishing)

For a vertex pair $\{a, b\} \subset V$, we say that a vertex $u \in V$ distinguishes a and b, or equivalently that u is a distinguisher of $\{a, b\}$, if $|\delta(u, a) - \delta(u, b)| > 1$. Let $D(a, b) \subset V$ denote the set of vertices $u \in V$ distinguishing a and b.

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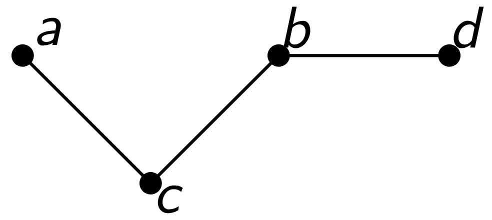

$D(a, b) = \{a, b, d\}$

Analysis of Simple

Lemma: Let B be the set of vertex pairs $\{a, b\} \subset V$ such that $\delta(a, b) \geq 2$ and $|D(a, b)| \leq 3n(\log n)/s$. We have $\mathbb{E}_S[|\hat{E} \setminus E|] \leq |B| + O(n^{-1})$.

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$= O(n^{-3})$

at most $n(n-1)/2$ vertex pairs $\Rightarrow O(n^{-3}) \cdot n(n-1)/2 = O(n^{-1})$

Analysis of Simple

$n \cdot s + |E| + |B| + O(n^{-1}) \qquad B = \{\{a, b\} \subset V | \delta(a, b) \geq 2, D(a, b) \leq \frac{3n \cdot \log n}{s}\} \qquad s = \log^2 n$

Structural Lemma. Let $\Delta = O(1)$ be such that $\Delta \geq 3$. Let G' be a multigraph corresponding to a uniformly random configuration. Let $\{v, w\}$ be a vertex pair in G' such that $\delta(v, w) \geq 2$. With probability $1 - o(n^{-2})$, we have $|D(v, w)| > \frac{3n}{\log n}$.

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$$n \cdot \log^2 n + \Delta n/2 + o(1) + O(n^{-1}) = \tilde{O}(n)$$

Proof Structural Lemma

- Configuration Model
- Construction of G'
- Interesting vertices
- Interesting vertices are distinguishing vertices
- Lower bound for the amount of interesting vertices

Configuration Model

$\Delta = 3, n = 6$

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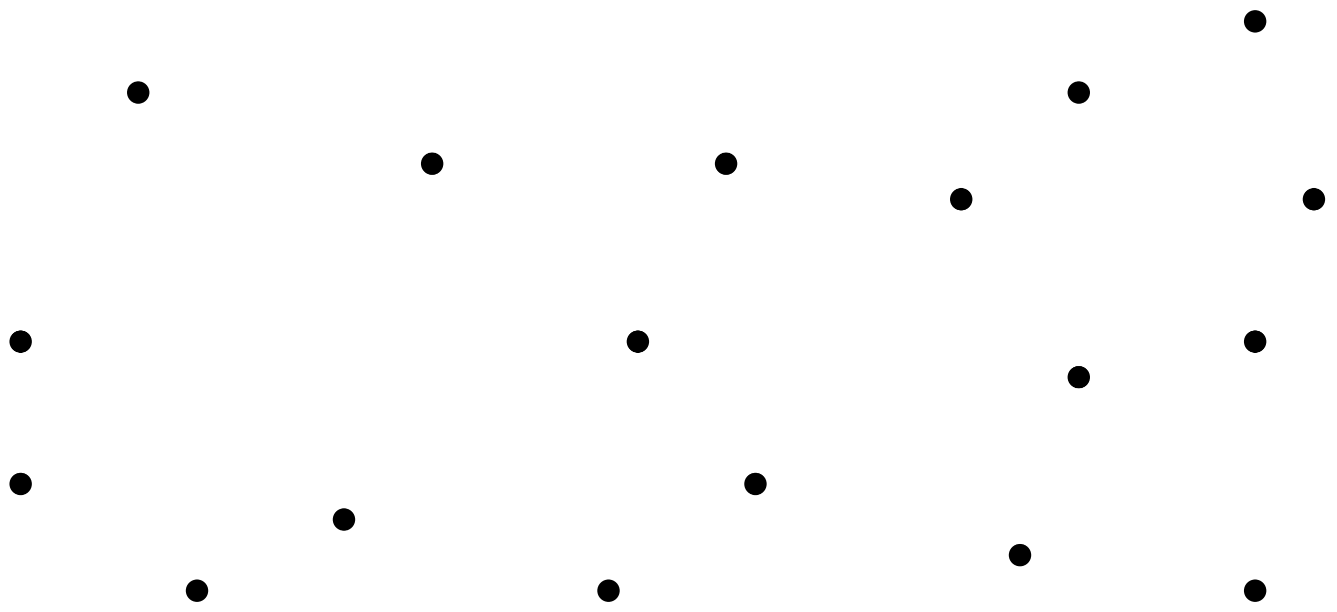
Δn points



Configuration Model

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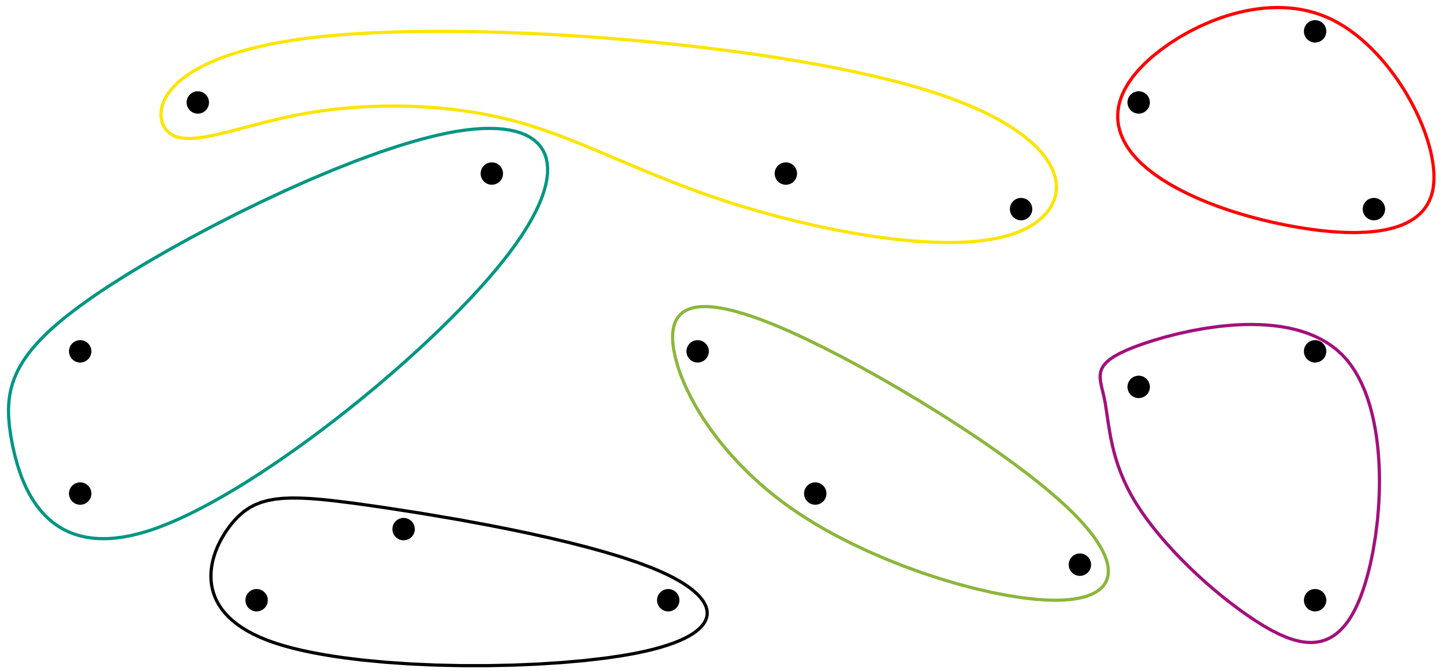

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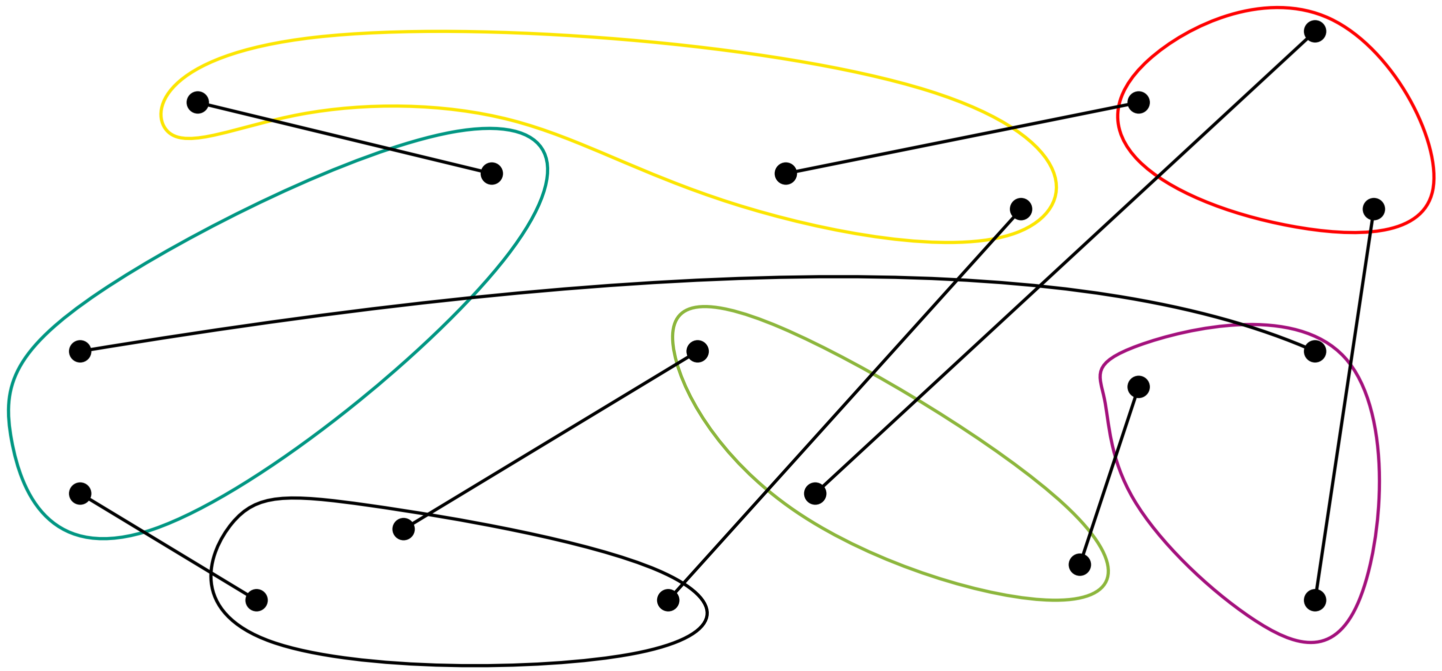

Configuration Model

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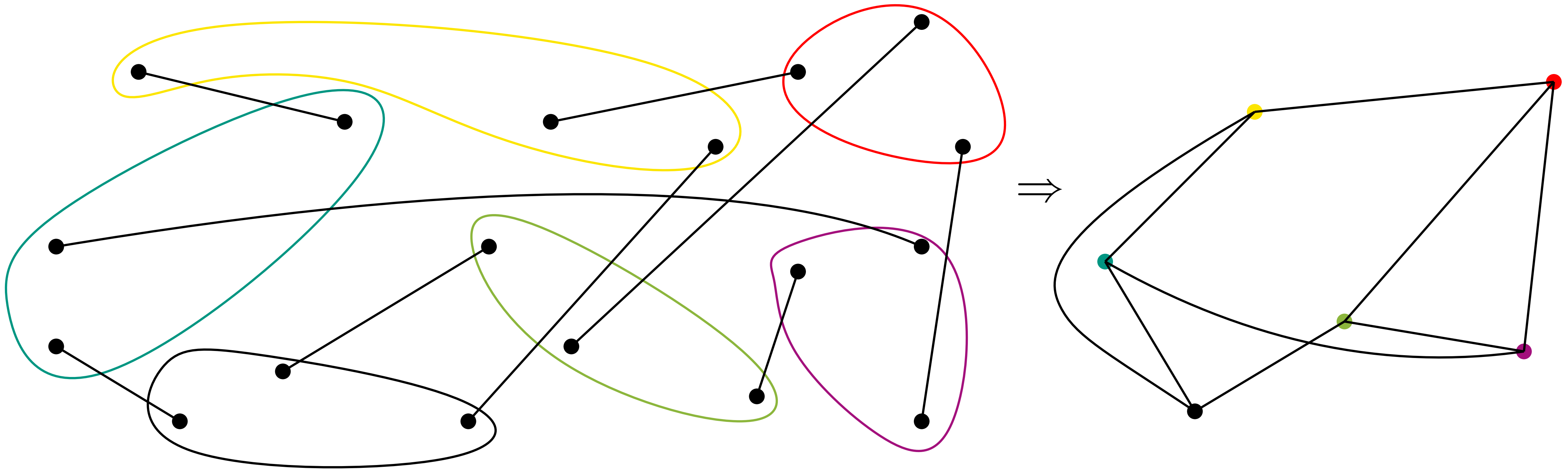
$\Rightarrow$ corresponds to a multigraph G' with cells as vertices



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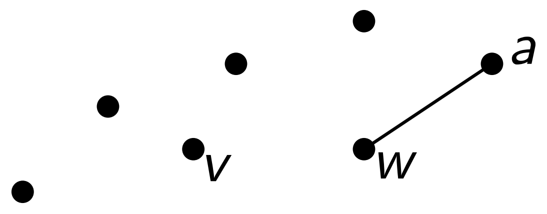

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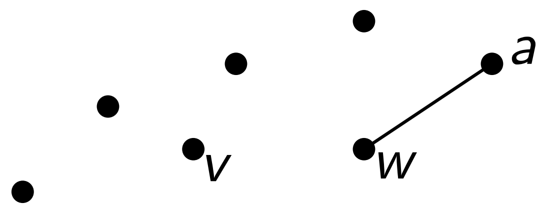

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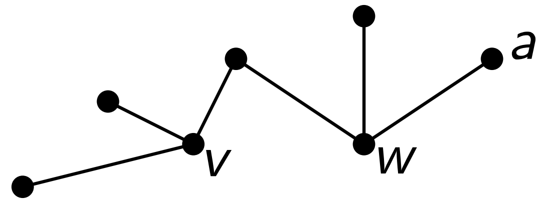

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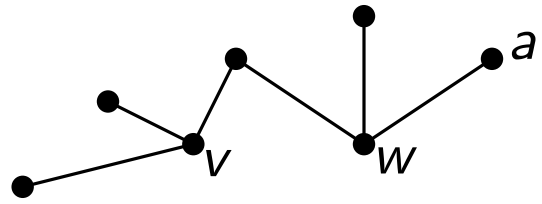

Fact. Neither v or w has a predecessor. For any vertex in V , its predecessor, if exists, is unique. If vertex a is the predecessor of vertex b, then $\ell(b) = \ell(a) + 1$.

G' Construction and Definitions

Interesting Vertices: A vertex $x \in V$ is v-interesting if, for all vertices $z \in V \setminus \{v\}$ with $\delta(v,z) + \delta(z,x) = \delta(v,x)$, the edges incident to z are indispensable. Similarly, a vertex $x \in V$ is w-interesting if, for all vertices $z \in V \setminus \{w\}$ with $\delta(w,z) + \delta(z,x) = \delta(w,x)$, the edges incident to z are indispensable.

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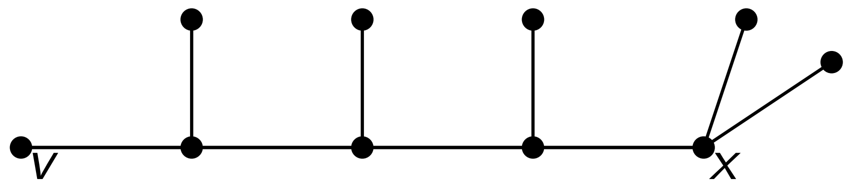

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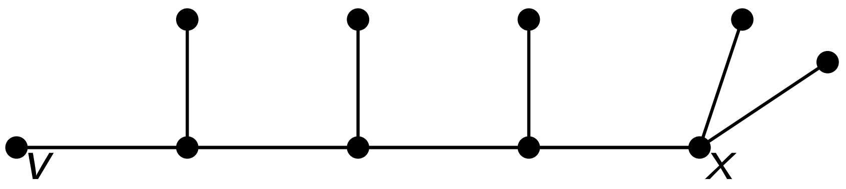

$I_k(v) \subset V$: v-interesting vertices, with $\delta(v, x) = k$,

Interesting vertices are also distinguisher

Lemma For any finite integer $k \geq 1$, we have $I_k(v) \cup I_k(w) \subset D(v, w)$.

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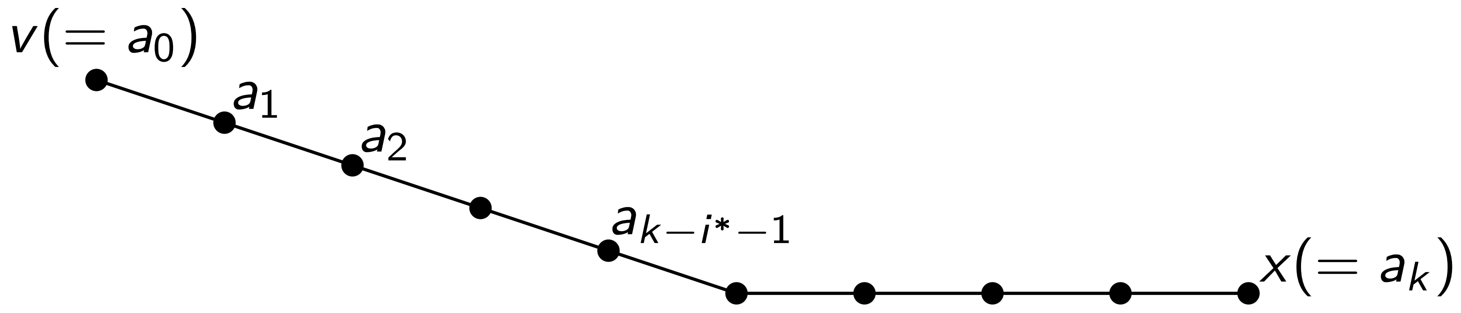

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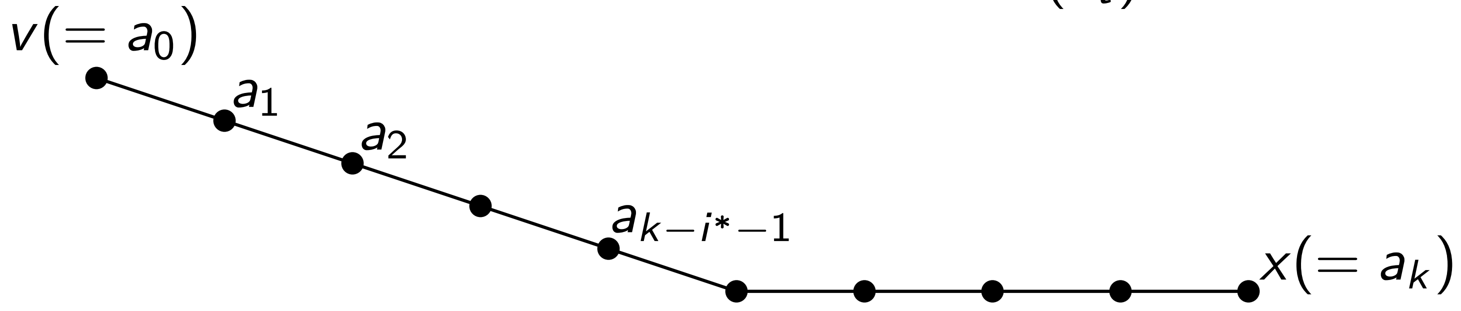
Fact: $\ell(a_t) = t$



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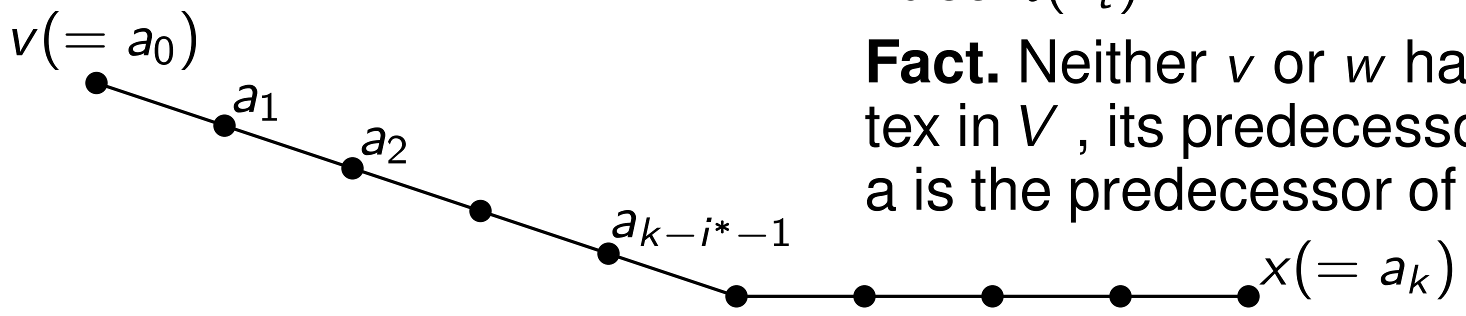

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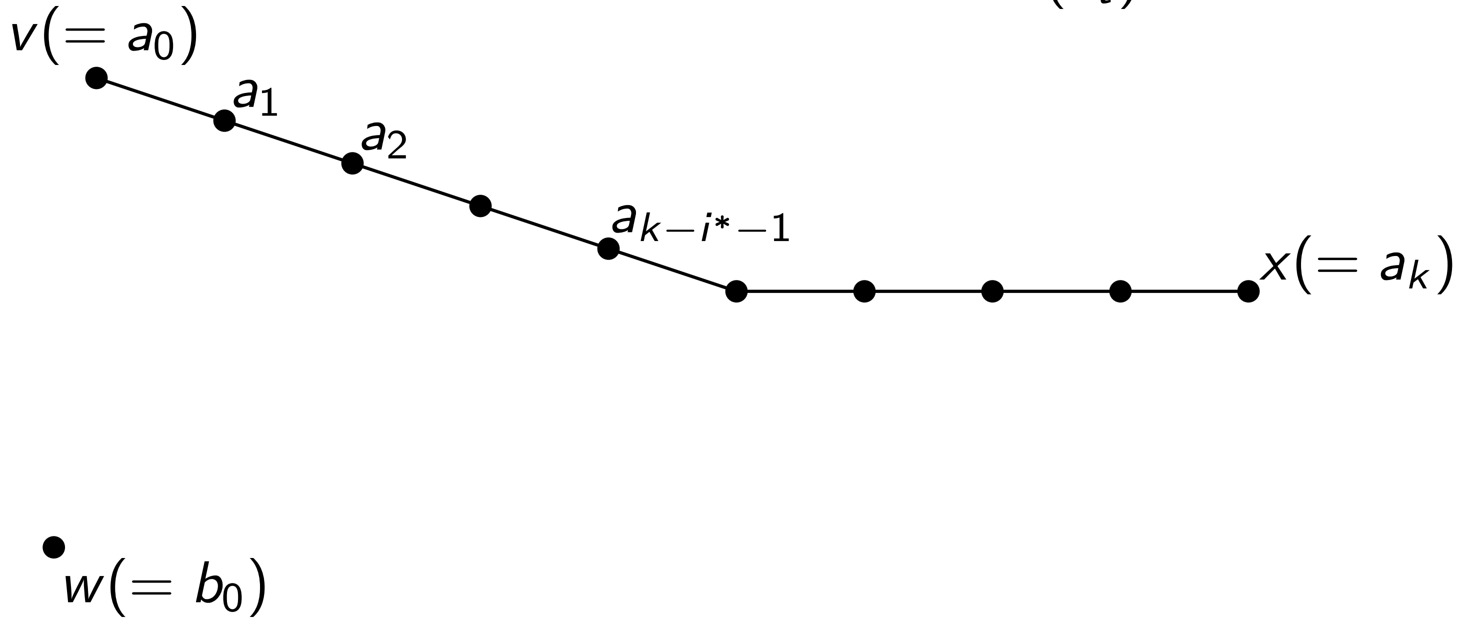

Interesting vertices are also distinguisher

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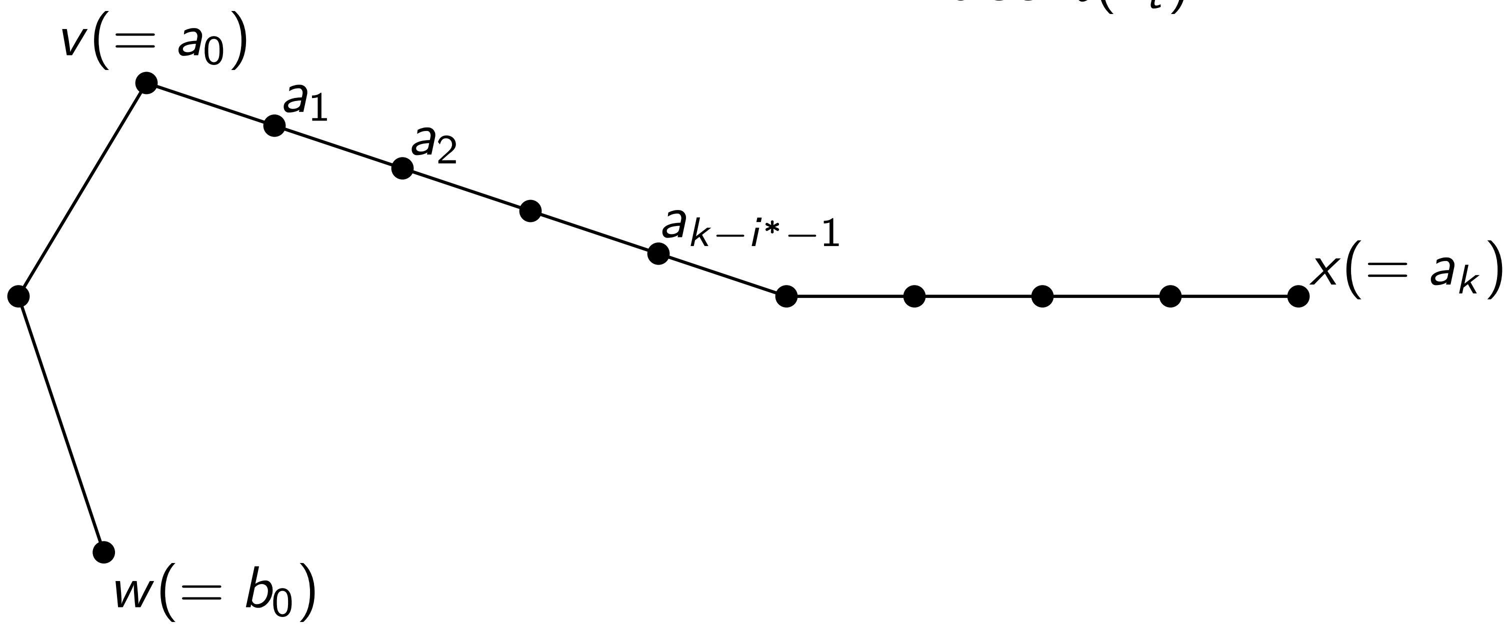

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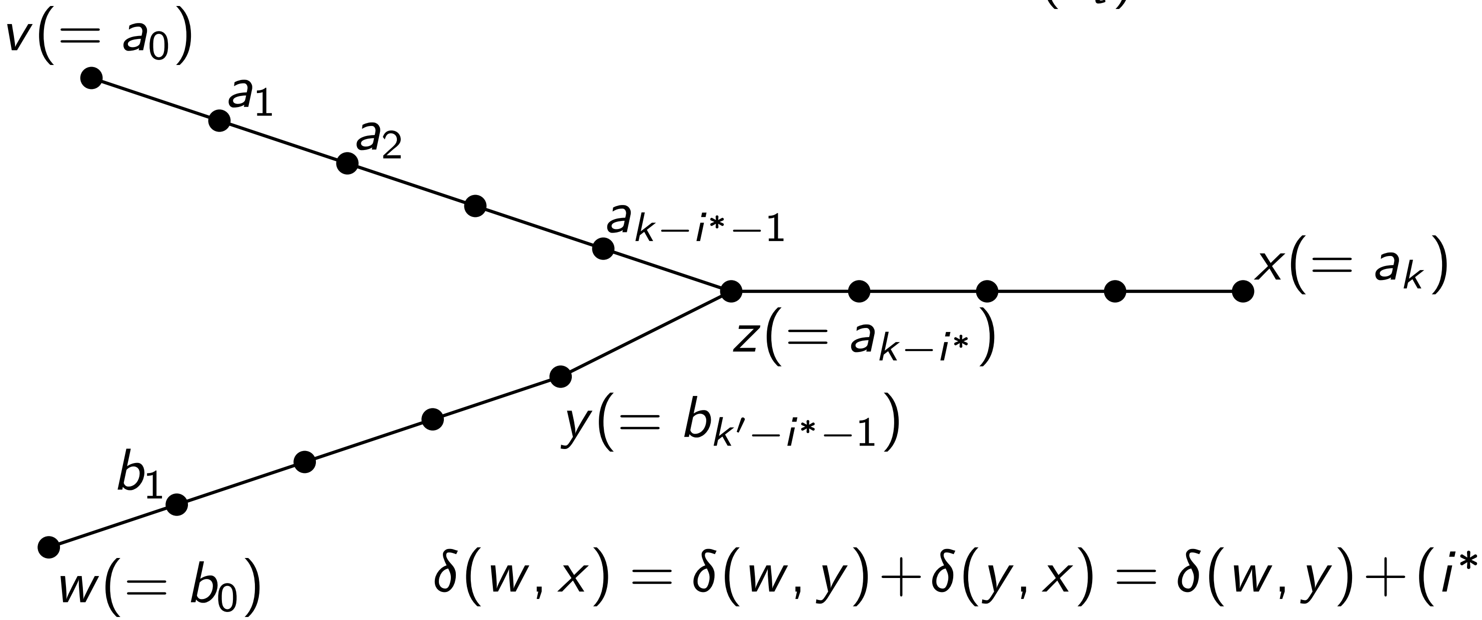

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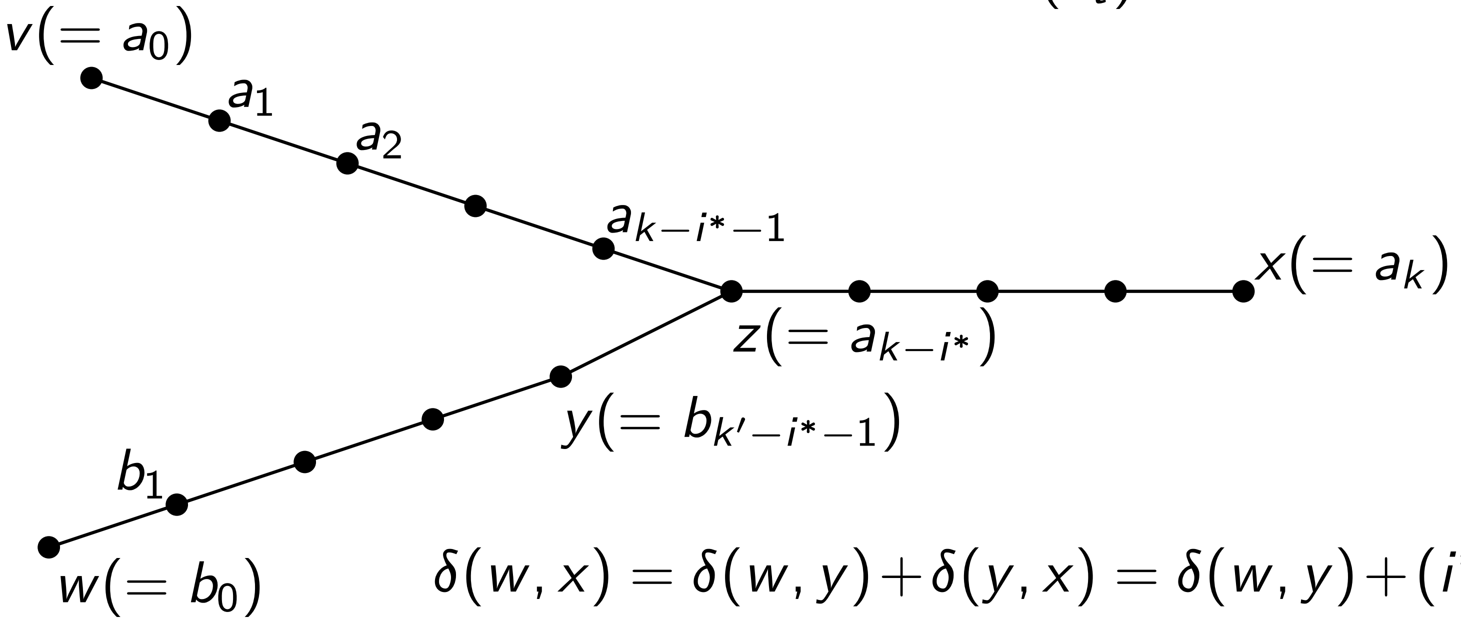

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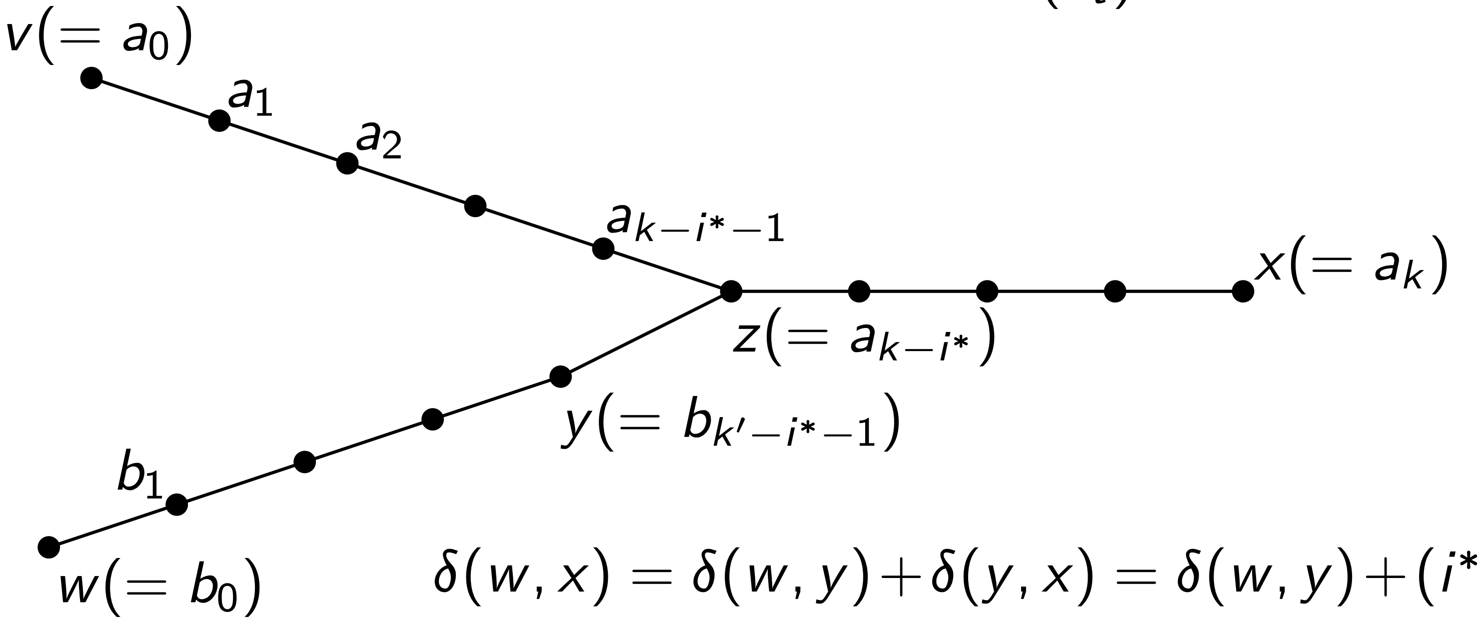

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$$\ell(y) \geq k - i^* + 1 \Rightarrow \delta(w,x) \geq (k - i^* + 1) + (i^* + 1) = k + 2$$

Lemma. Let $\Delta = O(1)$ be such that $\Delta \geq 3$. Let k be any positive integer such that $k \leq \lceil \log_{\Delta-1}\left(\frac{3n}{\log n}\right) \rceil + 2$. With probability $1 - o(n^{-2})$, we have $|I_k(v) \cup I_k(w)| > (\Delta - 2 - o(1))(\Delta - 1)^{k-1}$.

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Lemma 17. Let $M = \lceil \log\log n \rceil$. We can construct two non-decreasing sequences $\{g_i\}_{1 \leq i \leq M}$ and $\{L_i\}_{1 \leq i \leq M}$, such that all of the following properties hold when n is large enough:

1. $g_1 = 3$; and for any $i \in [2, M]$, $g_i = o\left((\Delta - 1)^{L_{i-1}/M}\right)$.
2. $L_M \geq \lceil \log_{\Delta-1}\left(\frac{3n}{\log n}\right) \rceil + 2$.
3. With probability $1 - o(n^{-2})$, for all $i \in [1, M]$, strictly less than g_i edges are dispensable among the edges incident to vertices in $U_{\leq L_i}$.

Lower Bound of interesting vertices

trivial edge: dispensable edge incident to v or w

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$F_0 \subset U_1$ vertices without trivial edges

$|F_0| \geq 2\Delta - 2|E_0|$

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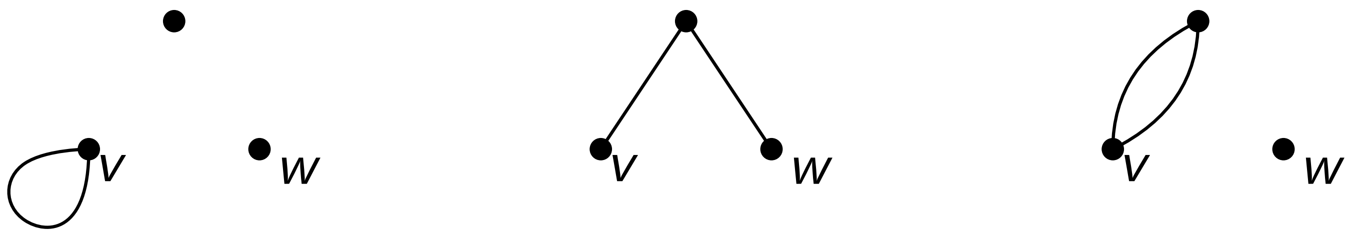

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$|F| \geq |F_0| - 2|E_1| \geq 2\Delta - 2|E_0| - 2|E_1| \geq 2(\Delta - 2)$

$g_1 = 3 \Rightarrow |E_0| + |E_1| \leq 2$

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$\Rightarrow |I_k(v)| \geq (\Delta - 2)(\Delta - 1)^{k-1}$

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Karlsruher Institut für Technologie

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$F' \subset I_{L_1}(v)$ T'(u) no vertex with dispensable edges complete $(\Delta - 1)$-ary tree

$|F'| > |I_{L_1}(v)| - 2g_2$

consider $y \in T'(x)$, with $\delta(v, y) = k \in [L_i + 1, L_2]$

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$$\begin{aligned} |I_k(v)| &> (|I_{L_1}(v)| - 2g_2) \cdot (\Delta - 1)^{k-L_1} \\ &\geq ((\Delta - 2)(\Delta - 1)^{L_1 - 1} - 2g_2) \cdot (\Delta - 1)^{k-L_1} \\ &= (\Delta - 2 - o(1/M))(\Delta - 1)^{k-1}. \end{aligned}$$

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$|I_k(v) \cup I_k(w)| \geq (\Delta - 2 - o(1))(\Delta - 1)^{k-1}$, for any $k \in [1, L_M]$.

Simple vs Multi-Phase

bounded degree

- Simple: $\tilde{O}(n^{5/3})$
- Multi-Phase: $\tilde{O}(n^{3/2})$

Simple vs Multi-Phase

bounded degree

- Simple: $\tilde{O}(n^{5/3})$
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regular

- Simple: $\tilde{O}(n)$
- Multi-Phase: $\tilde{O}(n^{3/2})$

Outlook

- other Oracles
- Simple Modified
- Parallel Setting
- Metric Dimension

Thank you for your Attention

Questions?