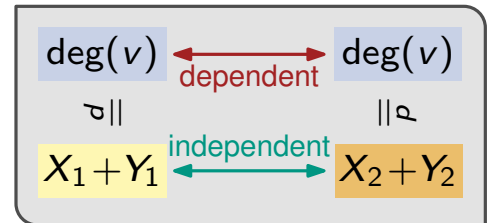


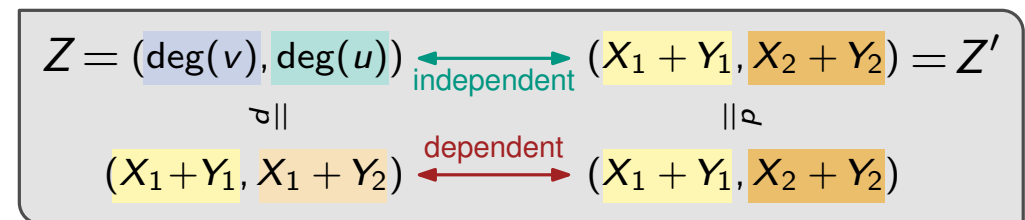
Coupling Degrees in $G(n, p)$

- Consider a $G(n, p)$ containing vertices u and v
- We are interested in the probability $q = \Pr[\deg(v) = d \wedge \deg(u) = d] - \Pr[\deg(v) = d]^2$
- Consider $Y_1, Y_2 \sim \text{Bin}(n - 2, p)$, and $X_1, X_2 \sim \text{Ber}(p)$ and the coupling

In a shared probability space and all independent



- Then $\Pr[\deg(v) = d] = \Pr[X_1 + Y_1 = d] = \Pr[X_2 + Y_2 = d]$
- Thus $q = \Pr[\deg(v) = d \wedge \deg(u) = d] - \Pr[X_1 + Y_1 = d] \Pr[X_2 + Y_2 = d]$
- Now define random variables as tuples of random variables: $Z = (\deg(v), \deg(u))$ and $Z' = (X_1 + Y_1, X_2 + Y_2)$, and consider the coupling



- Then $q = \Pr[X_1 + Y_1 = d \wedge X_1 + Y_2 = d] - \Pr[X_1 + Y_1 = d] \Pr[X_2 + Y_2 = d]$