The Impact of Heterogeneity and Geometry on the Proof Complexity of Random Satisfiability

Thomas Bläsius (KIT), Tobias Friedrich (HPI), Andreas Göbel (HPI),
Jordi Levy (CSIC), Ralf Rothenberger (HPI)

Satisfiability

k-SAT

- input: CNF-formula with k literals per clause

$$(x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_4) \wedge (x_1 \vee \neg x_3 \vee x_4) \wedge (x_2 \vee \neg x_3 \vee x_4)$$

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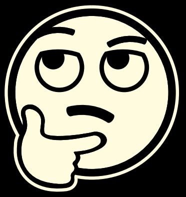
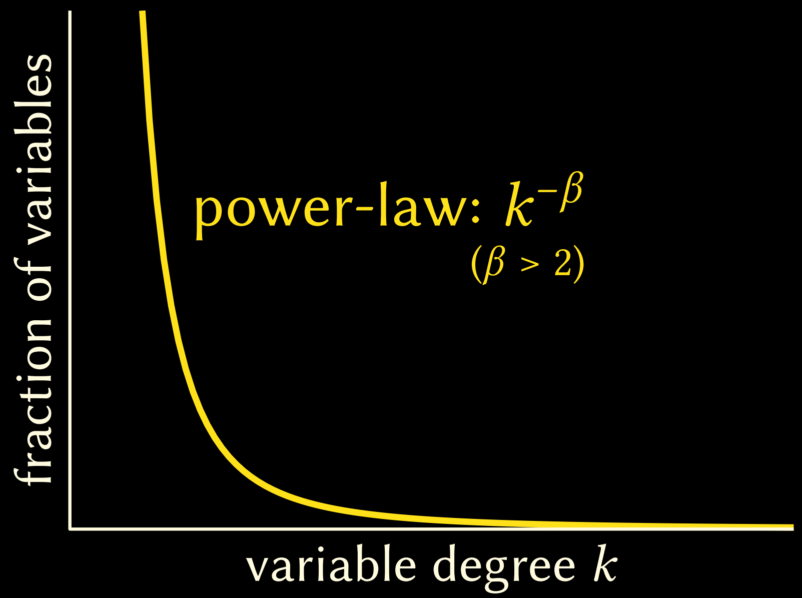
Theory vs. Practice

- NP-hard for $k \geq 3$
- industrial instances: efficiently solvable for millions of variables



Typical Properties of Industrial SAT Instances

Heterogeneity



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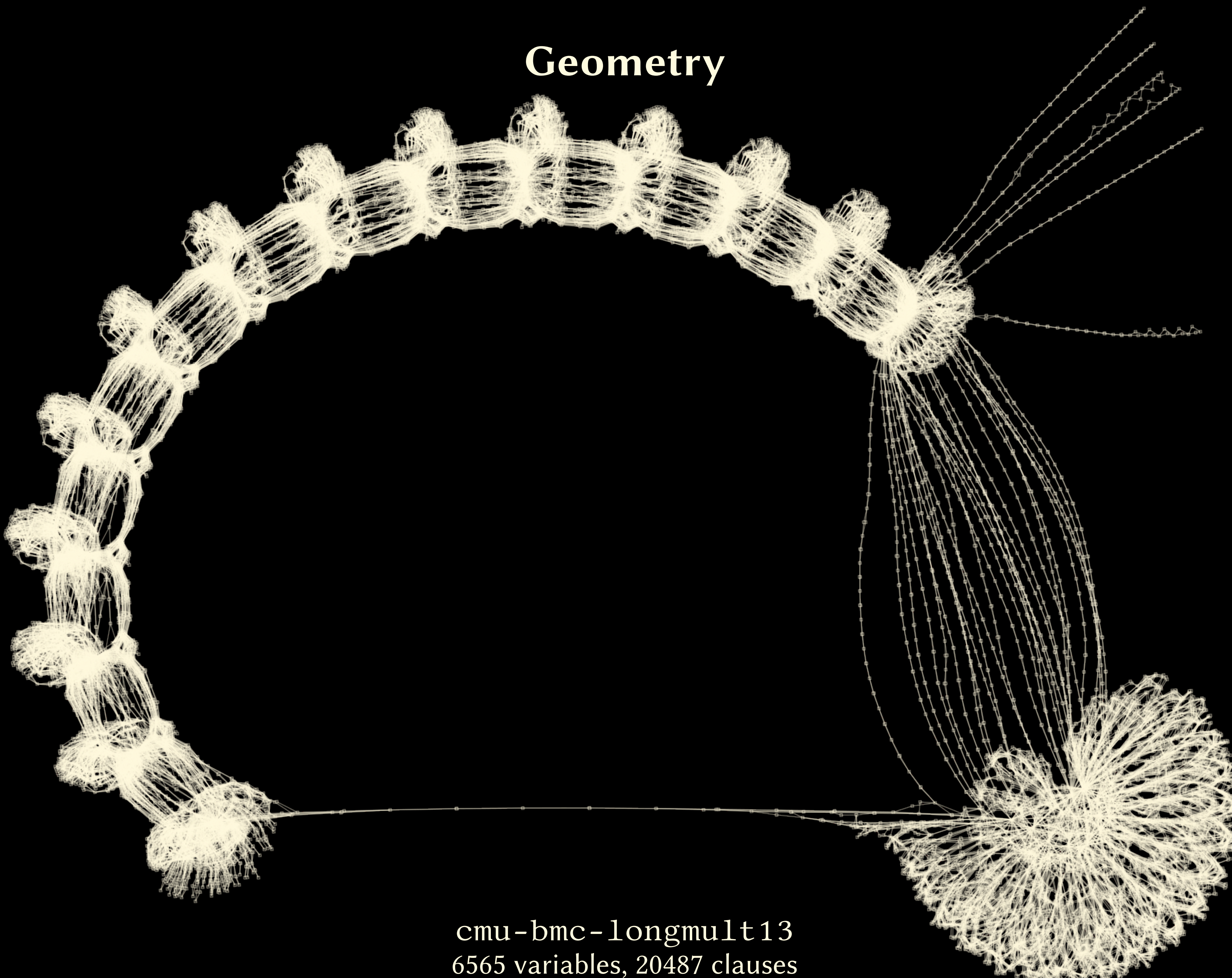

`cmu-bmc-longmult13`
6565 variables, 20487 clauses

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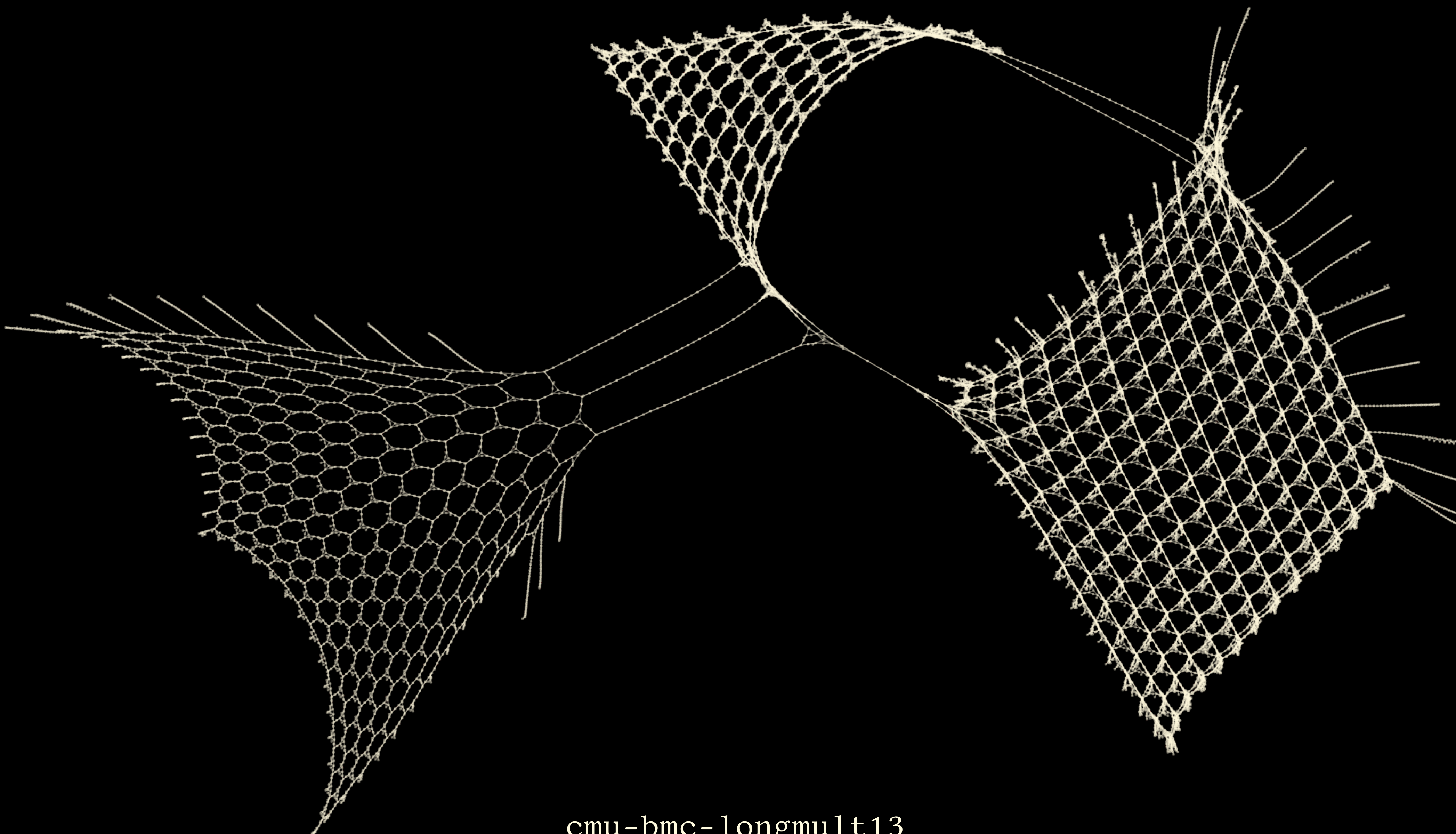

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Random SAT with Heterogeneity and Geometry

Heterogeneity: Power-Law Random k-SAT [**Ansótegui, Bonet, Levy** 2009]

- for each clause: independently draw k variables without repetition
- variable $v \in \{1, \ldots, n\}$ is chosen with probability proportional to $w_v = v^{-\frac{1}{\beta-1}}$
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- related to:
  - popularity-similarity SAT [**Giráldez-Cru, Levy** 2017]
  - hyperbolic random graphs [**Krioukov et al.** 2010]
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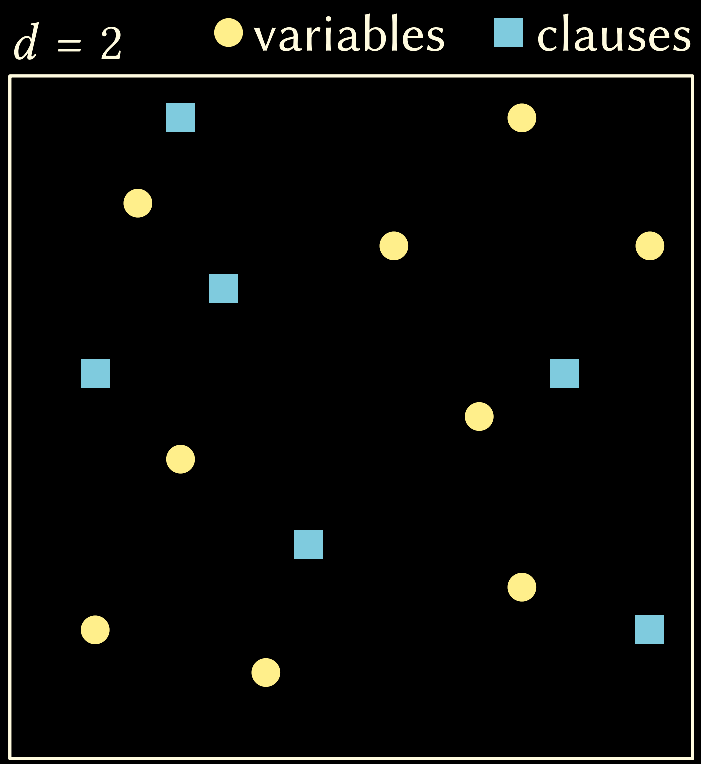

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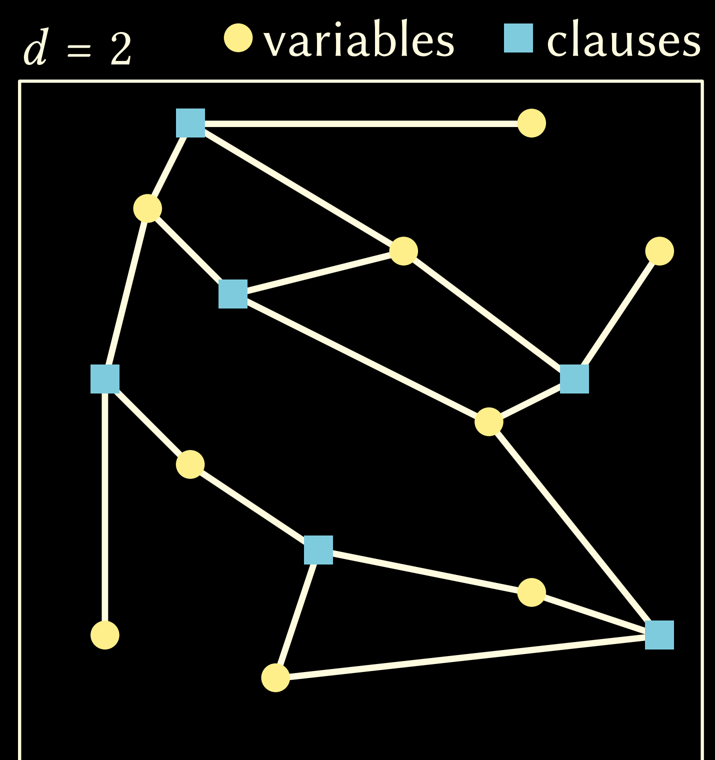

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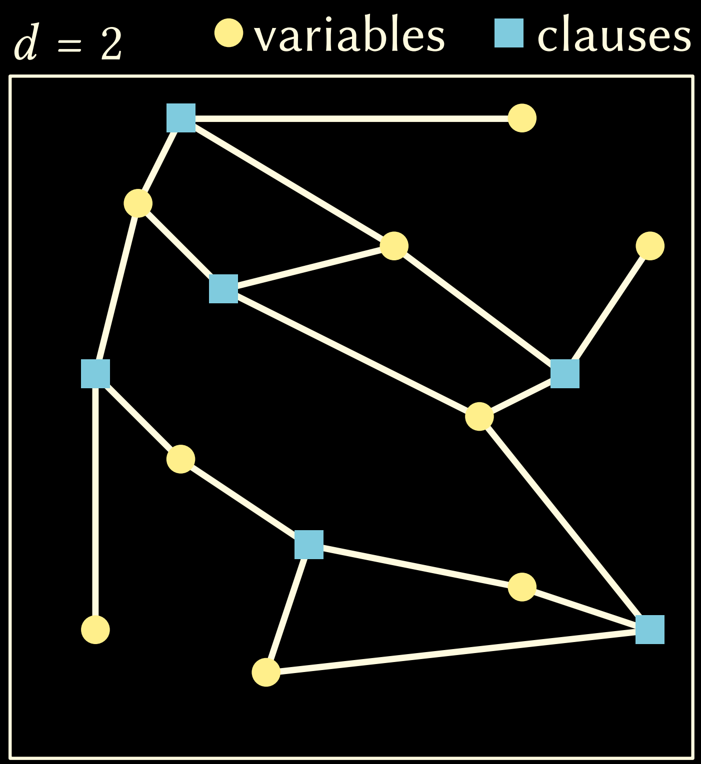
- sample d-dimensional positions
- connection weight for clause c and variable v:

$$X(c, v) = \left(\frac{w_v}{\|\boldsymbol{c} - \boldsymbol{v}\|^d} \right)^{1/T}$$

- for clause c: draw k different variables with probabilities proportional to $X(c, v)$



The Big Picture

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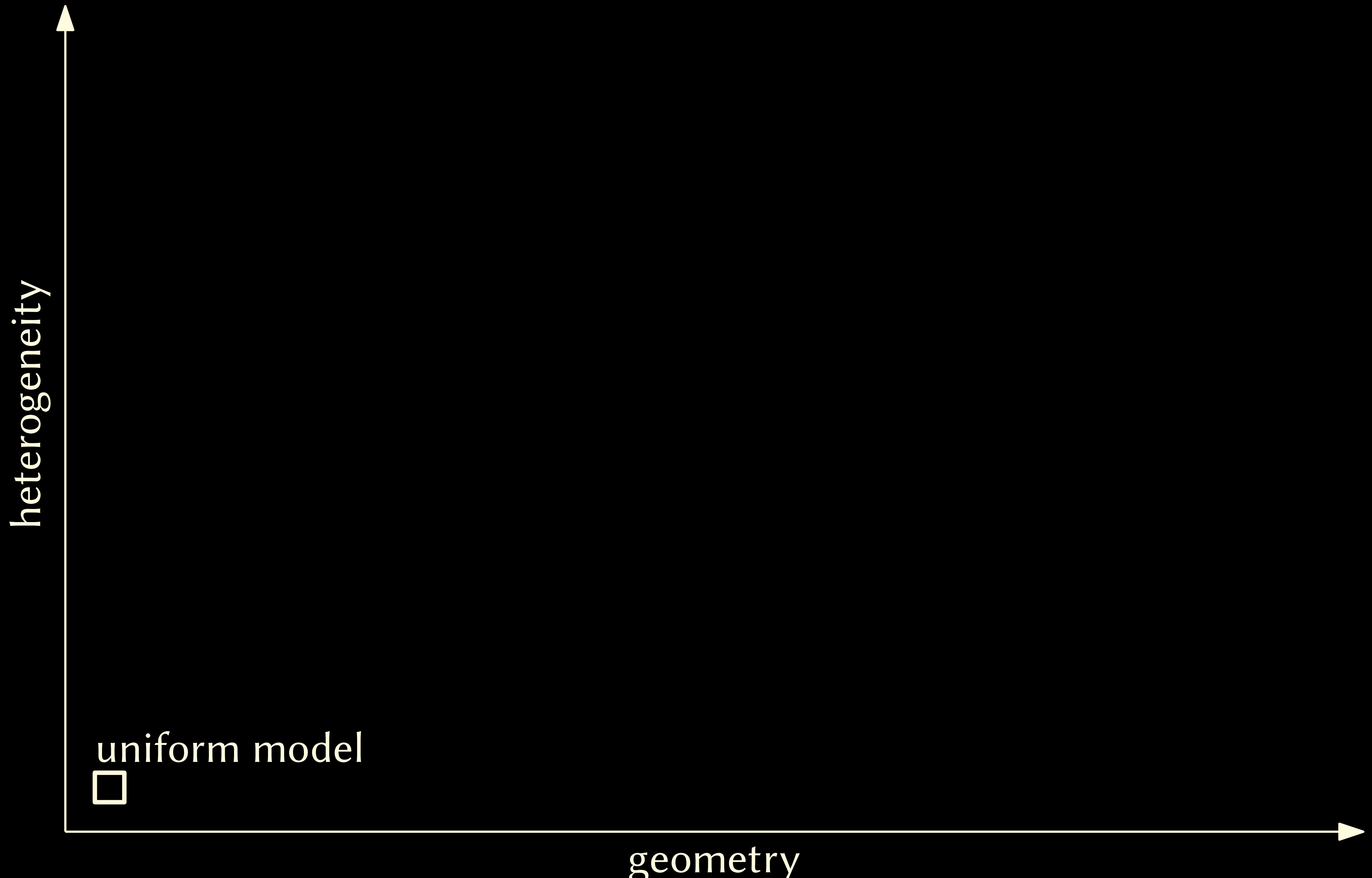

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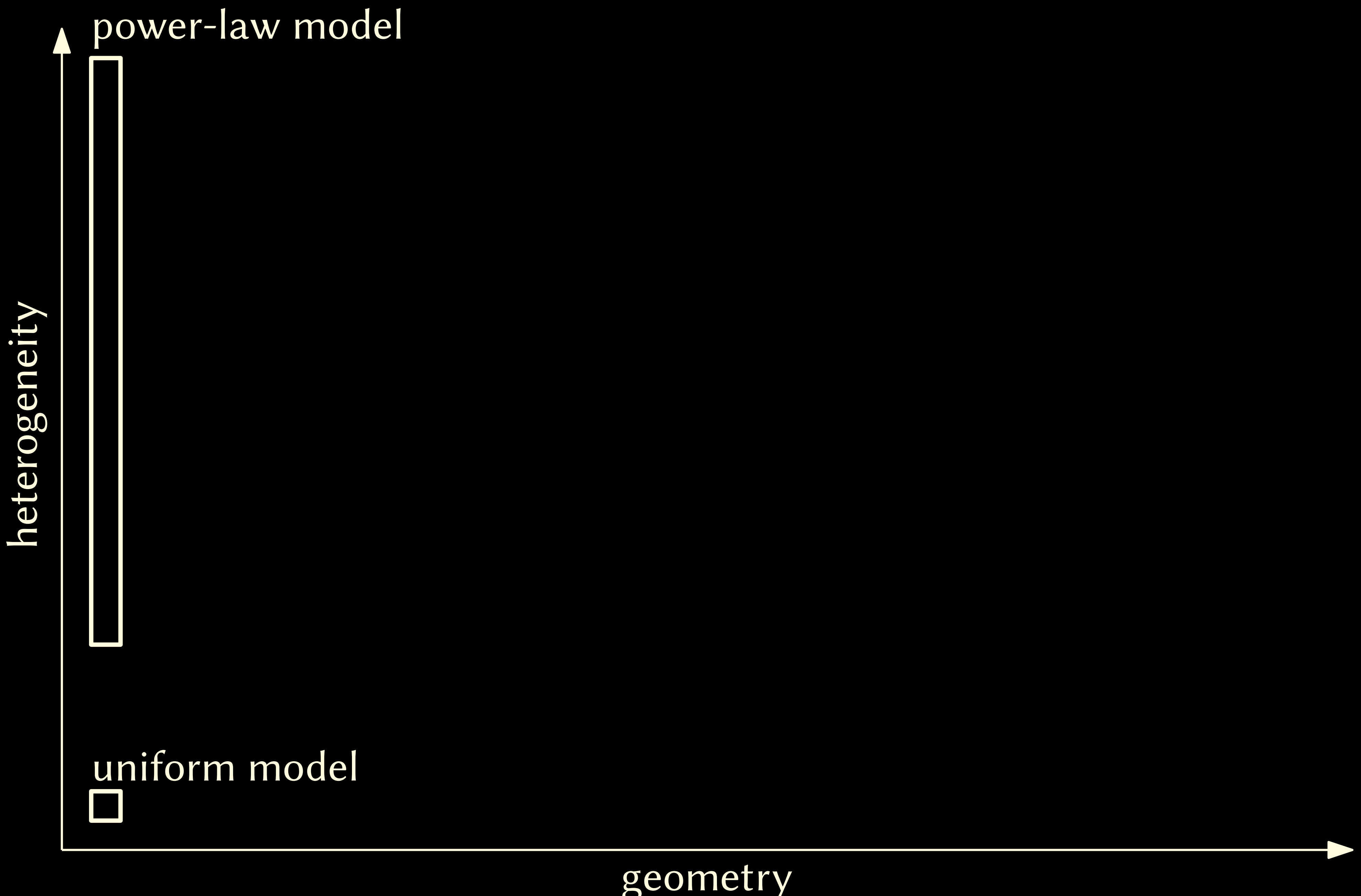

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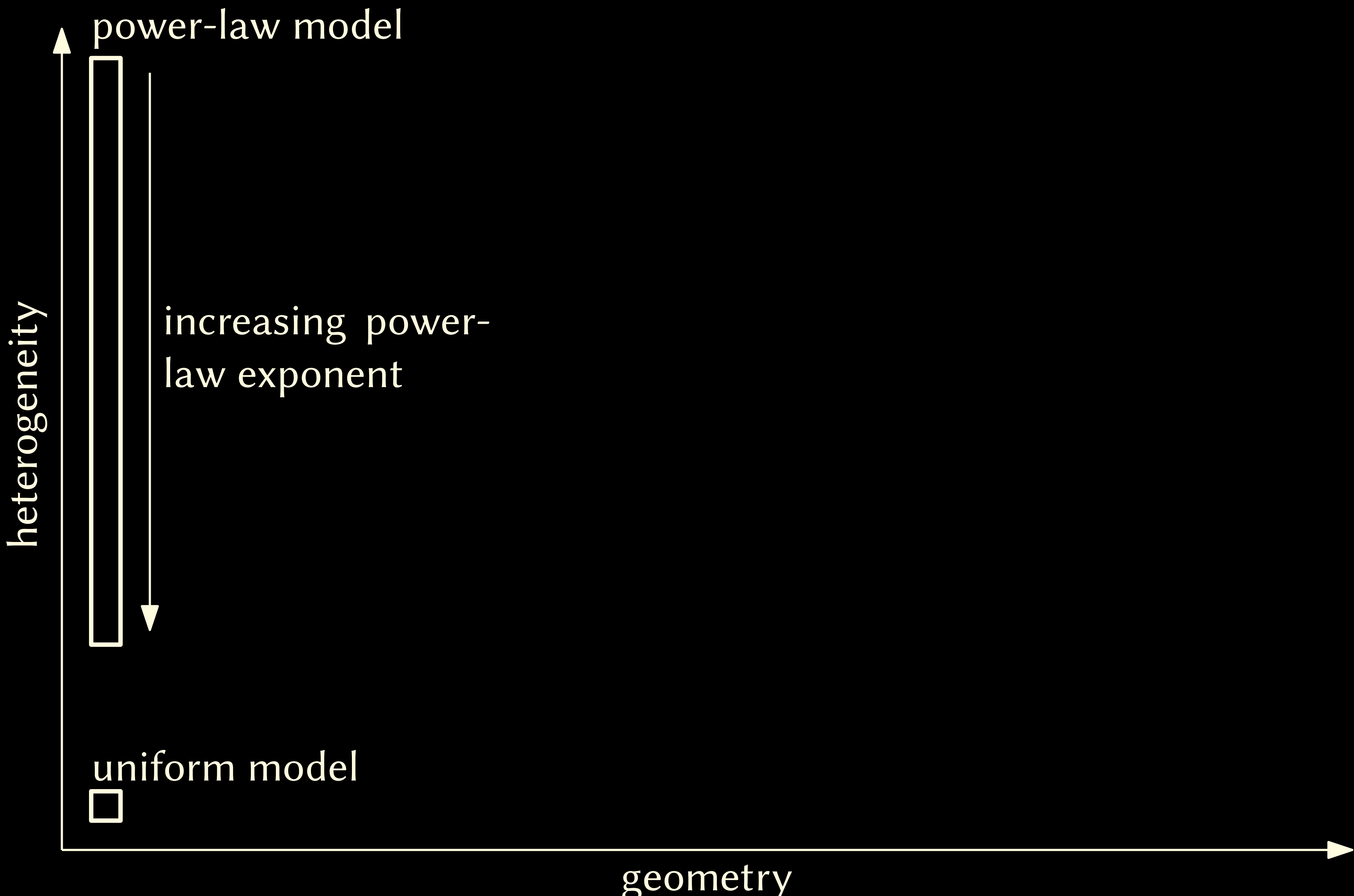

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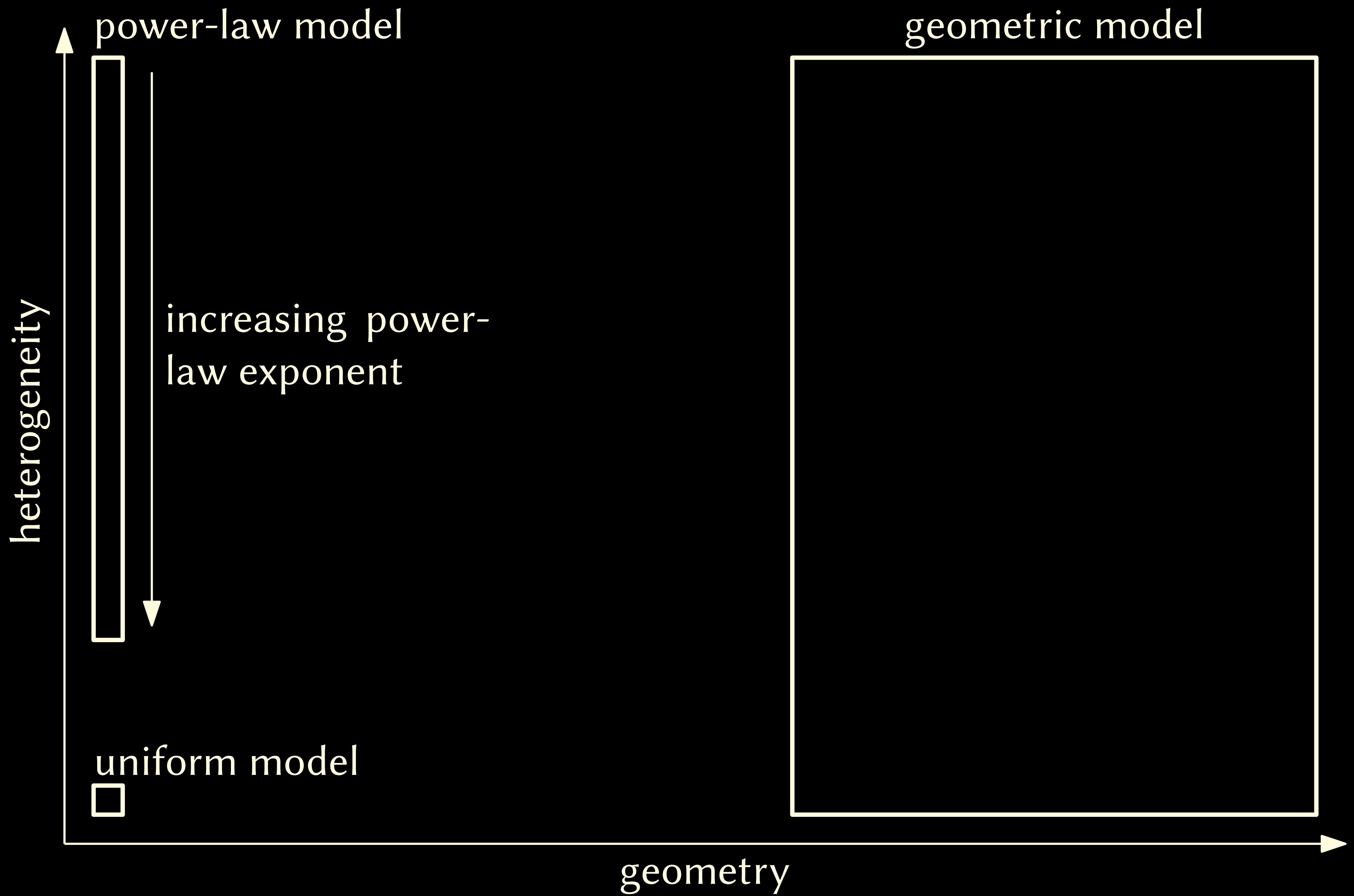

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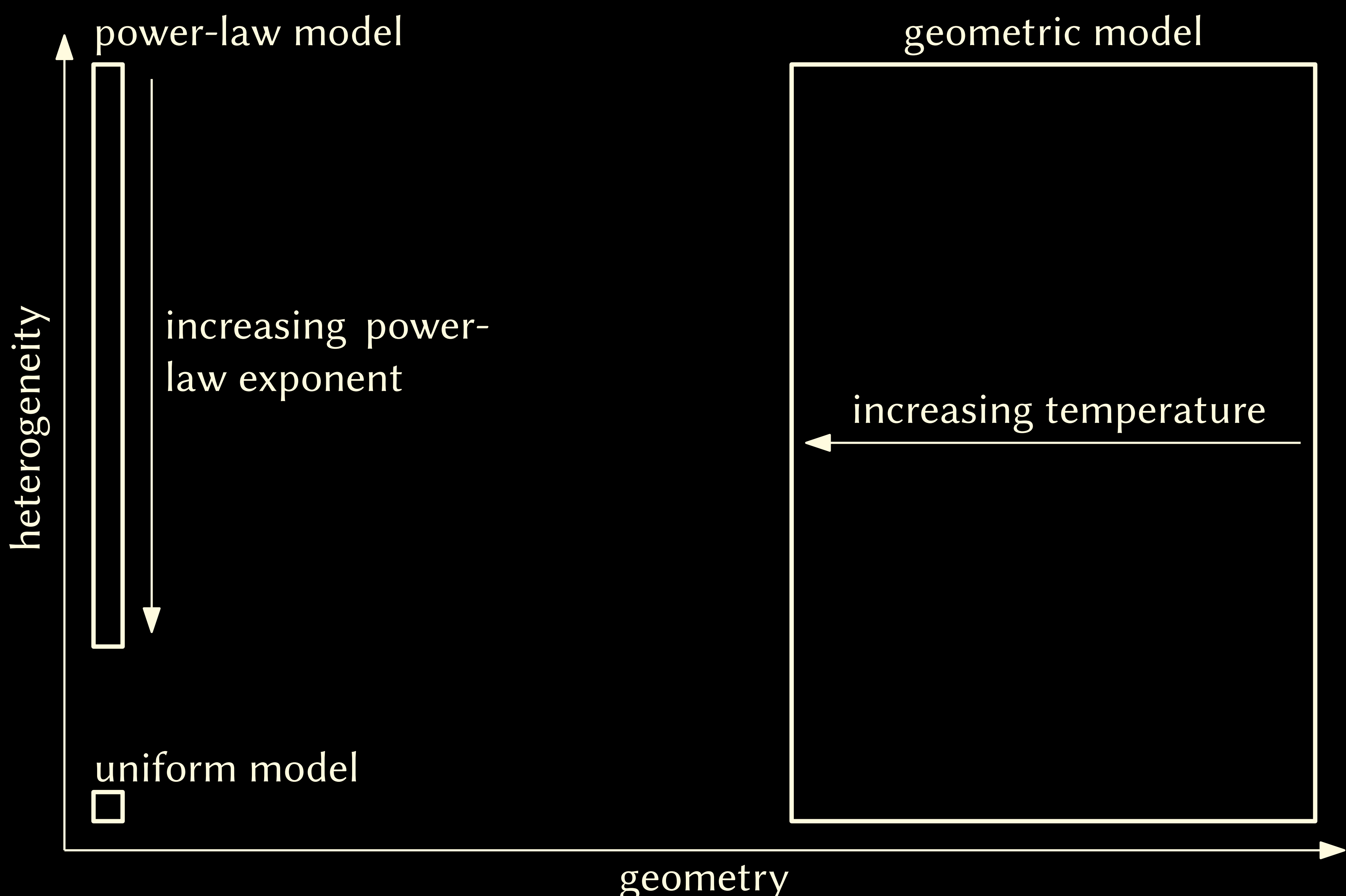

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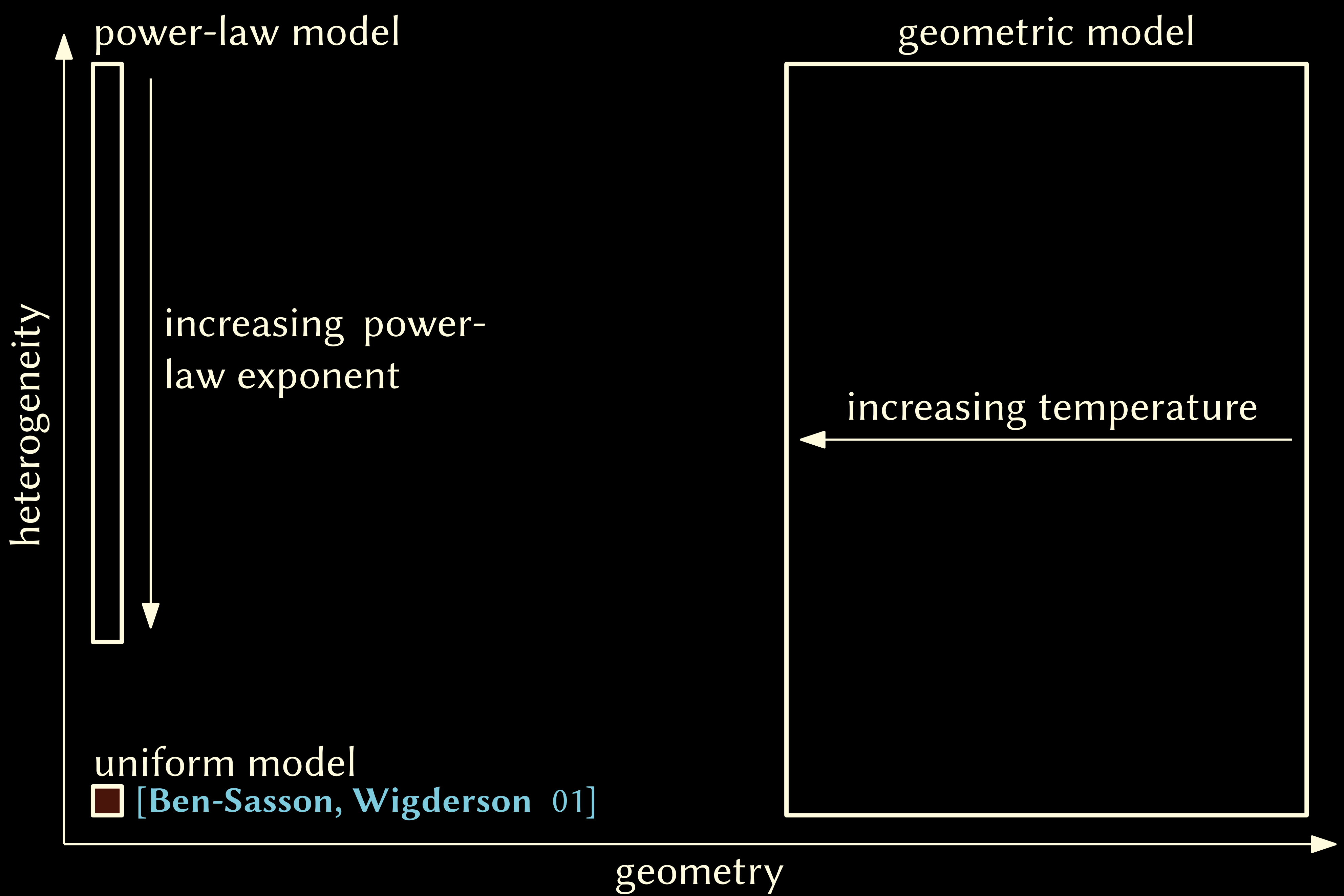

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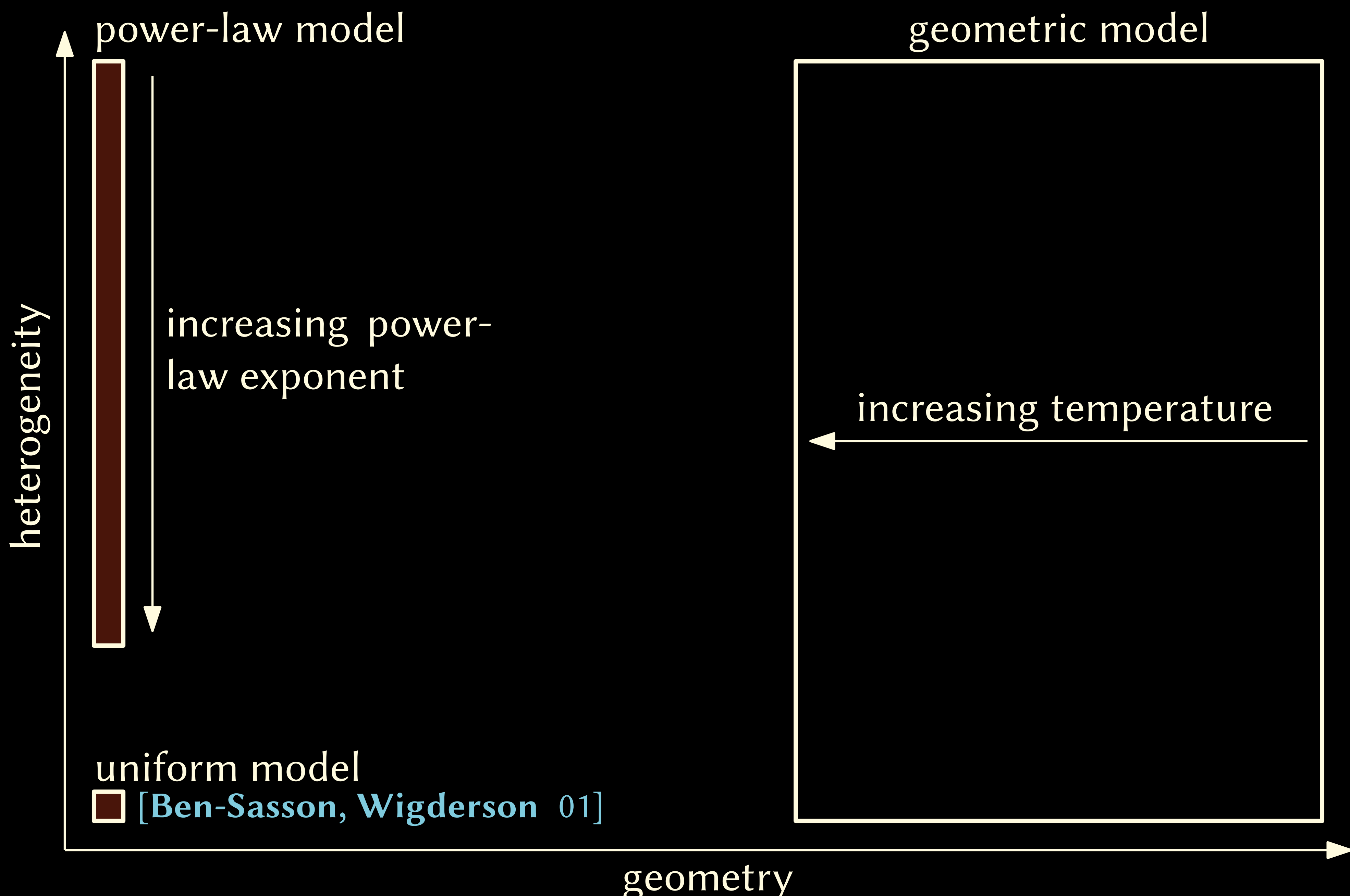

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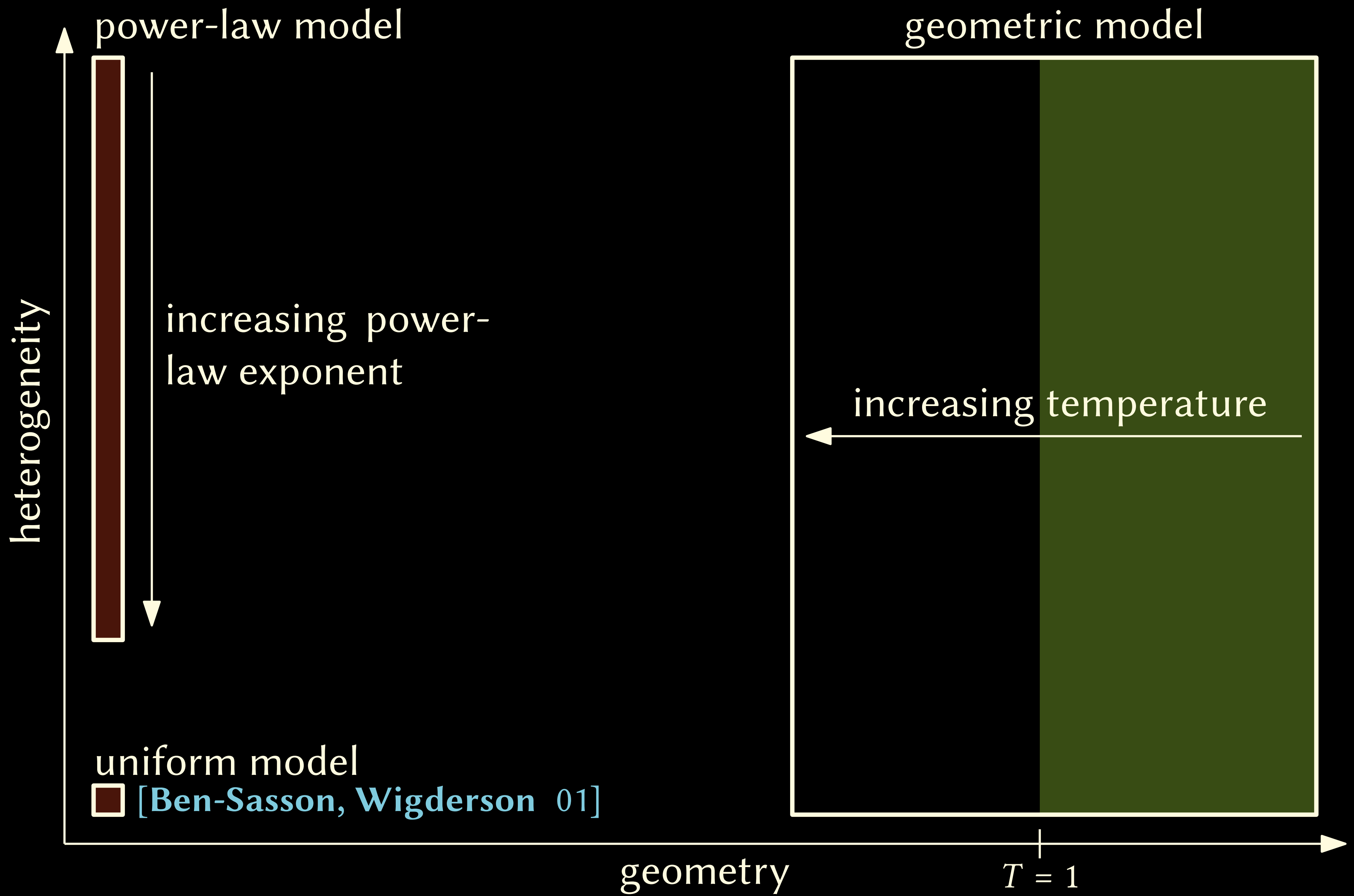

Power-Law Random k-SAT

Proof Complexity Lower Bounds [Ben-Sasson, Wigderson 01]

- *resolution width* w = largest clause in resolution proof (min over all proofs)
- lower bounds on the resolution proof size
  - $\exp(\Omega(w^2/n))$
  - $\exp(\Omega(w))$ for tree-like resolution

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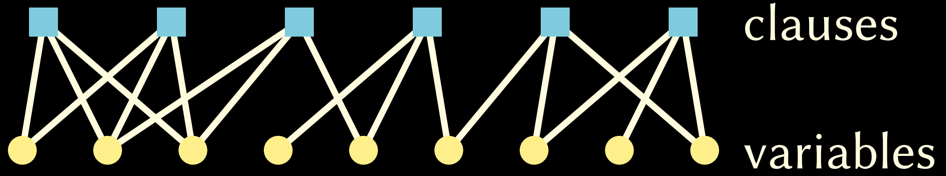

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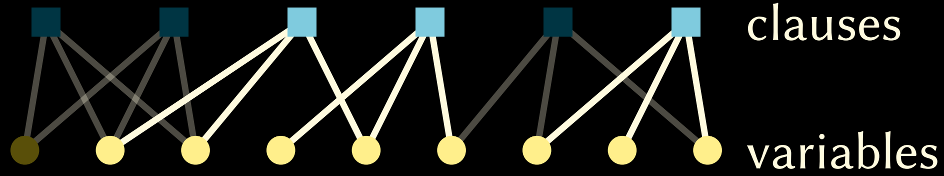

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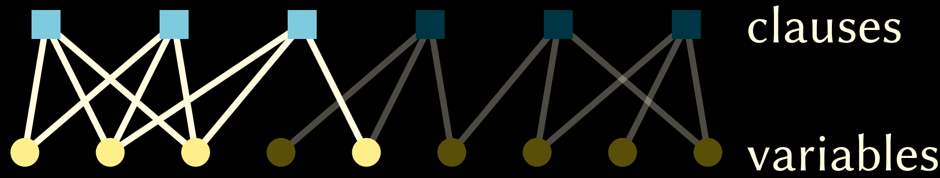

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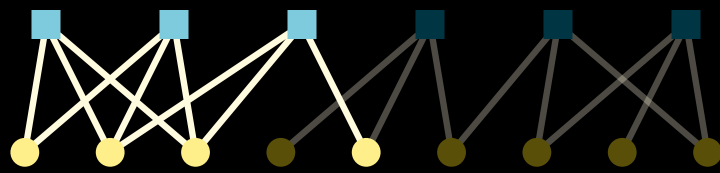
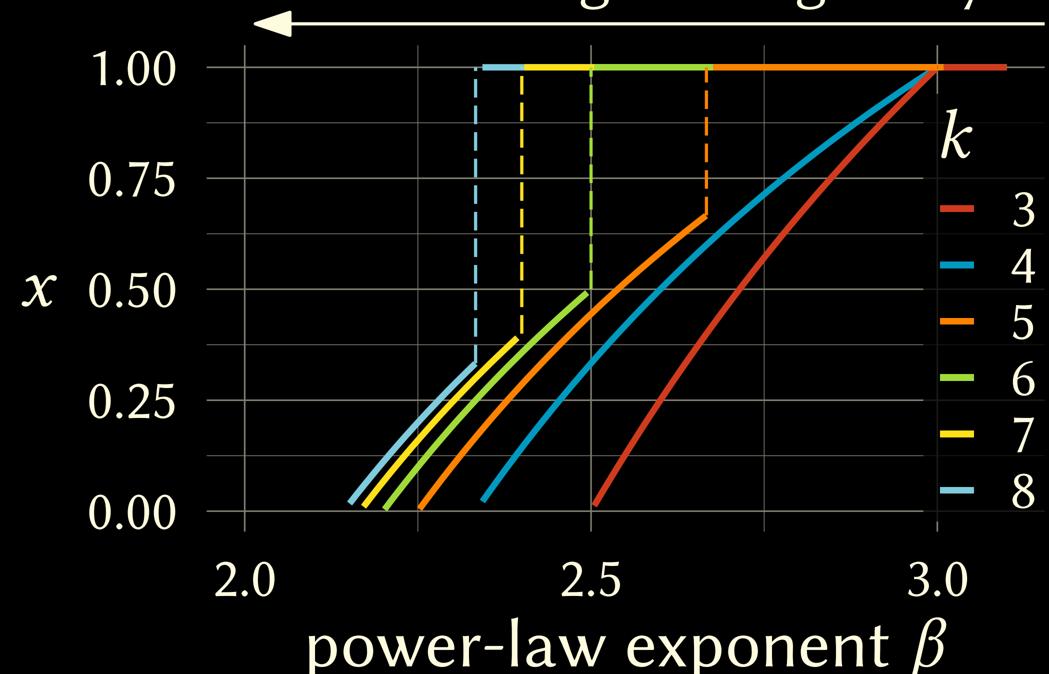


Our Lower Bounds (simplified)

- resolution width $w \in \tilde{\Omega}(n^x)$



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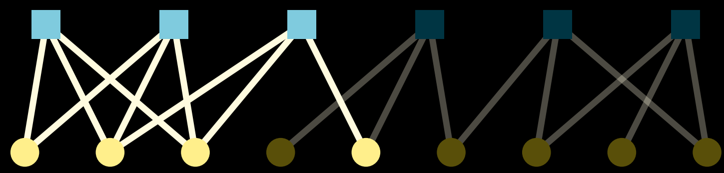

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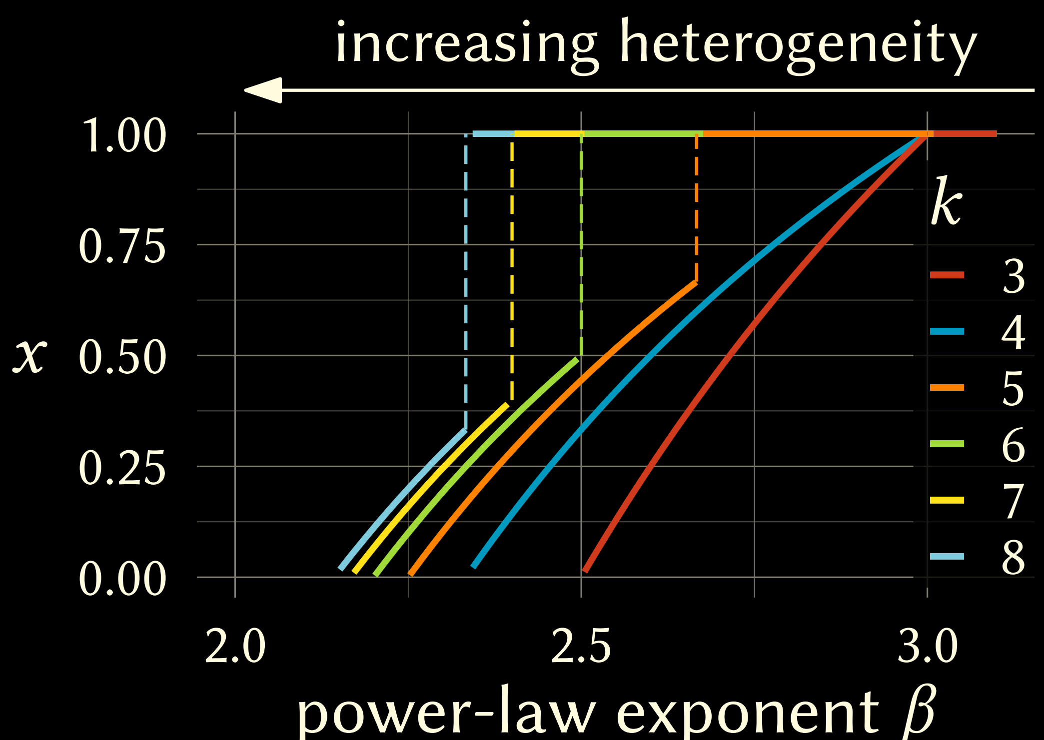

Power-Law Random *k*-SAT

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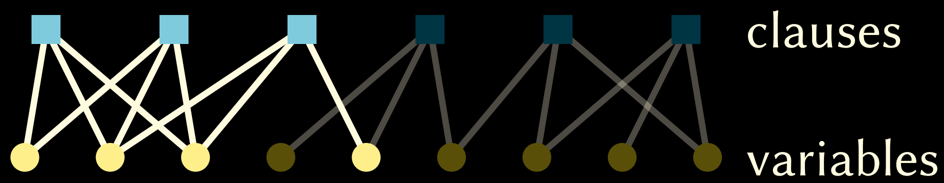

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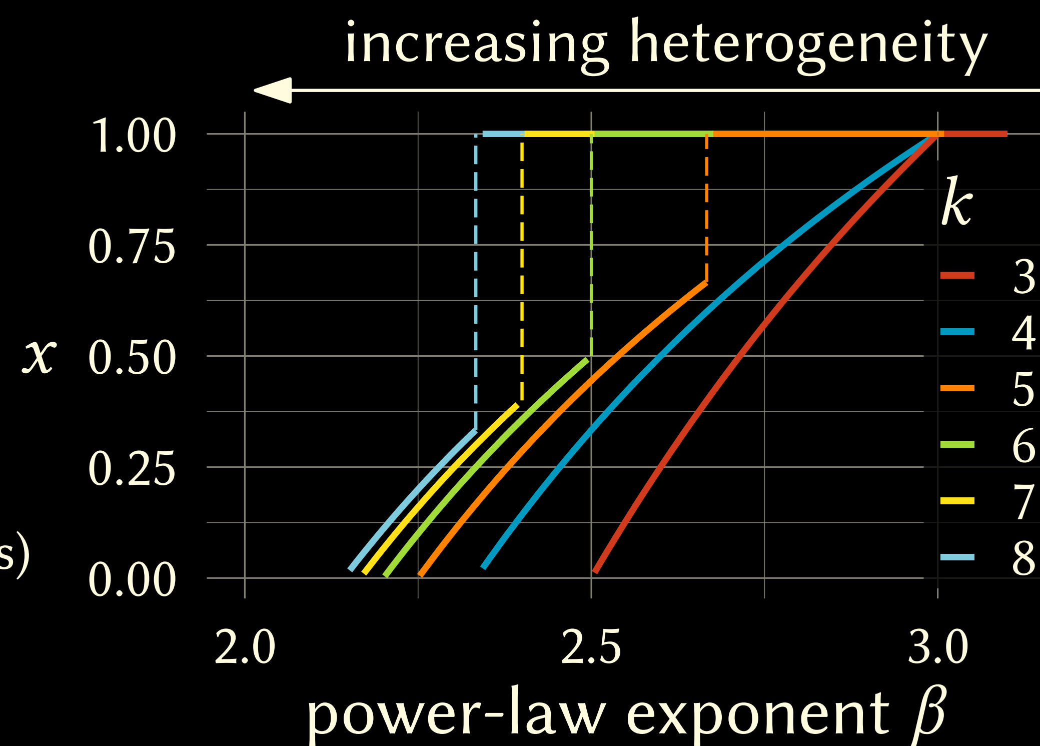

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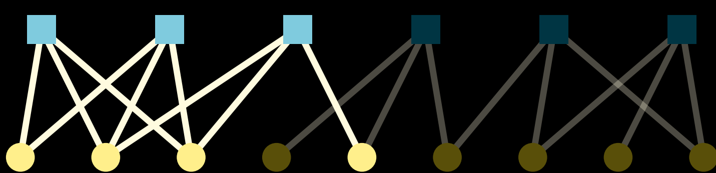

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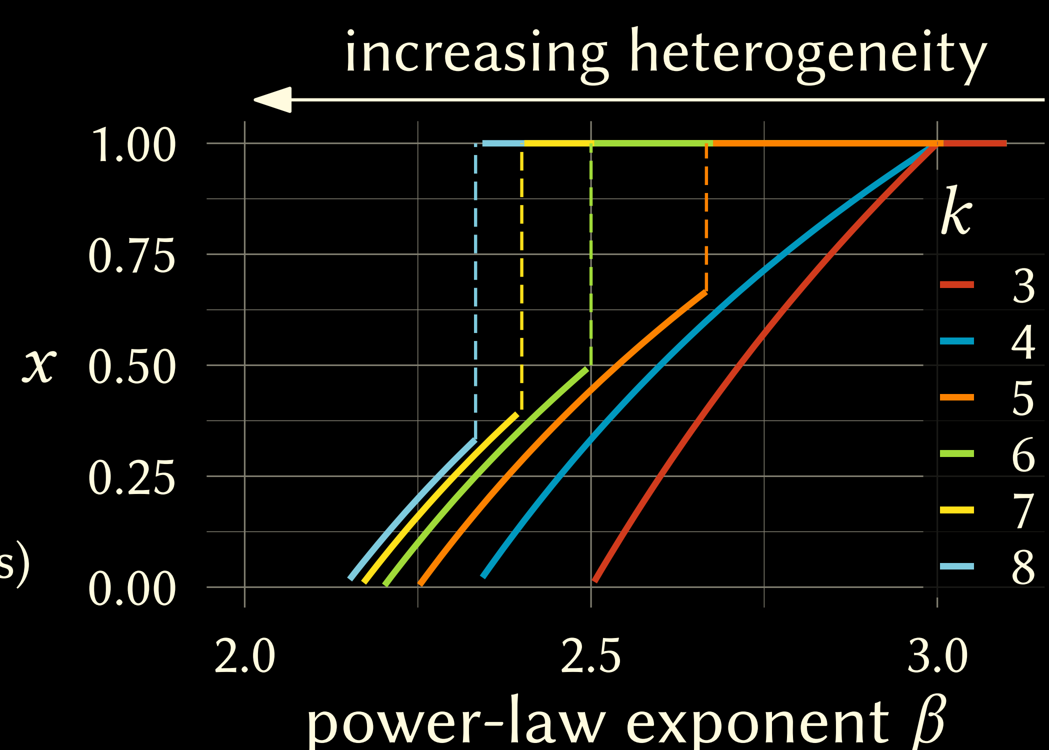
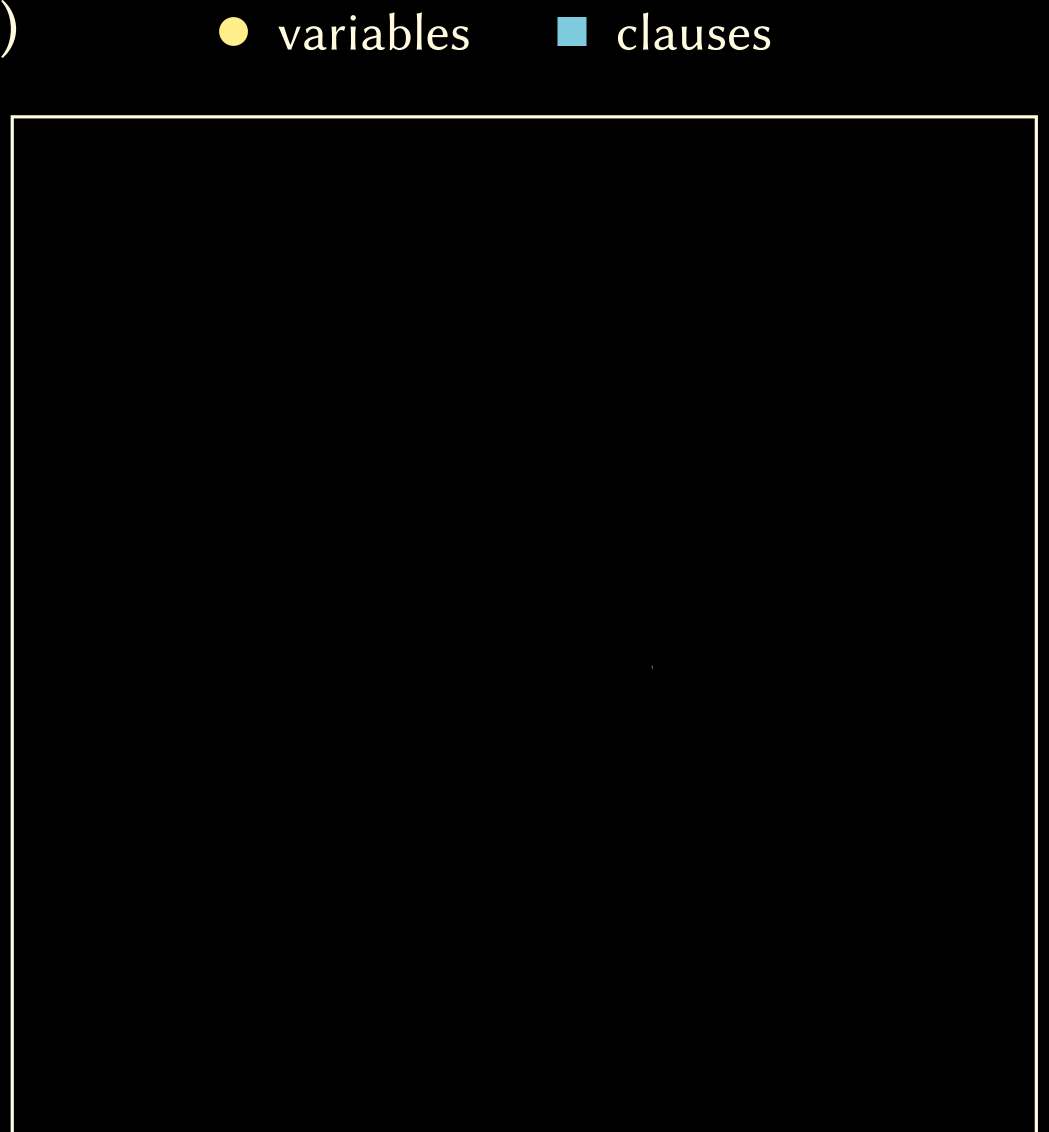
- β-range matches that of a constant satisfiability threshold



Geometric Random k-SAT

Simplifying Assumptions

- each clause contains the $k = 3$ closest variables ($T = 0$)
- 2-dimensional ground space (square)
- uniform weights



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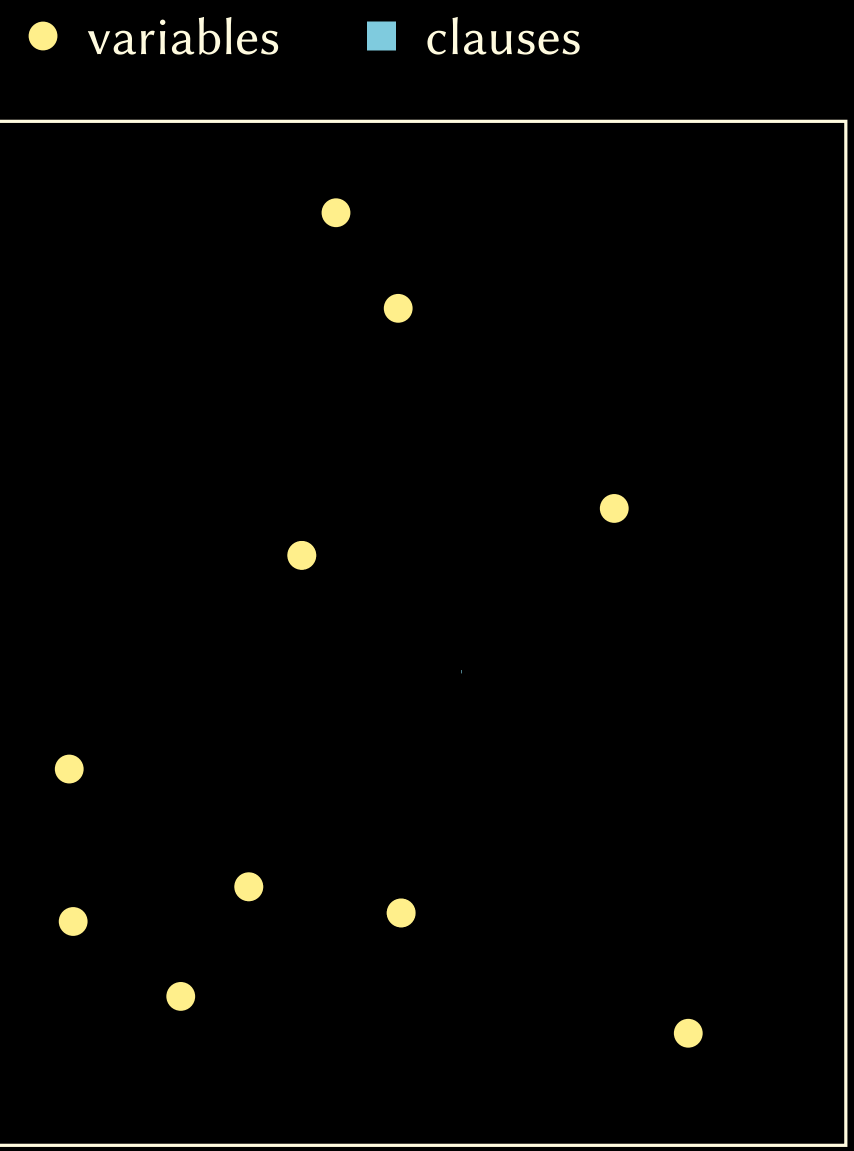

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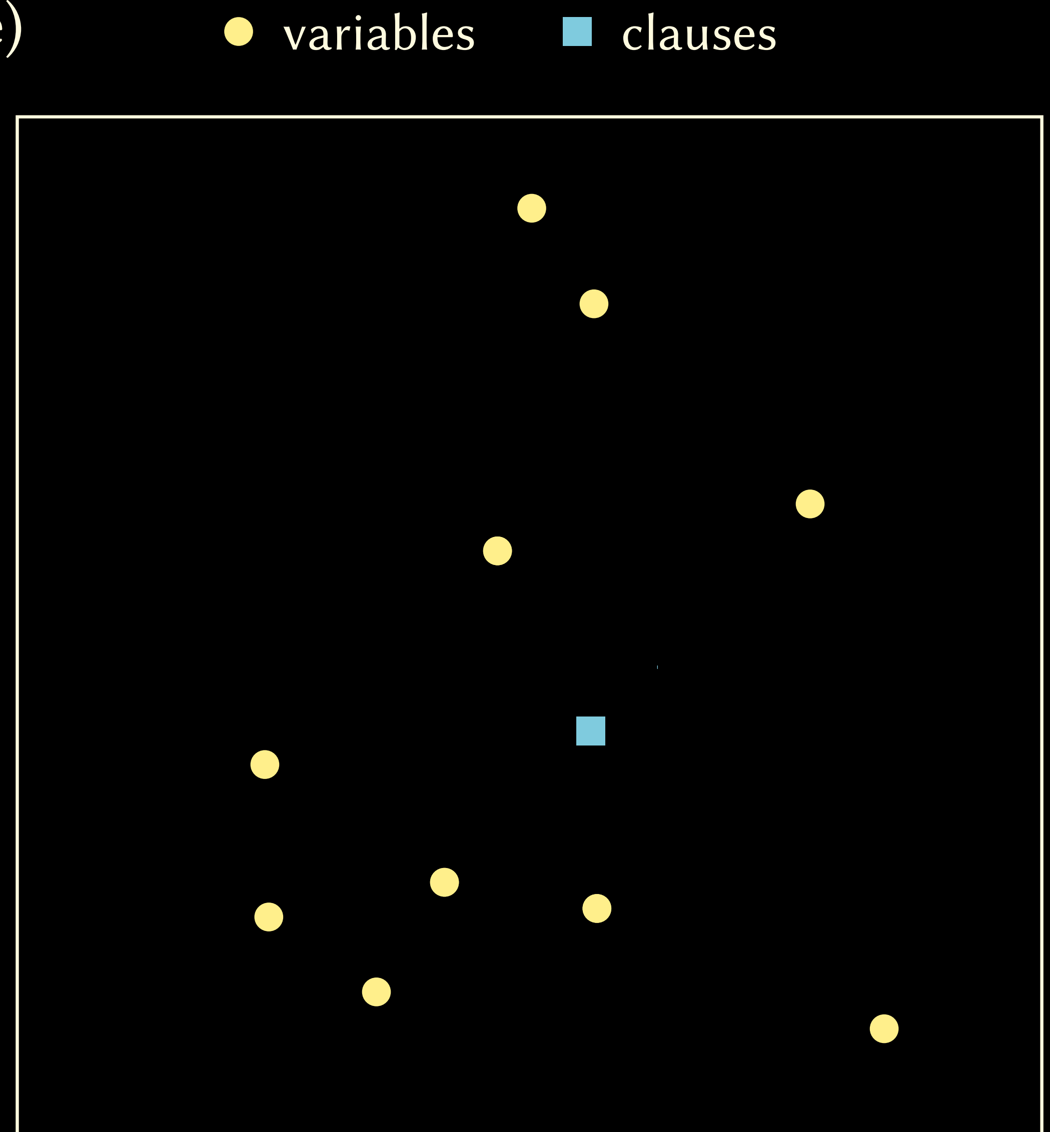

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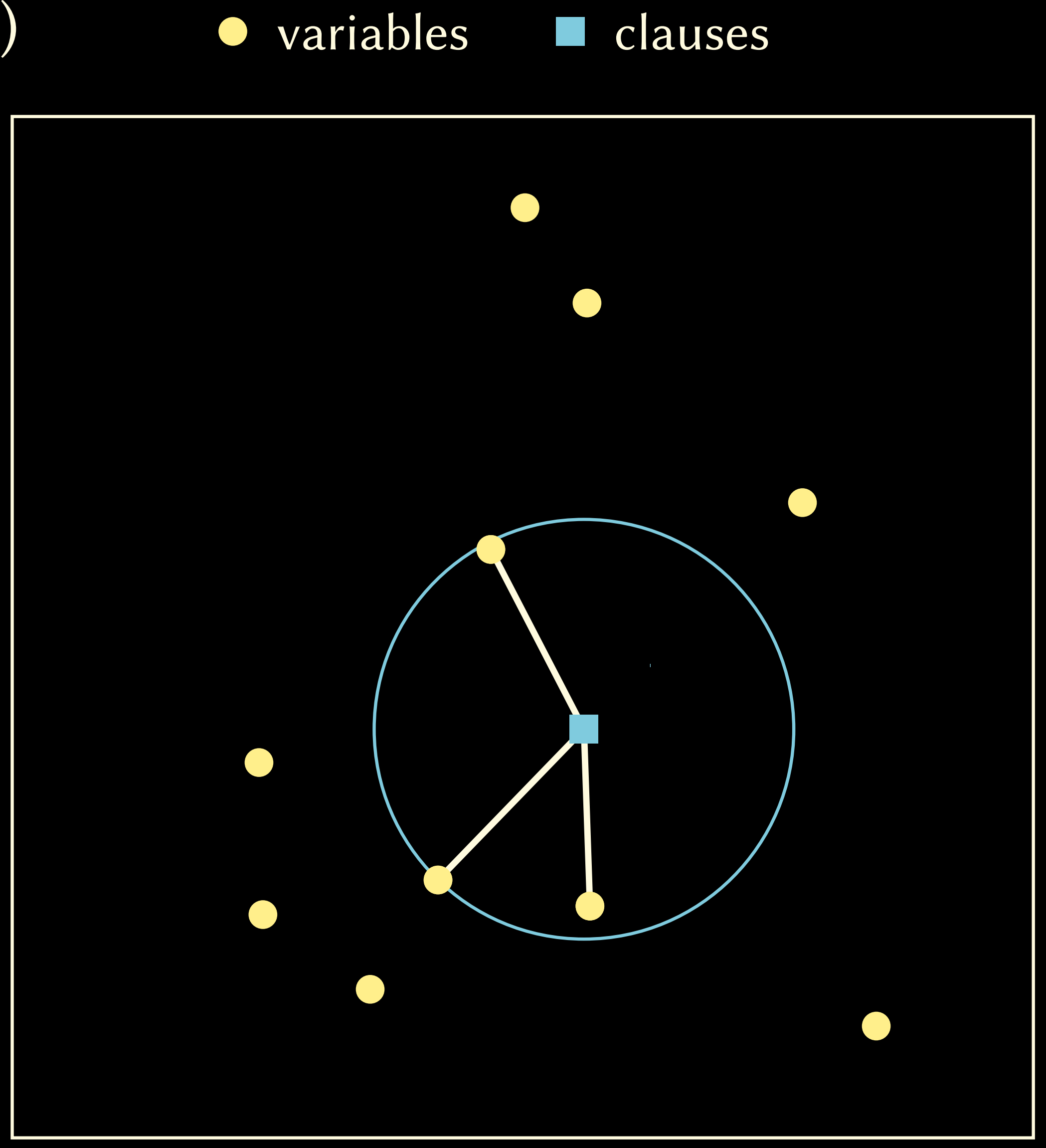

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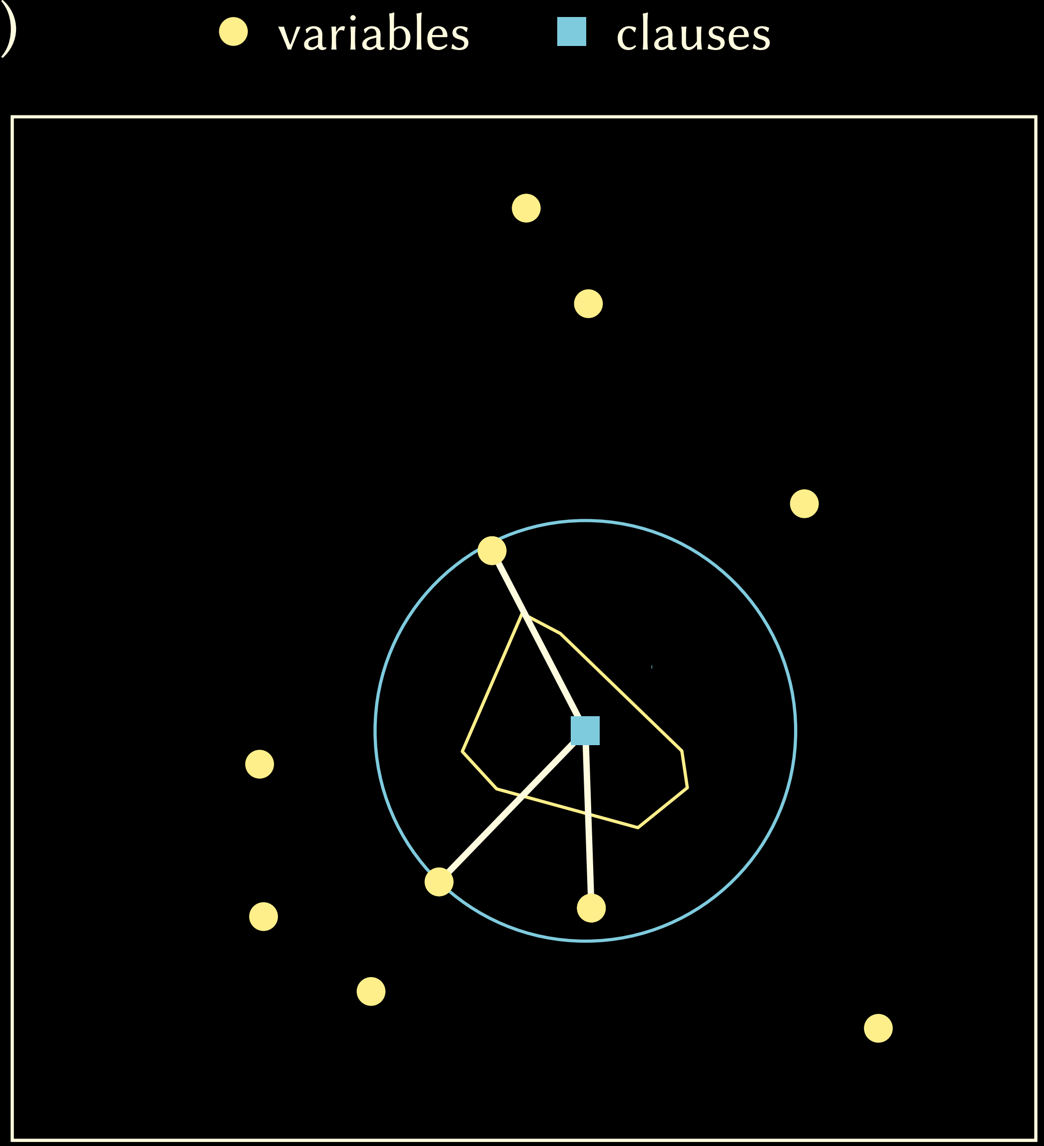

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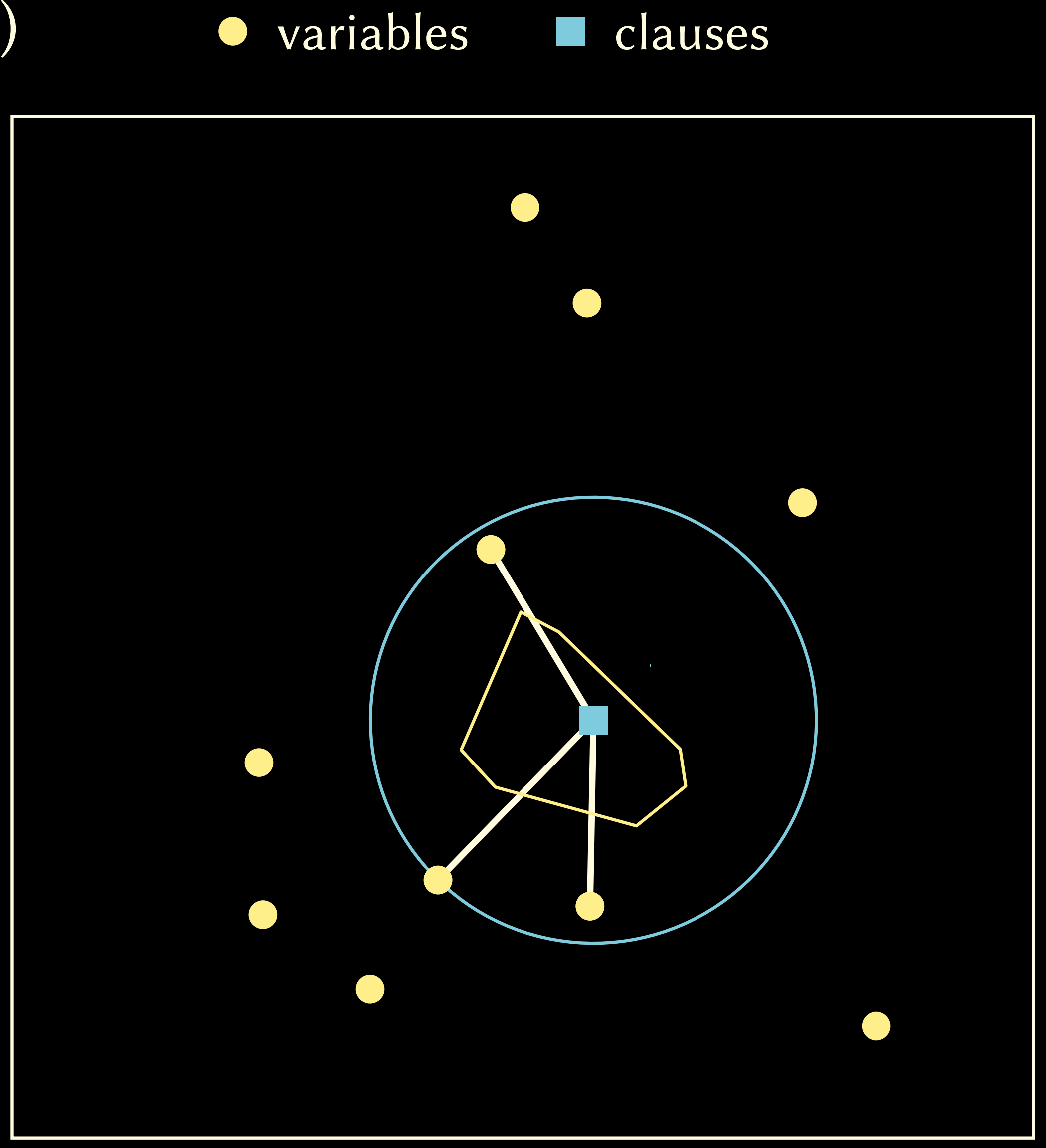

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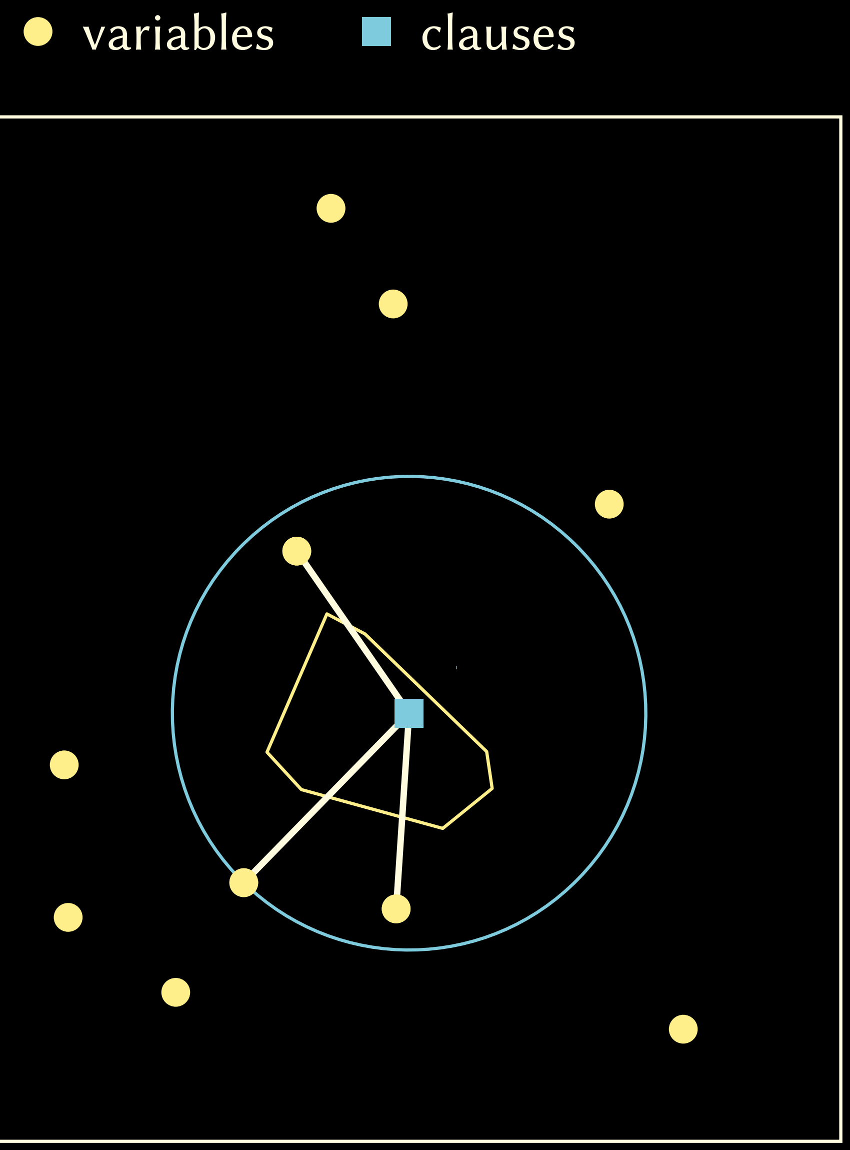

Geometric Random k-SAT

Simplifying Assumptions

- each clause contains the $k = 3$ closest variables ($T = 0$)
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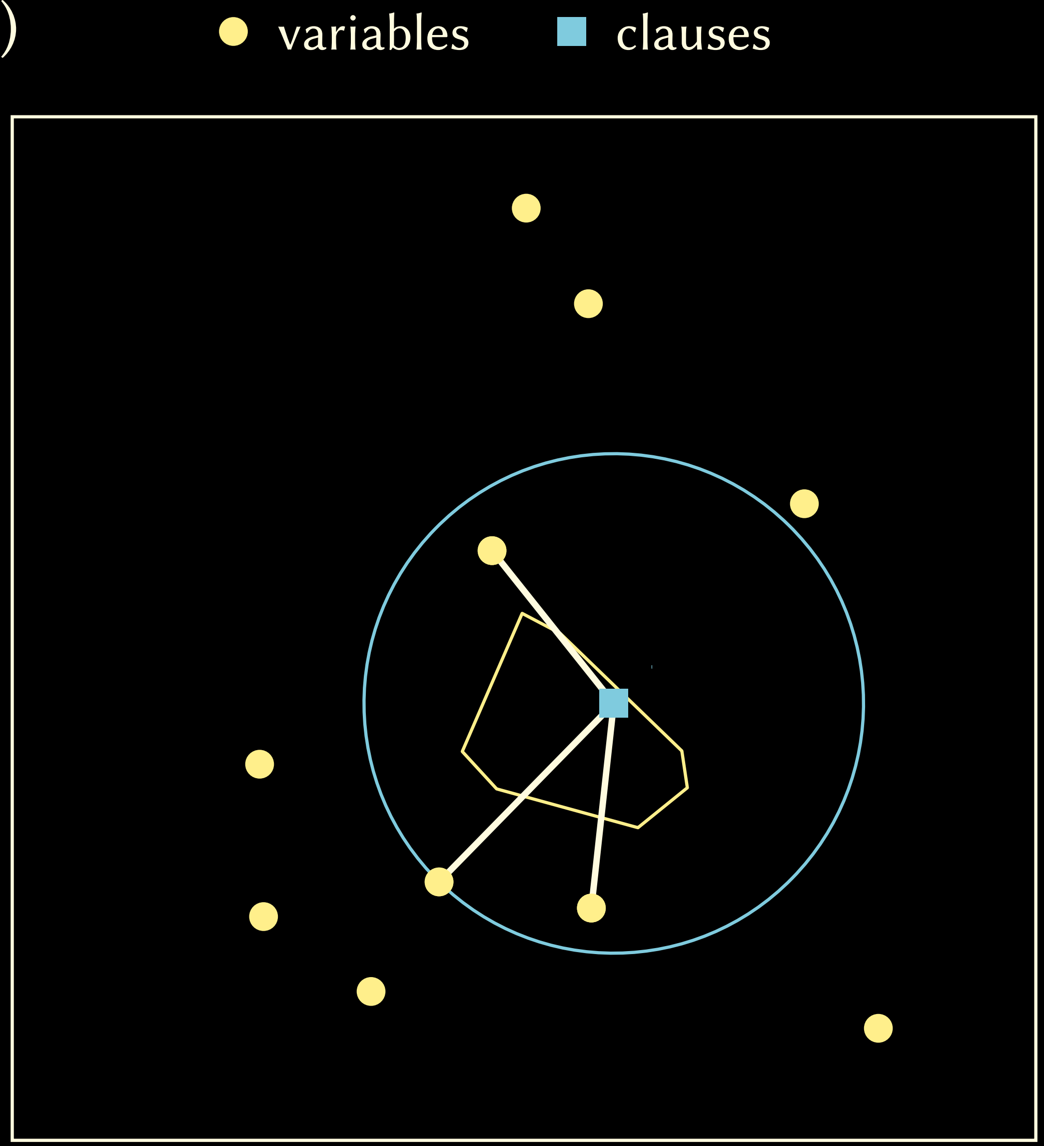

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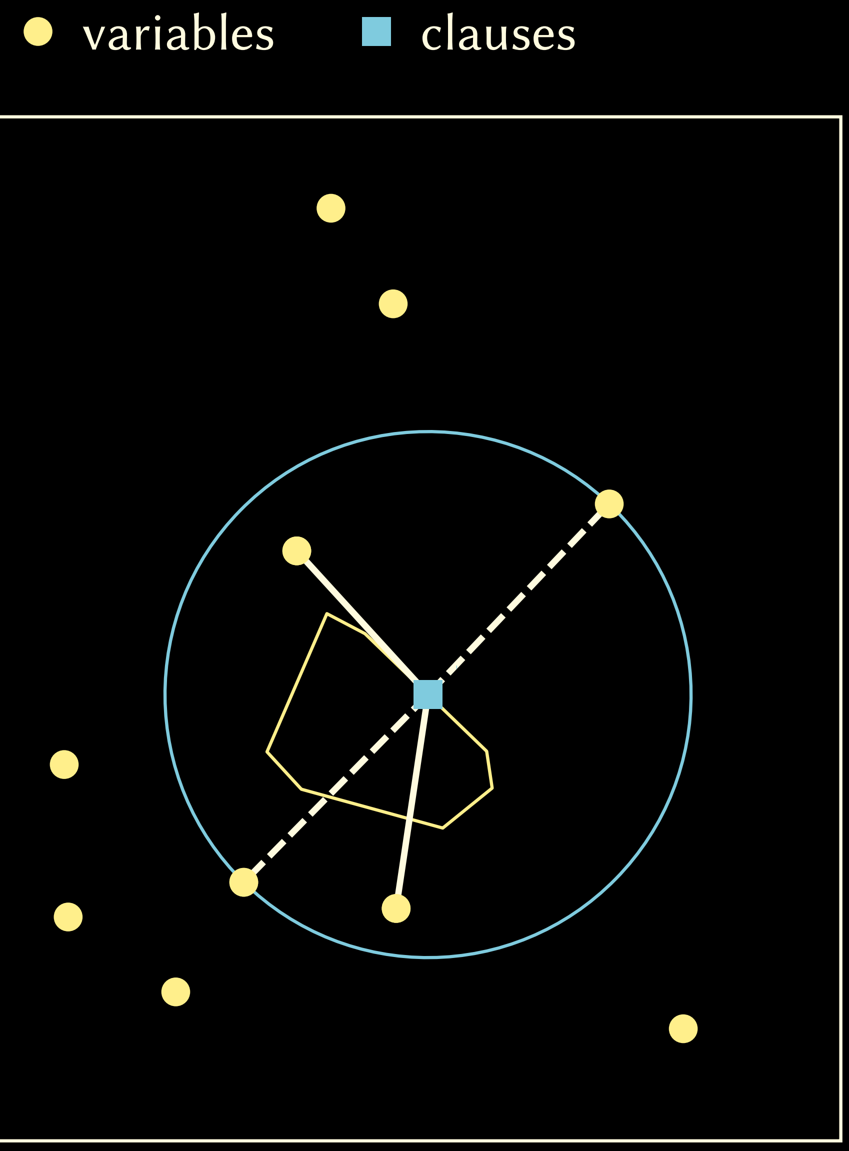

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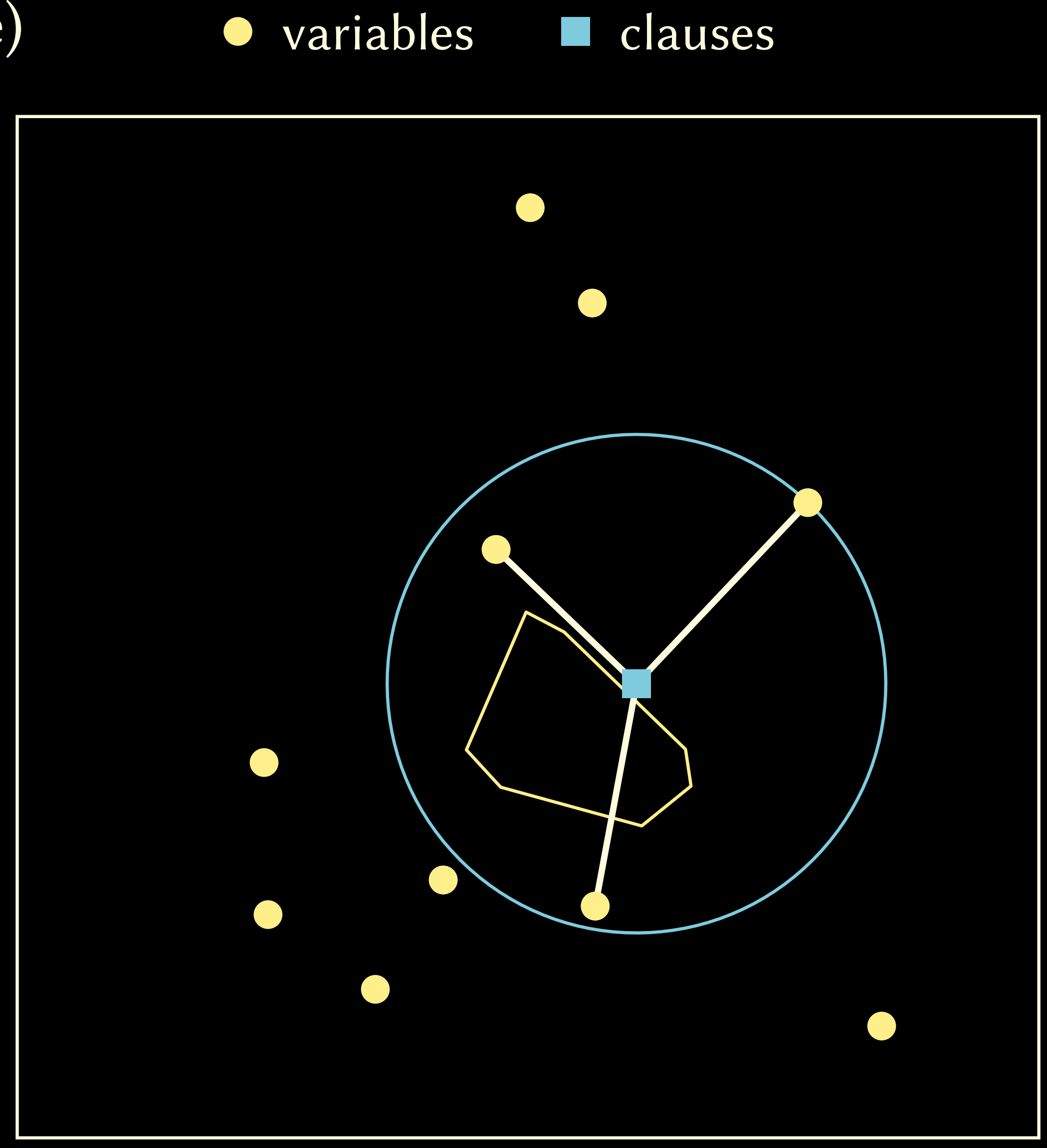

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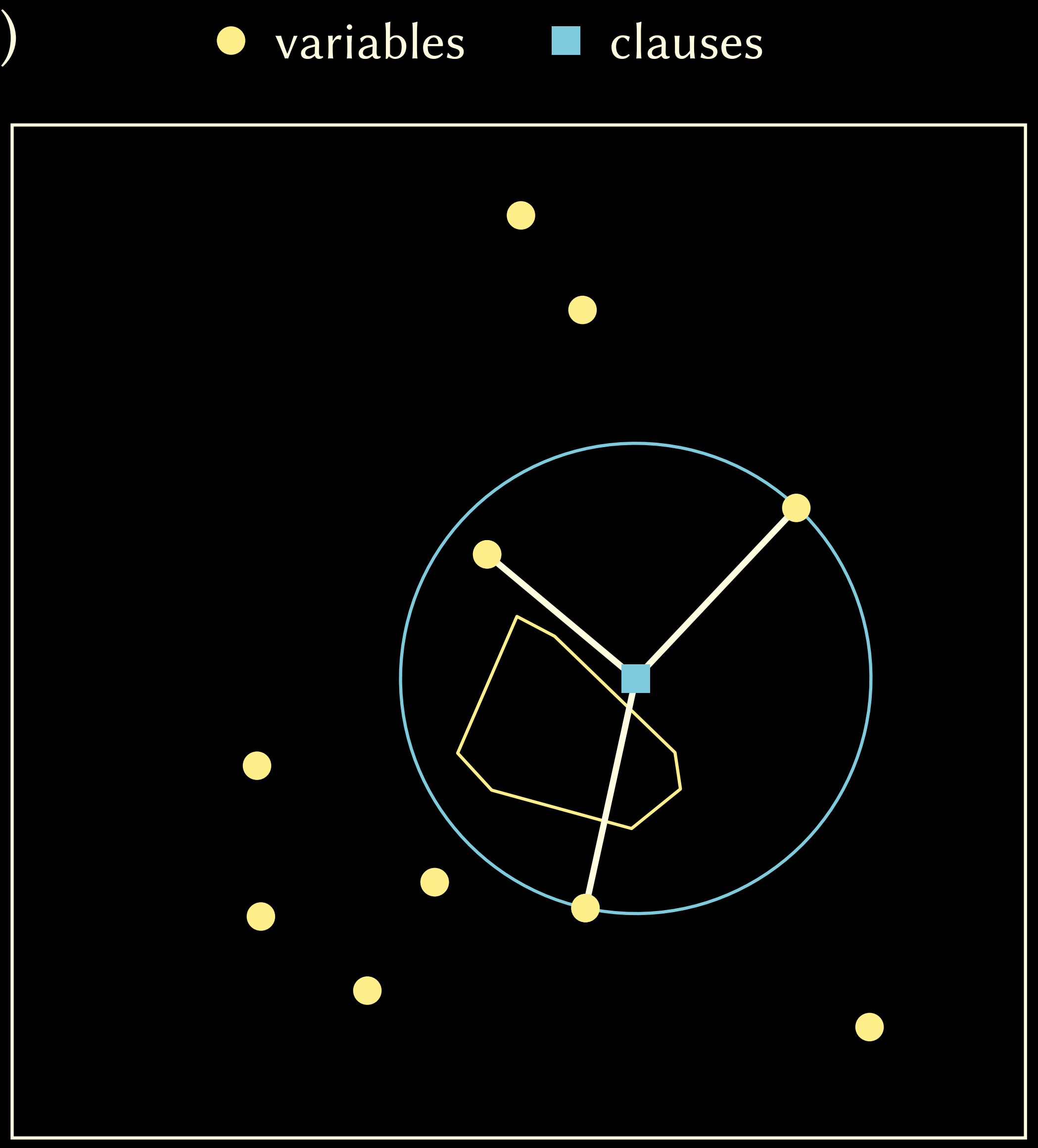

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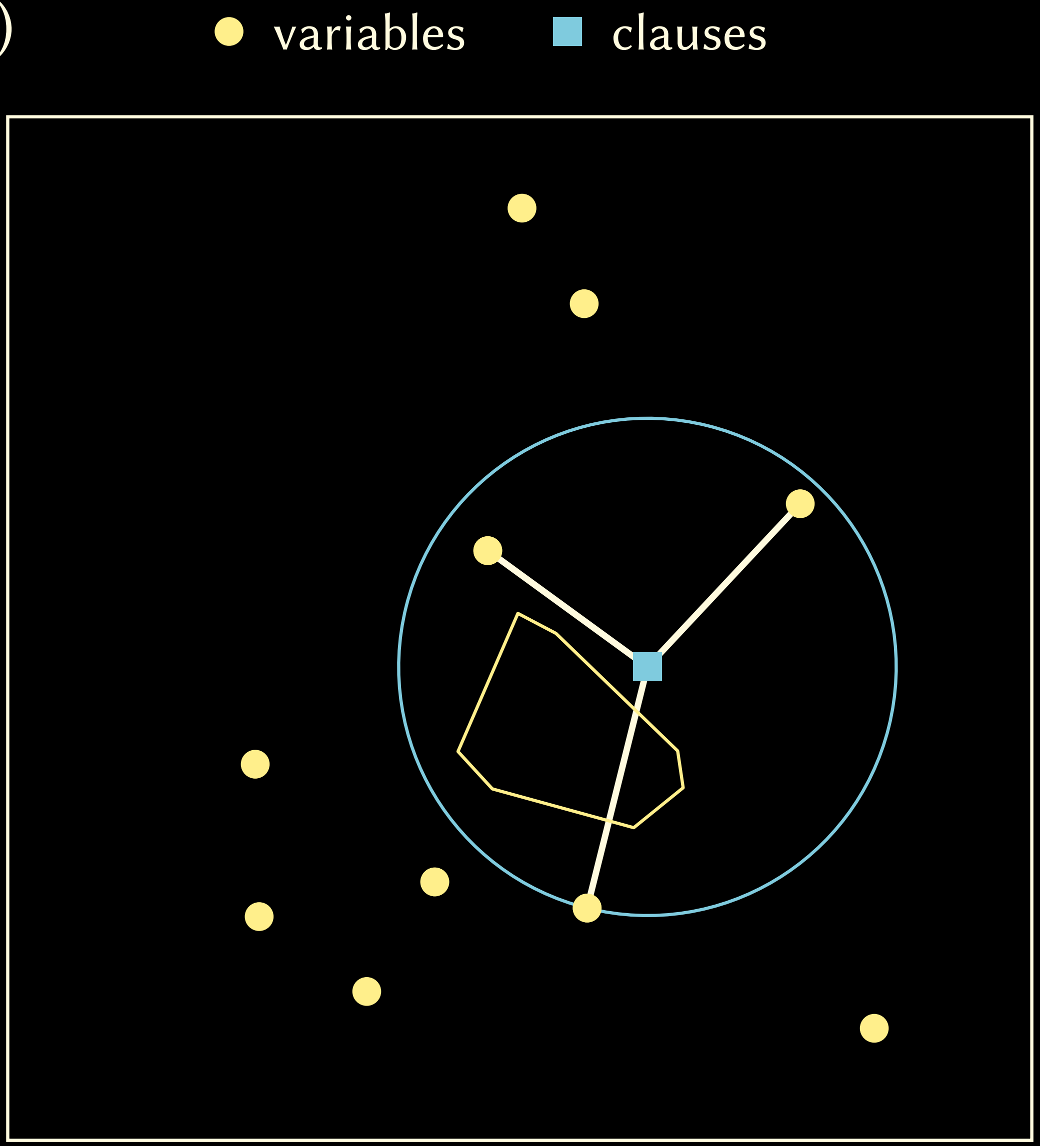

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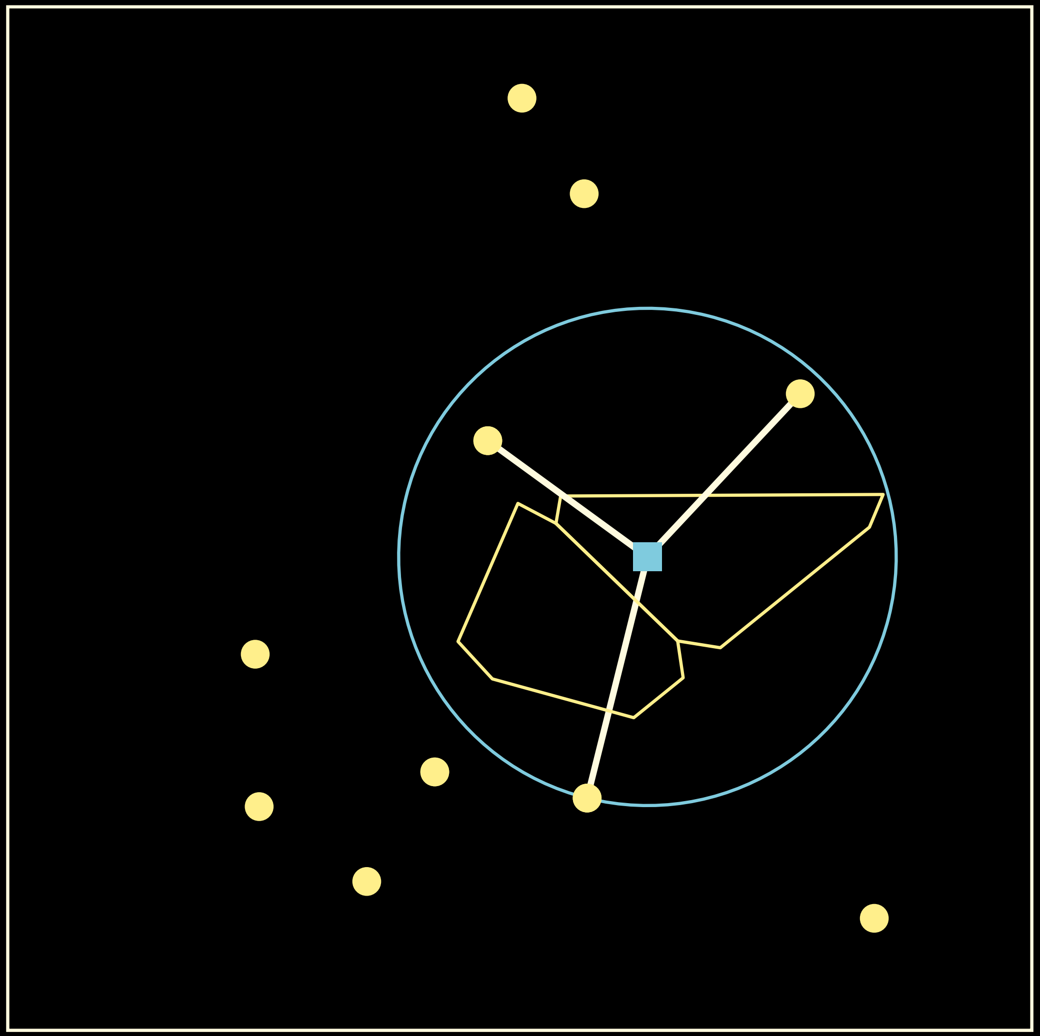

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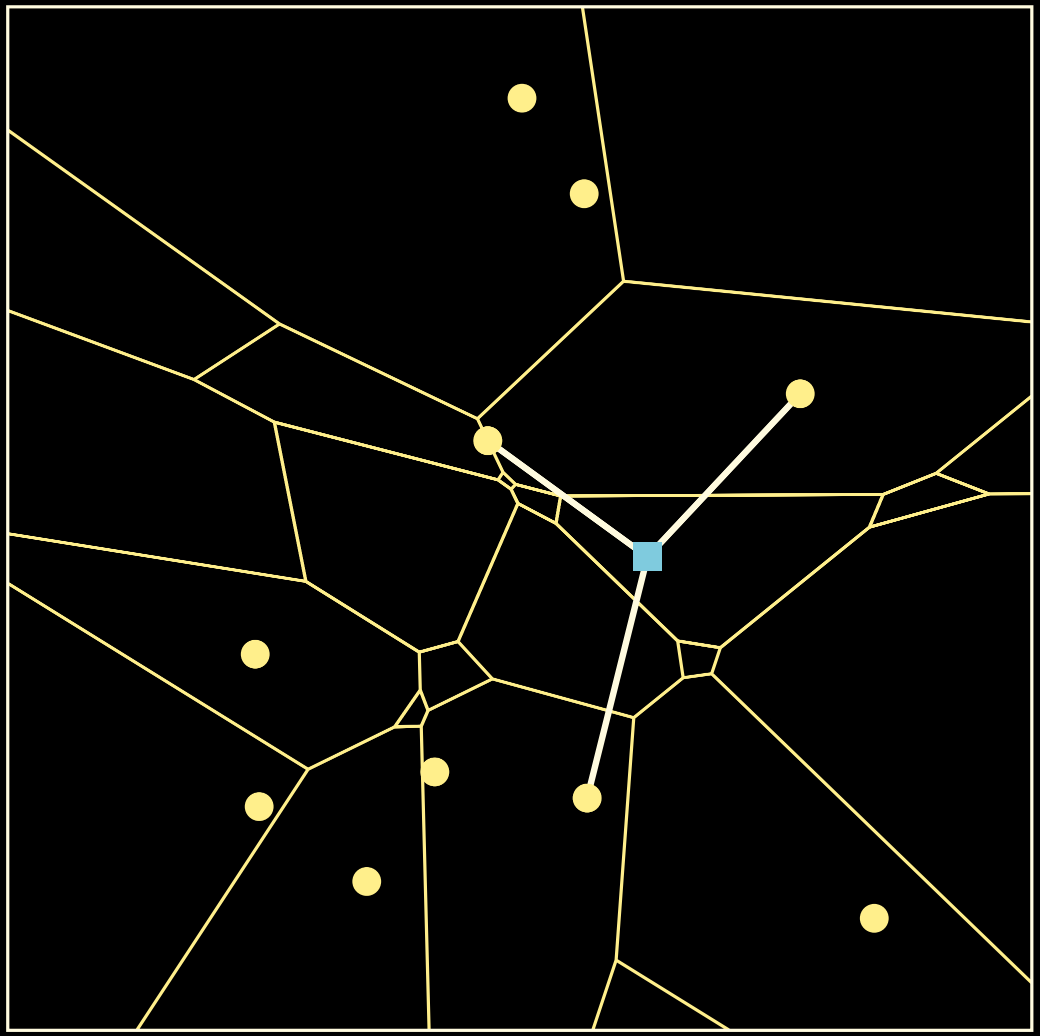

Order-k Voronoi Diagram

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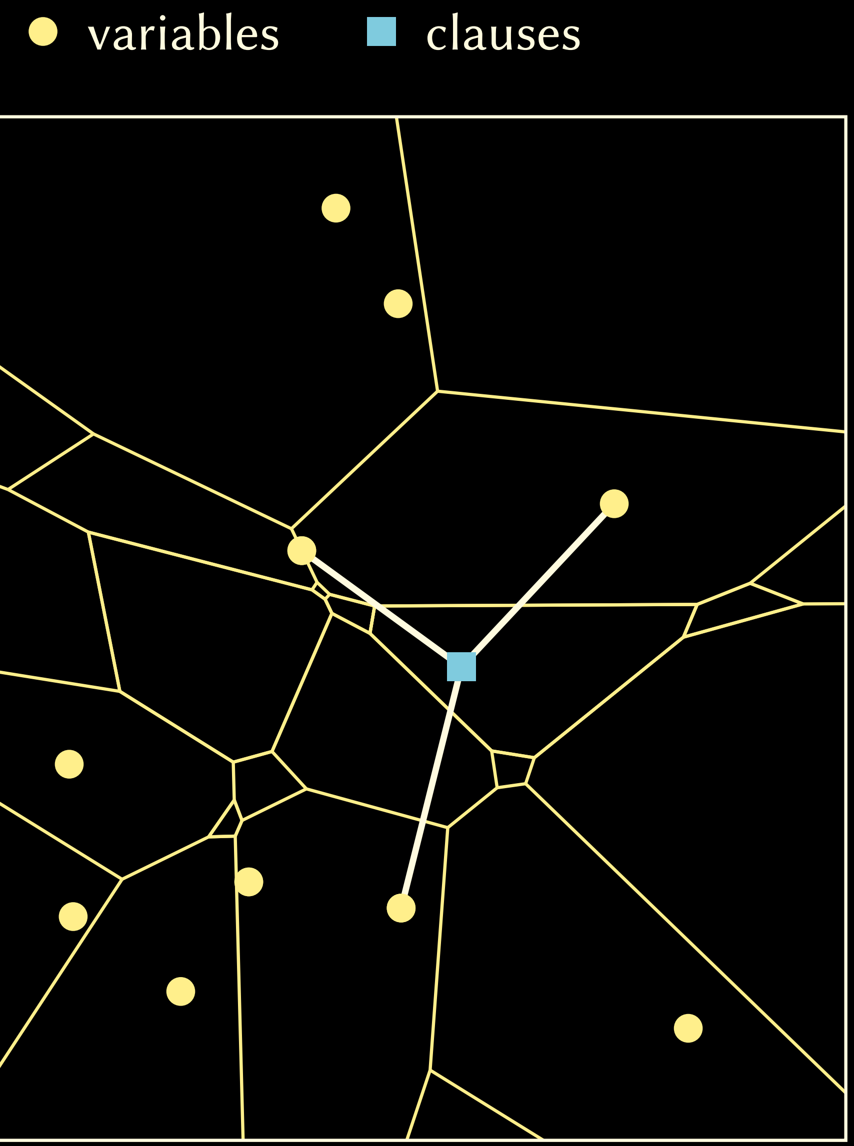

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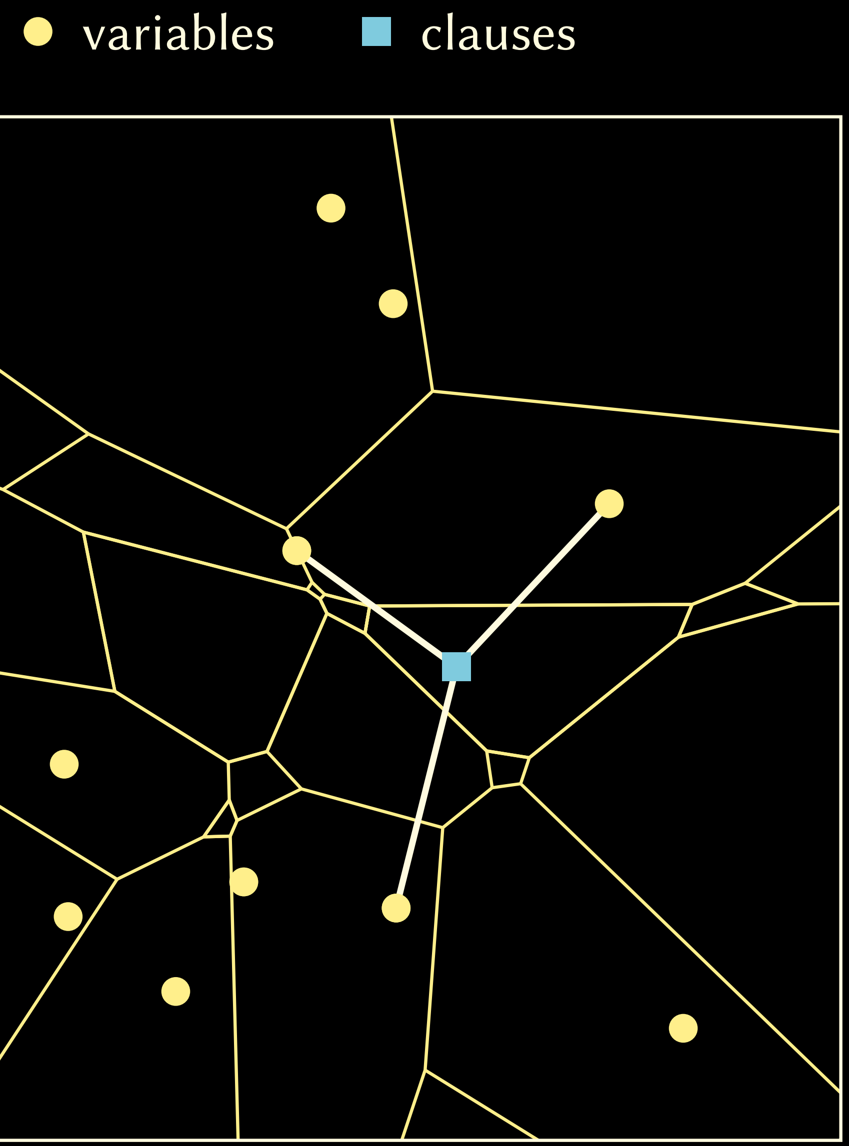

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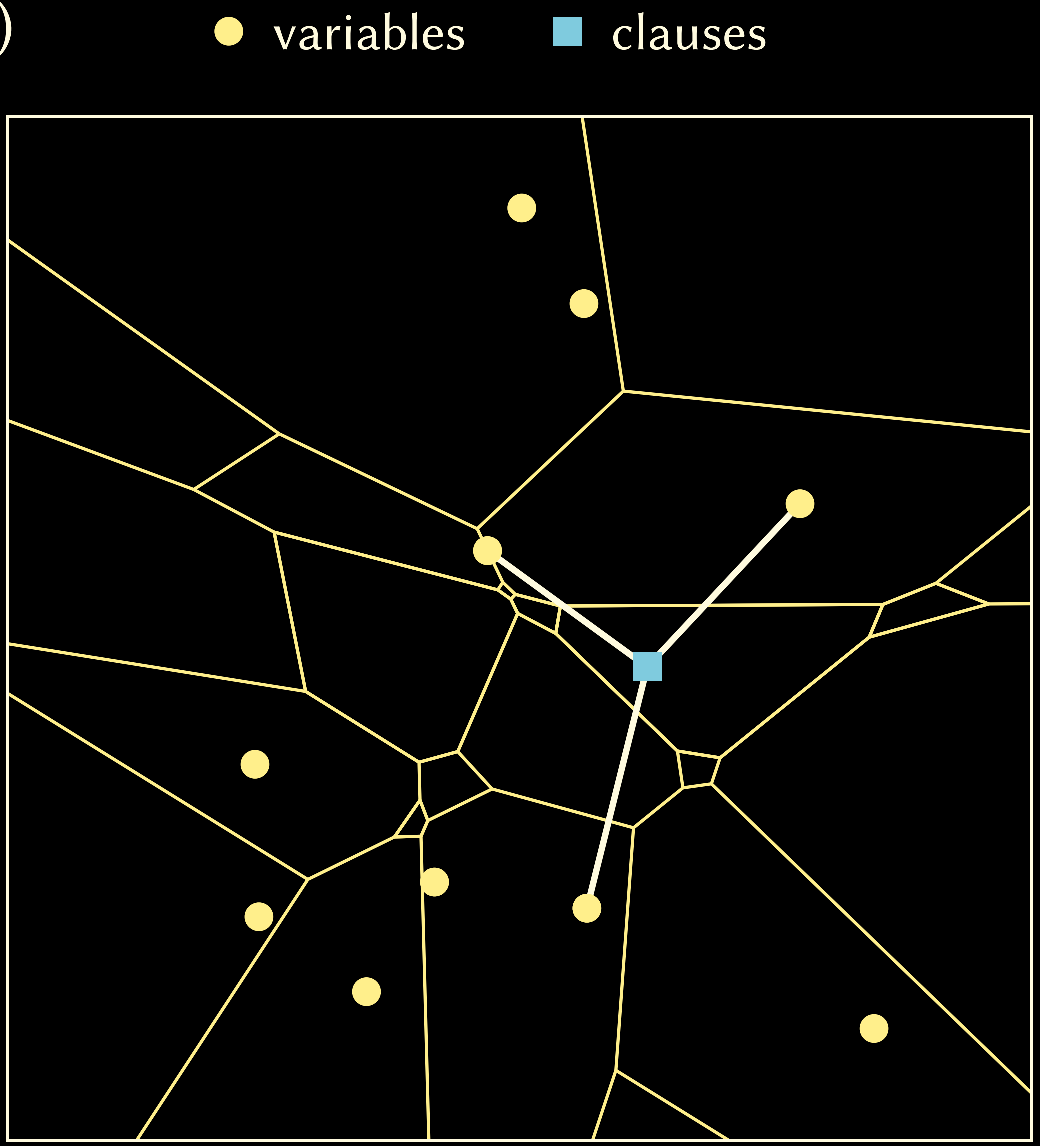

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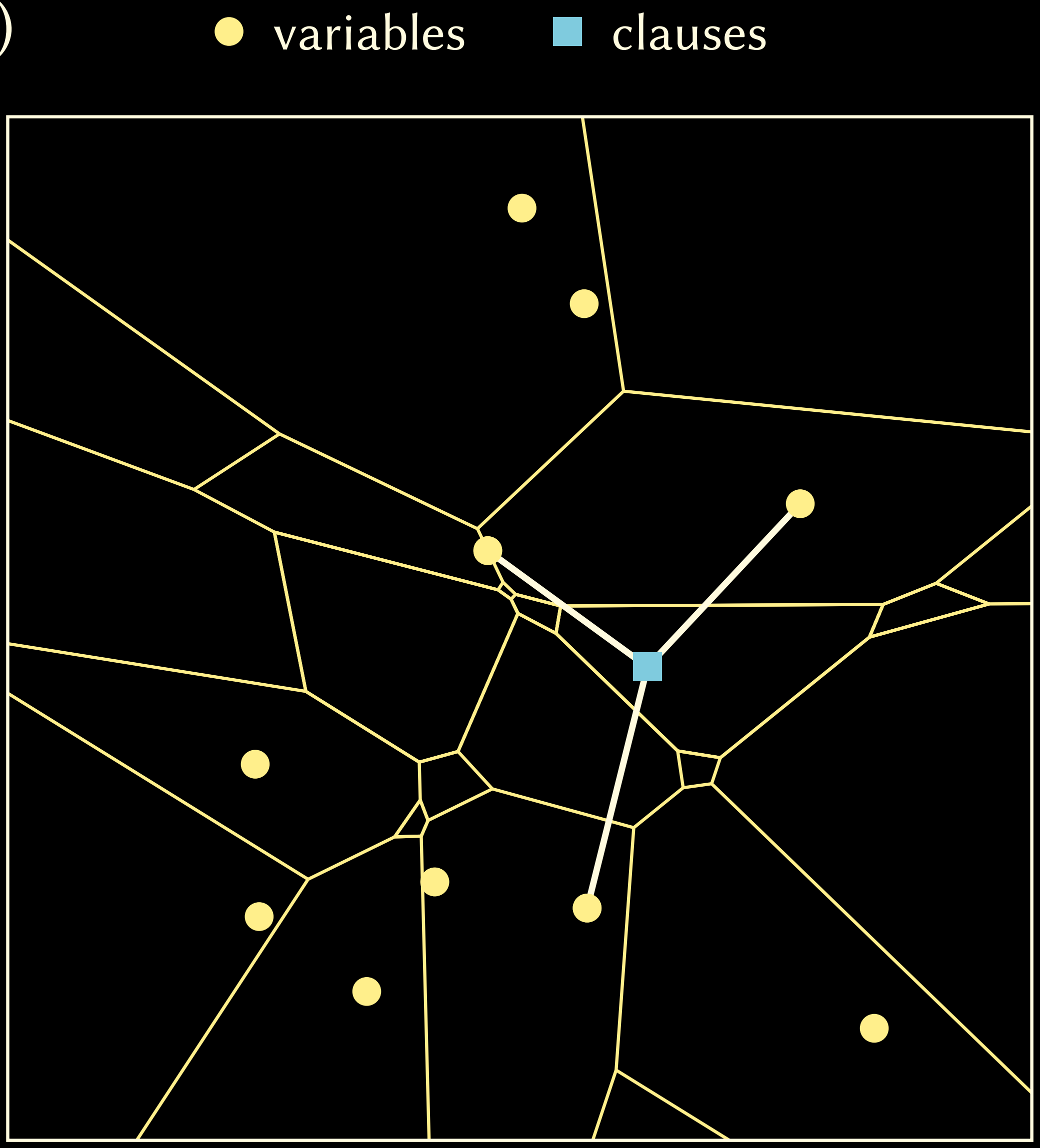

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Geometric Random k-SAT – General Setting

Weighted Variables & Higher Dimensions

- $\Omega\left(n^2\right)$ in each of these settings:
  - weighted, 2D, $k = 3$ [**Aurenhammer, Edelsbrunner** 84]
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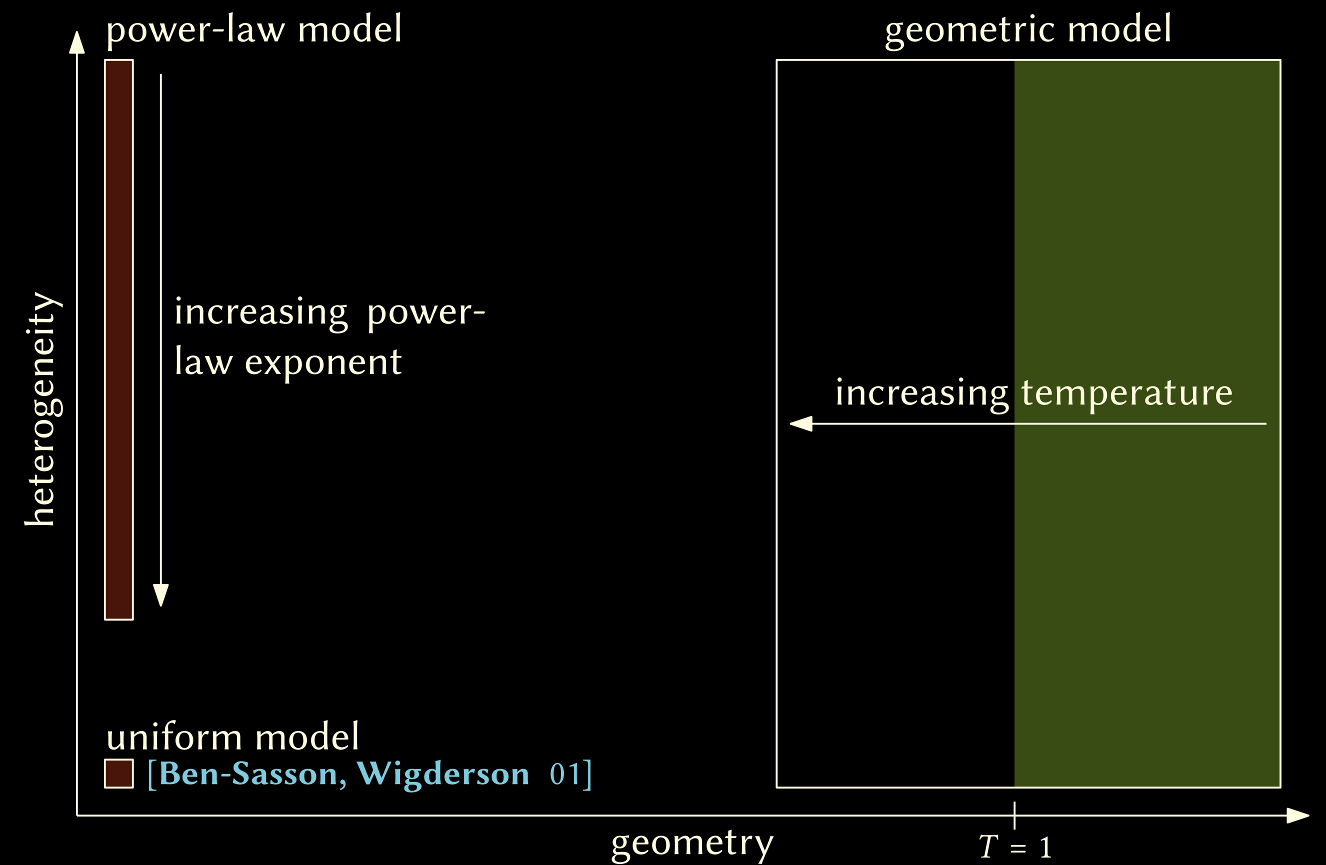
Conclusion: Why are SAT-Solvers so Fast?

The Big Picture

- heterogeneous variables: not very helpful
- underlying geometry: very helpful



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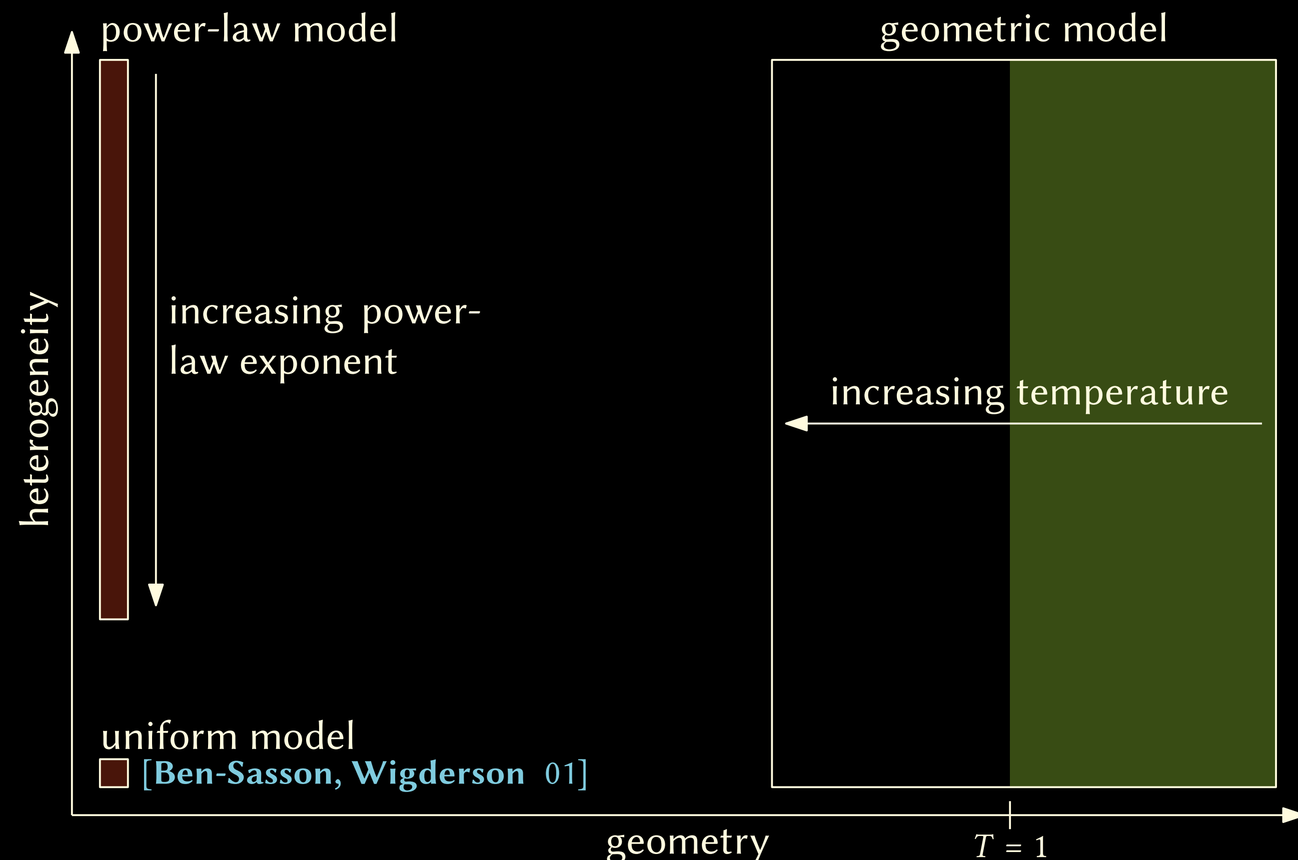

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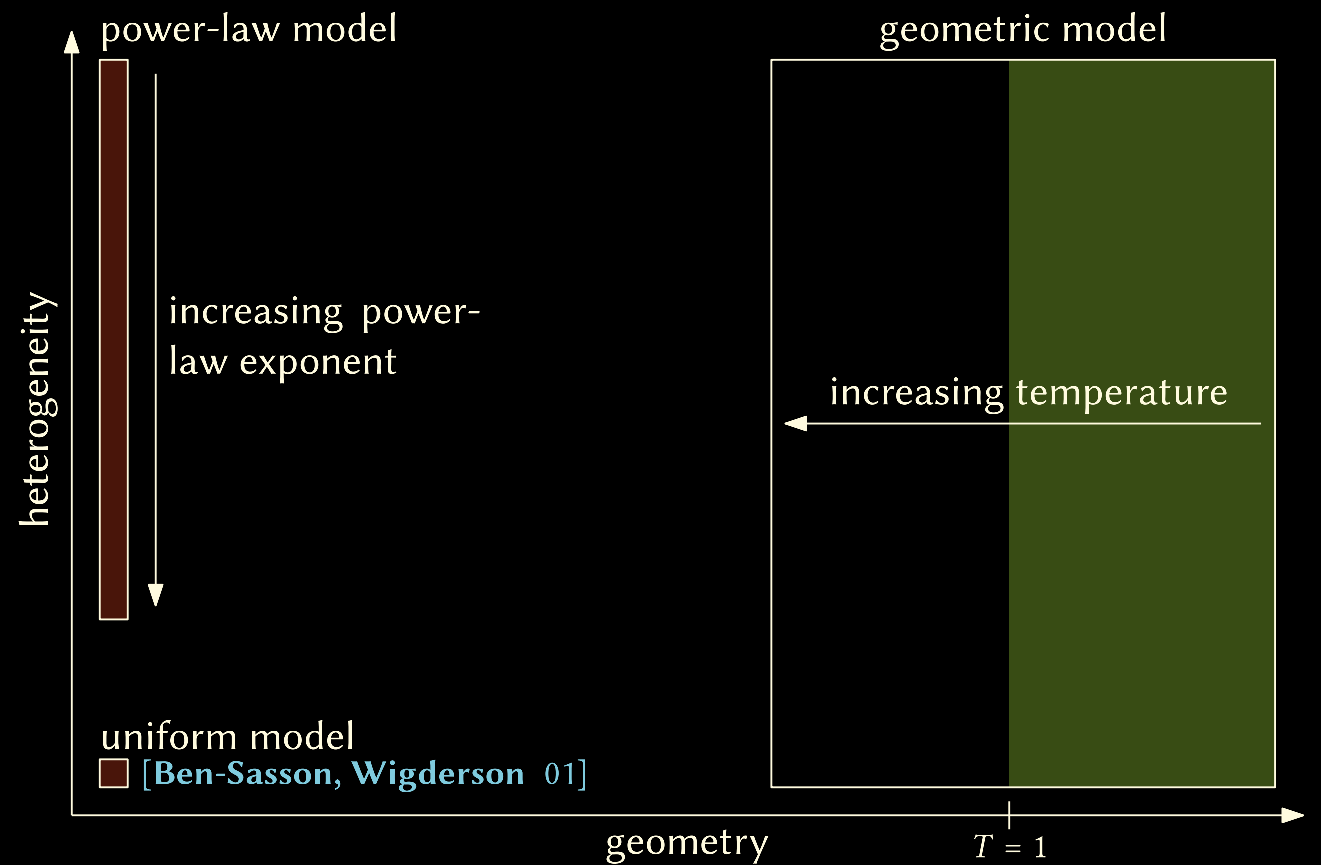

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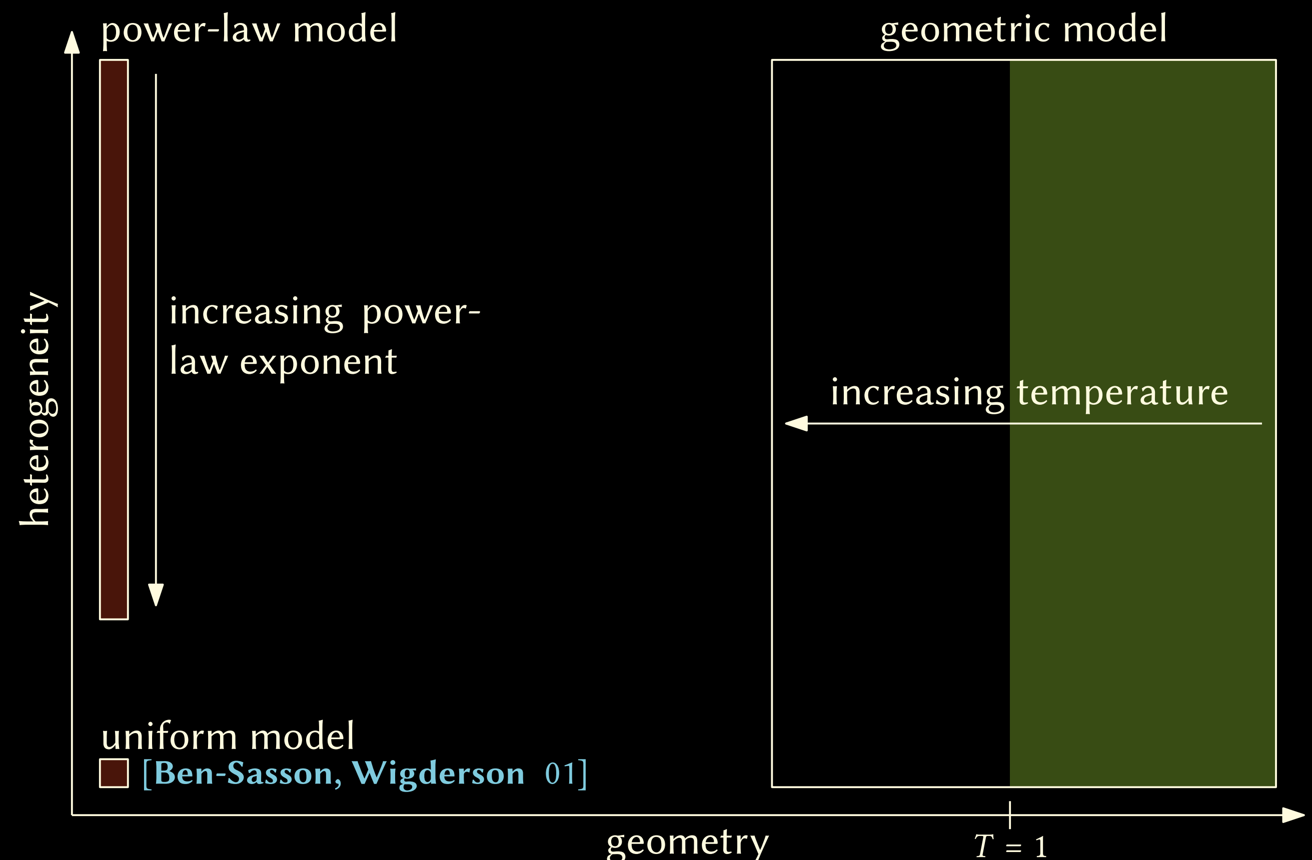

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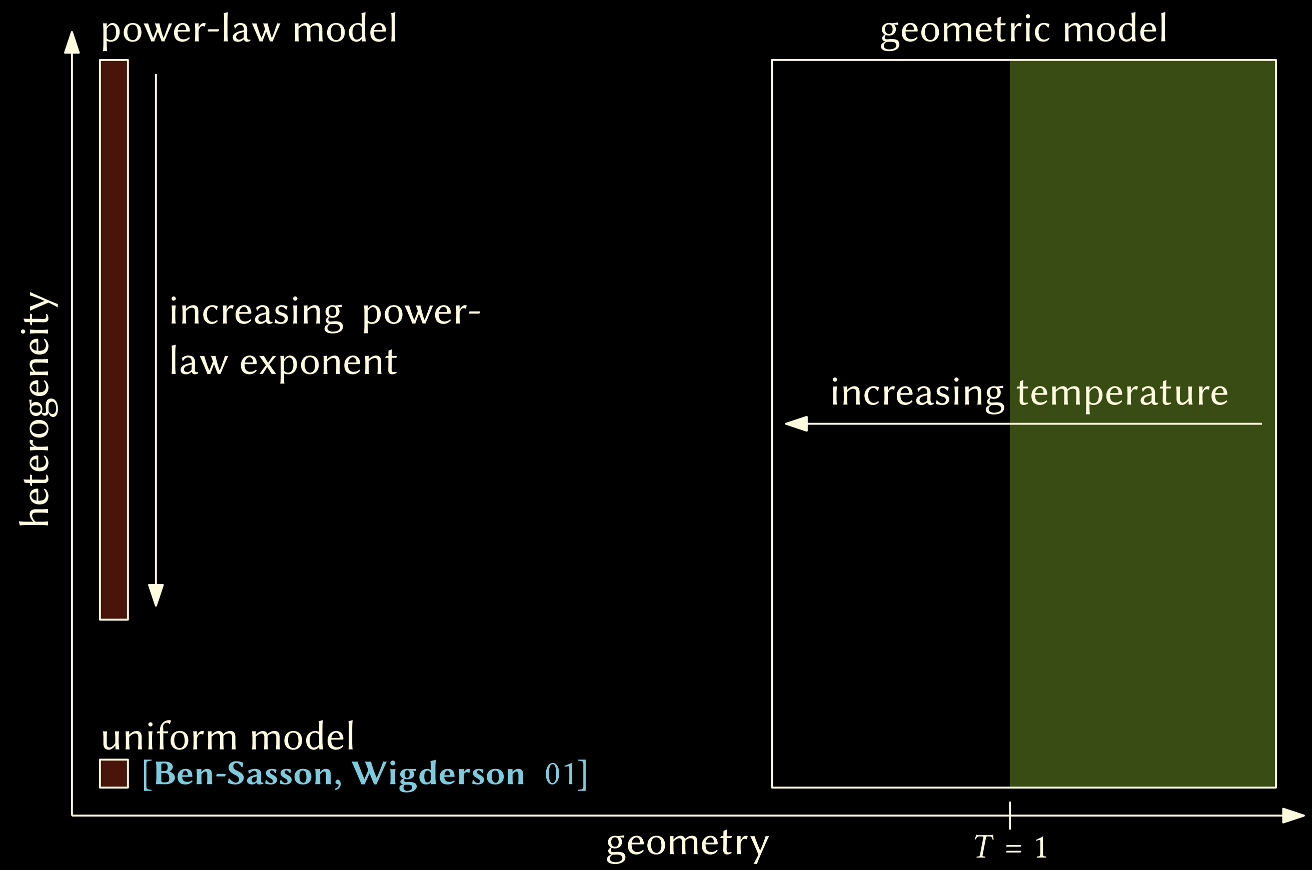

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- geometric model provides an easy average case
  $\longrightarrow$ the truth might lie between average and worst case