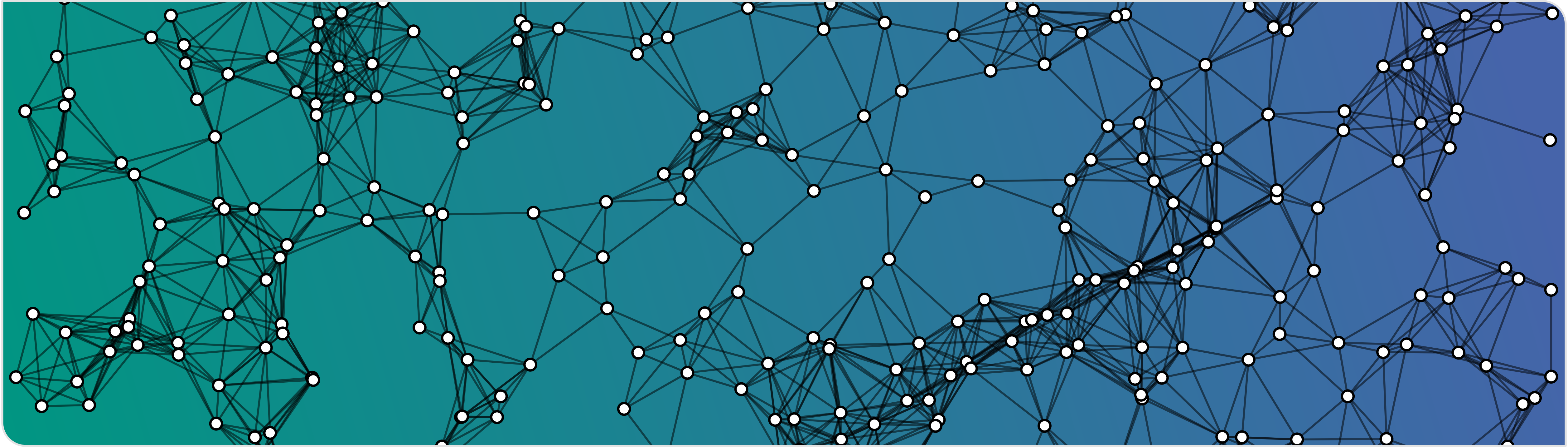
Karlsruher Institut für Technologie

Praktikum – Beating the Worst Case

Jean-Pierre von der Heydt und Marcus Wilhelm | 29.11.2023



Generierte Graphen

Wie gut funktionieren die Algorithmen auf den neuen Graphen?

Wie sieht es mit der Heterogenität und Lokalität der Graphen aus?

Wie sehen Graphen mit hoher Heterogenität und geringer Lokalität aus?

Übungsblatt 2

Echtwelt-Graphen

Verhalten sich die Echtwelt-Graphen ähnlich wie die generierten Graphen?

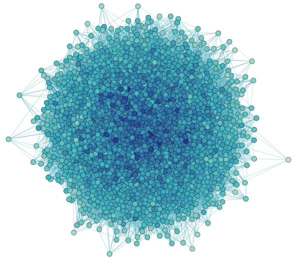
Könnt ihr herausfinden, wie wir die Graphen generiert haben?

Modelle für komplexe Netzwerke

Ziel: Modellieren und Erklären der Eigenschaften

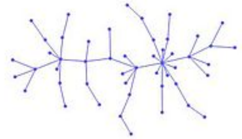
Drei Charakteristika:	ER 1959	Pref. Attach. / Barabási-Albert 1923 / 1999	Chung-Lu 2002	Watts-Strogatz model 1998	GRG	HRG 2010	GIRG 2019
heterogene Gradverteilung		✓	✓			✓	✓
kurze Wege / „small-world“	✓	✓	✓	✓		✓	✓
hohe Lokalität / Clustering				✓	✓	✓	✓

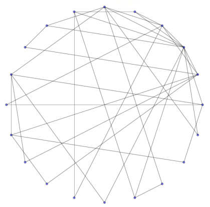
Erdős–Rényi model



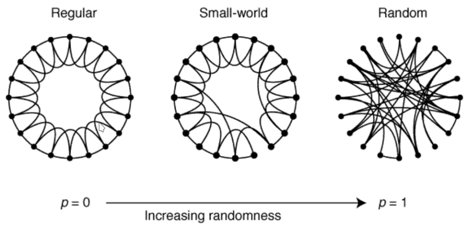
Preferential Attachment

iteratively add vertices, choose edges with probability proportional to current degree





Watts–Strogatz model

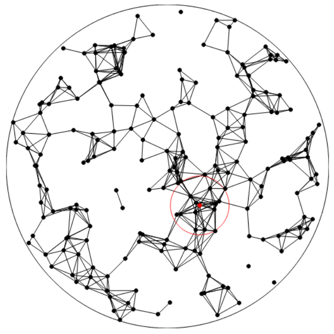


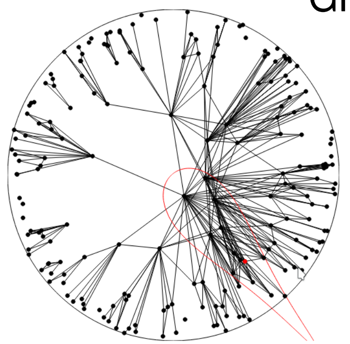
Chung-Lu / Configuration model / IRG

vertices with weights w_i (following power-law distribution);
$\Pr\left[\{e_i, e_j\} \in E\right] \sim \frac{w_i \cdot w_j}{W}$

Geometric Random Graph (Hyperbolic)

sample vertices uniformly in geometry, connect if distance below threshold

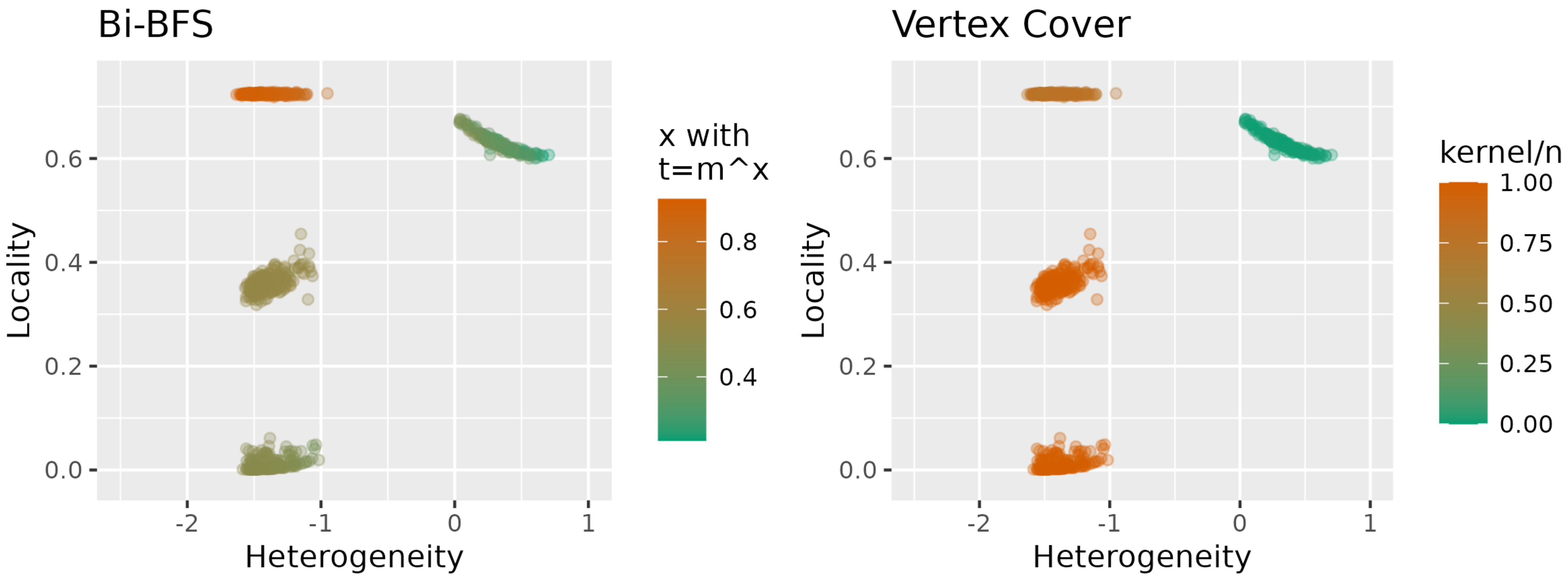




GIRG

GRG $\ast$ IRG

Auswertung von veschiedenen Graphen



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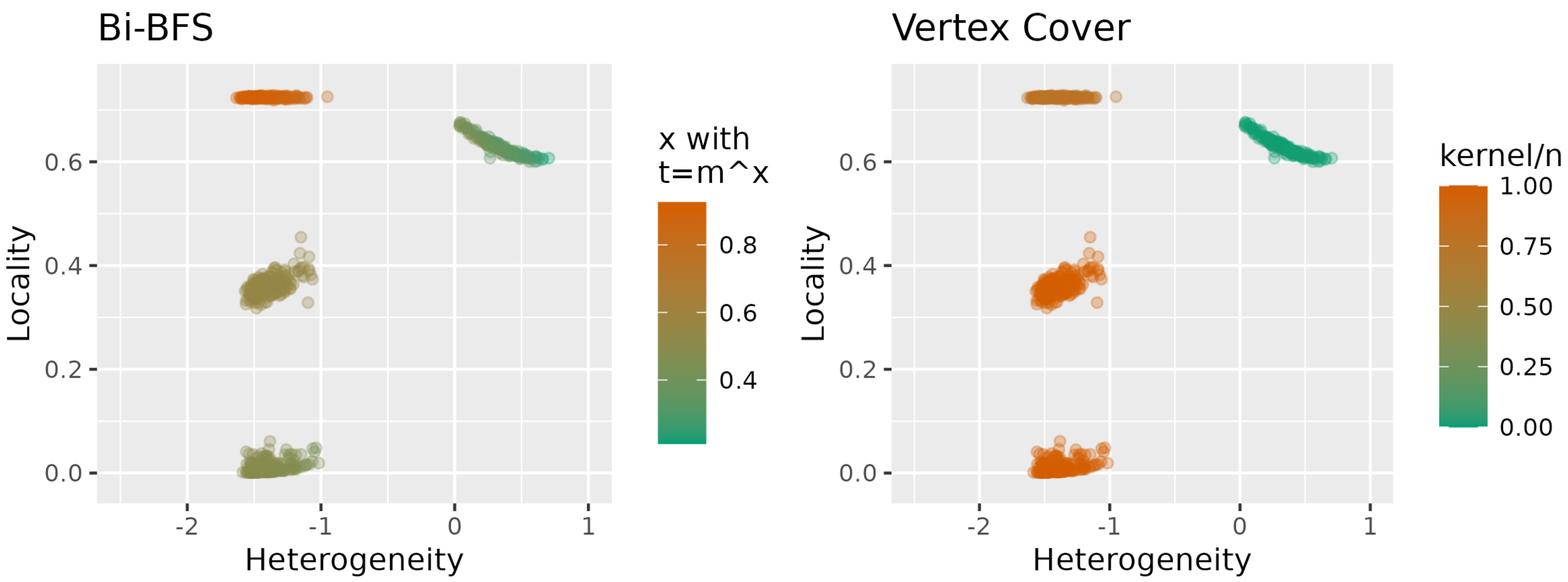

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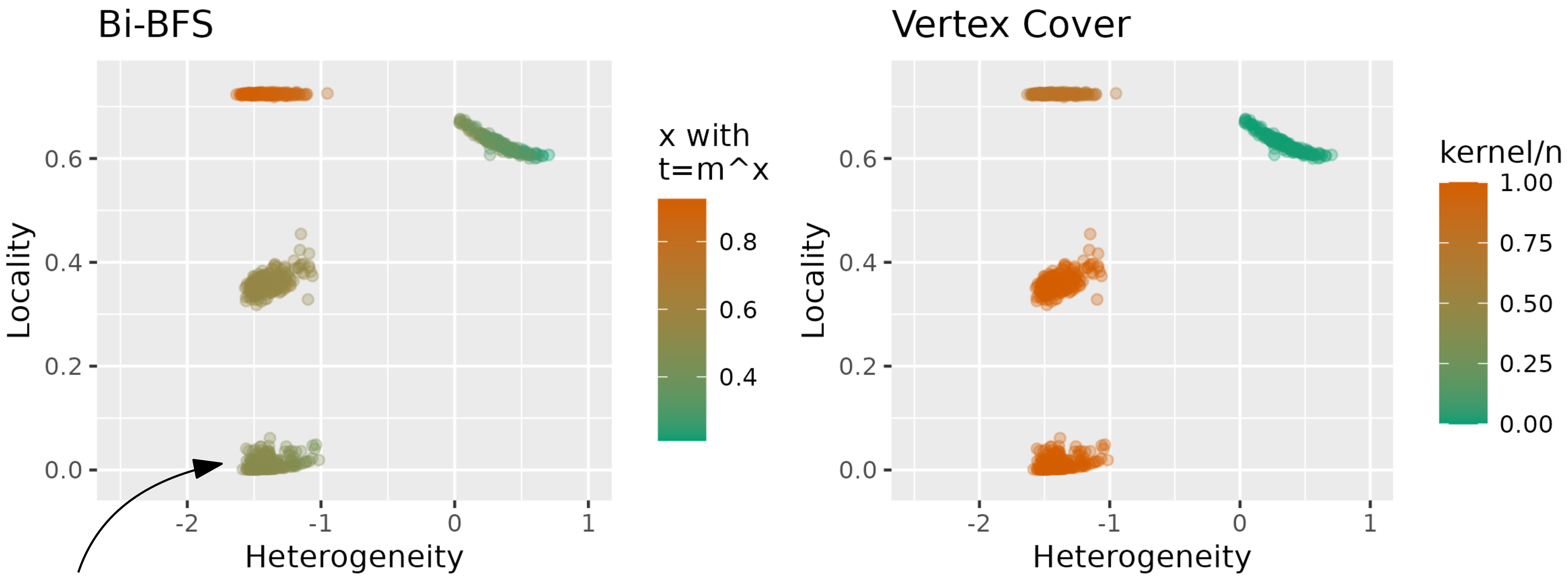

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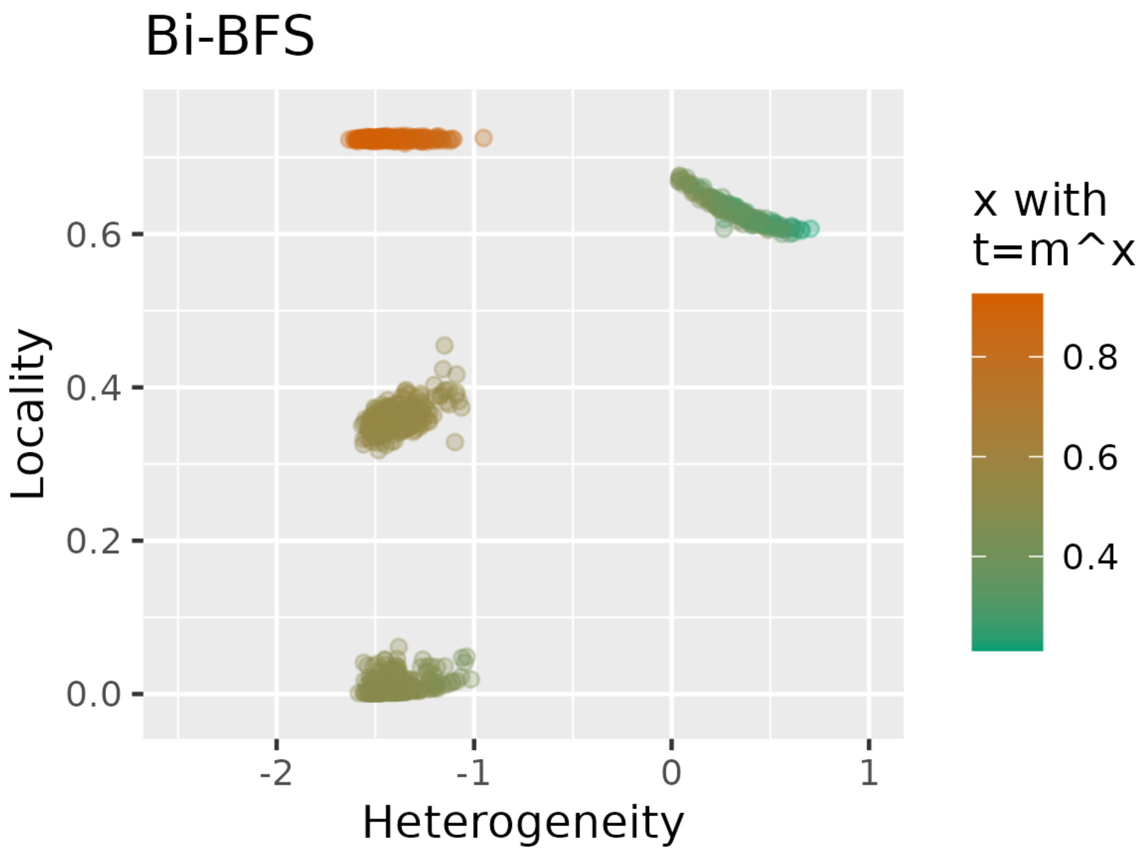

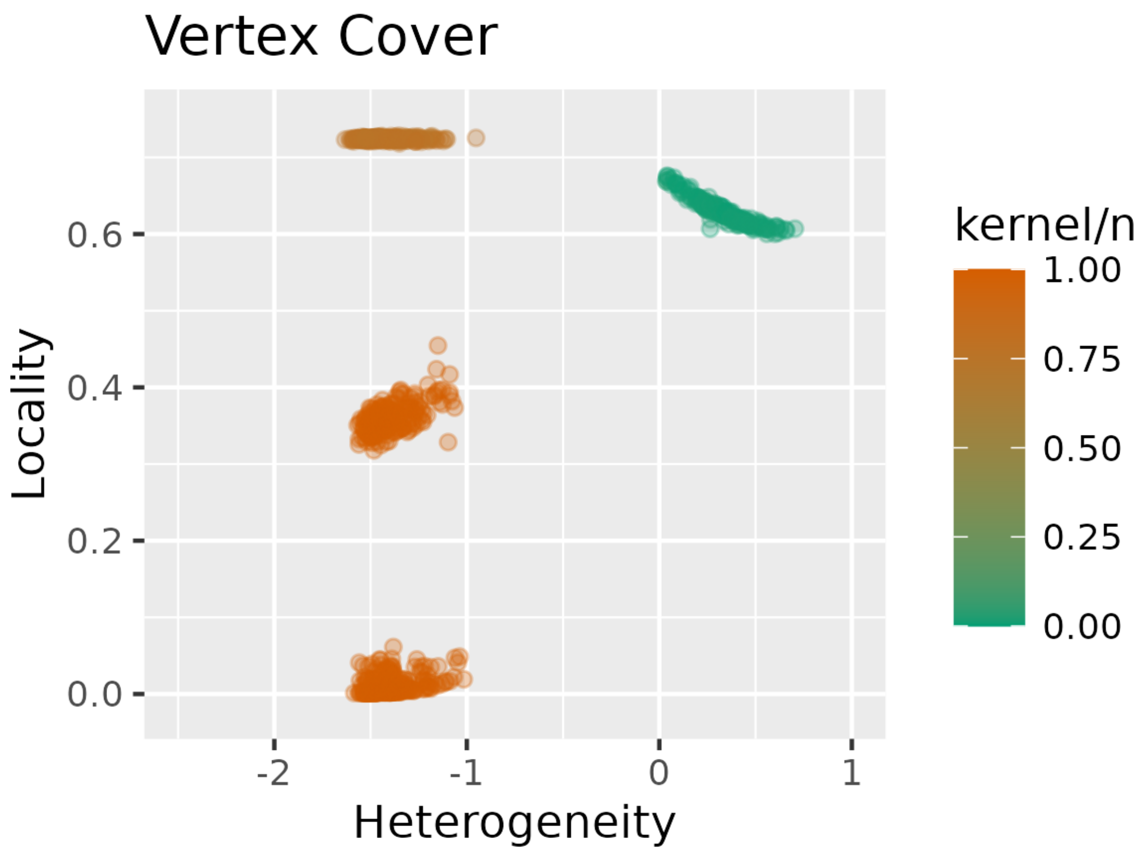

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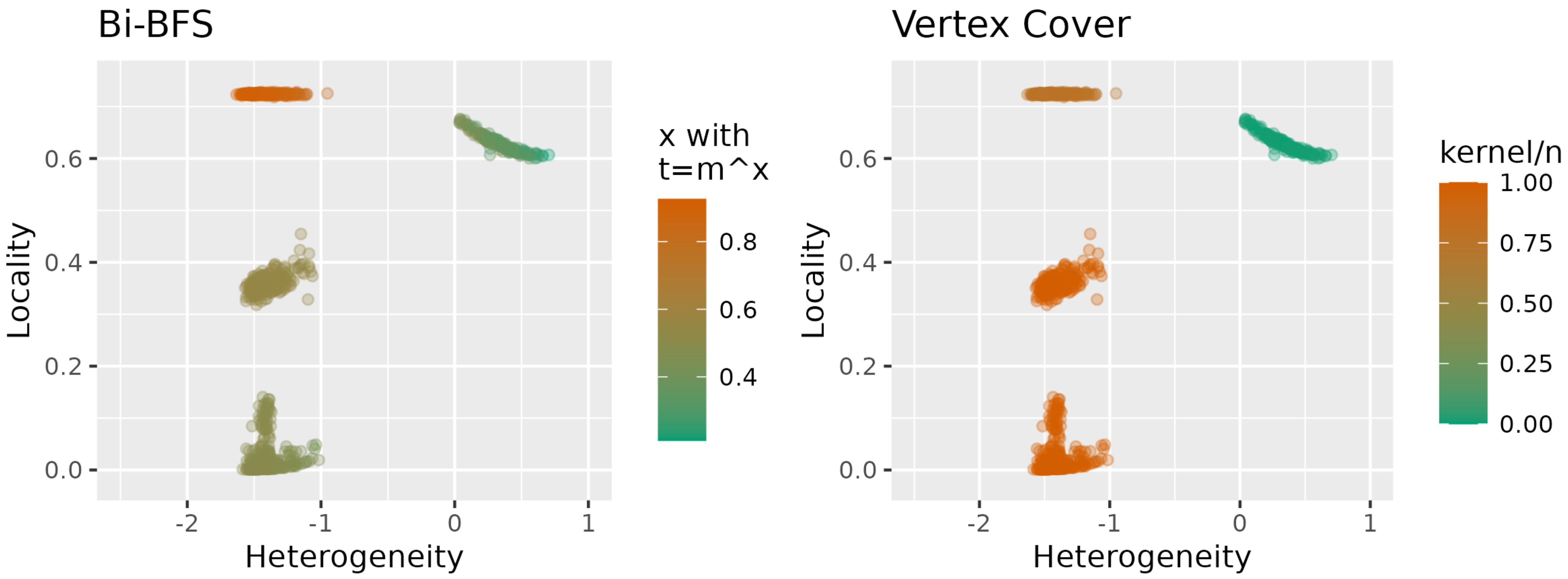

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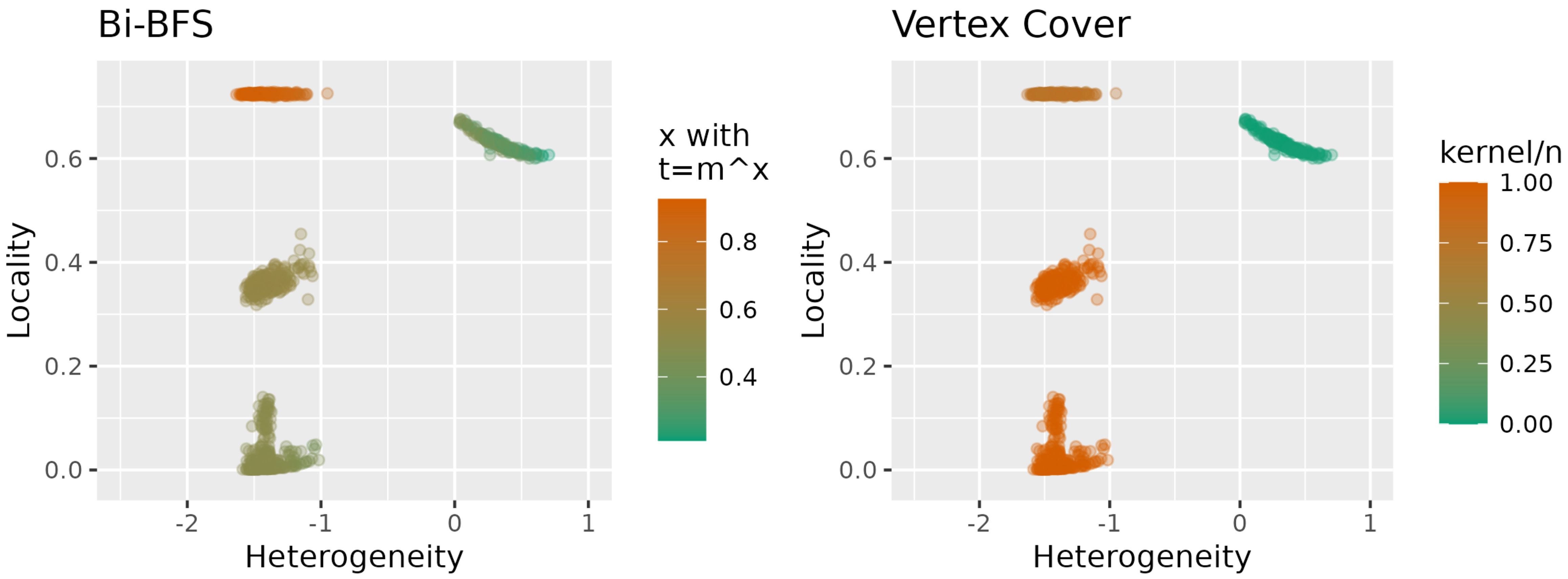

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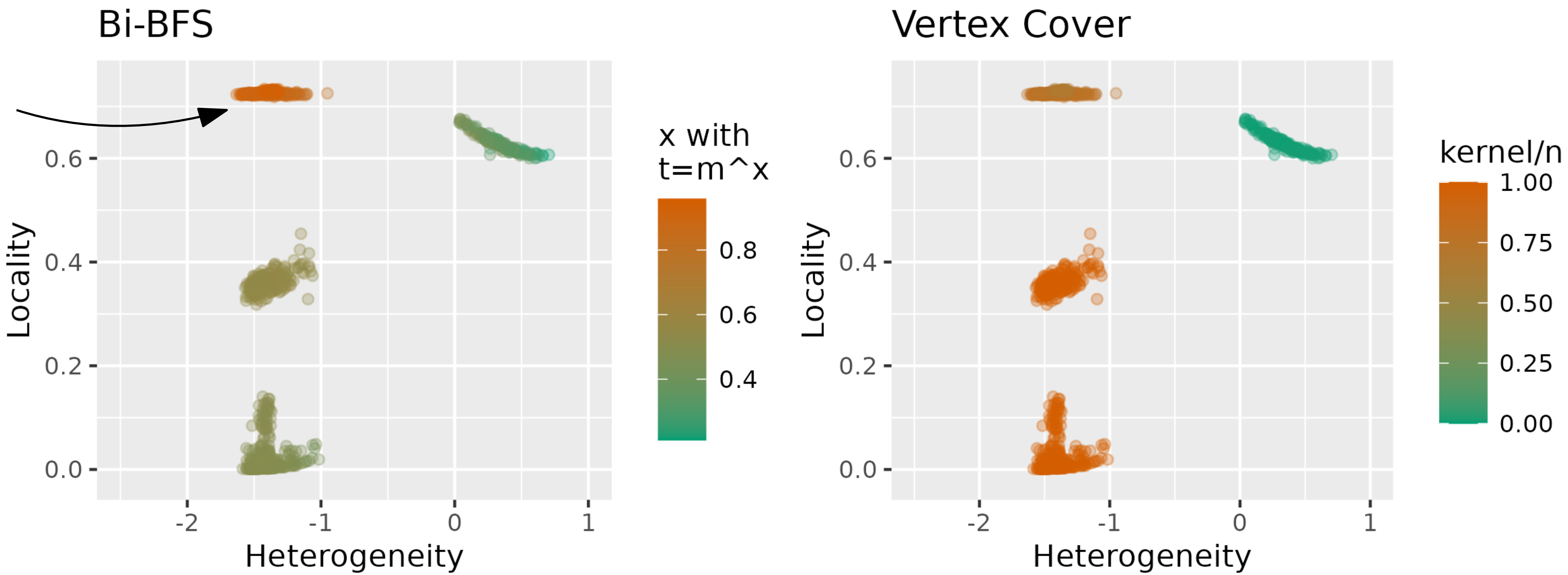

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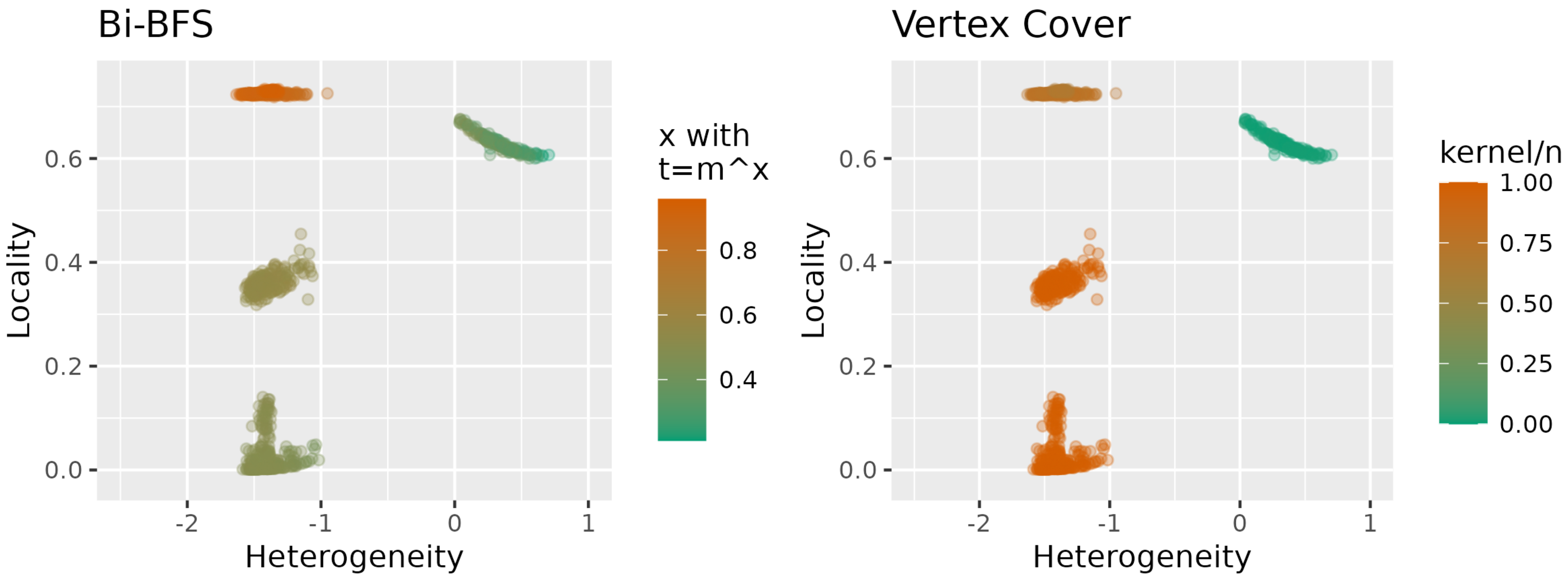

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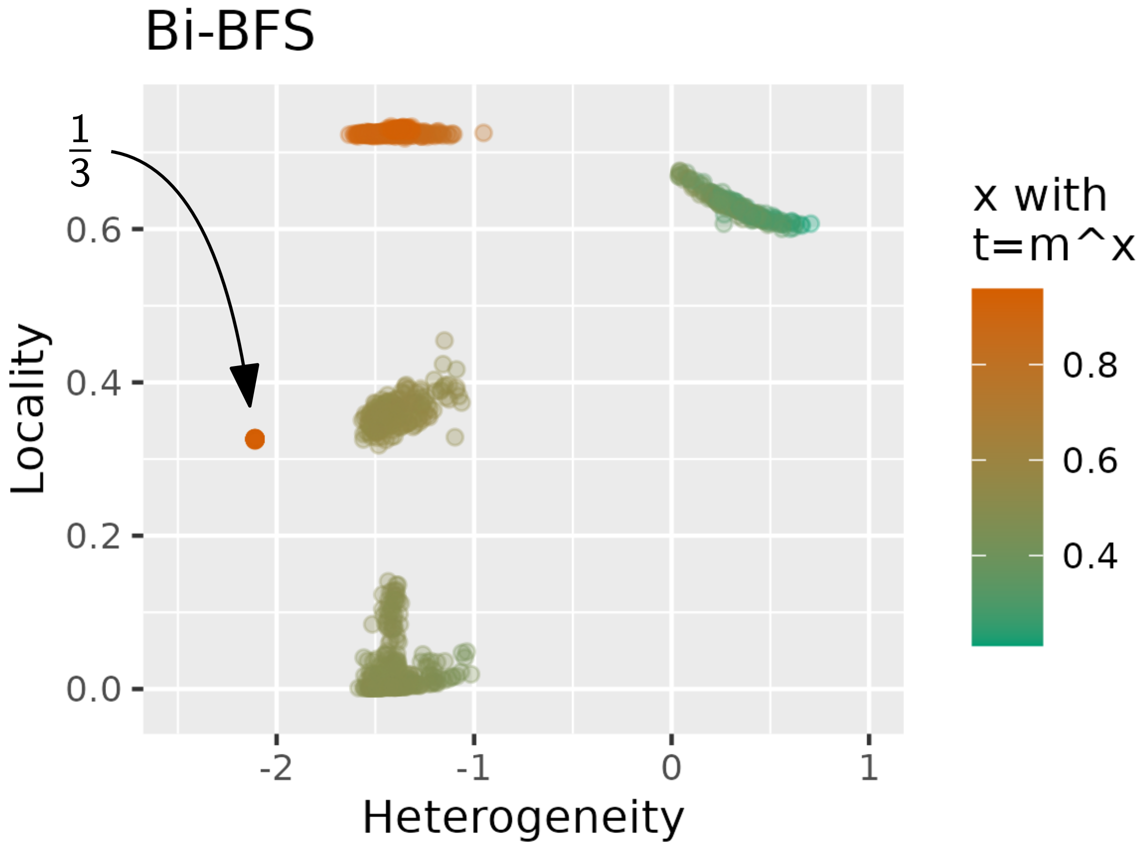

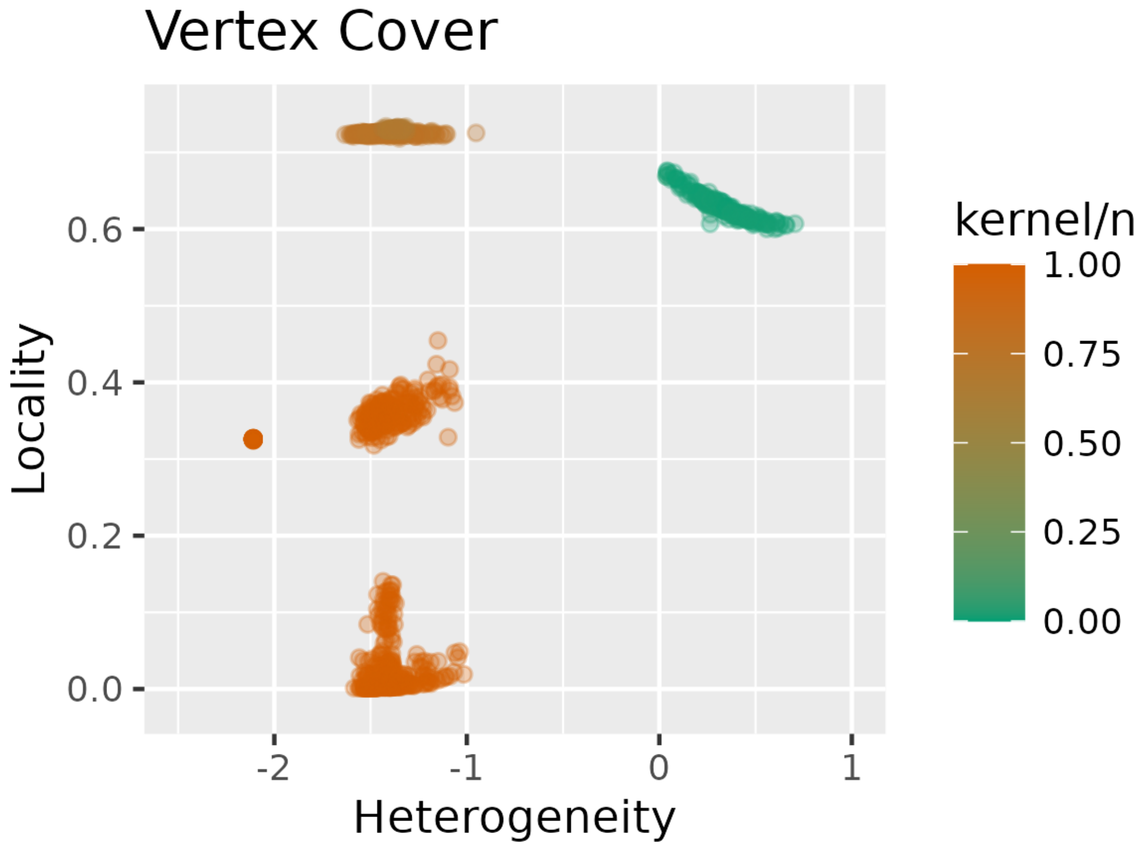

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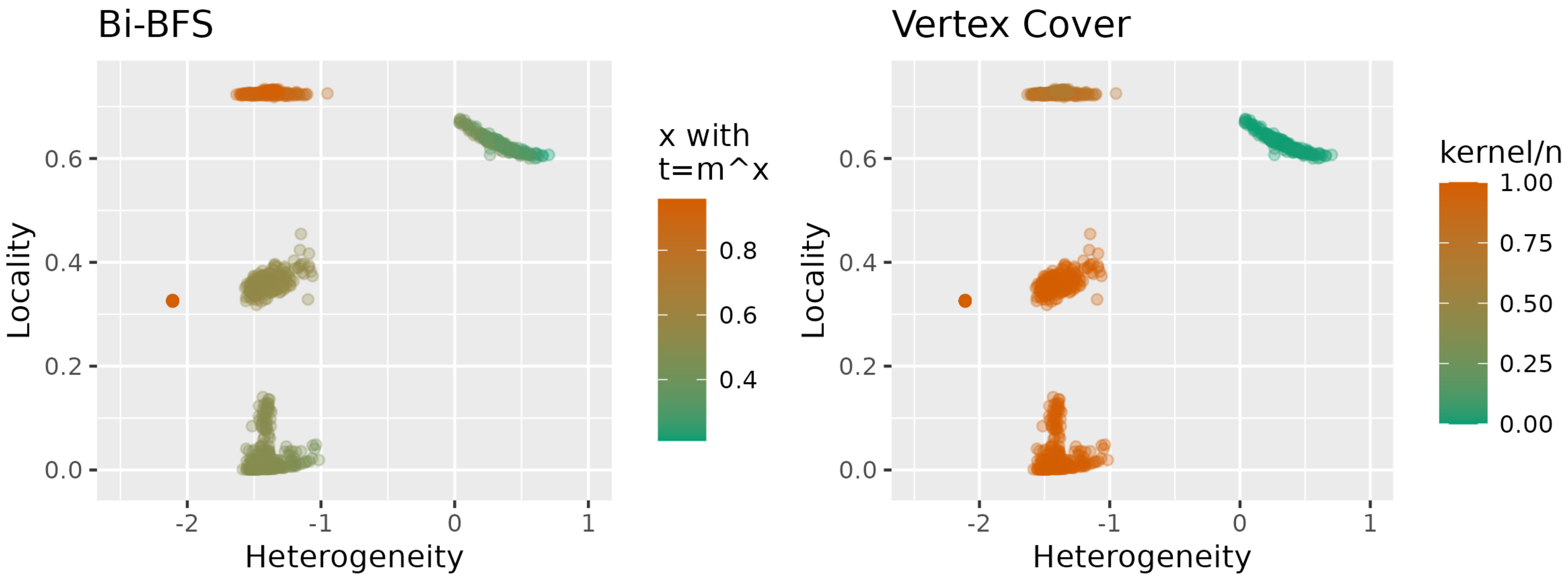

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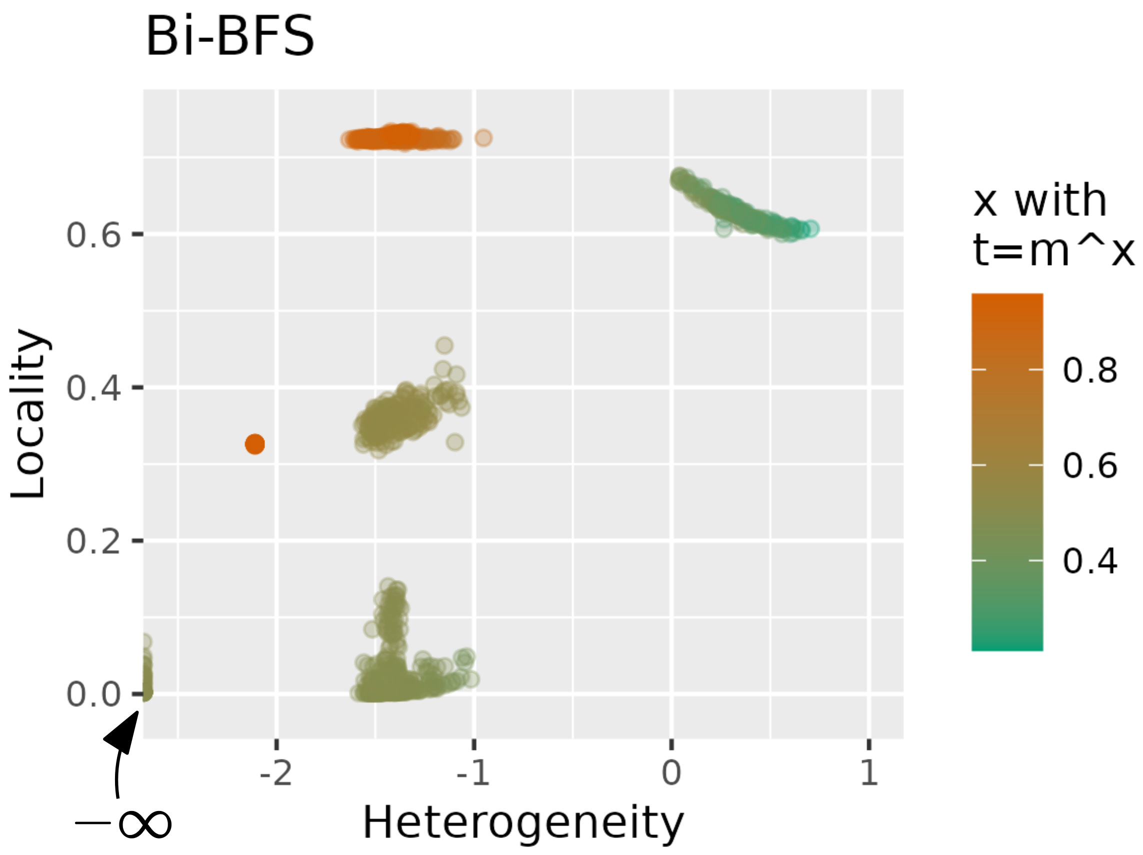

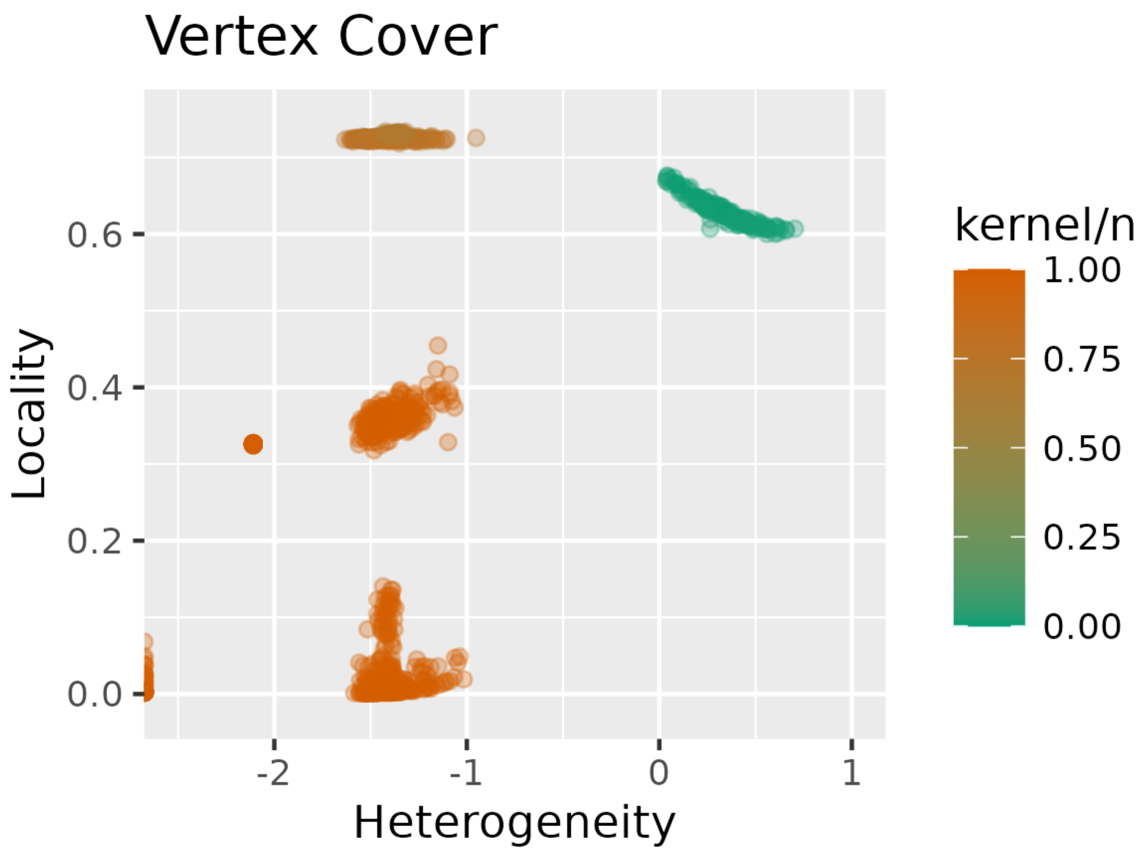

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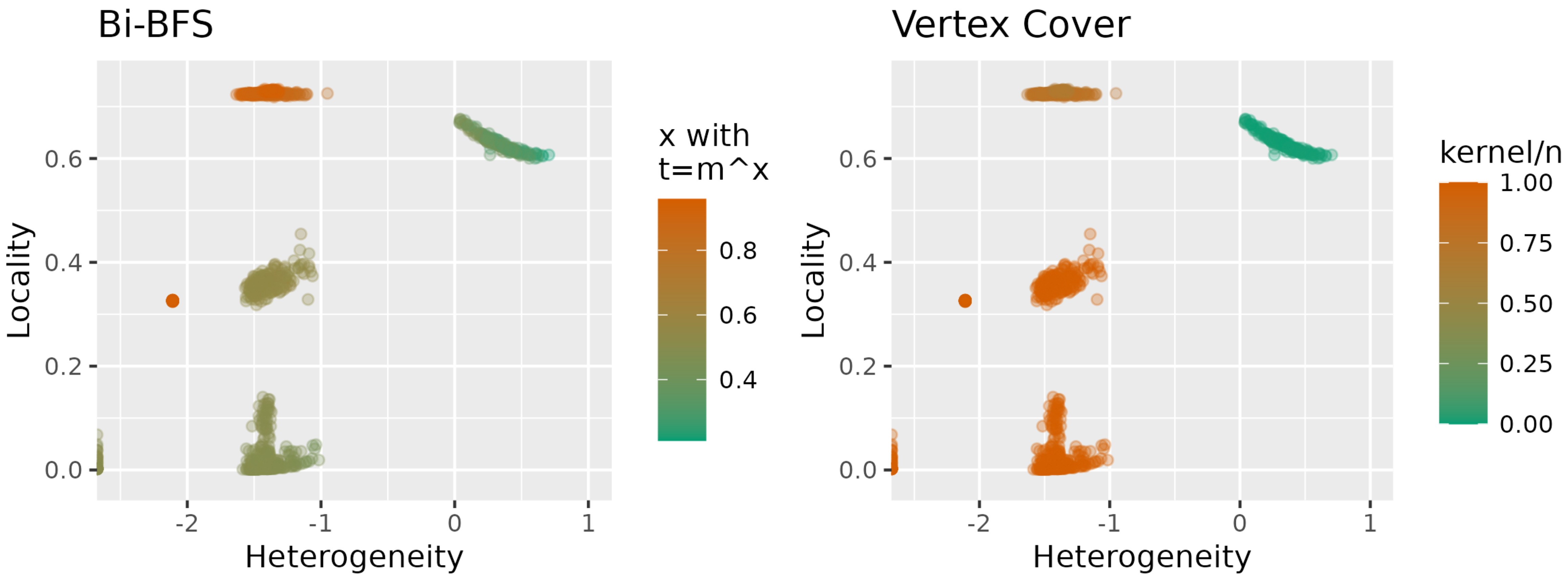

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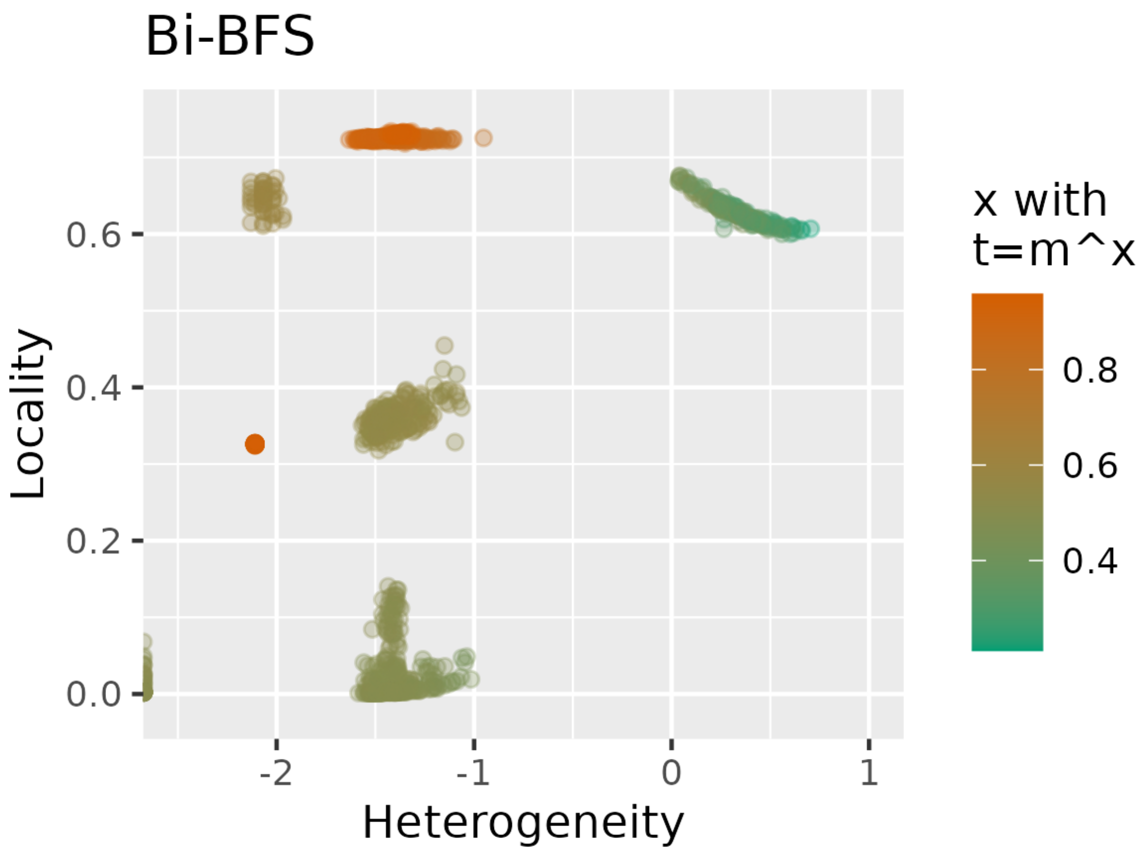

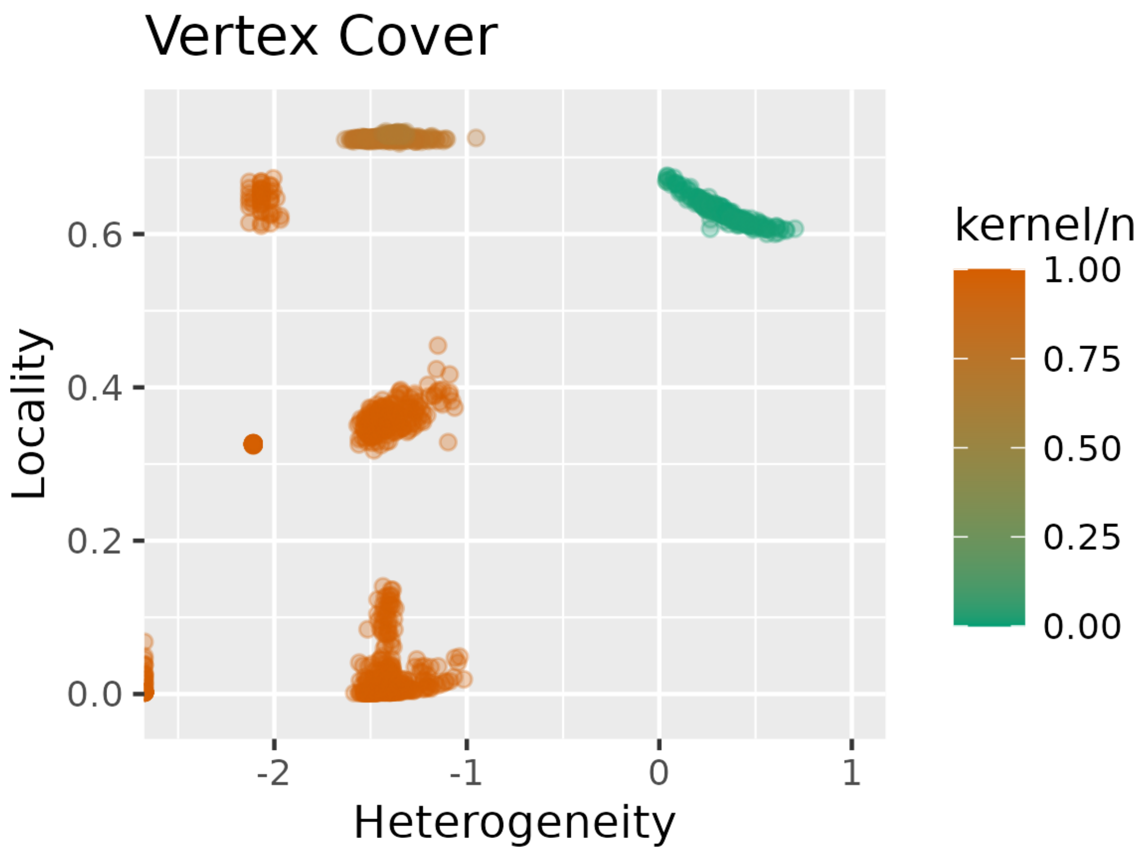

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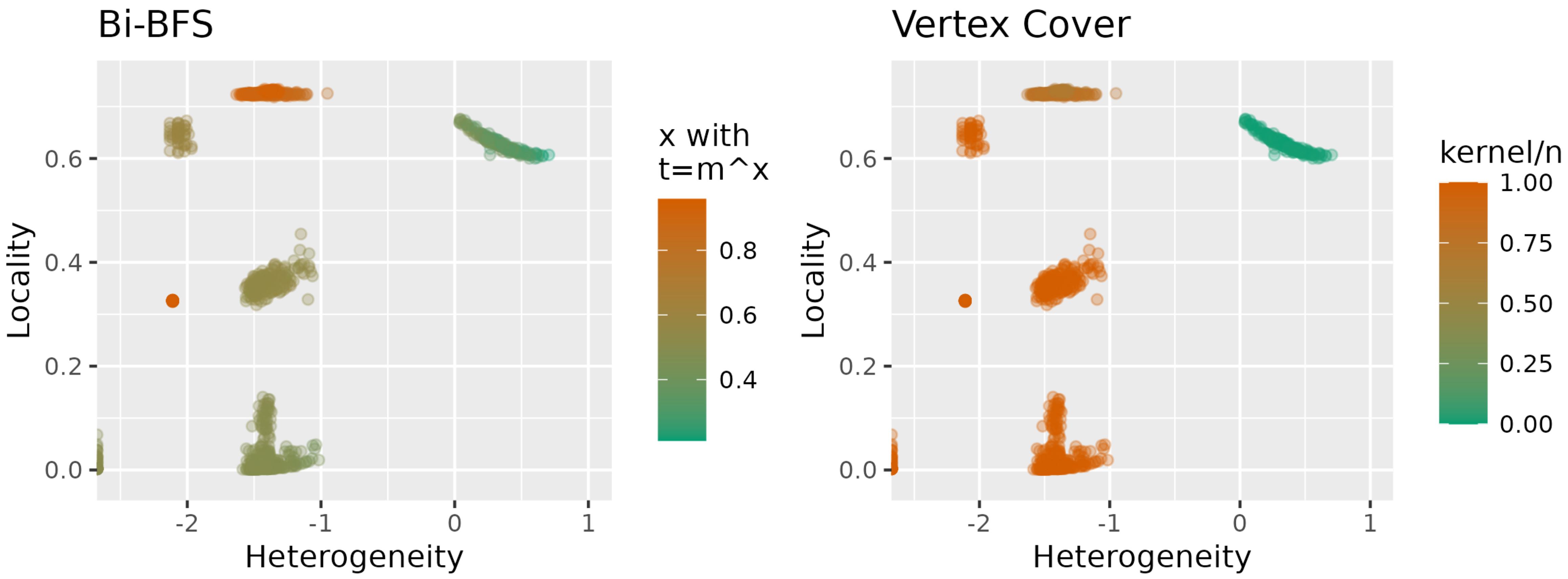

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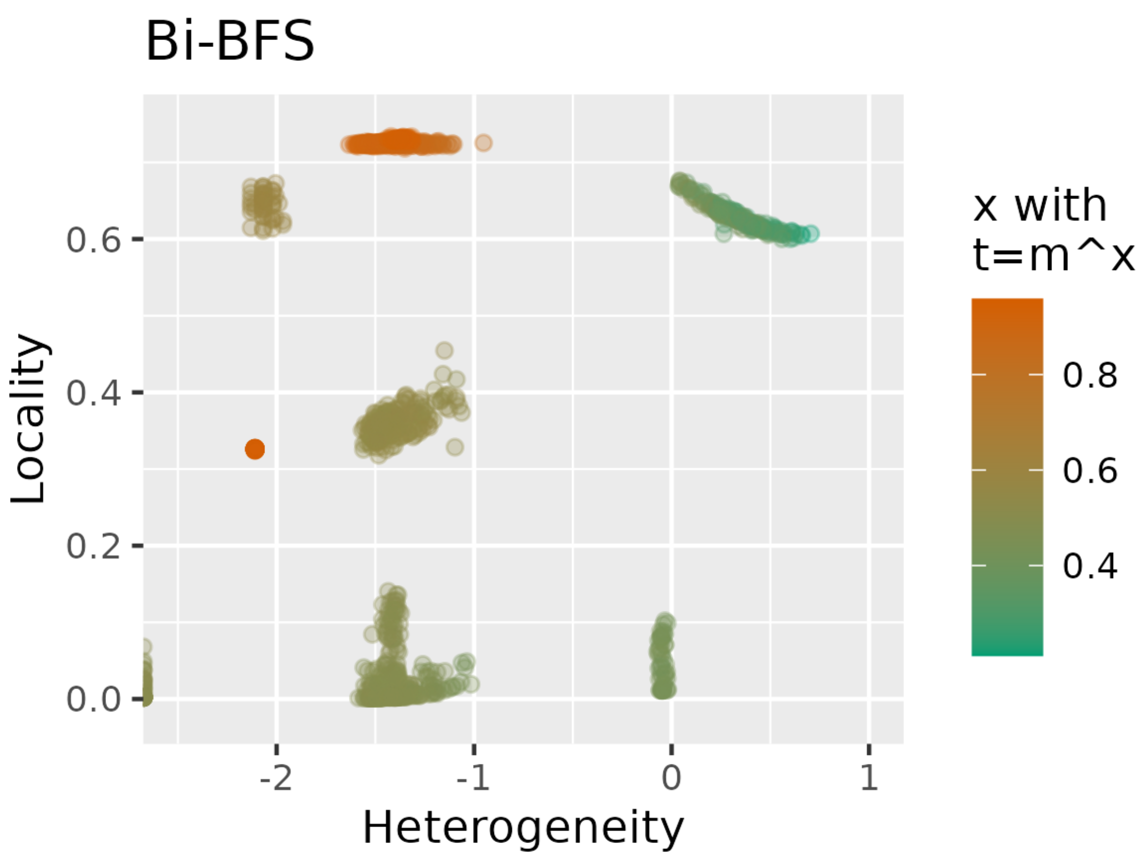

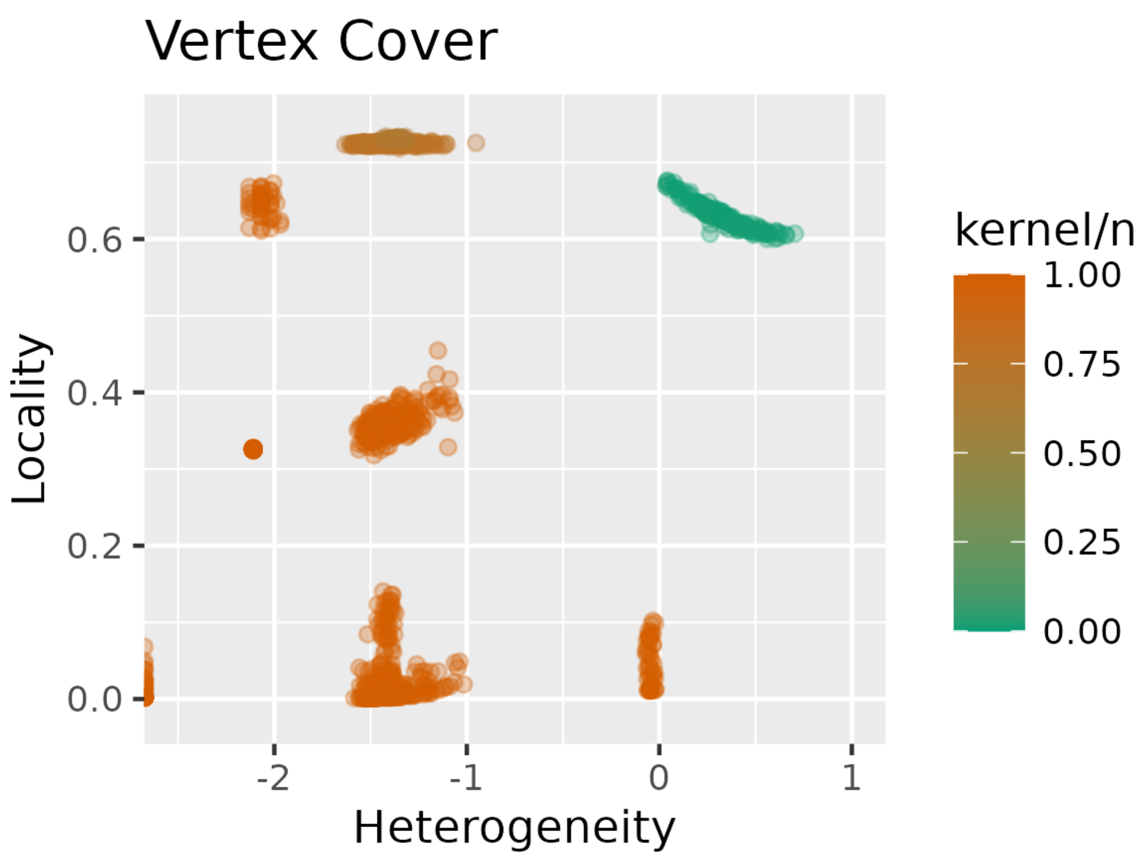

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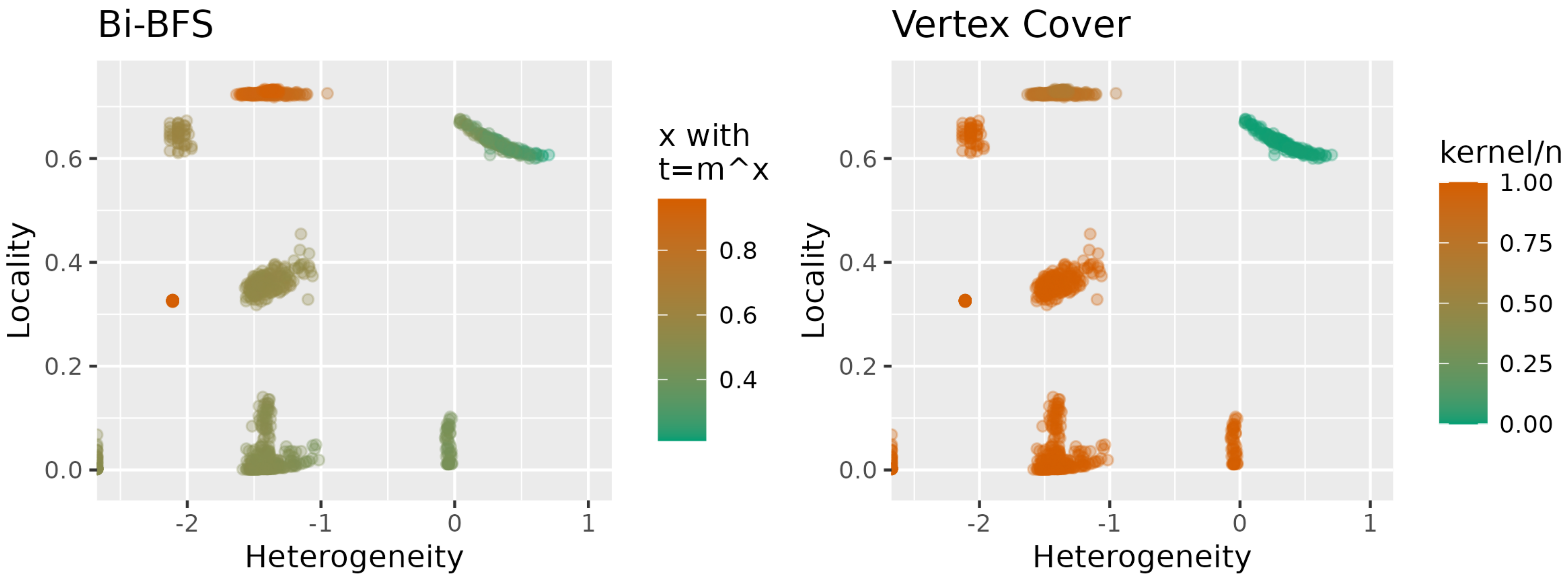
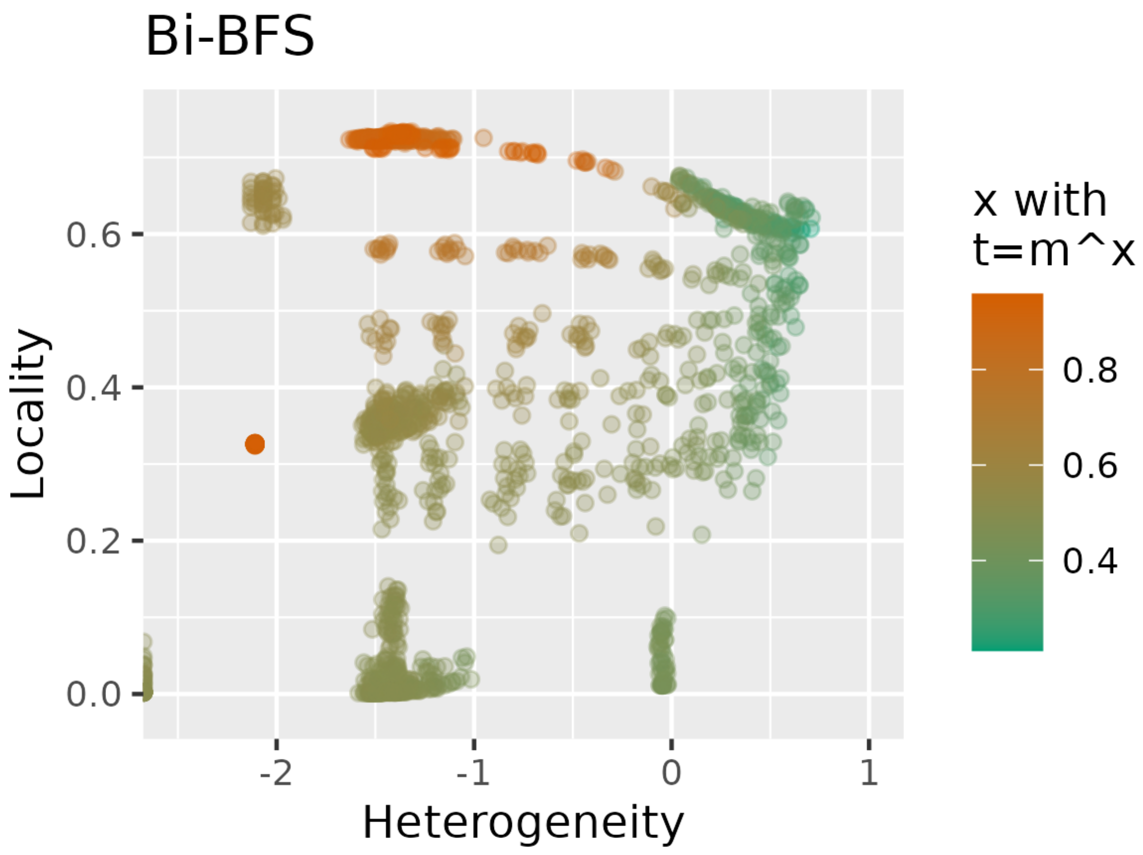
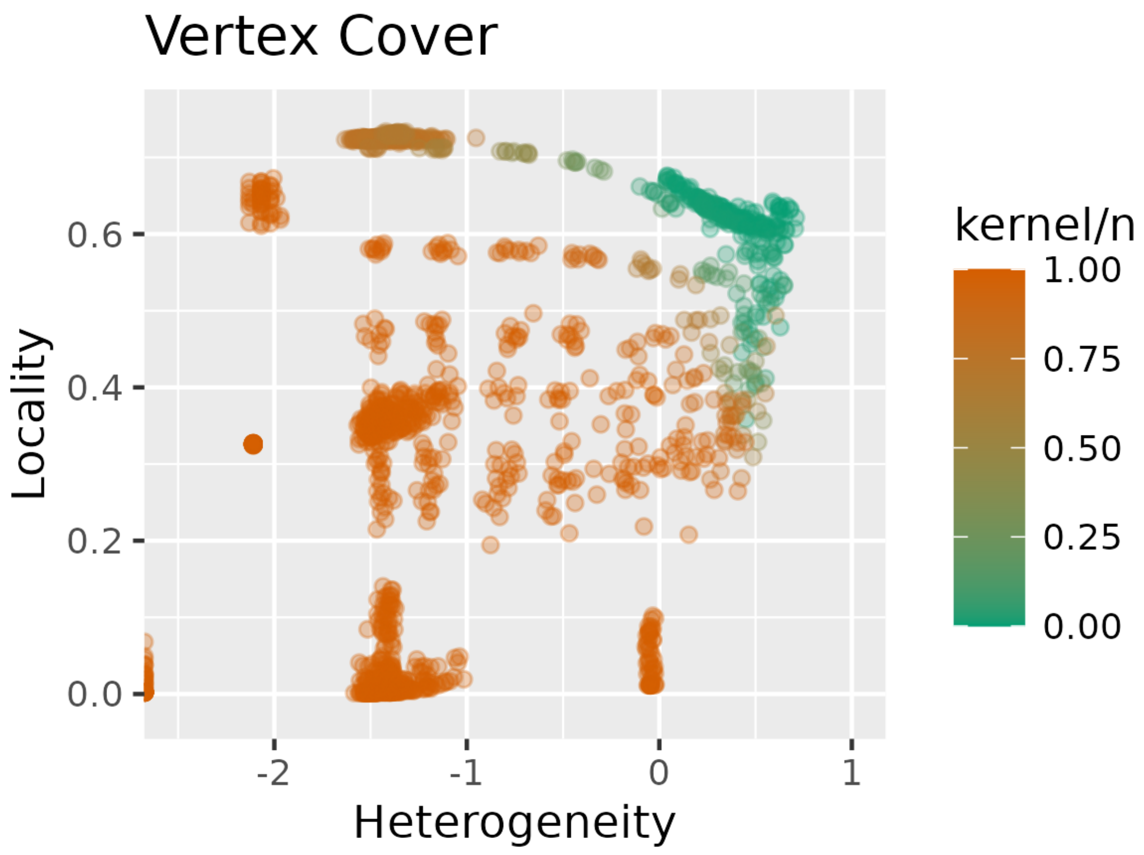


Auswertung von veschiedenen Graphen

- GIRGs?
- Hängt von den Parametern ab!





Overview: GIRGs and HRGs

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Geometric inhomogeneous random graphs
[Bringmann, Keusch, Lengler, 2019]

Geometric inhomogeneous random graph (GIRG)

- weights $w_1, \ldots, w_n$ (typically power-law distributed)
- random positions for the vertices (d-dimensional ground space)

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connection probability proportional to $w_u w_v$

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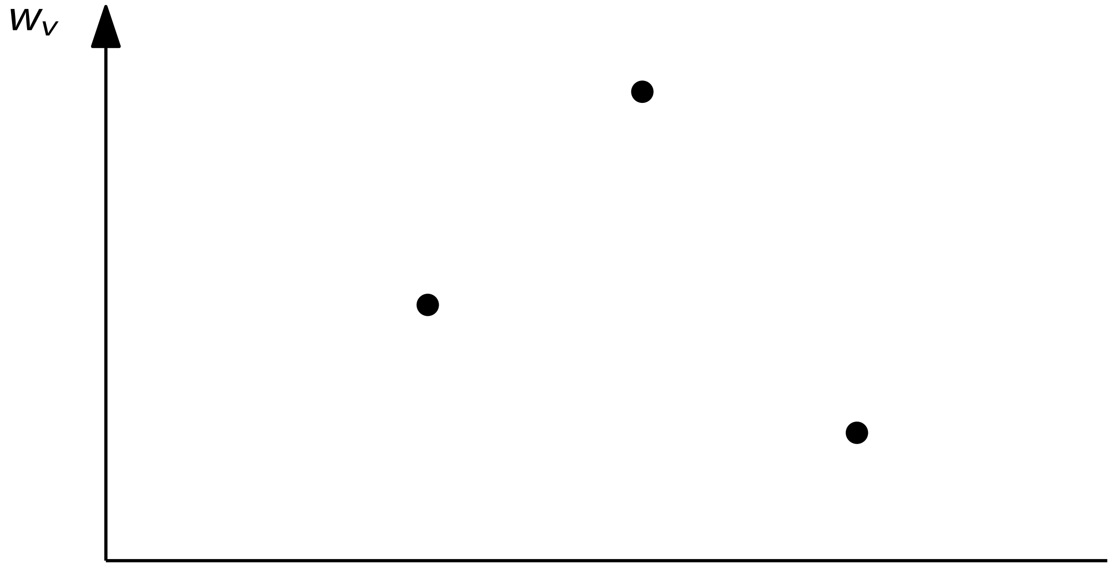

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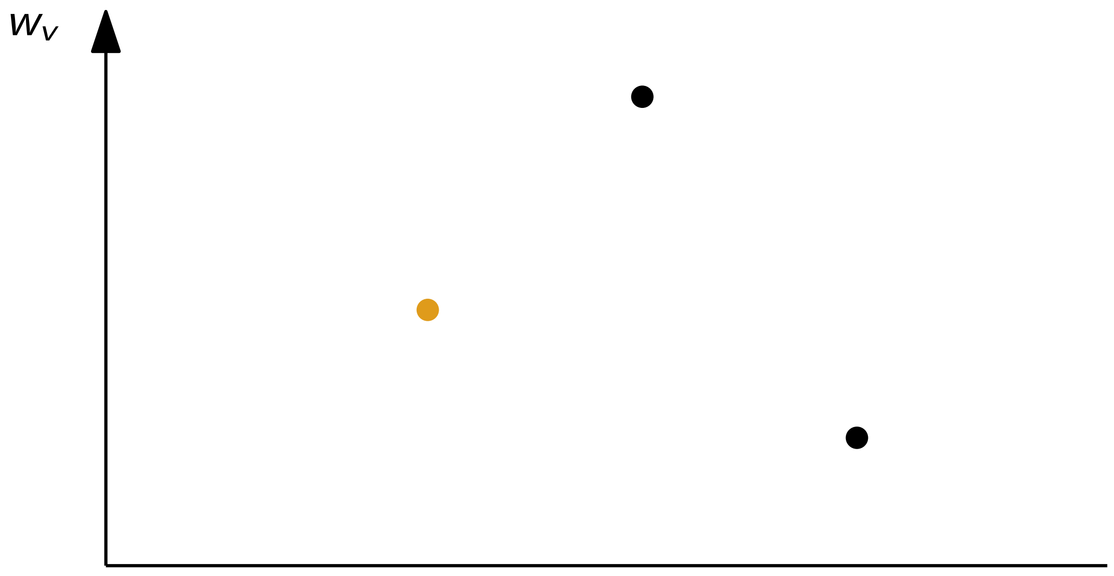

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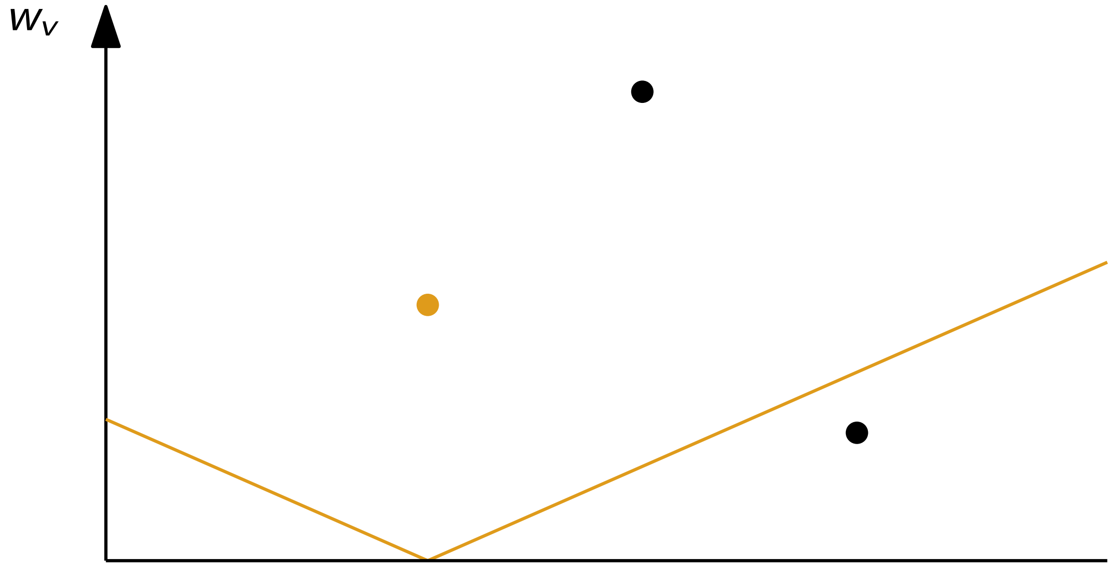

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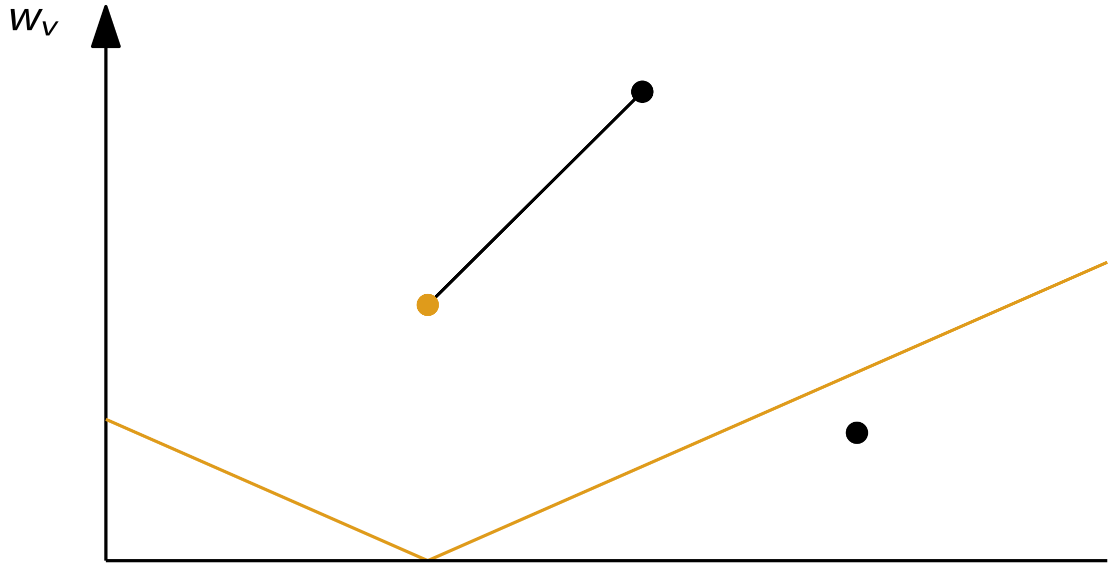

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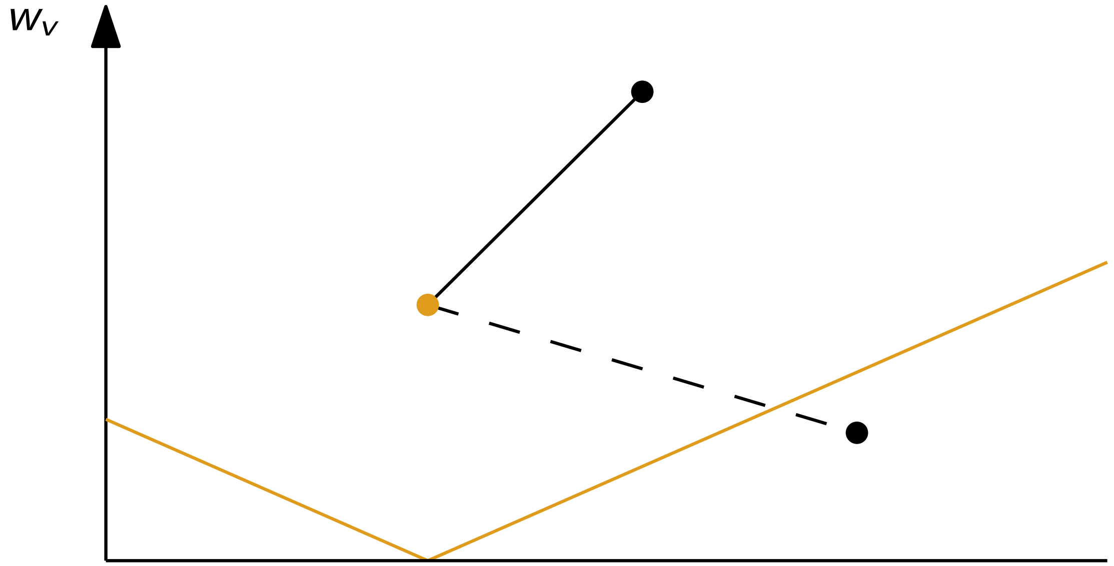

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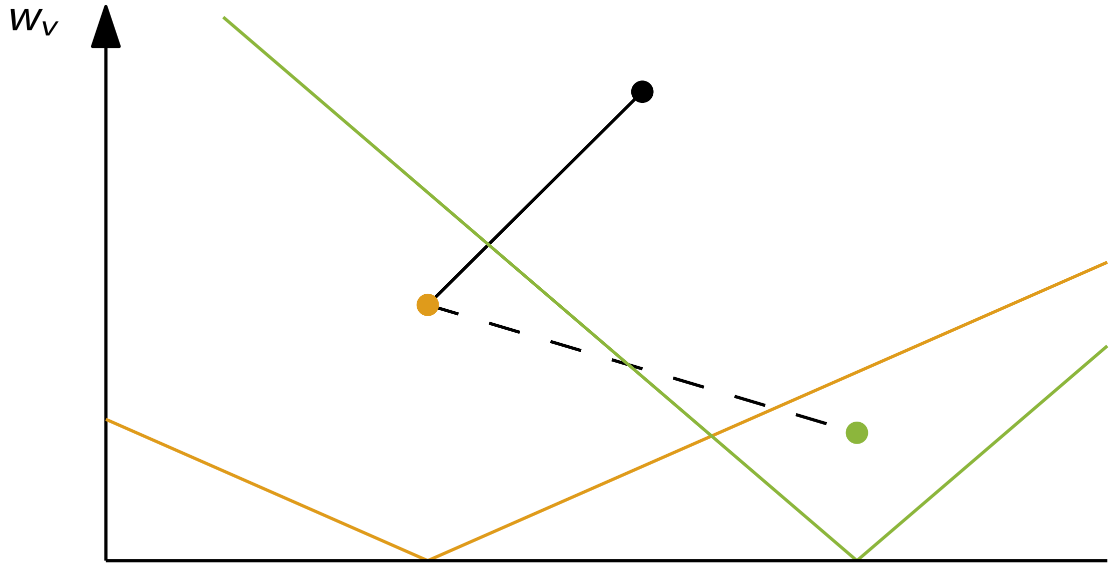

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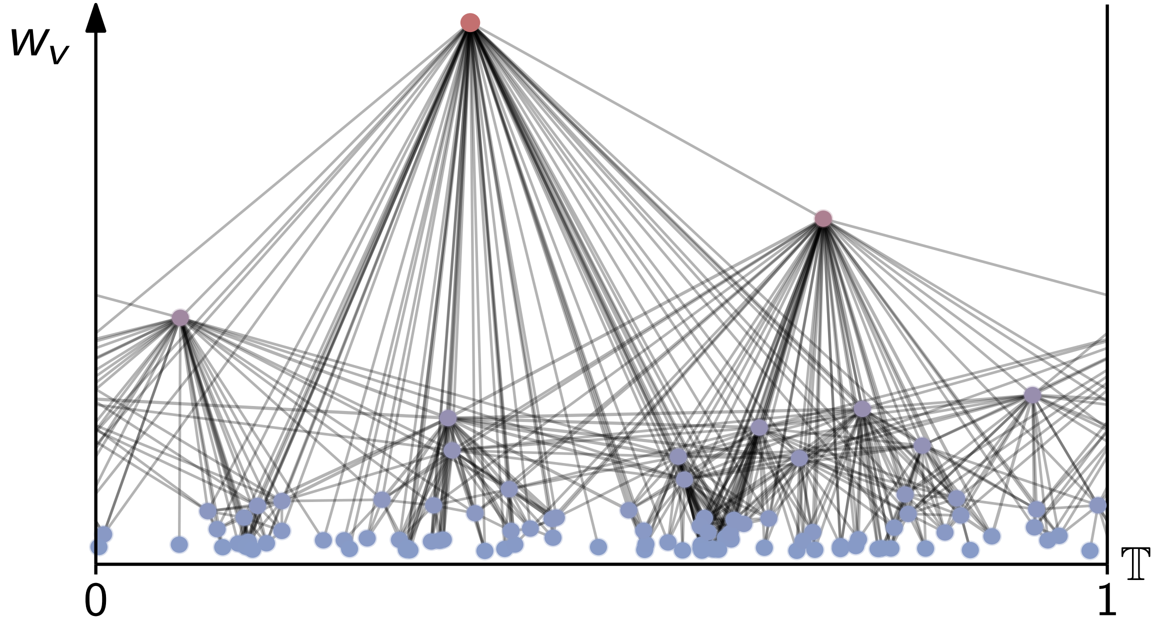

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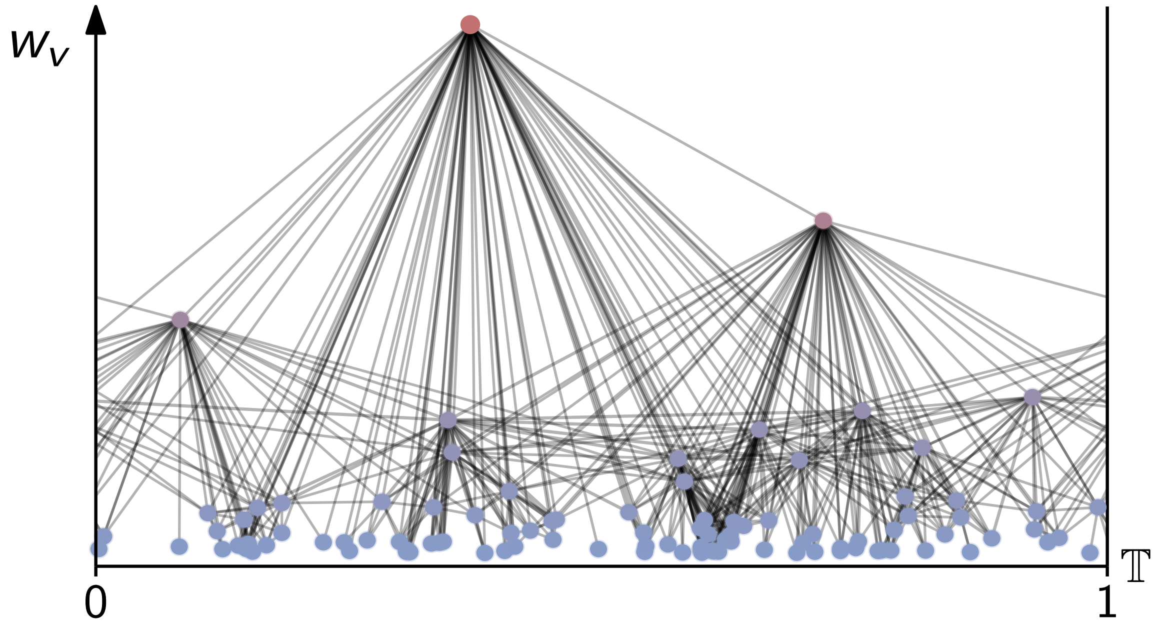
Popularity–similarity

Popularity versus similarity in growing networks
[**Papadopoulos, Kitsak, Serrano, Boguñá, Krioukov** 2012]



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Hyperbolic geometry of complex networks
[**Krioukov, Papadopoulos, Kitsak, Vahdat, Boguñá** 2010]

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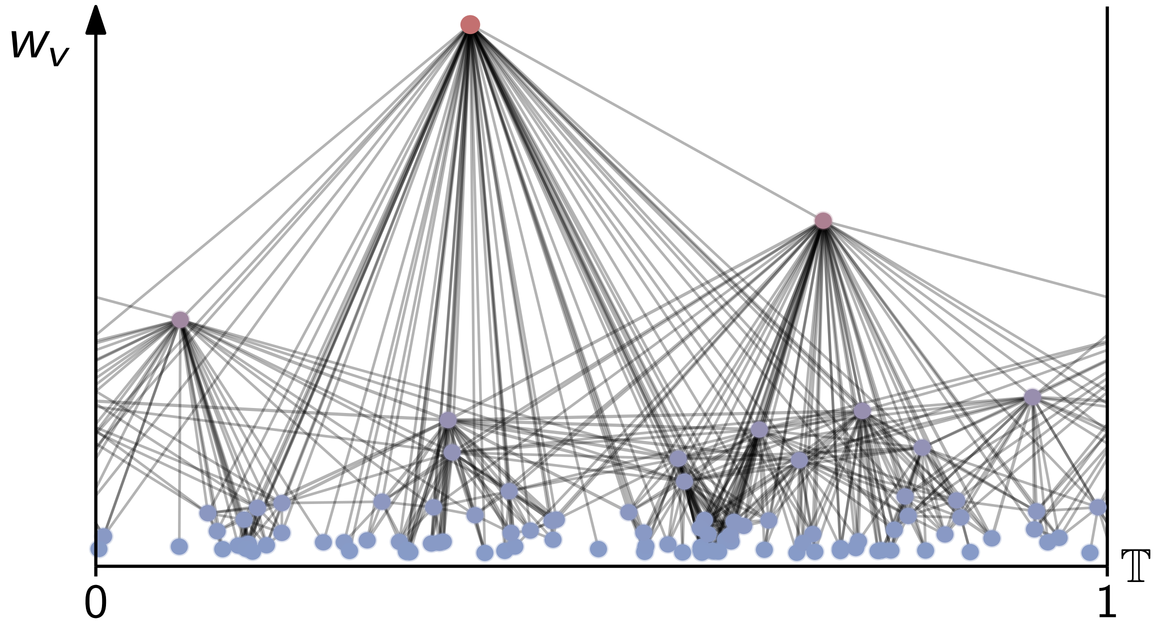

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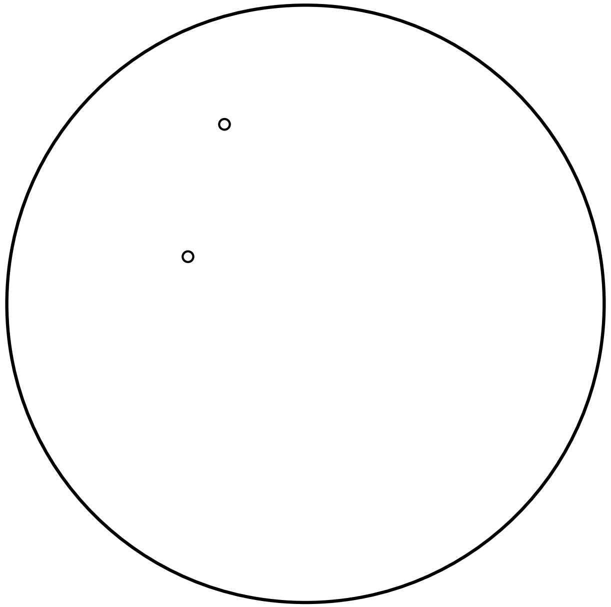

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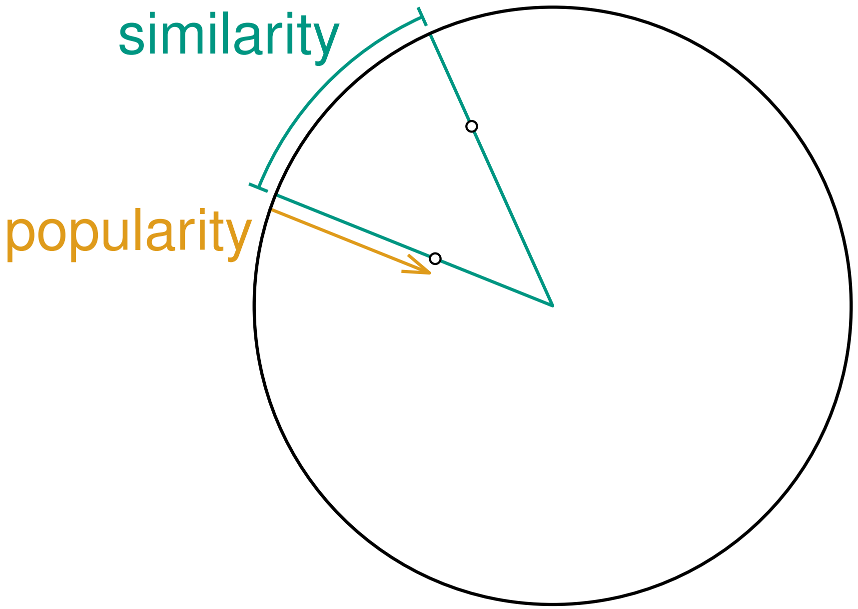

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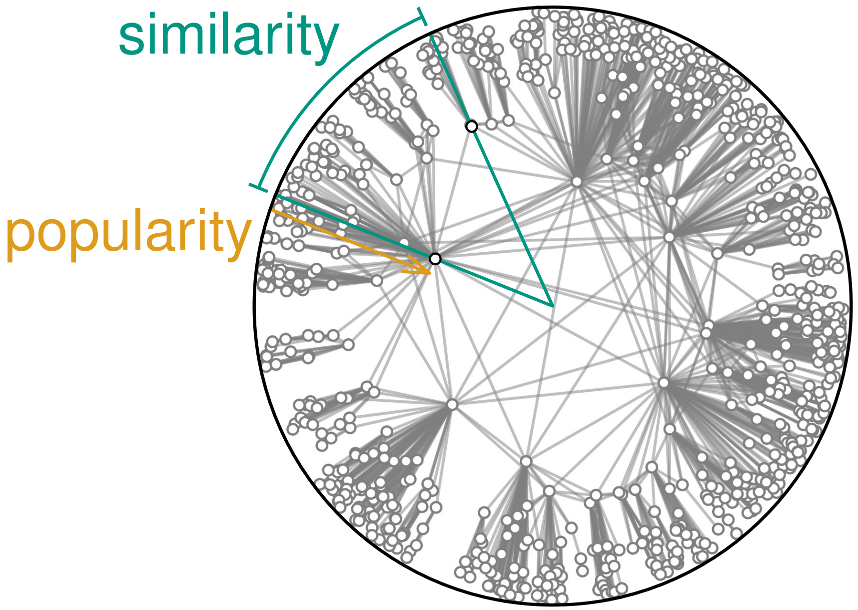

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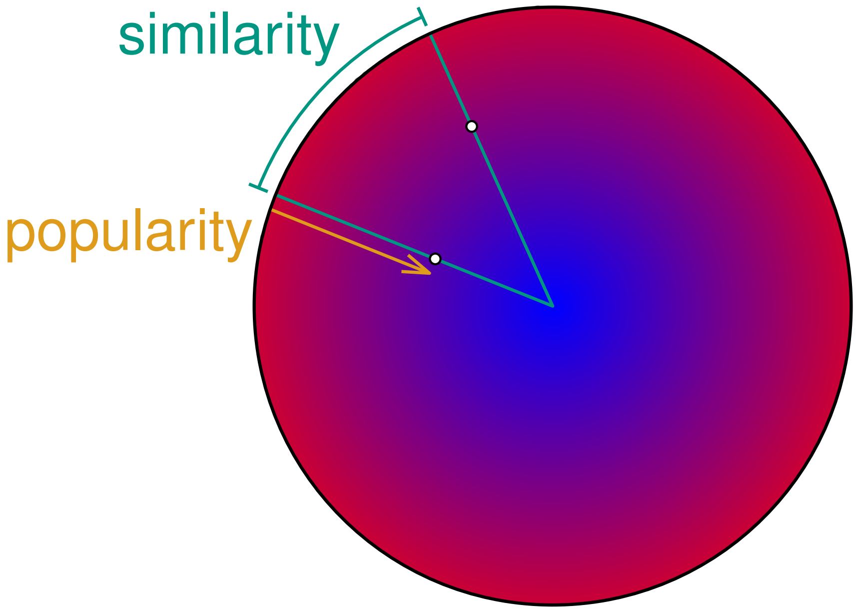

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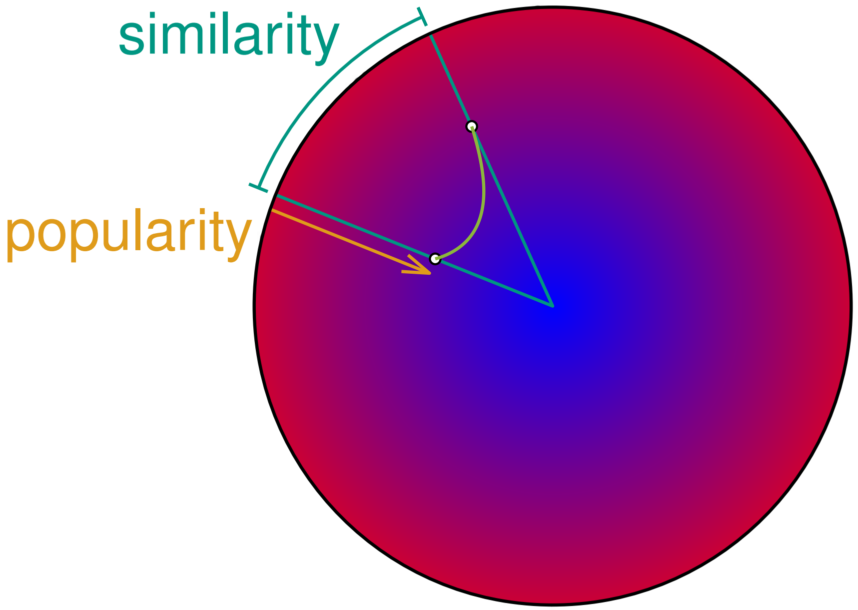

Overview: GIRGs and HRGs

Karlsruher Institut für Technologie

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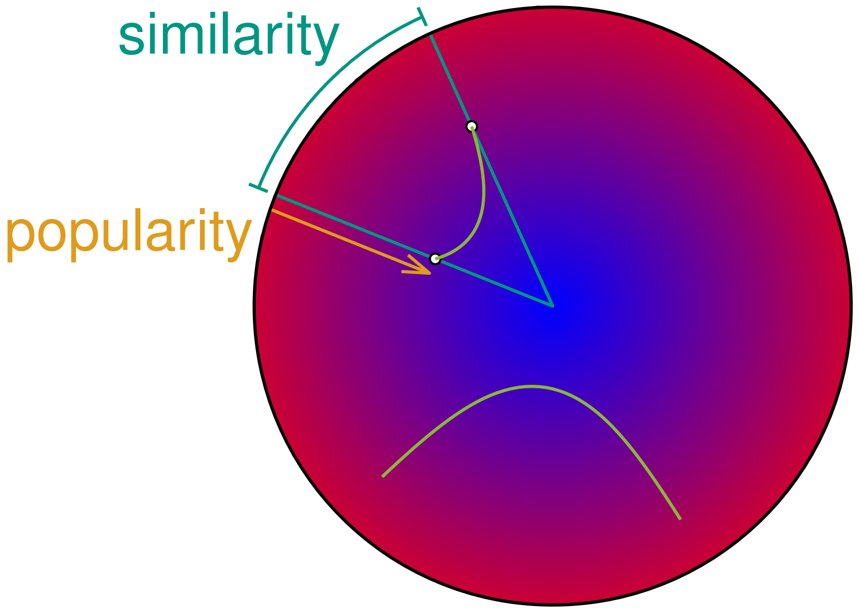

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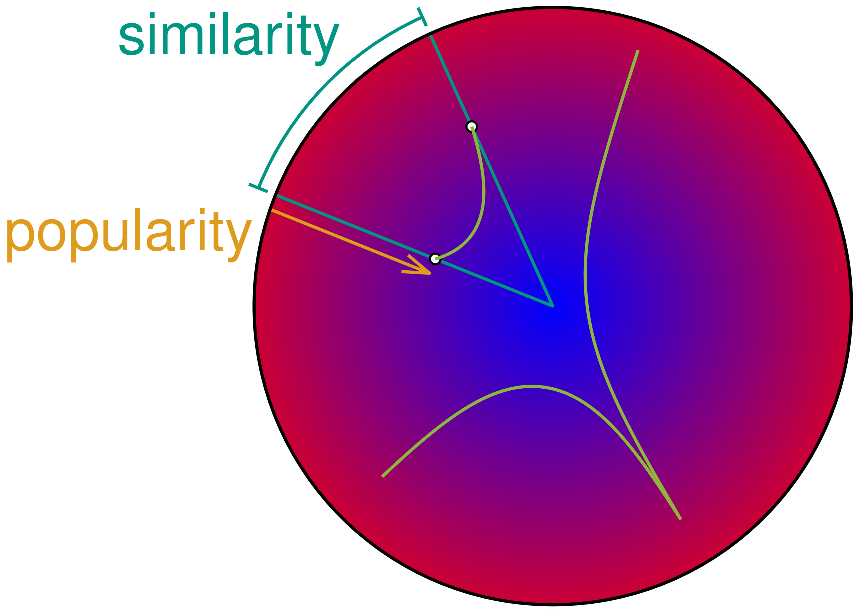

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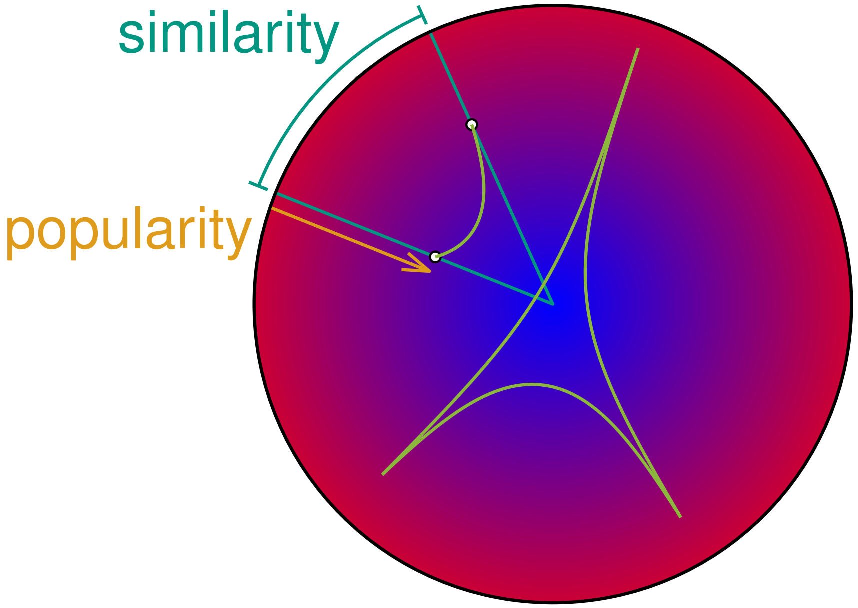

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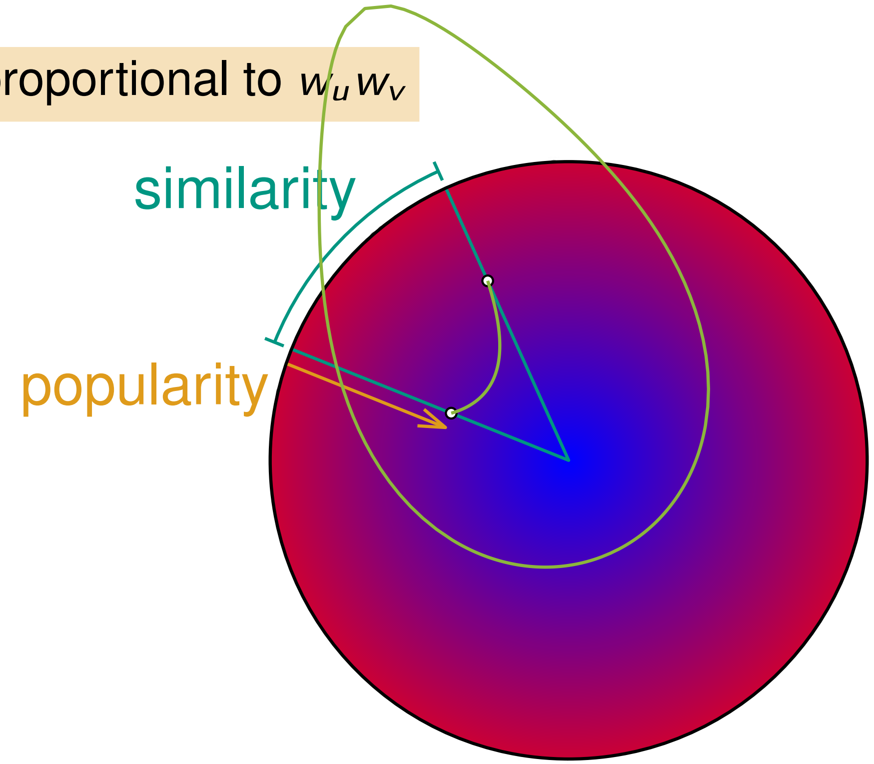

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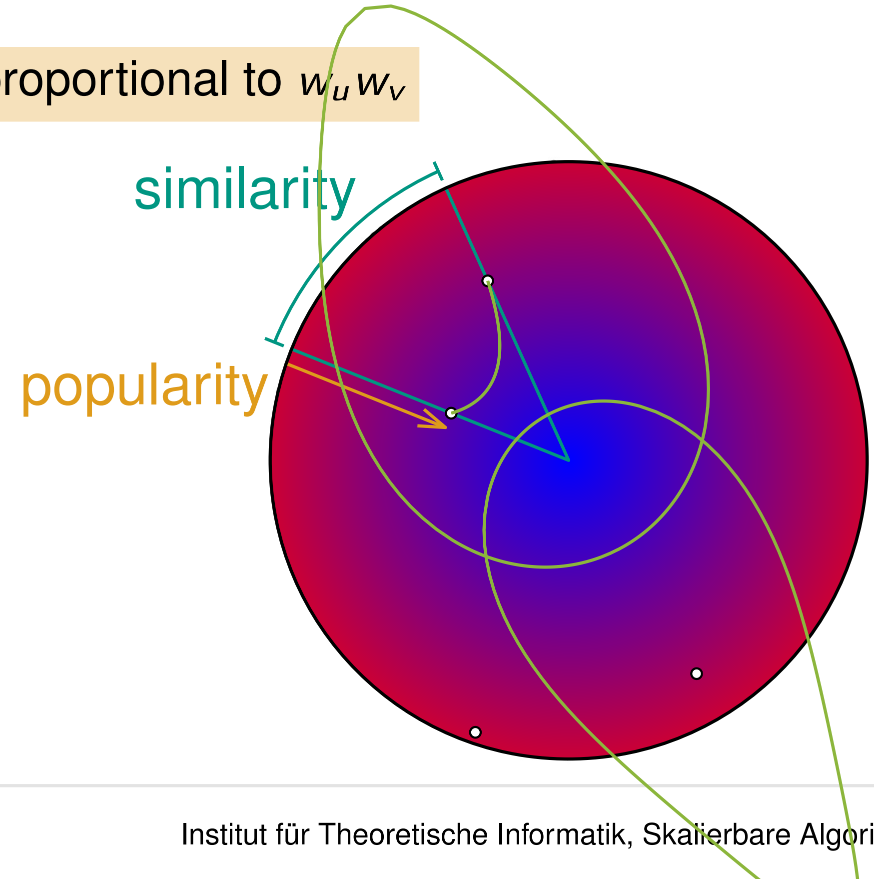

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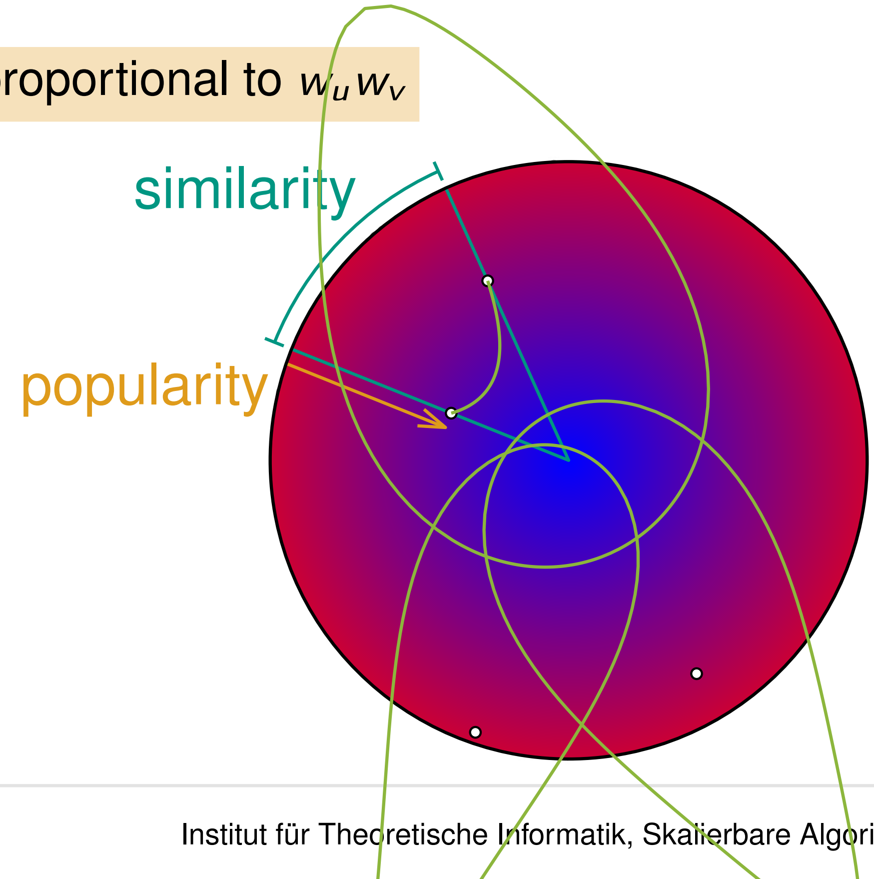

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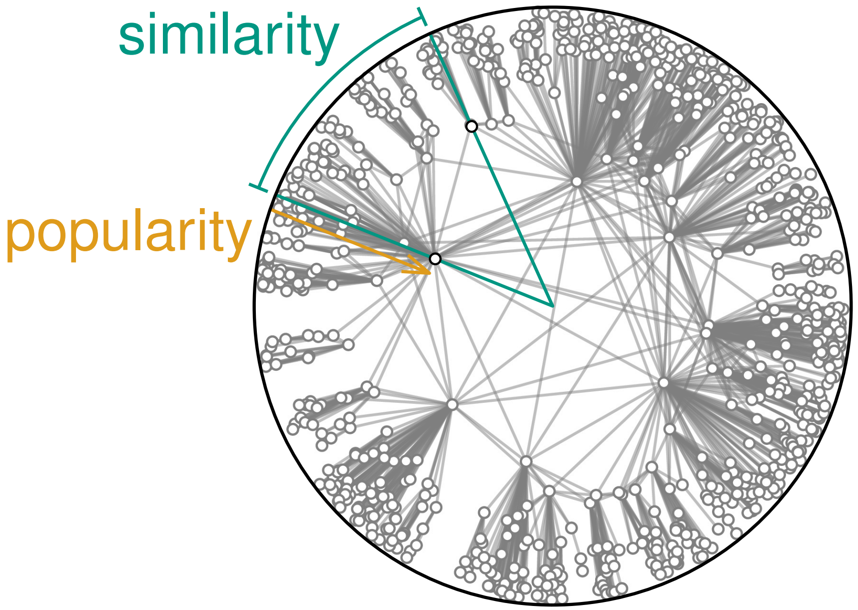

Geometric inhomogeneous random graphs (GIRG) – Details

Weights and positions

- random weights in $[1, \infty)$ with PDF $f(x) = cx^{-\tau}$ or deterministic power-law weights
- typical ground space: d-dimensional torus $[0, 1]^d$ with max-norm

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- it follows: $\Pr\left[\{u, v\} \in E \mid w_u, w_v\right] = \Pr\left[\mathrm{dist}(u, v) \leq \sqrt[d]{a\frac{w_u w_v}{n}}\right] \in \Theta(\frac{w_u w_v}{n})$
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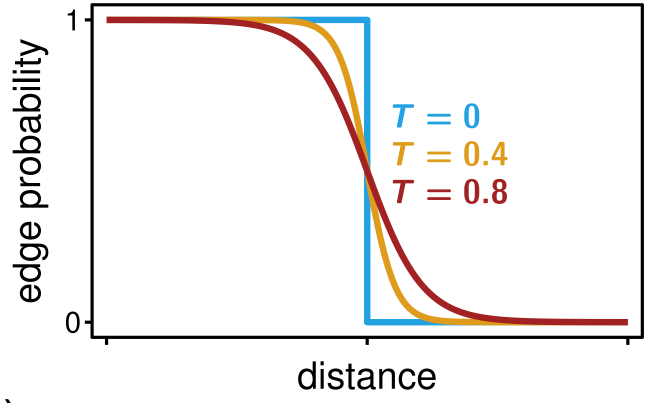
Temperature > 0

- additional parameter $T \in (0, 1)$
- connection probability $p_{uv} = \min\left\{1, \left(\frac{1}{\mathrm{dist}(u,v)^d} \cdot a\frac{w_u w_v}{n}\right)^{1/T}\right\}$
- interpolate between high locality ($T = 0$) and low locality ($T \to 1$)



GIRG Parameters

Two parameters

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- power-law exponent τ
- temperature T

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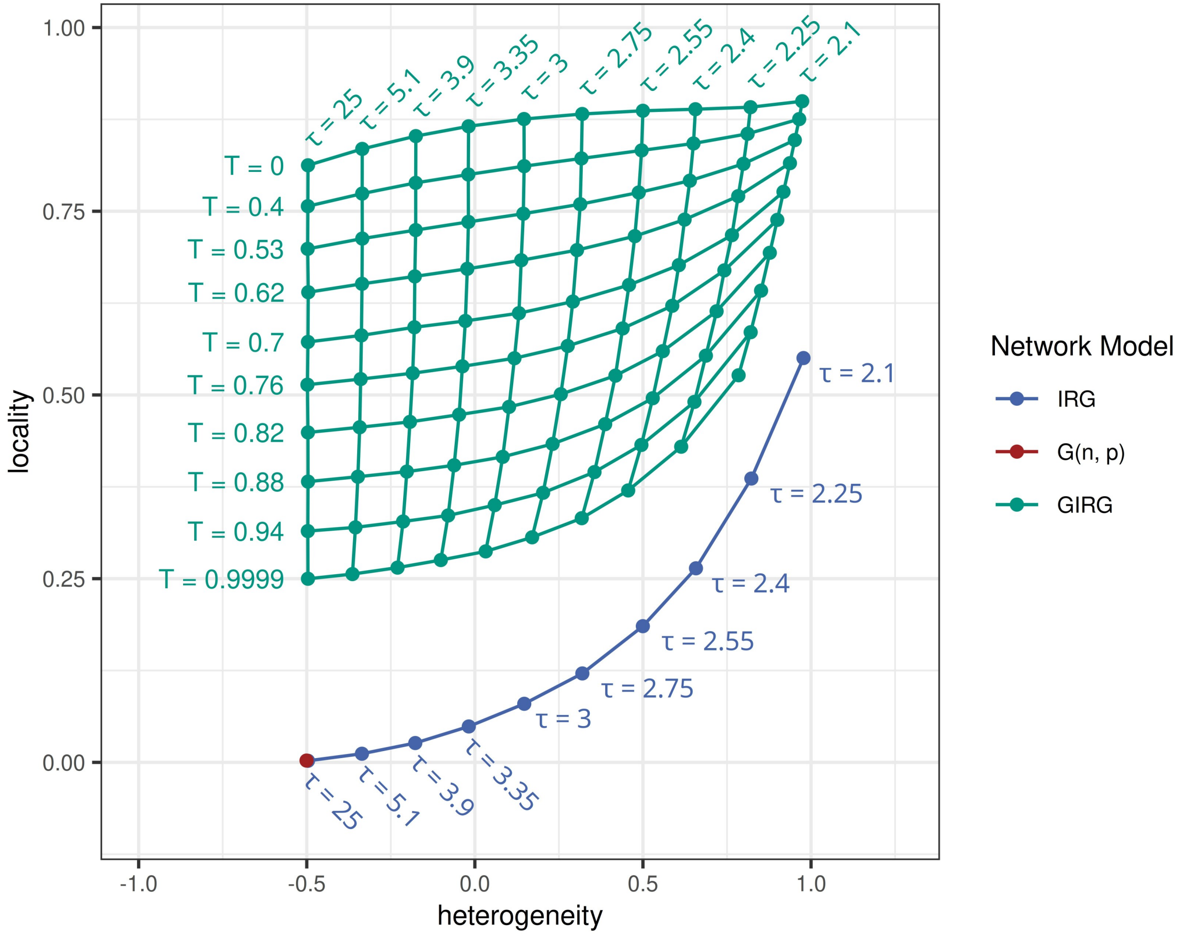
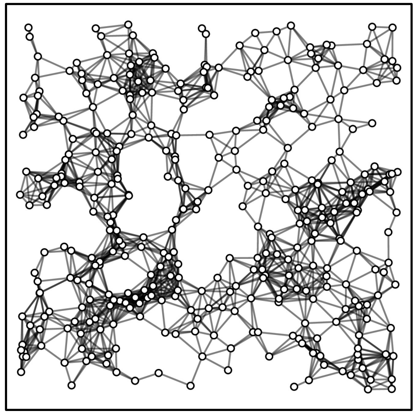
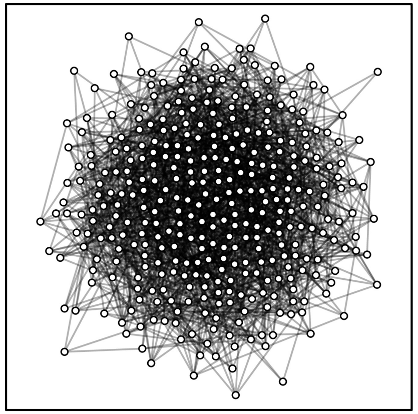
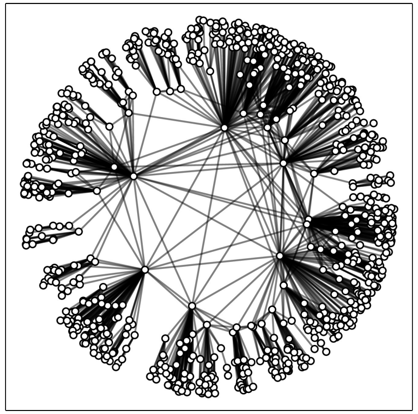


On the external validity of average-case analyses of graph algorithms [B., Fischbeck 2022]

GIRG Parameters







heterogeneity →

locality ↑

geometric ← τ hyperbolic GIRG

↓ T (geometric) ↓ T (hyperbolic GIRG)

$G(n, m)$
$G(n, p)$ ← τ IRG config

Wie geht's weiter?

Nächste Woche

- stellt eure Arbeit zu Übungsblatt 2 vor
- 5 Minuten, mit Slides (z.B. als PDF)
- Plots bitte mit Achsenbeschriftung!

Übungsblatt 3

- einen weiteren Algorithmus untersuchen
- Code und Workflow optimieren / aufräumen
- Zeitrahmen: nur *eine* Woche

Anschließend: Projekt

- Ziel: Forschungsfrage untersuchen und beantworten
- Präsentation und schriftl. Ausarbeitung